Conduction and Convection Heat Transfer
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Lecture 24
Review of Fluid Mechanics - V

Good afternoon. Last class, we were discussing the exact solutions of Navier-Stokes equation and we appreciated the fact that in certain special cases of steady incompressible laminar fully developed flow, the Navier-Stokes equation is converted to ordinary linear differential equation. This is because of the fact that the inertial terms in the left-hand side of the Navier-Stokes equation identically vanishes. So therefore, it is possible to have a close form solution by integrating the resulting Navier-Stokes equation and we have already discussed the situation like Couette flow.

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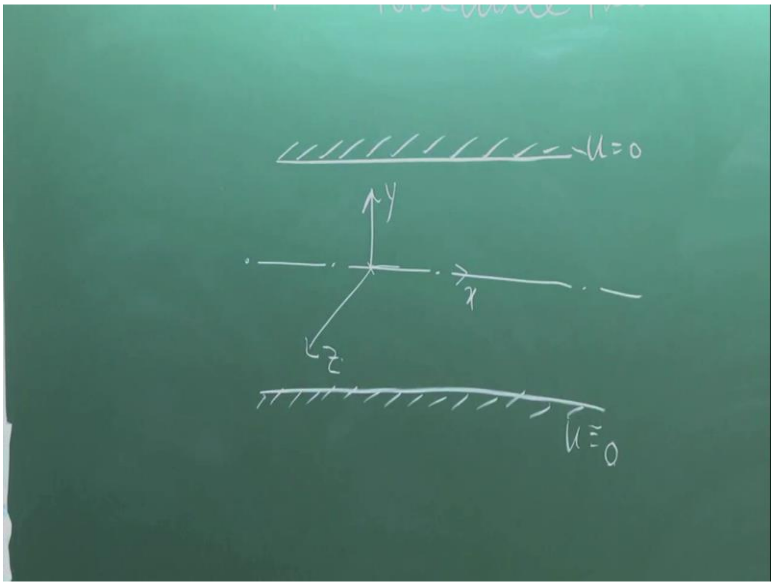

Today we will discuss Plane Poiseuille Flow we will discuss and it is very simple when we discuss the Couette flow, so it will automatically come. The plane Poiseuille flow, first of all we have to define what is plane Poiseuille flow. This is the name of a scientist. He is a great person in fluid mechanics. So, this flow is like this, this is again a steady incompressible laminar fully developed flow between two plates which are stationary, that means two fixed plates.

In Couette flow, one of the plates was moving with a velocity related to the other. So that the flow was caused by shear at the same time pressure gradient if it is imposed. But here since the two plates are held stationary, the flow takes place only by virtue of the pressure gradient. So, in this case if we remember that, if we take the axis like this, this is our x and this is our y. Now, since by geometry of the flow, this is symmetrical with respect to the central line.

Because the two plates are at risk, the identical kinematic bounded conditions. So therefore, it is convenient to use this as the axis x at the center line, y and z

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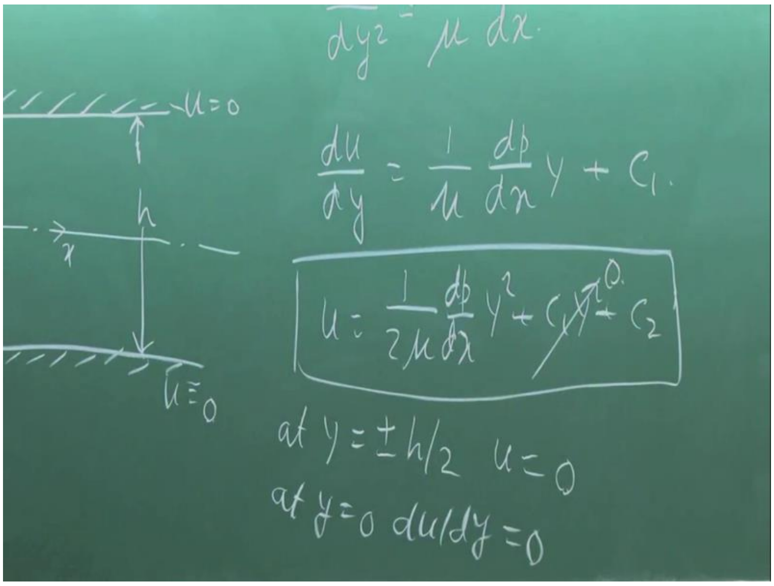

Now in general, we have already developed that in Cartesian coordinate system, for this situations of exact solutions our Navier-Stokes equation becomes d square u d y square, this already we appreciated 1 by mu dp dx. V0 w by definition 0, there is no velocity in the jet component and the width of the channel is so large compared to this spacing or the height and here we define this as h, total spacing, that the change of any parameter with respect to J is negligible compared to X and Y.

So, with all these assumptions we have found that u is a function of y only and p is a function of x only and this is the equation resulting from the x direction Navier-Stokes equation and since p is a function of x then u is a function y they are constant, all these things were derived the last class and if we integrate it we get du dy is 1 by m dp dx y, all these things we did earlier plus c1.

And the next one is that u is equal to 1 by 2 mu dp dx y square plus c1 y plus c2. So, therefore any such situation of steady laminar fully developed incompressible flow, this is a common equation with respect to a Cartesian coordinate system in general. So c1, c2 will change according to the definition of flow by geometry. So, plane Poiseuille flow is specified by the two fixed plates.

So therefore, our boundary condition will be at y is equal to plus minus h by 2 that means that 2 walls h by 2 and minus h by 2 since we are taking the x axis along the center line. Clear? No slip condition demands u is equal to 0. Since the flow is axis symmetric about this center line then we can write at y is equal to 0, du dy 0. And this comes from the physical strength that it is symmetry and if we take this type of axis that is plus h by 2 minus h by 2.

We have got, we have the same raisers which means that c1 has to be zero mathematically that y is 0 dy is zero. That means there cannot be any term containing the first part of y.

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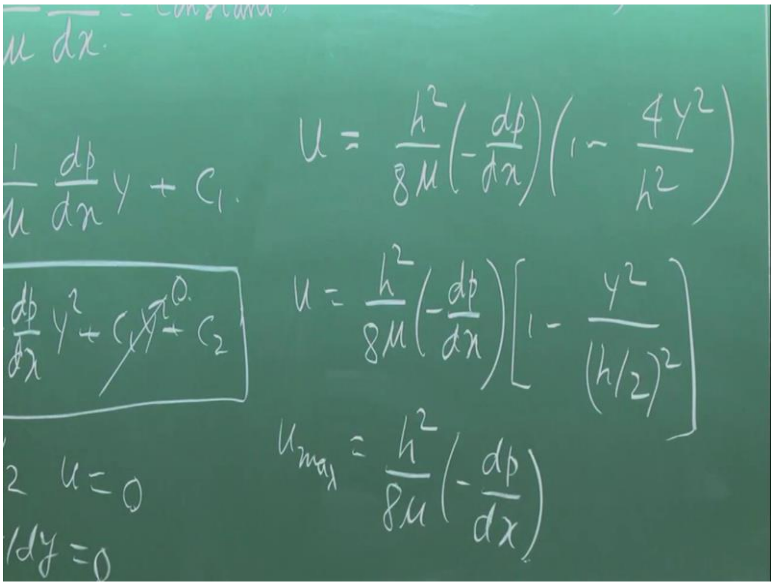

And if put this boundary condition we get, c2 is equal to h minus 1 by 8 mu. I write 8 square by 8 mu minus, this is a style of writing minus dp dx, so c2 becomes h square by 8 mu minus dp dx. So, if you substitute this c2 here, finally you get the velocity profile like this, h square and h mu you take this as common, minus dp dx into one minus 4 y square by h square. If we just put the value of c2 here.

Then you get this equation and it can be written in a form like this which looks bigger, dp dx into 1 minus y square by h by 2 whole square. That means square of the hub height or hub spacing of the channel. So, this is the u. Now it is clear from this equation that u is maximum. Now one thing, before that one thing you will see that if dp dx is zero, u is zero, unlike the Couette flow.

Because here the flow is driven by the pressure gradient and it is known as pressure driven flow. And it is at the same time apparent that the sign of dp dx and u are consistent. If we have any negative pressure gradient, that means pressure decreases in the positive direction of x which is known as favorable pressure gradient. That means the pressure force on a fluid element x in this direction, positive direction.

Pressure upstream is high, downstream is low and in that case u is positive. And accordingly, the reverse is true that dp dx is positive that when then downstream pressure is more than the off-stream pressure which we call as adverse pressure gradient when the pressure force on a fluid element these will be direction opposite to the positive direction of x. In that case this causes a velocity in the negative direction.

That means, flow is solely governed by the pressure variant and is known as pressure driven flow. Now the maximum velocity occurs at y is equal to zero and those value is, now it may be maximum minimum, the way you define it. If it is in the negative direction you call is a minimum and if it is a positive direction the dp dx negative you all it as a maximum. And accordingly, you see that du dy, d square u dy square sign depends on dp dx on the first governing equation.

If dp dx is negative d square u d y square curvature as in that means velocity has been maximum and if dp dx is positive, d square u d y square is positive that means velocity has been minimum. So, it is a question of sign convention. But the value of u max is this.

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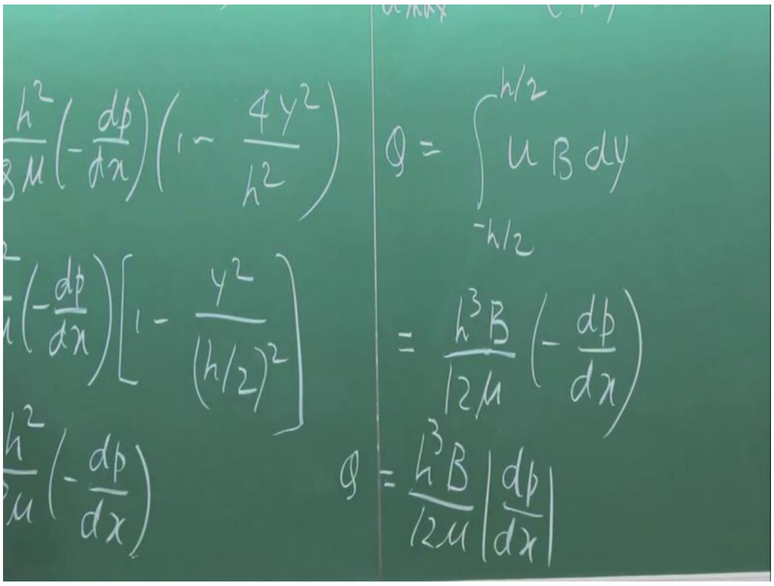

So sometimes we can write that u by U max this is another way of representing is one minus y square by h by 2 whole square. This is the velocity distribution. The next task is to find out the volumetric flow rate Q.

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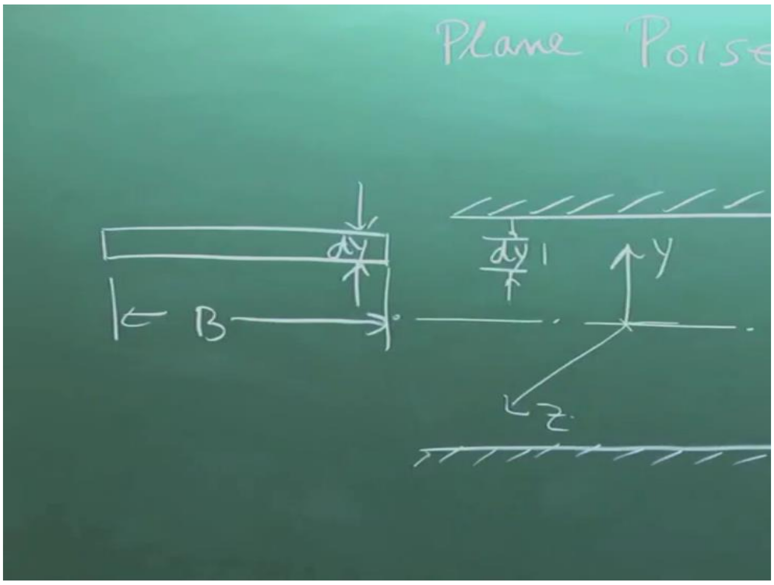

Now volumetric flow rate to find out, I have told you earlier that if I take a small element dy here and if you consider its width as b, a small element like this, cross sectional plane, that means if you see this view it is like this. This is dy and this is d. That means if we consider a small element of height dy and width d then through that the flow rate is du multiplied by B dy. That is the value of u prevailing at that particular y.

Now if you put this expression of view there and if you integrate to minus h by 2 to h by 2 that means across the channel high then you get a relationship like this, that is h cube B by 12 mu multiplied by dp dx. That means you just substitute this mu and simply we integrate with y. This will be dy, this will be y square dy y cube by 3 and accordingly you get this expression.

Now here one thing I like to mention very much which is not written in any book, I do not know that whether how many books have written it. Now volumetric flow rate is usually a scalar quantity. But if you deduce this way we come across with minus dp dx. So, if you think of the sign of dp dx because dp dx is special value, it must have a sign. So dp dx is either negative or positive.

That will indicate that pressure is increasing or decreasing in the direction of flow. So therefore, Q indicates a sign but it is better in my opinion to write Q as h cube B by 12 mu, the absolute or mode value of dp dx. I take only its value. And this represent a scalar quantity flow rate across the section.

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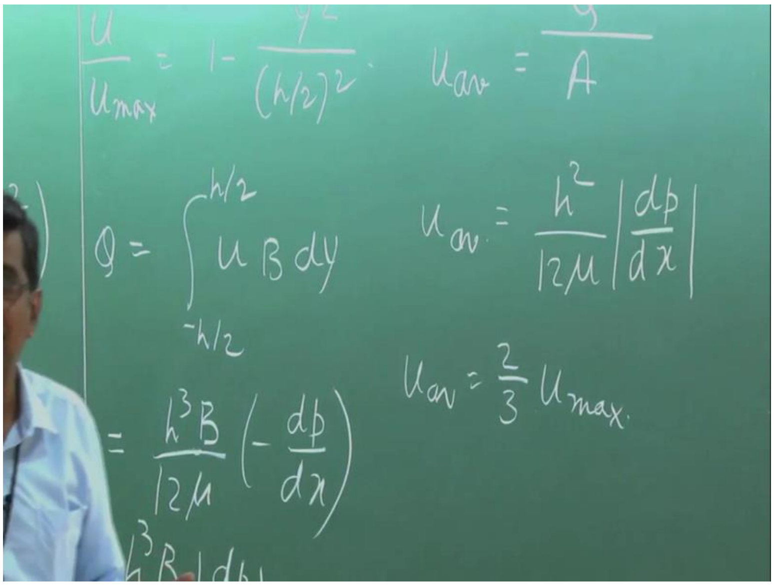

U average as average velocity in fluid flow is defined by the volumetric flow rate. That means for internal flow, flow through a duct, is defined as volumetric flow rate divided by the cross-sectional area. And if you do that cross-sectional area is B multiplied by h, then you get h square by 12 mu mode of dp ds. Of course, here you can put that minus dp dx type of thing to show the direction of Uaverage which will be inconsistent with the direction of U.

But if you think this way that average flow velocity I will consider as scalar quantity, it is a cross sectionally average based on the volume flow rate and which will simply represent the value whether in the distribution Q, local velocity will have its sign accordingly with the sign of dp dx. So therefore, you can keep this intact to show the U average as a scale of the Q, that is volumetric flow rate which is a scalar quantity.

So, it is simply h square by 12 mu dp dx. And if you see comparing with U max then we can write that U average is two third of U max, clear? Now the next task for us is to find out the shear stress at walls**.**

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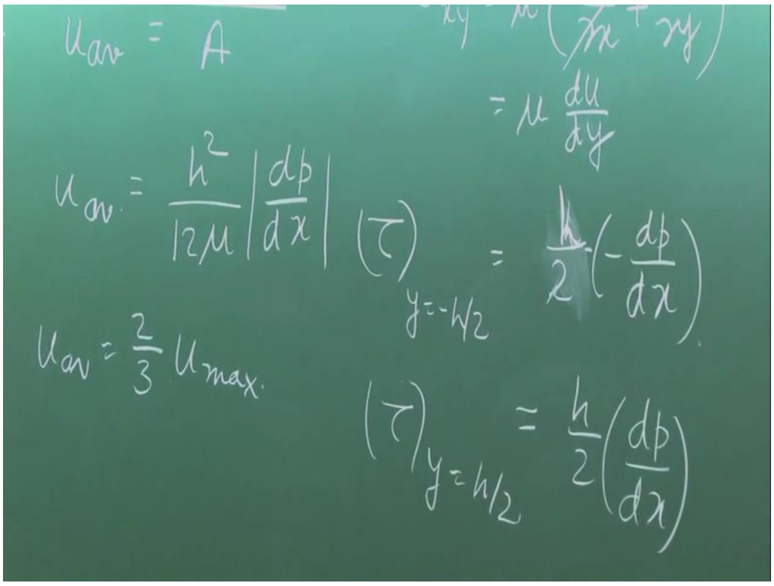

Now shear stress here takes place in the direction of x at any plane which is perpendicular to y whose normal is in the y direction. So, tau xy from a general relation we know mu multiplied b shear rate or rate of angular or shear strain in xy plane, that is del v del x plus del u del y, but at this moment, this is not necessary because we know everything here that there is no free component and U is a function of u y only, du dy.

So therefore, tau we can write, let us consider tau as y is equal to minus h by 2. Now what is du dy? du dy is this one. C1 is zero, one by mu dp dx multiplied by minus h by 2. So therefore, it is one b mu. Mu you divide, mu is cancelling each other, h by 2 multiplied by h by 2 into minus dp dx. If you put this value Y is equal to minus h by 2. Now tau at the upper wall Y is equal to h by 2 is what, h by 2 into dp dx.

This is the value of shear stress at two walls. The numerical value is the same because of the symmetry, but about the sign you see one thing that here first let us consider a flow taking place in the positive direction U positive, then what is dp dx, dp dx is negative. So, if dp dx is negative, so tau at the lower plate is positive, that means in the x direction. Whereas tau at the upper plate is negative in the opposite direction true from this two equations.

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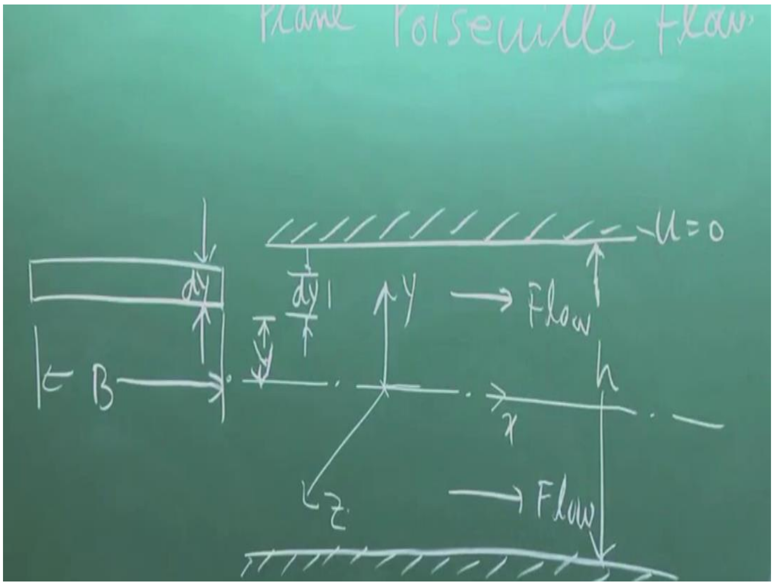

But physically it is not so because the flow takes place in this direction, this is the flow direction. So, shear stress at the lower plate is acting in this direction and the shear stress on the upper plate should act also in this direction. So where is the problem, where is my confusion. This is because the coordinate, the upper plate implies that tau x y is opposite to that. A positive tau x, y is in the opposite direction.

Because this is a plane where the normal is the negative y, not the positive y, I think this thing has been emphasized by Suman Chakraborty that when you specify the sign of a stress it is a second order tension. So, it has two sign, one is this direction and another is the plane, specifying the direction, outward normal, but the outward normal is in the negative y direction and that is why the sign convention is there.

So, with this thing, now I will go to the most important parameter of any fluid flow analysis which engineers are mostly interested is the drag on the surface, solid surface, for external flow and for internal flow. For internal flow, it is most important because it is the shear stress at the solid surface that determines the pressure drop that already you have done your basic mechanics class

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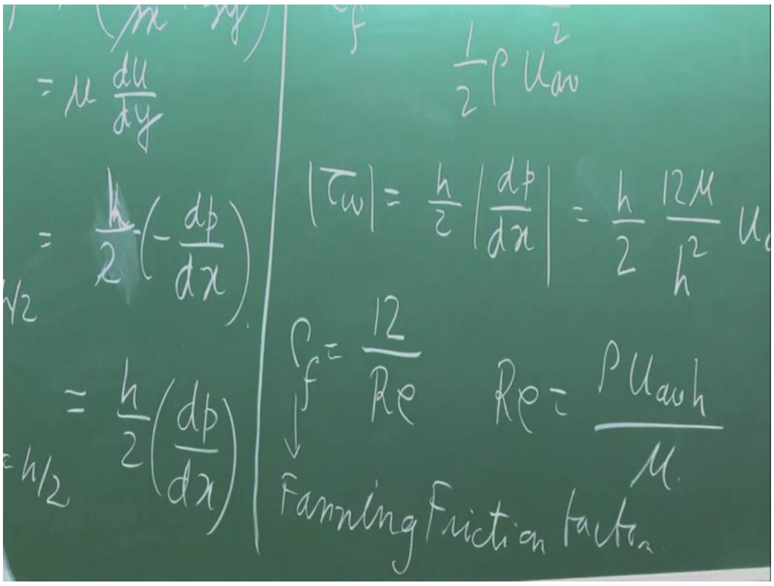

So therefore, shear stress at the solid surface is very important and this defines a scalar quantity known as coefficient of friction cf. I have already told earlier probably in one occasion that it is defined like this, half rho U average square, I already defined it earlier, so far if I recall that a friction coefficient in case of internal flow and drag coefficient in case of external flow is the representation of dimensionless wall shear stress or plate shear stress, surface shear stress which is defined as the shear stress, its absolute value.

Because this is a scalar quantity divided by the dynamic head based on the reference velocity which is U average in the present case. Present case means the internal flow it is the average velocity and for an external velocity it is the free stream time velocity. Now if we find out this one then you get tau W is equal to h by 2 dp dx. Now if you substitute that dp dx from the average velocity.

If you substitute dp dx, dp dx mode from the average velocity then you get 12 mu average velocity, it is equal to h by 2 dp dx, that means 12 mu by h square multiplied by U average. This is the, and if you substitute this, h will cancel and substitute d, you get an expression n12 by Reynolds number, where this Reynolds number is represented as rho U average h by mu. In any internal flow, the Reynolds number is defined based on the average velocity.

And hydraulic diameter which in this case is the height between the plates. This cf is known as Fanning friction factor. This is all about the plane Poiseuille flow. It is very simple up to this. This is common. This is a generic equation, C1 y plus C2. From there the geometry of the flow will allow. That means after this what happens is simply the algebra. There is no fluid mechanics.

And we have to understand the sign convention according to the algebra. Okay, clear?

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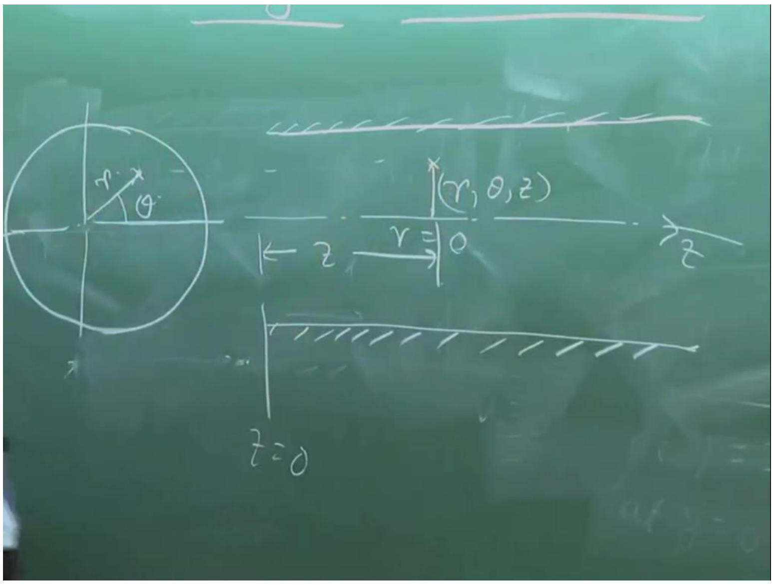

Now we will discuss another situation of exact solution which is more important than plane Poiseuille and Couette flow in engineering practice is Hagen Poiseuille flow. Incidentally these two are the name of the two scientists Hagen and Poiseuille like Bose and Einstein, Bose Einstein theory. Bose Einstein is not a same person, two persons. So that people know since his or her work.

But this is fluid mechanics without thermo fluids people, people do not know Hagen and Poiseuille they are very famous top personnel, it is in the field. What is the Hagen and Poiseuille flow again, I am telling you it is a telling steady incompressible laminar fully developed flow through a cylindrical duct through a tube, through a pipe. That is why it is very common in engineering practice.

That means in a duct with a uniform circular cross section throughout. But the flow is fully developed. The growth of Boundary layer from the surface, merges at the axis and steady laminar incompressible, but the geometry is that it is a circular tube. Now if we analyze the flow through a tube, through a duct with circular cross section it is always convenient to use the cylindrical coordinate system. Cylindrical polar coordinate system.

And in cylindrical polar coordinate system, a point here is represented by 3 coordinates. One is the r, and the central line is considered to the line at r is equal to zero and to represent this point in a cylindrical polar coordinate system we have to see the cross-sectional view of this duct.

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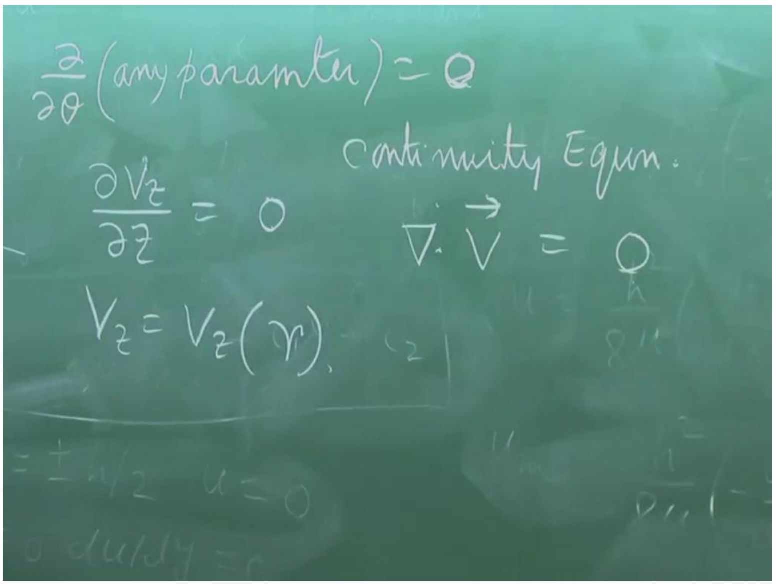

And if you see this cross sectional view it will be better that means this point comes here and this is our radial coordinate and with respect to any preferable direction this is the theta that you already know and z is the third-dimension perpendicular to this plane. That means in this view this is z. That means z is considered, some z is equal to zero plane, you consider this z, if z location is this. So that means r theta z is the coordinate of the point.

So, theta is the azimuthal coordinate, r is the radial and z is the axial coordinate. Now after specifying the coordinate system fluid mechanics stops here, only mathematics. Why? This is because this flow is a fully developed flow before that by the geometry of this flow it is axially symmetric or azimuthally symmetric. That means it is symmetric with respect to r is equal to zero which we call as an axi symmetric or azimuthal symmetric flow.

And this type of flow where the boundary conditions are also symmetric because around the entire cylindrical surface at values of r, it is a no slip condition, value u zero, that means it is azimuthally symmetric which can be mathematically written as del del theta of any parameter is zero, of any parameter this comes fast from the definition of the problem, that means the problem is azimuthally symmetric for which r is equal to zero is taken as the central axis.

Next is the fully developed flow and this fully developed flow condition as you know that del z is zero. That is the requirement of fully developed flow that z ceases to be a function of z. I have told earlier also it is function of r only. Because there is no dependence on theta. So therefore, the consequent is that vz is a function of r only. Well, now let us write the continuity equation.

Let us write the continuity equation. Now continuity equation for an steady incompressible flow as you know the divergence of the velocity vector is zero.

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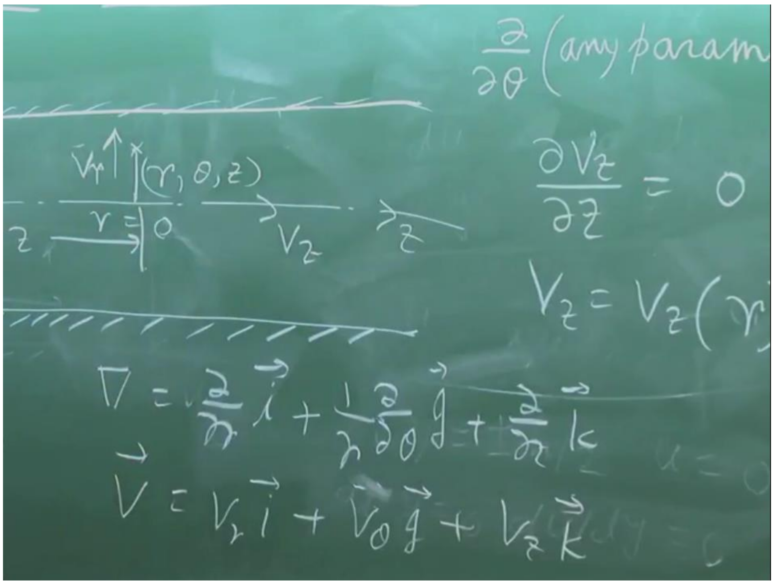

And in a cylindrical coordinate system I write the divergence is as you know is del del r, ijk if vr v theta, now I have to define vr. vr is this, in the r direction. Vz is along vz direction and v theta is along the theta direction. So therefore, this is one up on r del del theta j plus del del j k and v the velocity vector is defined as vr i j k are the unit vector v theta, sorry and vz k. Now this is a preliminary vector calculus.

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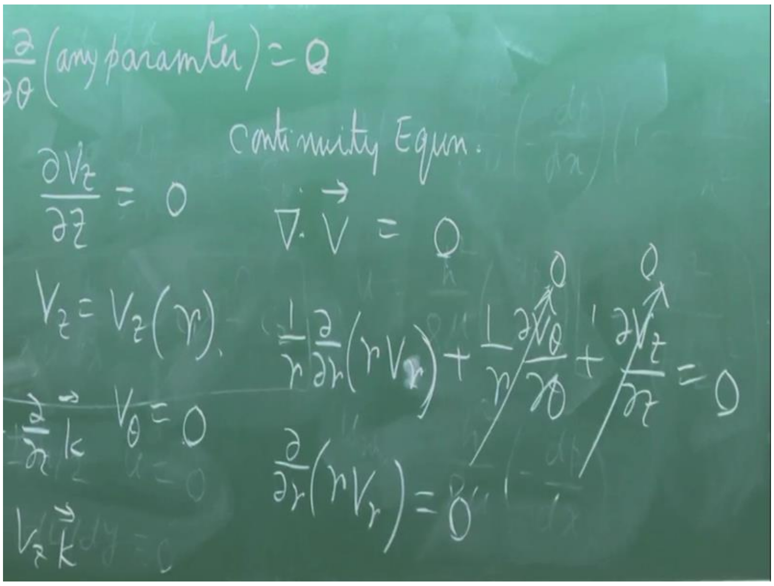

This becomes one up on r del del r of r vz, this has been told in your fluid mechanics class, r vr plus one up on r del v theta del theta plus del vz del z. That means it is counter part of del u del x plus del v del y plus it is the counter part of del u del x plus del v del y plus del w del z zero. So, this is the continuity equation. If we invoke the continuity equation this is zero because v theta is absent.

The flow is defined with v theta zero, there is no tangential component of velocity. It is not that only del del theta of any parameter is zero but tangential component of velocity is absent. But both the cases that this time is zero. Then del vz del z by the definition of the fully developed flow zero.

So, we have then the only option that mean this equation tells this is zero which means that del del of r vr is zero which means that, that r v r, del del r or r vr exactly, that means r vr is zero, why? This is because similar we have obtained that del v del y zero means v zero because impermeable boundary. So, if you apply this r vr from this point and go for each and every point in the flow you get the r vr zero. You understand me?

That means del del of any, del del y for example, del del r of any parameter is zero, means that parameter is zero provided this boundary condition stay, that two surfaces or along the surfaces this is zero. That means it will zero velocities in the entire field. So, this is a mathematical consequence and this has to be understood like that. If you put that condition at the surface, so you see that the immediate next point.

The immediate vicinity in that surface has to be zero if that gradient is zero, so that way next point is zero. So, it cannot initiate any value mathematically, it means that the field itself is zero. You understand? But if the surface, any one of the surface, here it is the single surface, but in case of two plates, the one of the surface is permeable there is a suction or blowing then this is not true.

That means even if this change with y is zero, the field is not zero. Clear to everybody? So, the similar way we obtain that del v del y zero v zero, r vr zero means, next say the r vr zero vr is equal to zero except the singularity at the axis where r is equal to zero, that singularity I will show you how to tackle with the velocity distribution equation. So therefore, we get that vr zero, v theta zero and vz is a function of r only.

Now you see that since del p del theta zero and there is no theta component of velocity, I do not want to invoke or call the Navier Stokes equation in theta direction. This is unnecessary. It is not required you can see in your book how it is written.

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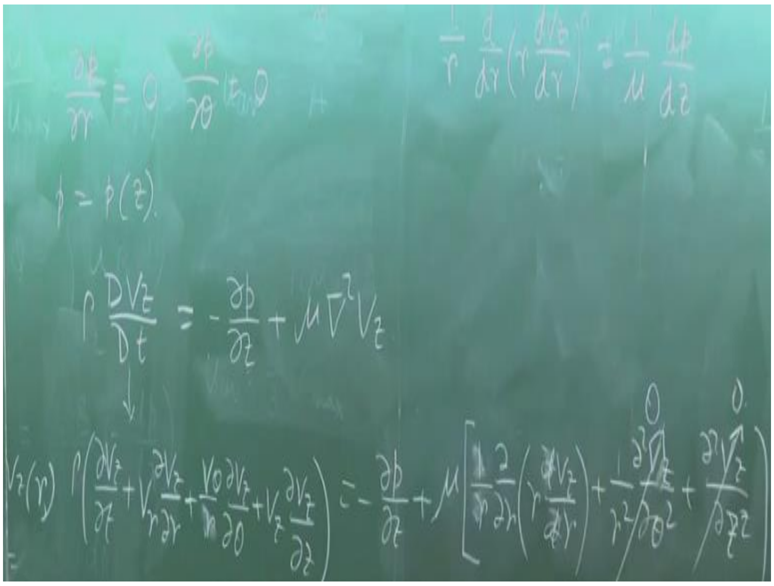

Now in our direction equation also, in absence of vr and v theta we will give you del p, there cannot be a pressure gradient. If you expand because in this short time I do not want to write it. This remains as your task that probably it has been taught earlier that if you explain the Navier-Stokes equation in r direction, you will see in absence of vr and vp that del p del r will be zero. It is similar to the Couette flow or plane Poiseuille flow in Cartesian coordinate del p del y is zero.

That means cross gradient is zero. And it is very easy to physically understand since there is no velocity in the r direction and no azimuthal velocity, no pressure gradient can exist. The question of azimuthal velocity comes because of the tangential velocity at radial pressure gradient is created which is responsible for the centripetal acceleration and the centripetal force.

So probably those had been taught at your fluid mechanics, basic fluid mechanics class. So therefore, in absence of vr vp that there cannot be any radial pressure gradient. So, del p del r is zero and already parameter with respect to theta is zero, del p del theta zero, we wrote it earlier the consequent, the speed becomes a function of z. It is in the similar lines we deal with the Cartesian coordinate system for plane Poiseuille and Couette flow.

Then I recall or invoke the r direction equation, Navier-Stokes equation. What is that? Rho dvz dt is equal to minus del p del j plus mu of laplacian and vz. Now I expand these for clarity, this temporal derivative though it is not required, you can tell sir we know that a steady flow temporal derivative is zero, however for the clarity then vr del vz del r, plus one by r, rather v theta by r, del vz, del theta plus vz del vz del z, equals to minus del p del z plus mu del square vz.

Now if we split these I think it is better to write in a compact form like this, this again if you follow the vector algebra, with this notation of del in the cylindrical coordinate system it is again the vector algebra that del del r, one by r, one by r if you write it is mathematics only, del del r, r del vz del r plus one by r square, del square v theta del theta square plus del square vz del r square.

That means it is counter part of the laplacian in Cartesian coordinate, del square u del x square plus del square u, del y square, sorry this is vz plus del square u del z square. So, it is the counter part of that, so it is in cylindrical polar coordinate system. Now, we know, first come here, this steady flow zero, we see how the z direction equation motion, the inertial term identically vanishes. vr absent, v theta absent.

And here though vz is there but del vz del z zero because of fully developed inertial term vanish. Now already this proof that v is a function of z, so it should be written in terms of ordinary differential equation and here also there is no dependence with theta for azimuthal symmetry and there is no dependence with del z square. There is no dependence with z. Del square vz del z square.

So only this term remains and also here I can replace this term as d, that means ordinary differential equation. That means from here, what I get, I get one by r, d dr of r d dj dr is equal to one by mu dp dj. From this equation, I get this. I think it is okay, so that I can finally get from all three Navier-Stokes equation in 3 directions r theta z this equation. The y-y obtained for Cartesian coordinate systems also for plane Poiseuille and Couette flow.

Now it becomes simple mathematics indication since v z is a function of r and p is a function of z equally the whole goods for a constant, which means from here only the logic plays that p is a linear function of z then u is a quadratic function of r, so that equality holds good. They becomes constant. That is second derivative of dz v with respect to r and first derivative of p with respect to z becomes constant. Now you integrate it.

The first integration was d dr of this. That means r d vz equals to, you just multiply with r and integrate, that means r square by 2 mu dp dz plus c1 and then next one is dvz dr is r by 2 mu dp dz plus c1 by r and the next one is vz is r square by 4 mu dp dz plus c1 lnr plus z.

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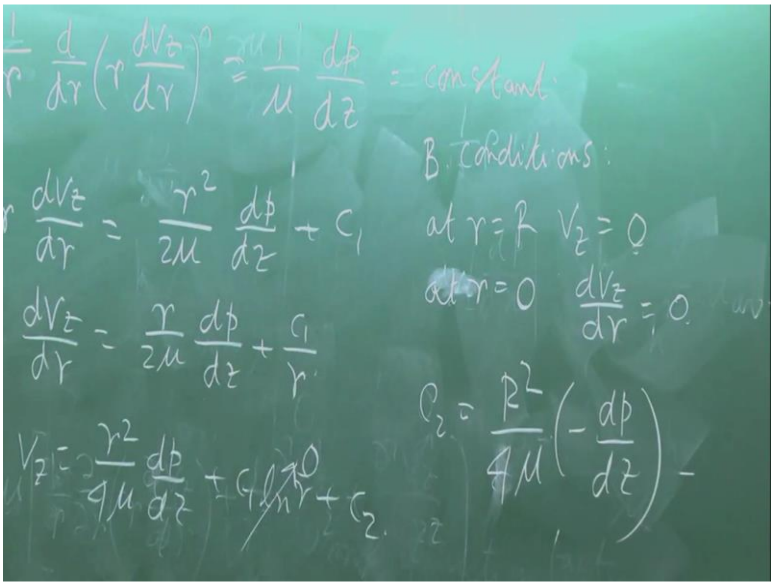

And here also the boundary conditions are at r is equal to r vz zero. We have two constant. Another boundary condition is the azimuthal or axi symmetry condition which means dvz dr at r is equal to zero, rather you write in the same fashion, r is equal to zero, dvz dr is zero. Okay? And this automatically says that this equation cannot have a term like this, lnr is, that is why I told that this will tackle the singularity at the center at r is equal to zero will be tackled automatically by allowing this c1 to be zero.

So that this term vanishes and similarly in the dvz dr term, this term cannot happen to have this singularity. So therefore, this term will not come at all and this becomes symmetric about r is equal to zero and if you put that you will get c2 is equal to R square by 4 mu multiplied by minus dp dz this is my technique of writing always minus dp dz to show that plus minus should be associated with dp dz.

We have an understanding with the favorable and the adverse pressure gradient depending up on the situation. Now after this our routine task is like this. We find out the maximum velocity. The maximum velocity at r is equal to zero obviously and that is equal to r square. Oh, I have not yet written this.

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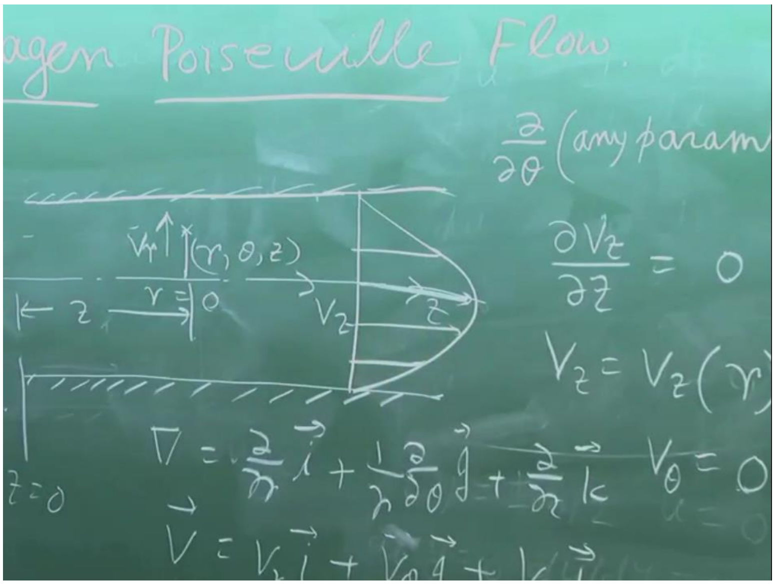

Vz then, if you substitute the c2 here, then you get vz r square, it is similar to Poiseuille thing 4 mu multiplied by minus dp dz multiplied by one minus r square by r square. The remaining part at school level thing, that vz maximum that is vz at r is equal to, at r is equal to zero that is r square by 8 mu multiplied by minus dp dz 4 mu into minus dp, why I am writing 8 mu, 4 mu multiplied by minus dz.

So, one can explain vz by vz max as one minus r square by r square. So therefore, the Poiseuille flow like this. Similar is the Poiseuille flow, this is the distribution of velocity for various pressure gradient. So, minus dp dz. So therefore, flow is purely driven by the pressure gradient, pressure driven flow, where dp dz is negative, flow is positive, dp dz is positive, flow is negative, that means in the opposite direction. Next task is what?

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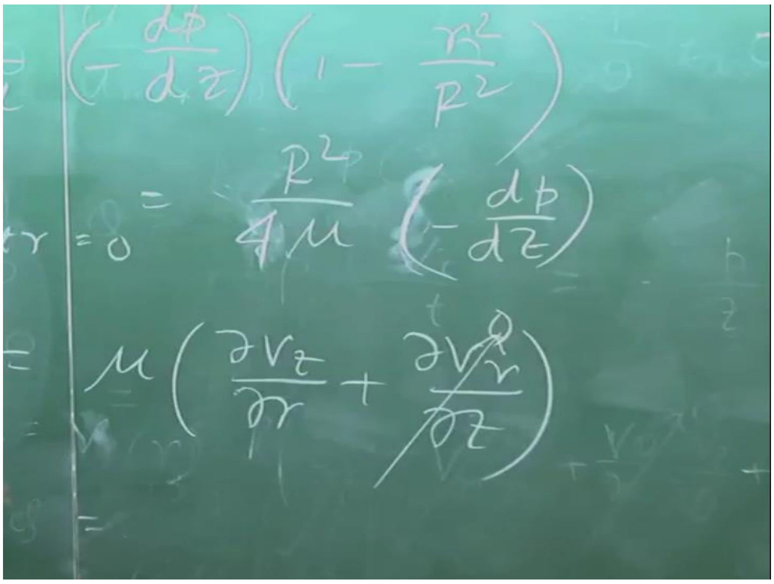

Next task is tau w which is tau rz. First of all, let us write tau. That means shear stress in the direction of flow which is in the rz plane. At any plane parallel to the flow that is rz plane which can be written as mu into corresponding rate of angular or shear strength, that is del vz del r plus del vr del j. This has been taught at your basic fluid mechanics class, since in this case vr is not there and vz is a function of r.

Therefore, tau is mu d vz dr. Well, therefore tau is mu dvz dr. Tau is mu dvz dr.

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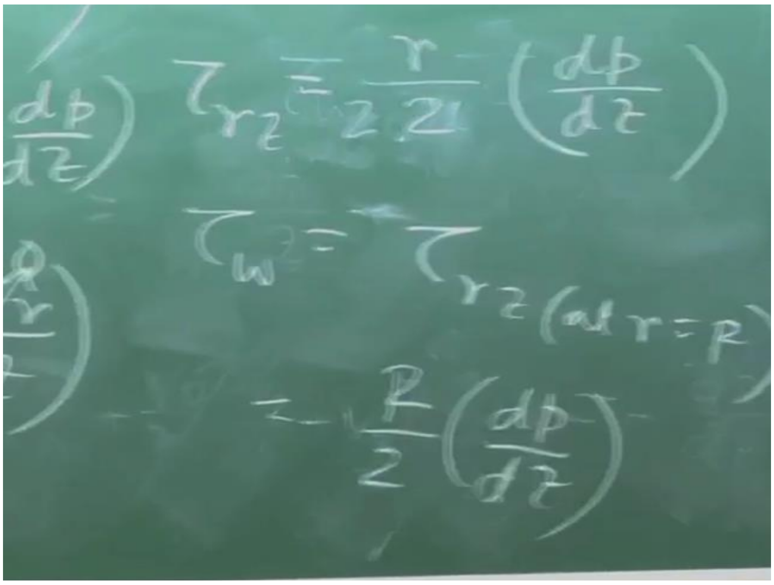

So therefore, tau rz in this case is mu dvz dr. Now rest part is very simple and routine. So, tau w is nothing but tau rz. Now let us find out tau rz, mu dvz dr. What is dvz dr? Now if you find out dvz dr it will be, now let us find out this also. It is r, twice r square, r square is cancelling, r by 2 mu multiplied by dp dj. Tau rj is r by, that means you get dvz dr from here. I have rubbed this, dvz dr was there in the solution.

So dvz dr if you may, then you will get this value that tau is equal to r by 2 mu. You get 2 r, 2-2 cancels and 2 mu r square, oh, also mu also will not be there, very good, r by 2 dp dz. There is no mu. Therefore, tau w that means r by z, what is tau w. Tau w is equal to tau rj at r is equal to R. That becomes is equal to R by 2 multiplied by, here again the same understanding, that dp dz negative gives a positive flow.

And shear stress physically acts in the positive direction but it gives a negative value mathematically because the plane is tau rz negative because r is down one, very good. So similarly, if you write the friction coefficient as this, half rho… before that I have to find out vz average. Sorry, I have not found out the vz average, before that I have come to shear stress. Now let us find out average velocity or volumetric flow rate.

Volumetric flow rate again, volume average flow in case of a plane geometry, the volume average flow is simply the arithmetic average. But here the volume average flow to find out we have to consider an elemental area, annular area with thickness dr at a radius r, thickness dr which is represented here like this, this is the dr in this diagram which is an annular ring with widthness dr.

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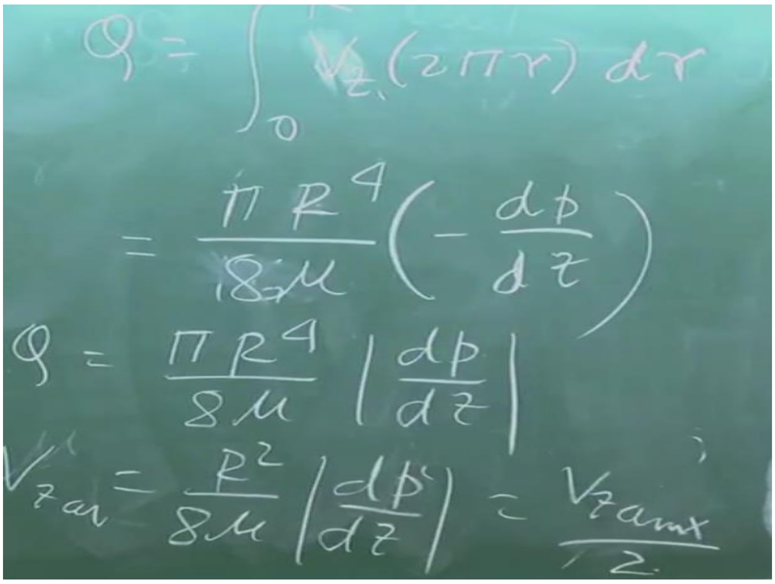

And to represent the q you can express this as the modulus of this. That means Q can be expressed as pie r four by 8 mu. I think this is more reasonable to understand because volume flow rate has no sign and vz average therefore is equal to then divided by the cross-sectional area, that means r square by 8 mu multiplied by dp dz which is vz max by two. Now I come to the coefficient of friction.

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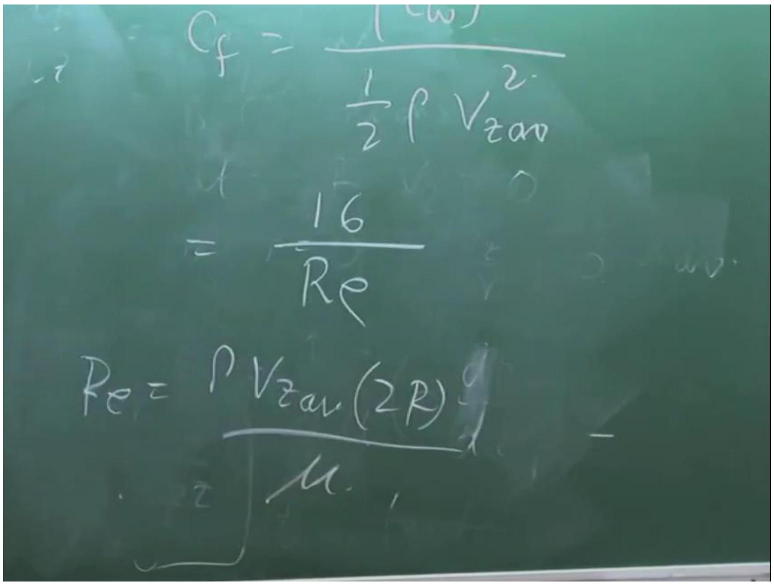

The same way as I did earlier for plane Poiseuille flow if you define the coefficient of friction, Cf as tau w by half rho vz average square and if you use this tau vz average from here, if you dp dz, dp dz if you, tau w is r by 2 dp dz. So, if you substitute tau in terms of dp dz and dp dz in terms of the vz average as I did earlier then it becomes equal to 16 by Reynolds number where Reynolds number in this case is defined as rho vz average two r by mu.

I do not want to spend time in such small school level calculations that I tell you that tau w is R by two dp dz. Here it is dp dz, you can use the modulus of that, which will be again substituted in terms of vz average, so that tau w will come in terms of vz average, rho will be there and this thing is denominated ultimately. This will give, this is a Reynolds number defined on the basis of the diameter 2r.

Diameter of the cylindrical duct which is the hydraulic diameter as you know it is defined as four cross sectional area divided by four times the cross-section area divided by the weighted perimeter and this comes out to be the diameter, 2 r rho average velocity. Mu is the dynamic viscosity. So here I aim this discussion on Hagen Poiseuille flow but at the end I would like to tell you one thing, which I ignored from the beginning.

Whenever I write the Navier-Stokes equation it is del p del x, del p del y or del p del z. What is this p, you know, p is the piezoelectric pressures? So, I all books, in my book also you can see that people use pu with a star. I think what Suman Chakraborty has told, when the Navier-Stokes equation is derived, we got rid of the body force balance. There is no body force we consider except gravity and we write the pressure gradient.

But gravity as the body force we can never neglect. We may neglect other body forces which are otherwise not imposed. Some electric field, some magnetic field may not be imposed on it. But gravity is there. So, gravity we never write as a body force, for example vertical flow in a vertical direction, y direction equation the gravity will come or in an inclined plane you have seen that liquid flowing gravity.

So that gravity is taken into account by piezoelectric pressure, this as we told it is a very preliminary knowledge at your basic fluid mechanics class. So, all this p what I have used, you consider this as a piezoelectric pressure. That is why in books usually it is written as p star, if you see in many books, it is written and in my book also p star.

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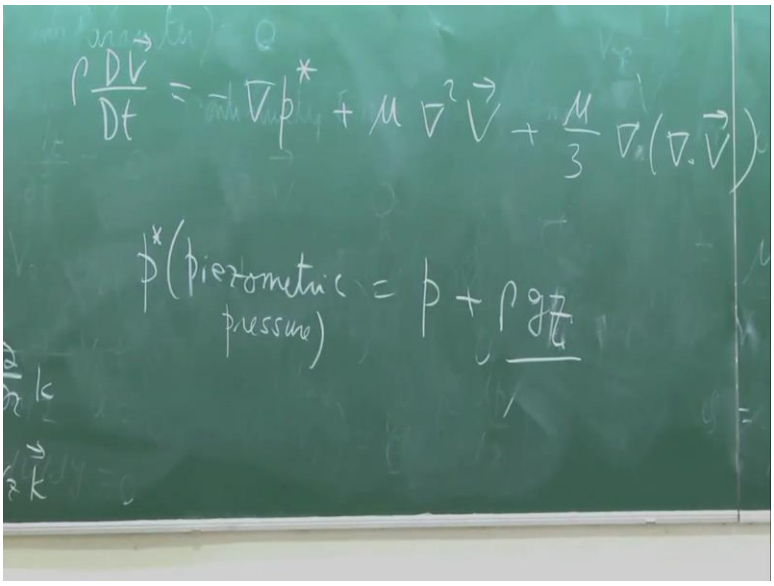

So, the pressure here which I write in the Navier-Stokes equation, for example if I write the Navier-Stokes equation, for example just this is for your concept, forget that exact solution is over, that when the Naviers-Stokes equation is written, let us write in a very compact vector from minus grad of p star, actually here we will not use star, we will read p as the piezoelectric pressure plus mu.

Forget about the next term, mu by 3 grad of that is not required, mu by 3 grad of del dot V. This is down for the comprehensibility. If incompressible flow, this is zero. But this p star is known as piezoelectric pressure. I think you know it takes care of the static pressure plus the gravity equivalent pressure. Not h, z where z is coordinate in the direction of gravity. It is elevation or whatever you get from a reference datum.

For example, when I told that the imposed pressure gradient, for an example like this, when this type of problem you have done earlier just for your concept I tell you.

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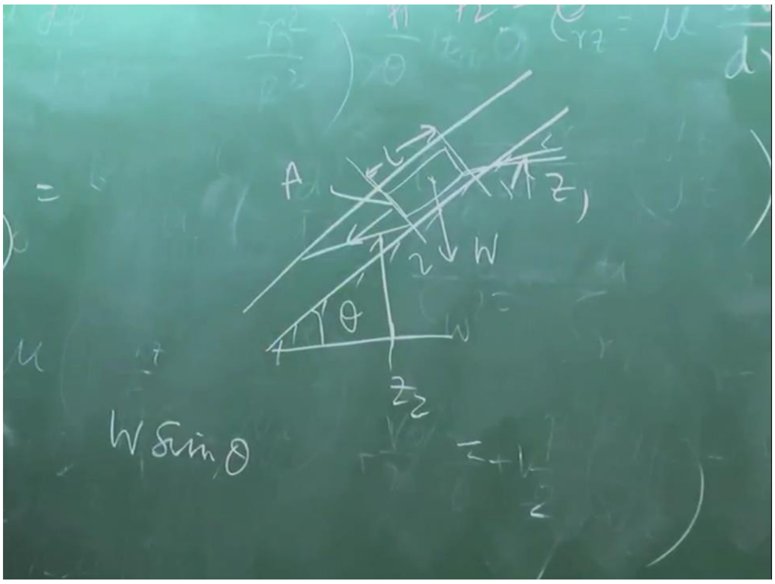

For an example if there is an inclined plane and liquid plane is flowing and this plane is inclined by an angle theta. So, if you take a fluid element, pressure here is all atmospheric at this and this. If this is section one and this is section two, p1 minus p 2 is zero, but then what causing the flow is the gravity, that means the component of its width in this direction and that is the difference of the piezoelectric pressure force.

That means in that case the piezoelectric pressure difference p star at one minus p star at two will be rho z, z1 minus z two. That time area, if you multiply our cross-sectional area you get the effective gravity. That means if you divide a cross sectional area A and if you define that this is z1 and if you define here this is z2, then you will get this equation. That means if you define this as length, so the effective gravity that is w sine theta.

That means this is in this direction which is causing the element to come, liquid to flow and that is giving by this rho z z1 minus z2 into the cross-sectional area will be the pressure force. That means when we deal with this type of very simple equation at very fundamental stage at the basic stage preliminary stage of starting we do not think of that piezoelectric pressure will simply make the force balanced, that the gravity is fully in this.

And if the shear force in the liquid at the wall balances the effective gravity then the film flows with the constant or uniform velocity. So therefore, this gravity is taking in terms of a piezoelectric pressure where rho z the gravity prevalent pressure is there. So therefore, all these Navier-Stokes equation without any imposed body force from outside implies the pressure as the piezoelectric pressure.

When we deal with flows in horizontal plane, then the del p is equal to del p star, that means the difference of piezoelectric pressure and difference that equation is same. So therefore, we can reduce, so that is simply static pressure So any questions? Alright. So, next class, we will start the Boundary layer theory, a brief introduction of the Boundary theory recapitulation like that. Many of you have already read it already in your fluid mechanics class, then that will be taken by the Prof. Suman Chakraborty. Okay. Thank you.