Conduction and Convection Heat Transfer
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Lecture - 19
Convection - I

So, today we will get started with the foundations of convective heat transfer, the mathematical foundations of convective heat transfer. Now, the mathematical foundations of convective heat transfer strongly depends on heat mechanics. And this is so because convection may be fundamentally perceived as advection assisted conduction. So, you have a situation, where conduction is a must, but that is assisted by something, which is advection or transport of heat by fluid flow.

So, to get a physical picture of why fluid flow is important for heat transfer, let us consider a situation like this.

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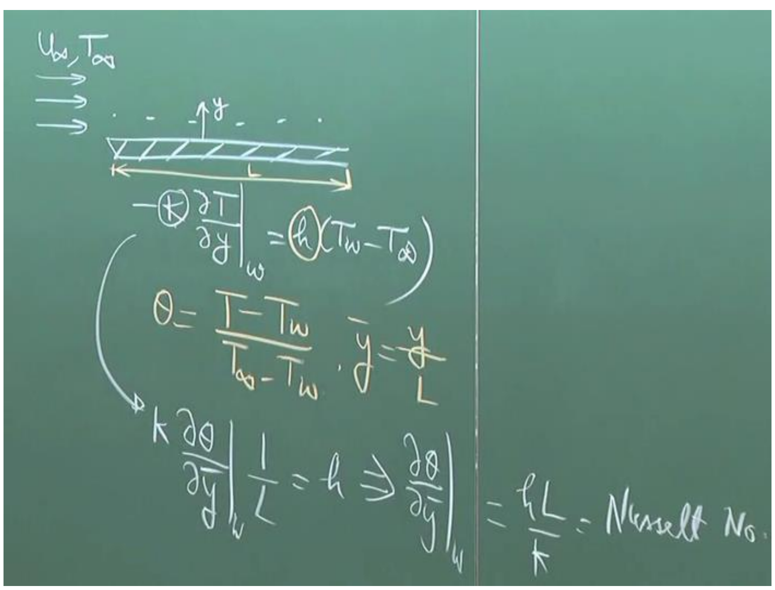

Let us say that you have a solid boundary and fluid is coming from fast stream with a velocity U infinity and temperature T infinity. Now, to write what is the heat transfer at the wall, we express it by this following boundary condition. We have discussed earlier that this boundary condition is valid for both steady state and unsteady state and only important assumption is that radiation is neglected, otherwise there will be a radiation term along with it.

Now, in convective heat transfer we are mainly interested to find out what is h. If we know what is h, then we know, what is the rate at which the heat is been transferred at the wall. The question is h will depend on what? What are parameters on which h will depend? So, you can of course, non-dimensionalise this equation and write, if you define a non-dimensional Theta, non-dimensional temperature Theta and non-dimensional y, y / L, where you can write this equation.

So, what you get is the non-dimensional temperature gradient at the wall, because this is non-dimensional, this is also a non-dimensional parameter, which is known as Nusselt number. It is very important to understand that there is a fundamental difference between this Nusselt number and the bio number that we have talked about in conduction heat transfer. We will come into that. Now, we will see what the parameters on which we depend and we will bring up context and we understand that how fluid mechanics relates to heat transfer.

So, my first question is what is this K? Is this K solid or fluid? This is my first question. You have a solid wall and you have fluid at the top of it. So, this k, is it solid or fluid? So, sometimes as I often discuss that these days it is easier to get an answer if we give in multiple choice mode. No 1. K are solid, No 2. K are liquid, No 3. K are either solid or fluid, No 4. None of the above. So, three choices K are solid, K are fluid, K are solid or fluid.

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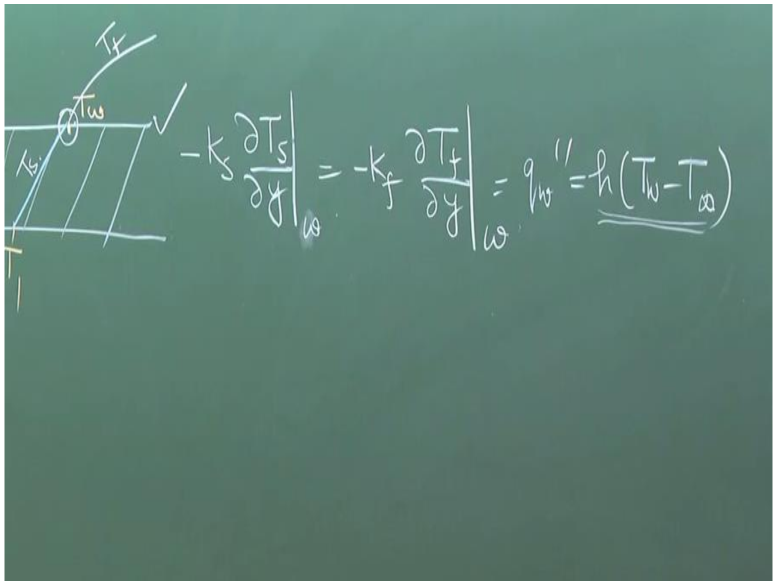

I am here enlarging a solid boundary, solid wall, this is a solid wall. Let us say this temperature is T wall (T w), which may be a function of x, this is x axis, T wall (T w) may itself be a function of x, may not be a function of x, so if this is T wall (T w), let us take an example that T wall (T w) is greater than T infinity. We can take two interesting examples, one is T wall greater than T infinity and another is T wall less than T infinity**.**

T wall equal to T infinity is not an example, because then there won't be any temperature difference between the wall and the fluid. Let us say this temperature is T 1. If it is steady state what will be the temperature profile between T 1 and T wall, liner temperature profile, right? Let us draw it with some color. What will be the temperature profile in the fluid? In general, not linear, if it is a system, where there is only conduction that means the fluid is stagnant, then it may be possible at the temperature profile in the fluid is also linear, but in general it is not so.

So, let us say that the temperature profile in the fluid is something like this, so, importantly what is common at the interface, what is continuous at the interface between the solid and the fluid, No 1. Temperature is continuous. Because at a given point, you cannot have two different temperature, one in the fluid, one in the solid. So, at a given point you must have unit temperature. So, there must be continuity of temperature.

And the second, the heat flux must be continuous. The rate at which it is transferred from the solid to the fluid here, is the same rate at which heat leaves to the fluid. The reason is that this is an interface, interface doesn't have any thermal storage capacity. So, whatever heat comes at the interface at the same rate it will leave. So, what is the rate at which it is coming at the interface? Minus K. S for solid. We will use S for solid and F for fluid.

This is the rate at which this temperature profile, this is T s and this temperature profile is T f. So, this is equal to the heat flux at the wall. Wall means interface not the bottom at this location. This is equal to h * T wall - T infinity. This is nothing but the so-called Newton flow of cooling. So, this is always valid except when radiation is present, a radiation heat flux has to be added.

Now, can you give an answer that whether it is K of solid or K of fluid? See it all depends on whether you are using the temperature profile of the solid or the temperature profile of the fluid. If here, you are using the temperature profile of the solid, then this will be K of solid. If here, you are using the temperature profile of the fluid, it will be K of fluid.

But, in convective heat transfer, our interest of domain is the fluid domain; therefore, we are interested to find out the temperature profile in the fluid and the temperature gradient in the fluid profile. So, if we are interested to do that means this is K of fluid. But fundamentally there is no restriction that it will be K of fluid. It depends on it is T of what?

If it is T within the solid, then this is K of solid and if this is T within the fluid, it will be K of fluid and you can see here that because in general K s and K f are different, therefore the slopes of these two lines are different. These two lines have same unique value at their points of intersection but you do not have the same slope because K s and K f are in general different.

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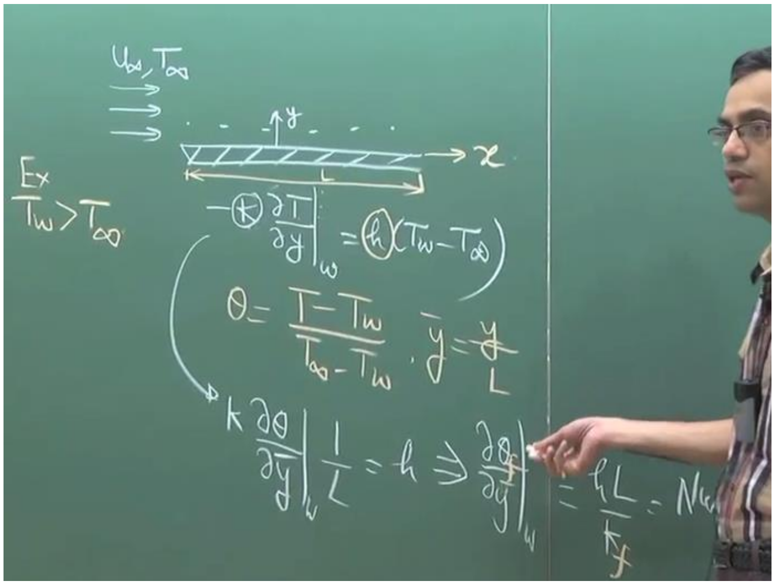

So, important thing is that if this Theta is Theta of the fluid. Now we write here the Theta of the fluid, then this is K of fluid. So, non-dimensional temperature gradient of the temperature profile within the fluid is equal to hL / K of the fluid. In the bio number definition, the K was K of solid and in the Nusselt number definition, K is K of fluid. So, typically we can say that Nusselt number is therefore a representation of non-dimensional temperature gradient in the fluid at the interface between the fluid and the solid.

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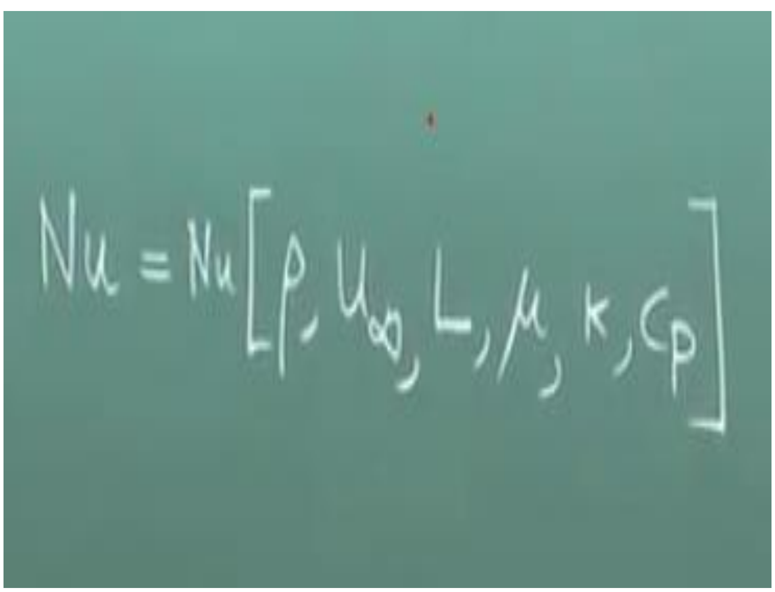

So, it will depend on what? Intuitively the Nusselt number will be a function of, these are the parameters like; out of these what are the fluid mechanic parameters? The fluid mechanic parameters are Rho, Mu infinity, L and Mu. And we will discuss about these parameters, why these parameters are important? And the non-fluid mechanic parameters are K, Cp and partly Rho also. Rho partly contributes to the fluid dynamic parameters and partly contributes to the thermal parameters.

So, the question is that why these parameters are important? These parameters are important because of the following reasons, let take an example. Let me ask you a question if U infinity is increased, do you expect a higher heat flux at the wall or lower heat flux at the wall? The question is if you increase U infinity, do you expect a higher heat flux at the wall or lower heat flux at the wall?

You expect a higher heat flux at the wall; the reason is like this, see what is happening with higher U infinity, there is a higher rate at which heat is getting transferred by fluid flow. To compensate for that to replace, more heat is to be supplied from the wall. So, let us say that there is some person who is a voracious eater, who always eats. And after eating the person becomes more and more hunger like a diabetic patient.

So then, to make that person happy you have to supply more fruits otherwise that person will not feel happy. So, this is something like this, the fluid flow is carrying heat at a very fast way. So, to make up for that more heat has to be supplied from the wall. So, that is one of the very important parameters. Then the other parameter L is of course important because the entire characteristic length scale of the problem is governed by the length scale.

Mu and Rho, the combination of Mu and Rho what does it play a role? So, it is essentially not Mu or Rho independently but Mu / Rho, which is known as kinematic viscosity. Now, the question is why Mu / Rho is important and why not Mu individually and why not Rho individually? So, what does Mu do and if you think of this typical problem, let us draw a separate sketch here.

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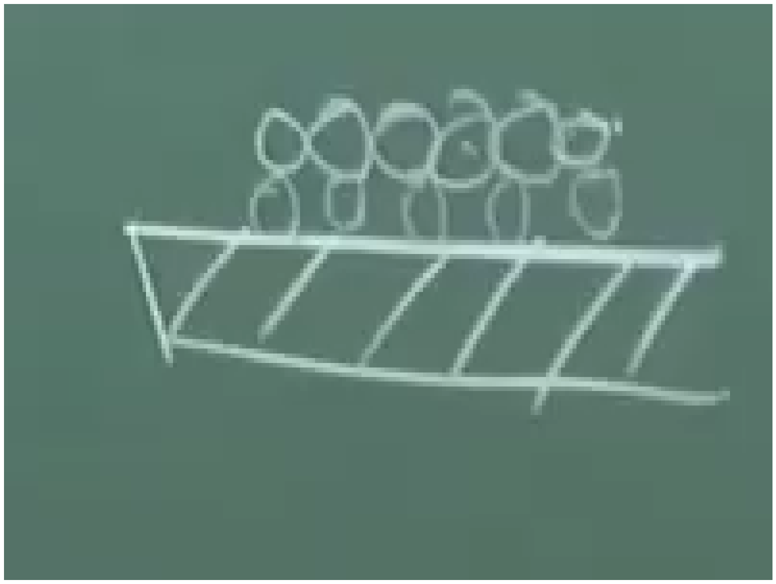

Let us say this is a wall, the first molecule, which are adhering to the wall will directly feel the effect of the wall so if the molecules are significantly compact, then they are almost like trapped by the attractive interactions of the wall and they cannot escape from the wall. This in fluid mechanics, in classical fluid mechanics is known as no sleep boundary condition. There is no relative tangential component of velocity between the fluid and the solid boundary.

Of course, there are many situations in micro fluidic and Nano fluidic, where this no sleep boundary condition is violated, but we will not discuss about that in the basic course. So, let us say that there is a situation, when the no sleep boundary condition is valid and the molecules are sitting to the wall. Now, these molecules are sticking to the wall because they directly understand that there is a wall. Now, consider the second layer of molecules on the top of these one.

These second layer of molecules, what happens to them? Does the second layer of molecules move with the velocity Mu infinity? No, it moves with a velocity much lower than Mu infinity but not zero. So, it moves with the velocity, which is much lower than Mu infinity but faster than this, that means why it is not moving at the velocity Mu Infinity. Because, it understands that there is a wall.

How does it understand that there is a wall, it is not in direct contact with the wall? So, how does it understand that there is a wall? There is a messenger in the fluid that propagates a message that, yes there is a momentum disturbance and that messenger in the fluid is nothing but viscosity of the fluid. So, viscosity of the fluid is physically a messenger for momentum disturbance. It gives a message that there is a momentum disturbance in the fluid.

Now, the fluid gets the message that there is a momentum disturbance, but the fluid has its own inertia. So, even though there is a momentum disturbance the fluid intends or tends to maintain its whole momentum. By virtue of which property of the fluid, by virtue of inertia fluid. By virtue of inertia of the fluid, the fluid tends to sustain its momentum and inertia of the fluid is proportional to the mass and mass is proportional to the density.

So, Mu is the tendency of the fluid to undergo a change in momentum or disturbance in momentum and Rho is the tendency to sustain the momentum. So, this is a relative ability of change of momentum with respect to the tendency to sustain its momentum. And that is why it is important as a basic feature. And you know it is very interesting that, there are different types of fluids which grossly vary in Mu and Rho, but in terms of the property Mu / Rho, there are many apparently different types of fluids which has close value of the kinematics viscosity.

If you go to steel plant, you will find that there is flow of molten steel in steel plants. Now to understand the fluid Mechanics of molten steel, many times in the R and D divisions of Steel plant what they do is they make water models that is instead of studying the behavior of Steel, they are studying the behavior of water in the transparent box. The question is that why it is being done like that?

Of course, the practical constraint is that molten metal flow at a very high temperature it is very hazardous to dilute it, it is very hazardous to handle. So, it is very difficult to do fluid mechanics experiments by visualizing that. Not only that molten metal, most of the cases they are not transparent medium. So, you cannot visualize the flow. So, there are practical reasons, but the question is that why you cannot visualize molten metal, how you can replace water with molten steel.

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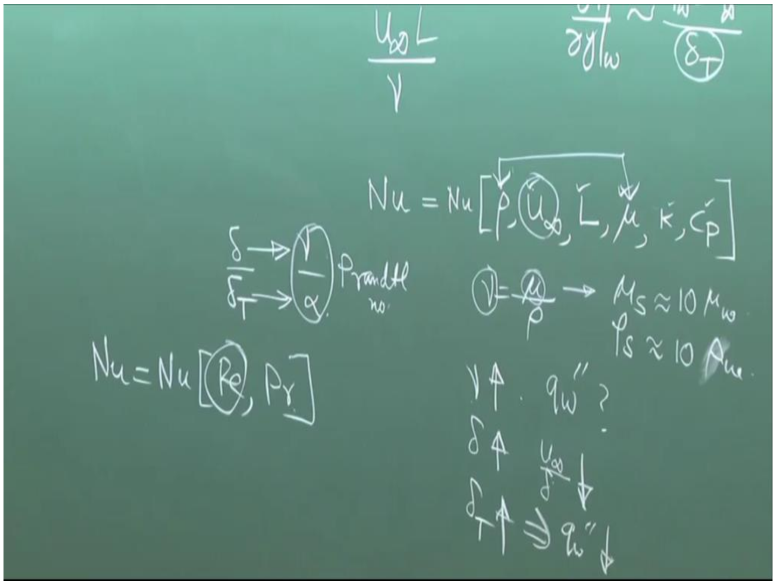

One of the basic reason is that typically Mu of steel is roughly 10 times Mu of water roughly, one order and Rho of steel is also roughly one order more than that of water. So, Mu / Rho of steel is roughly Mu / Rho of water. So, this is very remarkable behavior that all the density and viscosities of water and steel are grossly varying. But kinematic viscosity of water and kinematic viscosity of Steel are roughly very similar.

Now that make water models work but only when heat transfer effect is not very important. When heat transfer effect is important you also have to consider the thermal diffusivity, and thermal diffusivity of steel and water are grossly different. So, when heat transfer effects are very important just by these similarities you cannot say that whatever will happen for molten Steel be the same with water.

Because the thermal diffusivity wise, they are different. But, purely from fluid mechanics configuration this is the similarity. Now, we have to understand that what will be the role of this Nu that is, if you increase this Nu what will be the wall heat flux? Will it increase or decrease? See what we are trying to do, we are trying to understand physically first? influence of fluid mechanic parameters on heat transfer.

Because that will give us the motivation of why should we study fluid flow equations in the context of heat transfer. In the context of Fluid Mechanics, we study fluid flow equations, fine, but why in the context of heat transfer we should study fluid flow equations. So, U infinity role we have understood, so now we have coupled these two and we are trying to see that what is the role of this Nu? Now, if Nu is high, what will happen?

See Nu will dictate that how far from the wall you should go to assist, to attain U infinity. The distance from the wall that you go to attain close to U infinity that is known as the boundary layer thickness. So, boundary layer thickness can be anything from very small to very large. So, the question is that when Nu is increased then, what do you expect the boundary layer thickness to be large or small?

If Nu is large that means, if Nu is increased then what is the distance up to which the wall effect is faced? The distance is more. So, more and more fluid understands the effect of the wall if the fluid has higher kinematics viscosity. Because the momentum disturbance is very strong. So, if that means, if Nu is large then the boundary layer thickness is large.

If the boundary layer thickness is large, what is the velocity gradient within the boundary layer? If the boundary layer thickness is large, then the velocity gradient within the boundary layer for a given U infinity is small. Now it can be shown that, if we consider so this is related to velocity gradient. We will discuss about the velocity gradient later, but similar to the boundary layer within which the velocity gradient occurs, there is a boundary layer within which the temperature gradient also occur. That is known as thermal boundary layer.

So, in a thermal boundary layer the temperature will change from T wall to T infinity. Let’s the thickness of the thermal boundary layer be Delta T. So, Delta / Delta T, what is Delta related to? Delta is related to the kinematics viscosity of the fluid. Similarly, from analogy can you say what this Delta T related to? Thermal diffusivity of the fluid.

Kinematic viscosity and thermal diffusivity are very similar parameters. Thermal diffusivity is what? Thermal diffusivity is K / Rho Cp. K is just like Mu / Rho, Mu is momentum disturbance and Rho is momentum sustenance. Similarly, K / Rho Cp, K is thermal disturbance due to conduction and Rho Cp is the ability for thermal storage or that is thermal inertia. So very similar.

So, Nu / Alpha is related to Delta / Delta T and this Nu/ Alpha is the parameter known as Prandtl number. So, for a given Alpha if you have higher Nu, you have higher Delta, right? So, if you have higher Delta, because of Prandtl number is the property of the fluid you should also have higher Delta T. Because Delta/ Delta T is peak for a given fluid. So, if you have a higher value of Delta T then what will happen? If u have higher value of Nu, then you will have higher value of Delta T.

If you have higher value of Delta T, then what is the temperature gradient within the thermal boundary layer, is it more or less? Remember that we are essentially interested about the temperature gradient. So, what is roughly the temperature gradient within the thermal boundary layer T wall - T infinity / Delta T roughly, if it was linear, this is just order of magnitude. So, this is roughly Del T/ Del Y at the wall, order of magnitude.

So, if you have higher Delta T, you will have lower heat flux. That means the Nusselt number will be low. So, you can see we started with Nu we came with heat flux something very non-intuitive, right? We started with a pure fluid mechanics parameter and we came up with an increase of Nu will have a decrease of heat flux. That means decrease of Nu will have an increase of heat flux? So, decrease of Nu and increase of U infinity and also increase of L are of same effect. So, the net effect is of increase of U infinity L / Nu.

So, if you increase U infinity, the effect is like decrease of Nu and increase of L. And these together as a combination is Reynolds number. That all of you know, right? So, the Nusselt number for post convection will be a function of Reynolds number and these parameters Prandtl number. I just try to give you a pure physical arguments without getting into any mathematics.

Just simple physical argument but this kind of physical argument is very important. Because, we should first develop a motivation of why are we interested to go through the fluid mechanics equations to understand convection? So, here is the parameter which now is related to fluid mechanics. So, this gives us a motivation of why should we study fluid dynamics for convective heat transfer.

So, when we say fluid dynamics, we have to keep in mind that fluid dynamics, to understand fluid dynamics, we need to understand basic equations of fluid mechanics and what are the basic equations of fluid mechanics. The basic equations of fluid mechanics at the conservation equation like conservation of mass, conservation of momentum and in heat transfer, you also have conservation of energy.

So, when we say the conservation of energy in the context of heat transfer, we essentially talk about the conservation law dictated by the first law of thermodynamics. So, energy, where you have both heat and what involved. So, we will start with conservation of mass, but before that we will try to recapitulate a general theorem, which talks about conservation in fluid mechanics and then we will apply that theorem to address the issues of conservation of mass, conservation of momentum and then eventually conservation of energy. And that theorem is Reynolds transport theorem.

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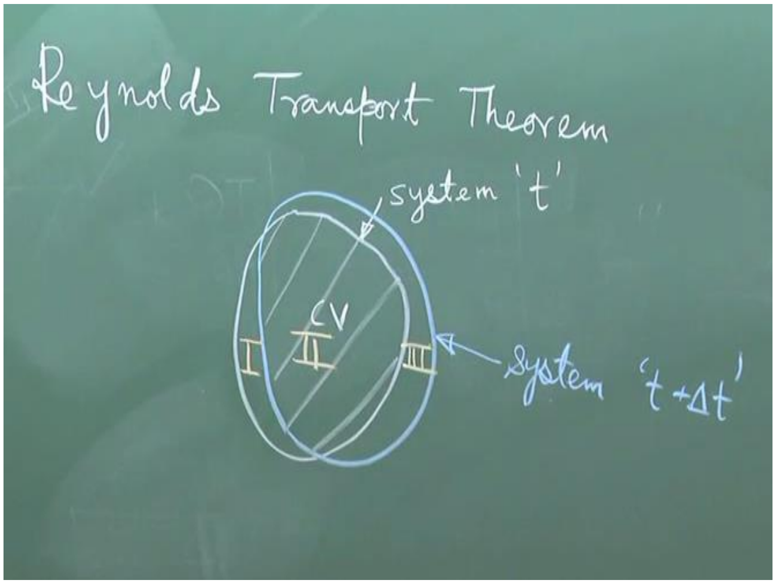

Again, everything requires a motivation. We should understand that, why are we interested about Reynolds transport theorem. I mean, as a teacher, I really say that any teaching should not start with, let us take a control volume, do this and do that all these things are fine. But, it is important to understand first that what you are trying to achieve with Reynolds transport theorem and why you are trying to do that.

See all the basic law of mechanics, classical mechanics are based on something on the concept which is control mass system, that is a system of particles of fixed mass and identity. If you think of Newton's Laws of Motion, the basic Newton's Laws of Motion they also talk about some identified particles on which you write, apply the basic law. So, you have a control mass system.

In a mechanics approach, it is also known as Lagrangian approach that is you track the motion of identified particles. The particles maybe solid particles, fluid particles whatever. On the other hand, in fluid mechanics you do not commonly rely on Lagrangian approach. Why? Because fluid has numerous number of continuously deforming particles and it is impossible to track the motion of individual particles just like that.

You may track the motion of few identified particles but the total description of motion of a fluid by identifying all the particles is difficult. So, instead what you do? Instead you focus your attention on a specific location. As if you were sitting with a camera and focusing the camera at a particular location and seeing what change is taking place across that. What fluid is coming in and what fluid is going out.

So, that approach is known as Eulerian approach or control volume approach. So, in fluid mechanics you are dealing with control volume approach, but the basic laws are all described on the basis of a control mass system. So, you require a transformation from a control mass system approach to a control volume approach, so that equations of motion can be written in terms of a control volume approach. And that transformation from control mass approach to control volume approach is given by a theorem known as Reynolds transport theorem.

That is why we need to study the Reynolds transport theorem. So, let us say that we have a system, which is basically a control mass system at time t. Let us say this blue colored outline is a system at time t + Delta t. This common region you can identify as a control volume that is a fixed region on which you focus your attention over the time interval Delta t in the limit as Delta t tends to 0. So, you can say that, this is your control volume. Now this entire region can be divided into several parts. Let us say this is zone number I, this is zone number II and this is zone number III.

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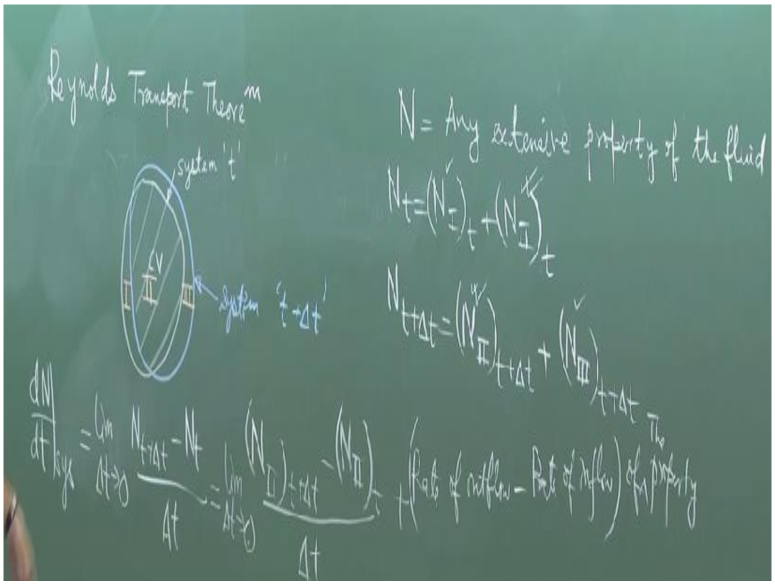

Let us say that capital N is any extensive property of the fluid. So, extensive property means a property, which depends on the extent of the fluid or mass of the fluid. So, N at time t is equal to N I at time t + N II at time t. Because at time t, the system is I plus II. Similarly, N at time t + Delta t is this. This is very straight forward. So, what we are interested is to calculate, what is dN, dt of the system?

So, by definition, this is limit as delta t tends to zero, N t plus delta t minus N t, divided by delta t. So, N t plus delta t, what is that, so it has two parts. One is two, another is three. And N t, it has also two parts, one is two, another is one. So, the two region is common for both, one has t plus delta t and another has t. So, we will isolate that. We will write limit as delta t tends to zero, N II t plus delta t minus N II t, divided by delta t plus N III at t plus delta t divided by delta t, what is that, that is the rate at which the property is flowing out of the control volume.

Remember that, as you are advecting in this way, this is acting like inflow and this is acting like outflow. So, this is plus rate of outflow minus rate of inflow of the property. Now the next job is to express this mathematically, rate of outflow minus rate of inflow of property.

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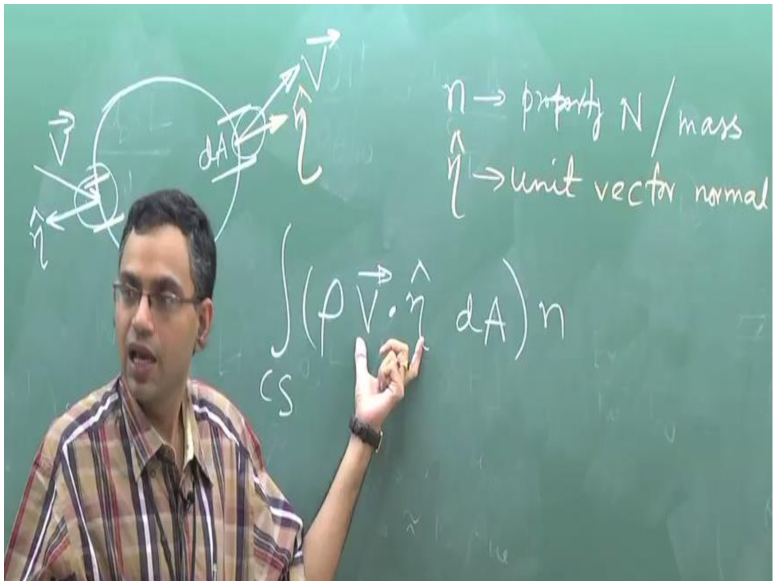

Let us make a sketch of the control volume, let us say this is an outflow boundary. Let us say dA is that elemental area on the outflow boundary. Let us say that V is the velocity of flow at this outflow boundary. And small n is the property N / unit mass. We also describe a unit vector, normal to the perceive. Let us say that Eta is a unit vector, which is, so what is Eta, unit vector normal (outward) normal.

Now can you tell what is the master unit to this dA. What is the volume flow rate to this dA. volume flow ratef is the normal component of velocity, time the area. So, what is the normal component of velocity? This is the normal component of velocity. Time the Eta is the volume flow rate. This time the density Rho is the mass flow rate. And the small n is the property / unit mass. So, this time n is the total rate of the property flow across the outflow boundary.

Let us think about an inflow boundary. Let us say this is an inflow boundary. That will be the difference between the inflow and outflow boundary. The only difference is that when you calculate v dot eta, the dot product will be positive for outflow and negative for inflow. That you can see from this figure. The dot product of these two vectors is positive here, but the dot product of these two vectors is negative here.

So, if you write outflow minus inflow, you do not have to separately take two terms. It is now integrity to what the entire control surface, a positive dot product will mean the outflow part of the surface and negative dot product will mean the inflow part of the surface. This CS is control surface, the surface that bounds the control volume.

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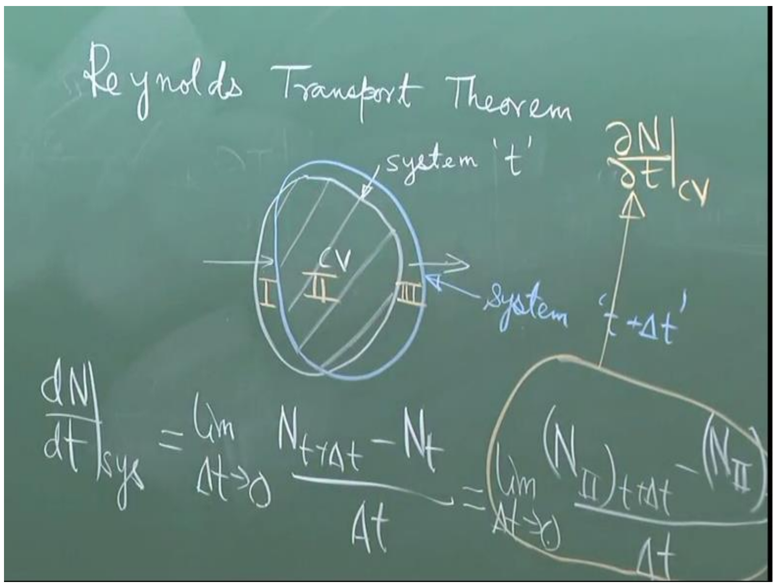

And the other term, which is this term, we can write this term as partial derivative works N with respect to time within the control volume. Why partial derivatives? Because you are fixing your location to and at the same location, you are finding the theory of property, with respect with to time. So, change of property, with respect to time, at a fixed location, that is why partial derivative and not ordinary derivative.

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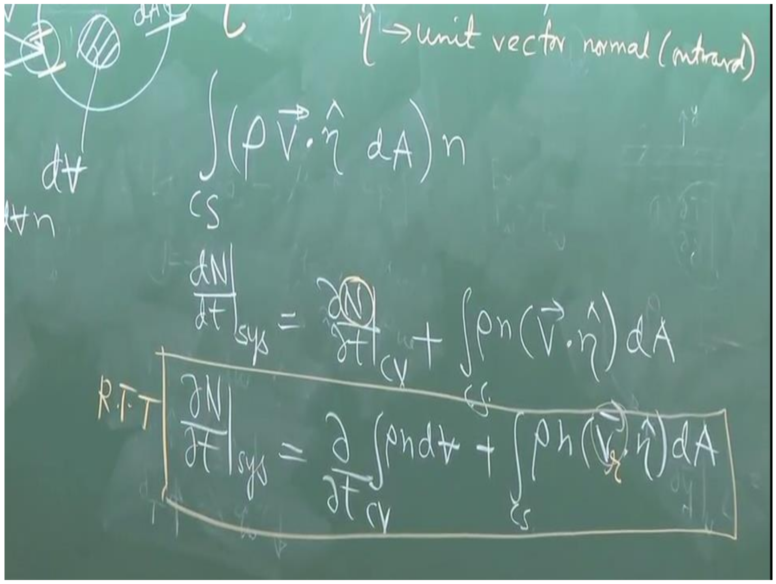

So, we can combine that and use in this expression to write dN, dt of the system. What is capital N of the control volume? Let us say this is a small element dC, a volume element that we get. What is the mass of this element? Rho * dV. What is the property for unit mass? Small n. So, the total property is this * small n.

So, this you can write as, there is one important point to note here, is that, what is this V. Let us say that it is control volume at the tank. Let us say that there is a tank, which is moving towards the right, with a velocity of one meter per second. And let us say water is also moving towards the right, at a velocity of one meter per second. There is a hole in the tank. Question is, what is the rate at which water is coming out of the tank?

There is a tank, in which there is a hole. The tank is moving towards the right with the velocity of one meter per second and water is also moving towards the right at a velocity of one meter per second. Then what is the rate at which the water is coming out of the tank. Zero. Because, the tank is moving at one meter per second, in the same direction water is also moving with one meter per second.

So, there is no relative velocity of water with respect to the tank. So, the flow across the control surface depends on not the absolute velocity, but the velocity of the fluid relative to the control volume. So, this V, must be V r, velocity of the fluid relative to the control volume, not the absolute velocity. So now, see we have been successful, in deriving a general relationship, where the rate of change with respective to a sys scale, this is the Lagrangian parameter, is described in terms of rate of change with respect to a control volume. That is an (()) (50:48) parameter. And this is known as Reynolds transport theorem.

In the next five minutes of course, we will illustrate the use of Reynolds transport theorem for the law of conservation of mass. We will start with, as I told that in fluid mechanics, we are interested about some basic laws of conservation and the most fundamental law of conservation is the conservation of mass. So, we will start with that. The first example is conservation of mass.

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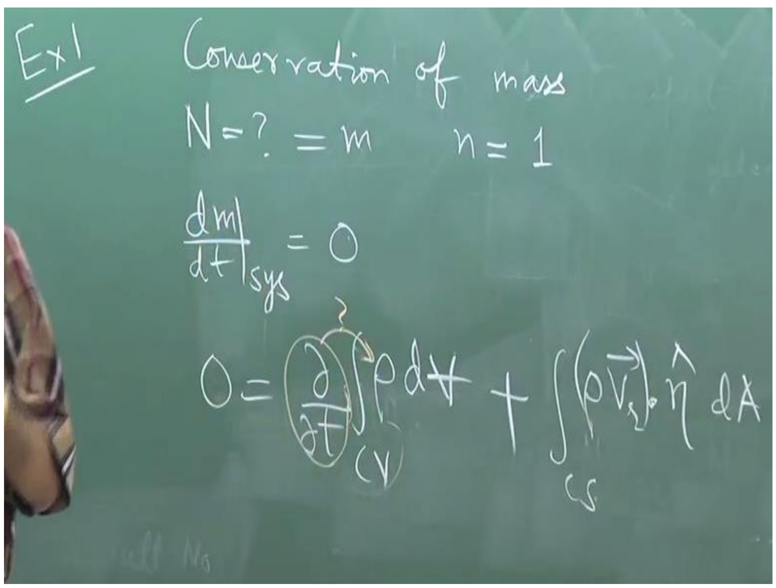

For conservation of mass, what is capital N? It is the total mass of the system. What is small n? One. Let us look at the previous state. So, what is dn, dt of the system? By definition, a control mass system, has a fixed mass. So, what is the rate of change of a mass of a system? Zero. So, this is zero. Now to simplify in the next stage, there is an immediate tendency of taking this derivative inside the integral. Question is, can we do it or not?

The answer is very straight forward. We cannot always do it. When we can do it? We can do it, only when the volume of the control volume is not a function of time. Then you can put this time derivative, outside and inside the integral, as per your wish. Otherwise, you have to correct, when you differentiate outside the integration and when you differentiate within the integration. They are not the same and they have to be corrected by an additional term, which is given by the Leibniz rule, of differentiation under integral time.

So, it is not that we can do it always, we can do this, we can put this inside, provided the volume of the control volume is not a function of time. So, when the volume of the control volume is the function of time, let us say that there is a balloon, which is continuously expanding or contracting whatever, so for solving a fluid mechanics problem in that, we cannot put this derivative within the integral.

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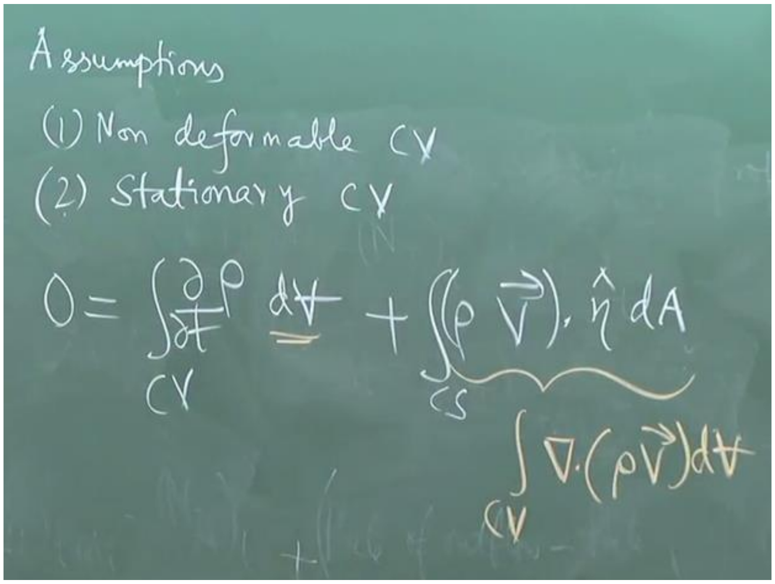

So, we will make certain assumptions. One is non-deformable control volume. That means d is not the function of time, volume of the control volume. Second is stationary control volume. Stationary control volume means that the control volume is fixed. If the control volume is fixed, then relative velocity or absolute velocity are the same. So, under those conditions, you can write.

Finally, this is the volume integral. So, we can convert this into a volume integral, by giving the divergence theorem. So, we can write. So, if you have a vector function s, s dot, say ds is equal to divergence of s dv, where the surface is over with the integral is made, is a surface that completely bound the control volume. By definition, the control surface is the surface that completely bounds the control volume.

So, this is a very important pre-requisite for the divergence theorem to be applied. So, when that is there, you can convert the area integral to volume integral. Why? What is the motivation? The motivation is that you, both of this trans are now having the volume integral.

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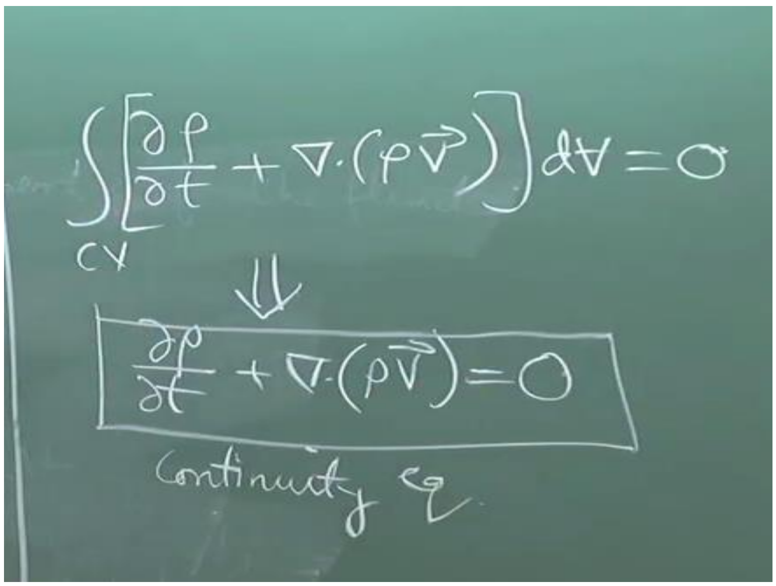

So, you can write the integral of. Now let us look into a situation. You have integral of fx, dx equal to zero. Does it necessarily mean that fx has to be zero? If it is a definite integral, not. For example, you may have a sinusoidal function, so y is equal to sin x over a period, integrated over a period, the integral will be zero.

That does not mean that sin x is equal to zero everywhere. But when this is true, for any arbitrary choice of the domain, then the function must be zero. So, this is true, for any arbitrary choice of control volume. That is what is very important. Not just a specific choice, but for any arbitrary class of the control volume and that means that, this must be equal to zero. This is nothing but the continuity equation in fluid mechanics. So, the continuity equation, physically talked about conservation of mass.

We will discuss in the next class, some implications of the continuity equation and then some special cases, what does it lead to for incompressible flow, may be for steady flow, unsteady flow with certain situations, we will discuss and then we will use the Reynolds transport theorem for a second example, which is conversation of momentum. And from that, we will derive something which is known as Navier-Stokes equation. That we will take up in the next lecture. Thank you.