Conduction and Convection Heat Transfer
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Lecture No - 18
Introduction to Convection

Ok. Welcome you all to this session, conduction and convection heat transfer. So far, we have discussed, the conduction heat transfer in a stationary medium, which is a solid. This is because, I told you earlier that in fluid, if there is a temperature gradient, as we know that heat transfer is always associated with the temperature gradient. With the temperature gradient, motion sets in a fluid, even if there is no imposed motion by the external source.

Therefore, solid is the only medium, which can be considered as absolute dress or stationary, through which the conduction mode of heat transfer has been studied in detail. And we have analyzed, both steady and unsteady heat transfer and steady one dimensional and two-dimensional heat transfer. So, now we will start the convection heat transfer. Today I will present before you a brief introduction of the convection heat transfer.

Convection heat transfer is referred to the phenomena, when heat is transferred between a solid surface to its adjacent fluid, which is in motion. Now, fluid motion may be imposed by an external agency, fan blower, while we call it as a post convection. Or if the fluid motion is self-induced by the temperature gradient, due to the buoyancy which we tell as buoyancy induced flow or buoyancy driven flow, then, we call it as a free or natural convection.

So, they are good. These phenomena of heat transfer from a solid surface to its adjacent fluid in any direction, depending upon the relative values of the temperature, from solid to fluid or fluid to solid, is referred to as convection heat transfer. So, therefore, to understand the convection heat transfer, let us consider a simple case of flow of fluid past, a flat surface, flat plate for example.

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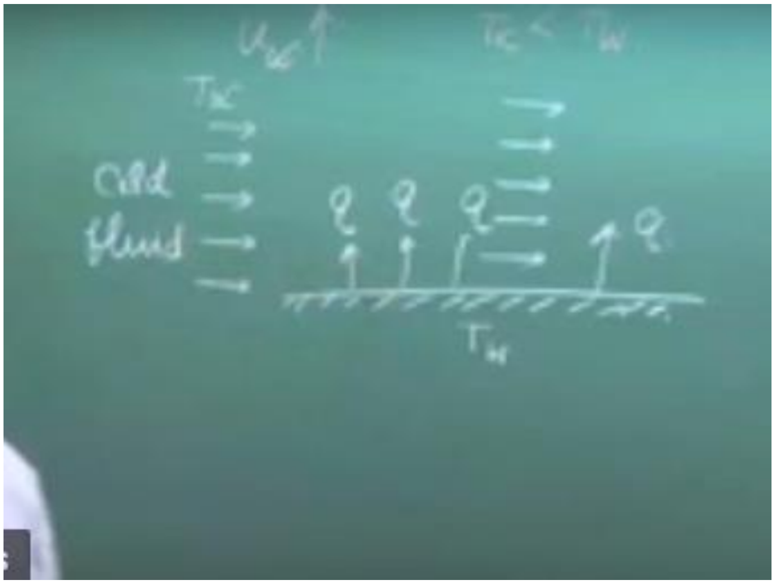

Let us consider a flat surface, at a temperature T w and stream of cold fluid, flowing through these past these surface, with the temperature of T infinity, which is less than T w, stream of cold fluid. It is cold fluid. As it flows past the solid surface, it will take the heat from the solid and it will be heated. So, therefore it is a common expectation that as the cold fluid flowing past the solid surface, surface temperature is had.

Heat is flowing from the solid surface to the cold fluid in this direction. Each and every point, we can represent this as the heat flux Q, heat flow per unit area. So, it is our common experience. Everybody knows it that if we increase the flow of the cold fluid, if we consider the cold fluid is flowing with the velocity of U infinity, the plate is cooled faster. That means the rate of heat transfer or the heat flux is enhanced from the surface to the fluid, which means that these flow of the fluid has got some effect in the transfer of heat from the surface.

So, this is the phenomena of convection, where the heat is transferred from the surface to the adjacent fluid. And at the same time, the rate of heat transfer or the rate of heat flux, depends on the flow of the fluid (()) (4:45). If we decrease the velocity, the rate of heat transfer will be decreased. The plate will be cooled at a slower rate.

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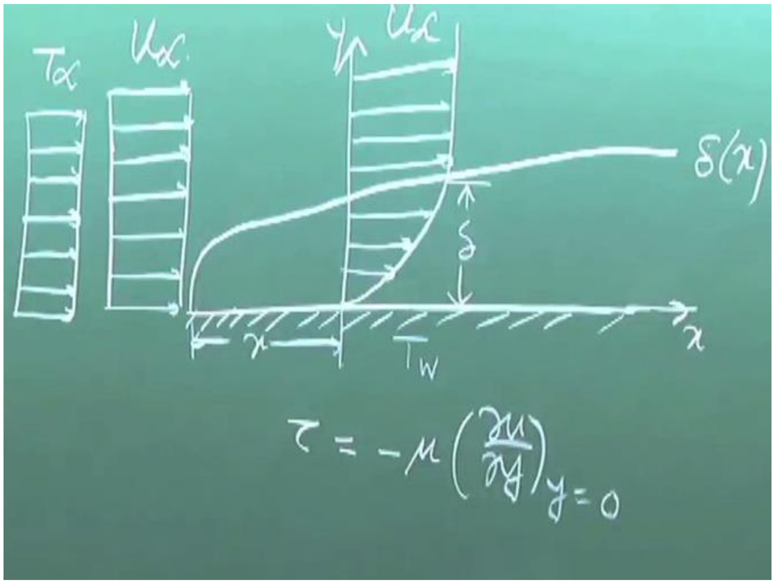

Now to understand the mechanism, what happens here, what the heat transfer, let us consider this, in a much deeper sense that, let us consider that plate again with the temperature T w, at a fixed uniform temperature T w. Now, let me denote this U infinity as this scale, the velocity with which the fluid is approaching and also I show a temperature scale of uniform temperature T infinity, though temperature is a scalar quantity.

I am showing it by the arrow, just to have a representation like this, that with the temperature uniform T infinity, which is lower than T w and do the infinity approaches this way. As you already know, in fluid mechanics, which will be again told in details afterwards or we will have recapitulation of the basic fluid mechanic that the interaction between the fluid and the solid surface and in consequence of the momentum transport in the fluid.

Because of its property like viscosity and density. We know that there is a development of a layer that is known as boundary layer, which is a function of this, if we consider this as x, in a sense that at any distance x from the leading edge, if we draw the velocity profile, it loops like this. That means the velocity changes in the direction parallel, perpendicular to the plate, let us consider y, within a thin region, which is known as the boundary layer thickness.

In fact, the velocity never approaches, never reaches the U infinity free stream velocity, it approaches asymptotically and we consider a thickness by giving a reference value of 99% of U infinity. But for our understanding, we consider better understanding that delta is the thickness, where the U infinity is reached. So, this is the velocity profile.

It is always observed because of the two things, one is the surface fluid interactions, which usually for usual surfaces in the domain of continuum, is a no slip at the surface. That means, the fluid velocity relative to surface is zero. If the surface is at rest, fluid velocity is zero. And, it is observed that the velocity suddenly increases and within a short distance asymptotically marches to U infinity and that is known as the boundary layer thickness.

And, this is because of the momentum transport and the retention of momentum played by the two important properties of the fluid, Mu viscosity and the density Rho. And the ratio Mu by Rho, as you know the kinematic viscosity that decides, how far the presence of the plate will be felt, in terms of the velocity gradient. And as we know, it is little recapitulation of your basic understanding of hydro dynamic boundary layer.

The Tau that shear stress at the plate is given by minus Mu, Del u, Del y, at y is equal to zero. Similarly, let me write this thing that side. Tau is minus Mu, Del u, Del y, at y is equal to zero. At y is equal to zero is the flip, at any layer Tau, Del u, Del y. That means the c axis is confined within the layer, boundary layer thickness, where Del u, Del y is there. But beyond the boundary layer thickness, Del u, Del y is zero, no shear stress is felt.

This is the basic boundary layer. This is known as hydrodynamic boundary layer, because of the momentum transport and the interaction between this solid surface and the fluid. Similar is the case for the heat transfer, due to the temperature T w, which is differing from T infinity, differing from T w.

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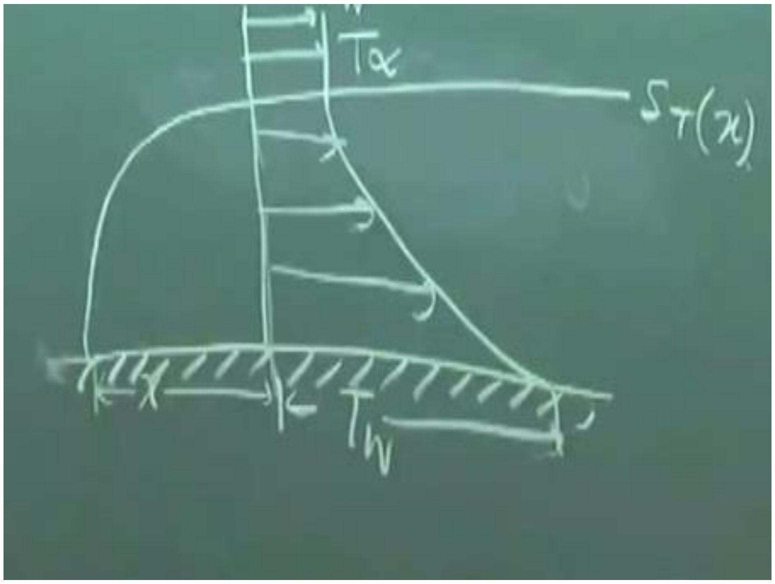

In the similar fashion, if we draw the plate, we will see that a thermal boundary layer will grow, which is also function of x and like the velocity boundary layer in a sense, that at a distance x, from the leading edge, that this is the scale of T infinity, that means it is the scale T w, that means the temperature variation from this surface to the same way I draw it, the temperature also.

The temperature variation is restricted within a small layer, amplified to what I have shown you. That is known as thermal boundary layer. So, that beyond to these, the temperature variant is zero and the temperature marches with the free stream temperature T infinity, from T w. So, this is the growth of the thermal boundary layer. This is the consequence of two things. Here also, the surface to fluid interaction is that, it requires the thermal equilibrium, so that at the surface, the fluid temperature has to be the surface temperature.

And the heat transport or the diffusion of heat in this direction, which is determined by thermal conductivity K, along with the heat capacity, how much heat or thermal energy can be retained and how much can be transported, so that the temperature of the plate is felt away from the plate or influenced. The plate is influencing the free stream at what distance that is determined by thermal conductivity mainly and the Rho cp that is the specific heat and the density.

So, depending upon these factors, a boundary layer, thermal boundary layer develops, which gives the temperature profile like this.

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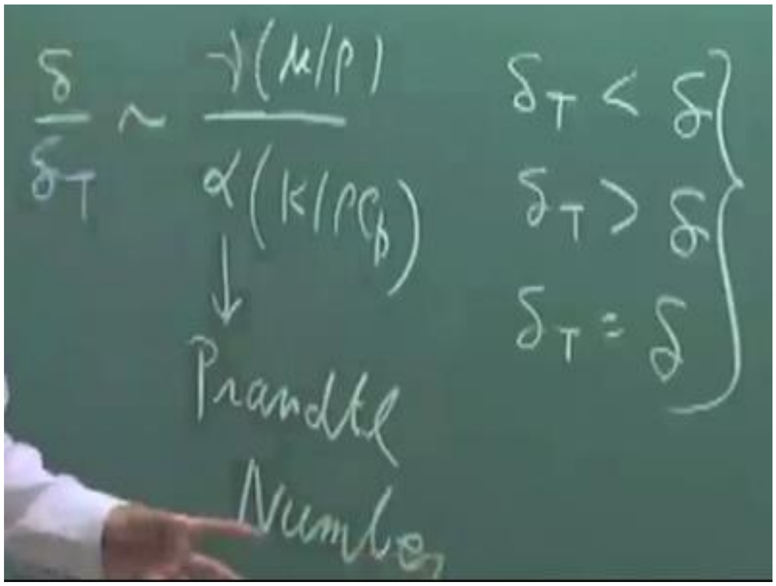

Now this thermal boundary layer delta T, may be less than delta, may be greater than delta hydro dynamic, may be equal to delta. All cases are there. Here I have shown, a case, where delta T is greater than delta. And the ratio of this two, that is delta by delta T, is governed by the two important properties, just as I have told you or explained you, it is proportional to Mu, the kinematic viscosity by alpha. What is that? Mu is nothing but, Mu by Rho.

And alpha is k by Rho C p. That means, it is the ratio of the kinematic viscosity, which is Mu by Rho, that means both Mu and Rho combinedly decide the diffusibility of the momentum, so, the distance up to which, the influence of the plate will be felt. In the similar way, it is the thermal diffusibility, which is the ratio of thermal conductivity and row C p, its capacity, which decides the diffusion of it, how much it will diffuse, so that the influence of plate is felt. And their ratio, will determine this ratio.

This ratio may be greater than one, less than one or equals to one, depending upon that type of the fluid, is the fluid property. This is known as Prandtl number. At this moment, this is just an information for you and this will be told in detail afterwards. Do not get discouraged that something is information, because in this class, nothing will be information. All will be known in depth, with the concept. But at the present moment, it is the information that this ratio depends upon that.

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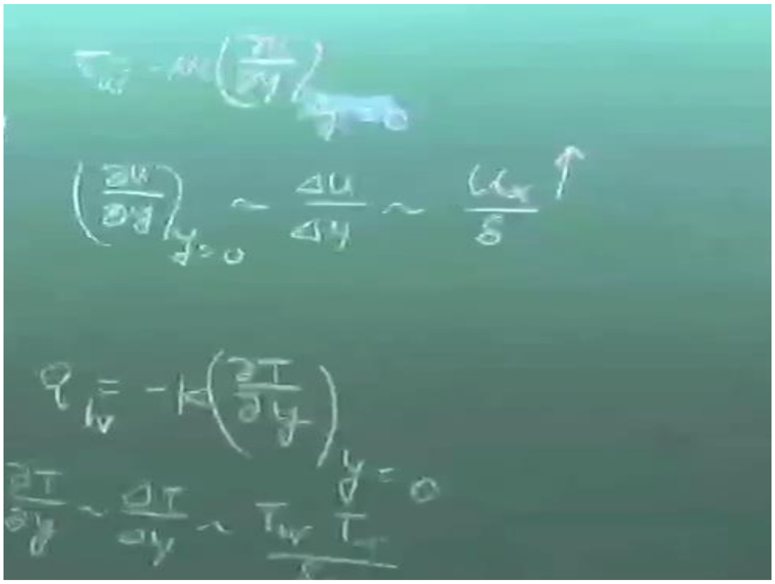

Now you see that, here in this case, the heat flux Q, at any location x, is written as minus k, Del T, Del y. Now, in convection heat transfer, we are interested at the wall, heat flux at the wall, so this is at y is equal to zero. And, in fluid mechanics also, we are interested at with the shear stress on the wall, not at any other plane, because that gives the total drag exerted by the fluid on the surface. So therefore, this y is equal to zero.

Now you see the two equations look very similar. That the shear stress is proportional to velocity gradient (()) (15:18) proportionality Mu, co-efficient of viscosity and it is the thermal conductivity. So, therefore you see for a fluid with given values of Mu and K, both shear stress and heat flux, depends on velocity gradient and temperature gradient. So, that controls the rate of heat flux and also the shear stress.

For comparison, I am writing this. Now rate of heat flux that means heat flux at the solid surface, it is given their Q. Q is by conduction. And by the Fourier heat conduction law, Q w is minus K Del T by Del Y. Now, consider a K is so and Q infinity is increased. If Q infinity is increased, what will happen? Now, this can be retained as Tau is Mu Del u, Del y. Now you write here, Del u, Del y, at y is equal to zero, scales as change in u, change in y and that scales as Q infinity by delta.

This is the scale of Del u, Del y, Q infinity by delta. Similarly, here, the Del T, Del y, which scales as Delta T, Delta y, which is equal to T w minus T infinity, divided by Delta T. Now consider, the case when the flow of velocity is increased, that means if Q infinity is increased, what will happen? We have already read in fluid mechanics, so when flow velocity is increased, the hydrodynamics boundary thickness gets decreased.

That means, with more flow velocity, the diffusion of momentum is such that the influence of the free stream comes close to the plate. That means plate is incapable of giving its influence signal far away from this surface. So, therefore an increase in Mu infinity, delta is decreased.

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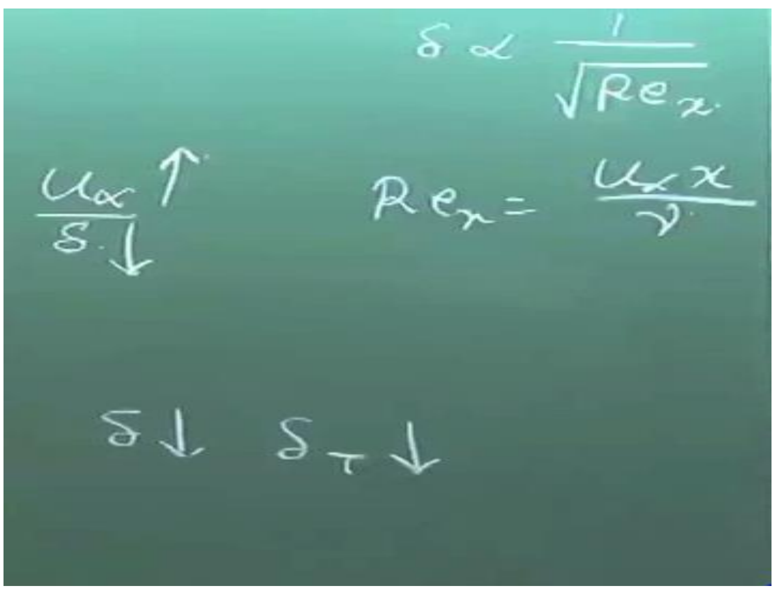

I will give you for your recapitulation, which will be again done in this class as a review of fluid mechanics that this Delta hydrodynamics boundary line thickness is proportional to one by root over Re x. x is the local Reynolds number. Delta is the local boundary line thickness. If you remember, this Re x is U infinity x by Nu. So therefore, an increase in Nu infinity, decreases the hydrodynamics boundary layer theory.

And, for a given Prandtl number, delta T is also reduced. So, now first let us see here what happens, it is reduced. That means, we see there is a tremendous reduction, increase in the velocity gradient. Velocity gradient becomes much more steep or increase means shear stress is increased. That is why, when the flow velocity is increased, shear stress is increased.

Here what happens, since T w and T infinity remain same, so delta T is reduced, with the increase in U infinity, because when delta is reduced, delta T is also reduced for a given fluid. So, therefore the thermal boundary layer is reduced, which gives then results in an increased temperature gradient. So therefore, we see clearly that in an increased velocity, both the temperature gradient and the velocity gradient is increased and we have more shear stress and we have more heat flux at the surface.

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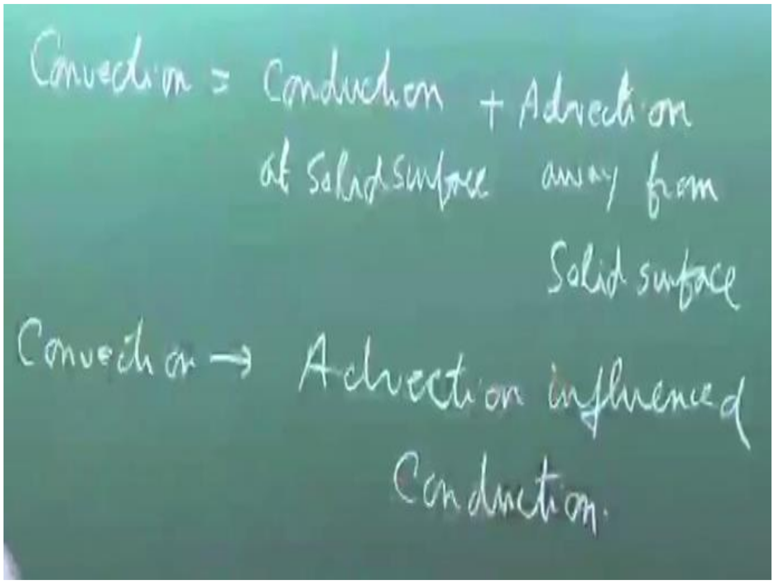

And this is the reason for which convection is considered, the phenomena of convection, therefore considered to be consisting of two things. One is conduction at the solid surface plus advection due to flow away from solid surface. So therefore, the convection phenomena is consisting of conduction at the solid surface and the advection away from the solid surface not only that and they are related.

So, if we increase the advection, conduction is increased, enhanced and if you decrease the advection, the conduction is decreased. The conduction heat transfer is decreased and this is the reason for which I tell, this you will not get in any book, I usually tell that convection mechanism wise is advection influenced conduction.

In fact, I tell you, I don't know, you may find it in many books or in some books you get this that the convection is a phenomena which comprises conduction from the solid surface plus the advection in the fluid flow, main stream, away from solid surface, but I add this thing, convection is nothing but advection influenced conduction because the heat transfer is basically by conduction, which is being affected by the advection.

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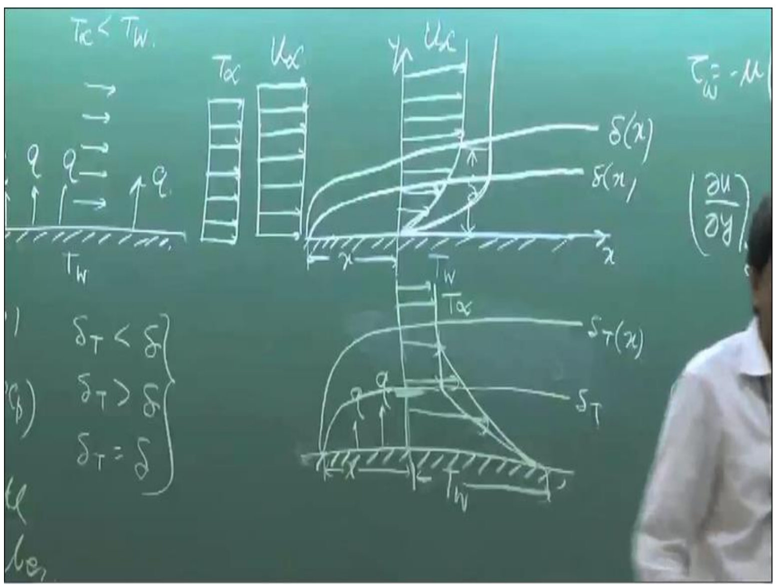

You can find it geometrically also, here if you draw a boundary layer which is lower in thickness, for example, here in the second case the boundary layer thickness is reduced and also this is the second case boundary layer thickness and also your free stream velocity is increased. So, you see this will be your velocity distribution. This line, so that this curve is steeper than this curve with respect to the Y axis that means Del u Del y for this curve is more than this one.

Similar is the case for Del p Del y that can be geometrically seen that if you have a Delta T in the second case, then what you have? You have this thing and the temperature free stream will reach here at the age of this and then it will, so the slope of this curve is more than this with respect to Del T Del y is more in the second case as it is found mathematically.

Now, after this, the very important, this is the basic mechanism of convection heat transfer, what happens is that it is basically conduction at the surface which is affected by the advection that means heat is being pumped from the solid surface by conduction and this pumping by conduction is influenced by the advection of the main stream.

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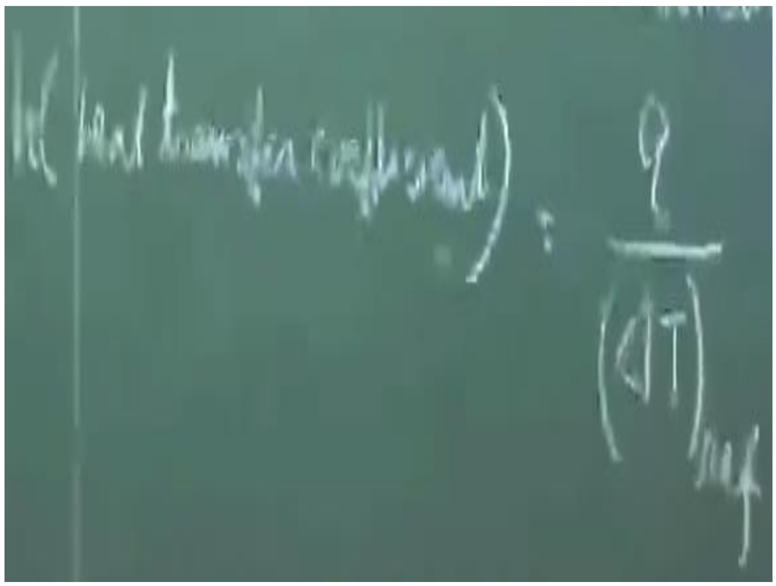

Now, in convection heat transfer we define a quantity known as heat transfer coefficient. Now, come to certain conventional terms and formality of convection heat transfer, heat transfer coefficient. How the convection heat transfer is present, this is the presentation of convection heat transfer, I will tell you in more detail about its physical concept so that so much of it also we will follow after this class in theory.

But heat transfer coefficient is a terminology we define in convection heat transfer coefficient as this heat flux divided by some reference temperature difference. If you make an analogy I will tell you it is similar to, I am not writing here, the keen friction coefficient with respect to fluid flow which is defined as shear stress divided by a reference velocity head, half Rho v square, reference velocity v.

In the similar way, the heat transfer coefficient that we have already taught at, you have been taught at fluid mechanics, q by delta T reference, this q is the heat flux and the delta T reference is a reference temperature difference.

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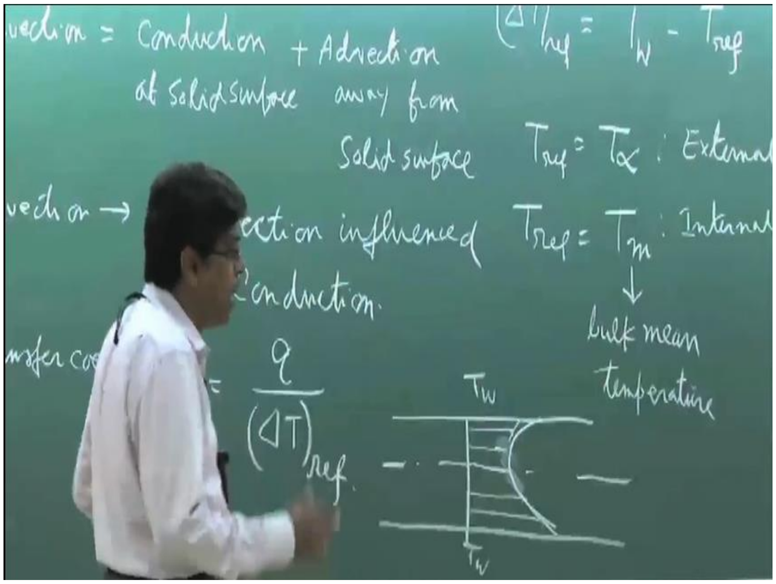

And this delta T reference is actually this solid surface temperature, which is represented as T w wall, wall means solid surface which maybe constant and may not be constant. For our understanding, we make it constant, in the most of the cases we deal with a constant solid surface minus a reference temperature T reference and this reference temperature, this is your whole life prescription that this reference temperature is the free stream temperature in the case of external flow like this which today we are discussing as an example.

And this T reference, the reference temperature is the T m. T m is the bulk mean temperature. This again will be taught afterwards. Today, along with the information, along with the concept some information has to be there, how the convection is presented for internal flow that means flow through ducts. Now, the bulk mean temperature concept is like this. I tell you, for example, a fluid is flowing through a cylindrical duct which has got a temperature T w at this surface and fluid is cold initially.

At any section, we expect qualitatively a temperature gradient like, a temperature distribution like this. T w is higher than the fluid temperature. So, the bulk mean temperature is the cross sectionally average temperature based on the total thermal energy flux crossing a particular section. That means it is that constant temperature which is multiplied by the flow rate and the C p will give the actual thermal energy crossing that section.

The (()) (26:17) define the average velocity. How do you define the average velocity? Sorry, this diagram is not, the average velocity when there is a velocity distribution based on the volumetric flow that is volumetric flow based average velocity. Similarly, it is the thermal energy flux based mean temperature which is known as the bulk mean temperature which is the reference temperature. However, in our case it is not much important, it is for your information.

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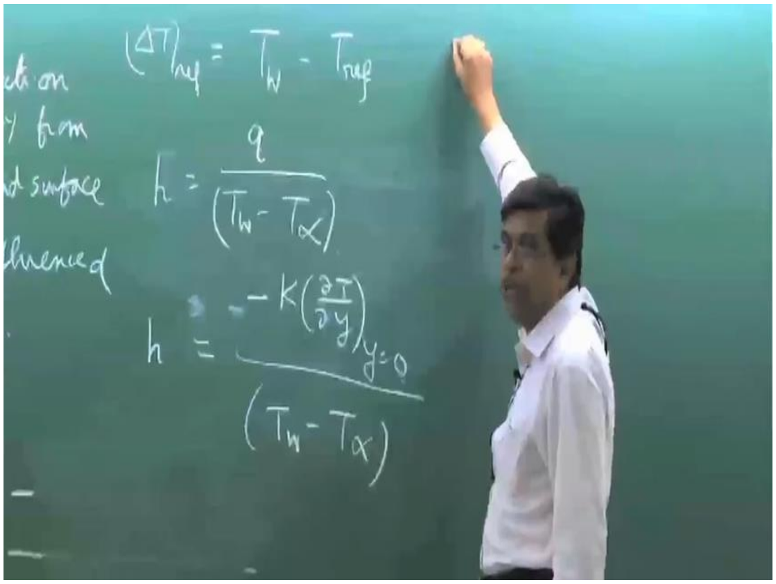

Now here, T w minus T reference, in this case is T w minus T infinity. So therefore, in this case I can write that h is q by T w minus T infinity and since q, heat flux, is a local property which varies from point to point. So, therefore h is a local property which varies from point to point. Therefore, heat transfer coefficient at any location is defined as the local heat flux, heat flux at that location that is q by a, that is heat transfer per unit area divided by this.

Now, if we relate this with the heat flux from the actual mechanism of heat transfer by Fourier heat conduction law then it becomes equal to – K Del T Del y at y = 0 in this case, divided by T w minus T infinity. Again, I am writing h that means if you substitute q by this it becomes like that. Therefore, the local heat transfer coefficient, which usually people speak in case of heat transfer, convection heat transfer, this is nothing but A is related to the temperature gradient that K Del T Del y at y = 0, this is a local property, temperature gradient divided by T w minus T infinity.

This is a very important formula that how the heat transfer coefficient is related to the temperature gradient. This is because the basic mechanism of heat transfer from the plate to the fluid is by conduction only. Now, we define an average heat transfer coefficient.

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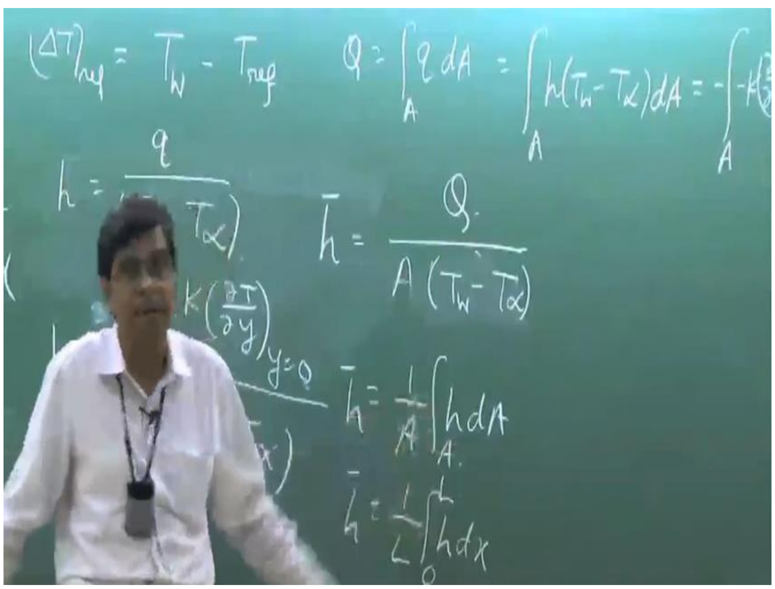

Now, if we have to find out Q, total heat transfer, what we have to do? We have to integrate q * dA. That means in terms of h if you have to integrate h * (T w – T infinity) * dA over the area, A, or over any area, any finite area or it can be written in this way also (- K Del T Del y at y = 0) * dA. This is very simple because I know per unit area and then I have to integrate with area. Now, final integration will depend how the area varies. That becomes the problem of mathematics.

So, this is the total heat transfer for a finite area A. So, average heat transfer coefficient over a finite area A is defined as the total rate of heat transfer over the area A divided by A * delta T reference. That means if you put this delta T reference, I will not use here, this is in general reference terminology, I will straight forward put here T w – T infinity, in this case. So, if you put now Q like this, then h bar is area averaged heat transfer coefficient.

In a flat plate here, with a constant width in the direction perpendicular to the plane of the board, it is simply and arithmetic mean over the length h * dx 0 to L because here dA is dx * b, b is the width of the plane. So, it depends upon the type of the surface, so rest part is nothing but mathematics. So, the way we define the average heat transfer coefficient over an area it boils down to the fact that the average heat transfer coefficient is nothing but the area averaged local heat transfer coefficient.

So, this is the way we define the heat transfer coefficient in a convection heat transfer, which is related to this temperature gradient. Now this is true for both forced convection and free convection. Let us now consider a free convection case. Exactly the entire thing is similar only this picture will be different.

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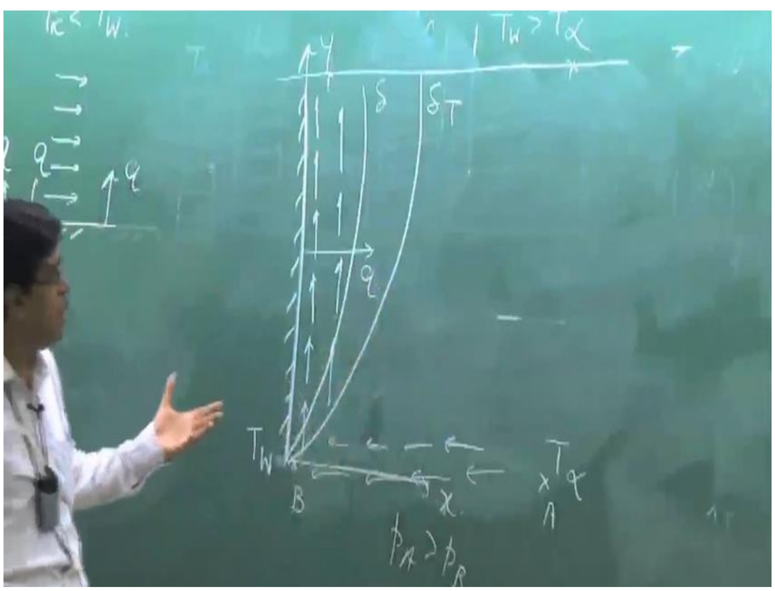

If you consider a free convection, there are several situations for free convections but I give you a classical example which I already told earlier. If there is a vertical plate, which is at temperature T w different from the temperature of the surrounding fluid in quiescent state, T infinity. And now let us consider that T w is greater than T infinity. I told you a motion like this will be set up because of the buoyancy due to the temperature difference.

A motion which is shown like this, the air will flow like this upwards. How it can be explained, it is the chimney effect. Very simple that if you take two points at the same horizontal line here A and at the same point very close to B the pressure here, the hydrostatic pressure is higher than this point because of the cold column of air, that means any arbitrary reference or horizontal plane above this.

If we take two points, vertically above B and vertically above A, the pressure here is the pressure at this point plus the Rho gh, the hydrostatic head, with respect to this cold column, of the cold column of fluid which is at higher density. Whereas here this pressure and this pressure is the same at the same horizontal plane, hydrostatic pressure is same. It is this pressure plus the hydrostatic head the Rho gh where Rho part is to the density of a hot column of fluid.

So, therefore this pressure is lower. So therefore, pA is greater than pB and this is the mechanism by which this type of flow takes place. If this is at lower temperature than the surrounding fluid, that is the reverse direction, the flow will come from top to bottom. So, what happens because of the buoyancy induced flow whenever there is a flow, the formation of a boundary layer and thermal boundary layer because of heat transfer will remain same.

So therefore, there will be the formation of hydrodynamic boundary layer and there will be formation of thermal boundary layer also. So, the picture remains the same. Here the coordinate may be different. If this is vertical plane y, it is always reasonable to give the y coordinates in the vertical direction, it doesn’t matter you can give x. So, if you give y then x direction heat flux is q. So, therefore the question remains, the thing remains same and it will be Del t Del x.

So, therefore whether it is a free convection that is a buoyancy induced flow or the forced convection, it is a forced induced flow that means flow is induced by external agency. Ultimately it is dominated by the flow field and the heat transfer from the surface is because of conduction and that's why we relate the heat transfer coefficient with temperature gradient where heat transfer coefficient in convection is defined like this.

Now, we try to write this temperature gradient in non-dimensional form. You know the reasons for non-dimensionality of the variables which has been taught in detail in fluid mechanics class and it will be again repeated. So, it is done from both theoretical point of view that means in the theoretical equations and also, we can do it by the help of dimensional analysis.

What are the non-dimensional terms that come into picture because it is the non-dimensional term which takes care of the physical similarities and if you express any phenomena by non-dimensional term, it can take care of several situations with several variables by dumping them into few less number of variables. So, with this philosophy we make a non-dimensional approach. So, therefore our first attempt will be, let us make a non-dimensional suggestion of this thing, K Del T Del y divided by (T w – T infinity).

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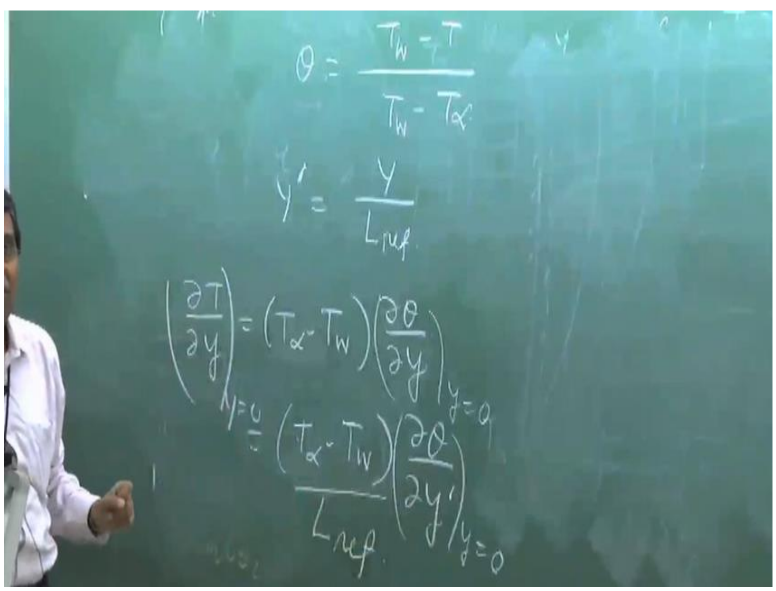

That means let us first try to non-dimensionalise the temperature. Let us define a non-dimensional temperature in this way, which is the convention for external flow is the surface temperature minus the temperature in the fluid divided by (T w – T infinity) which varies from 0 at the surface to 1 at the age of the boundary layer. It is not dependent upon the actual value of T w and T infinity. So, it is normalized. Non-dimensional variable means normalized variable.

Let us make y non dimensionalised as y dash by using a reference length scale. Let us do that. Then, what is the value of Del T Del y? Del T Del y is (T infinity – T w) * Del theta Del y. And what is Del y? It is Del y dash * L reference. That means it is (T infinity – T w) * Del theta Del y dash divided by L reference. So now, if I put this, and it is valid for y = 0 also, in this equation that means Del T del y in terms of a non-dimensional temperature gradient but the non-dimensional temperature is defined like this and the non-dimensional y coordinate is defined like this y by some length scale, some reference length scale.

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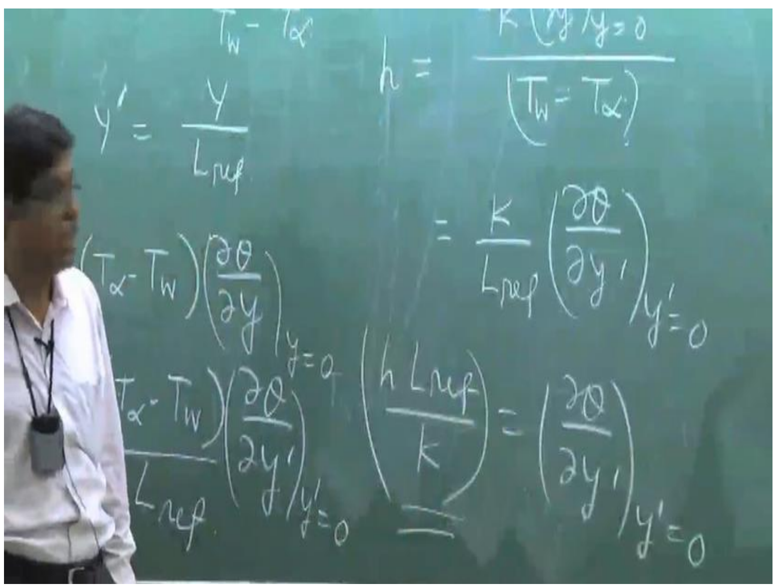

Now, if I put there then I get h is equal to that - K Del T Del y at y = 0 divided by (T w – T infinity). Again, I write – K, it will be easier if I write it here, Del T Del y at y = 0 divided by (T w – T infinity). If I now write this then what you get? You get K by L reference * Del theta Del y dash at y = 0. Now, y = 0 means y dash = 0, because I want to have all one-dimensional parameters here.

So, therefore y = 0 means y dash = 0 because y dash is y by L reference. It is scaled with L reference. Then we can write h * L reference by K is equal to Del theta, in non-dimensional temperature gradient at the wall or at the surface. So, this is non-dimensional temperature gradient that means the left-hand side is also a non-dimensional parameter and if you write the unit, what is the unit of heat transfer coefficient?

Heat flux divided by temperature that means watt per meter square Kelvin and if you use that watt per meter square Kelvin and k is watt per meter Kelvin and L reference is meter, then it is dimension. And this dimension less quantity is known as dimension less heat transfer coefficient.

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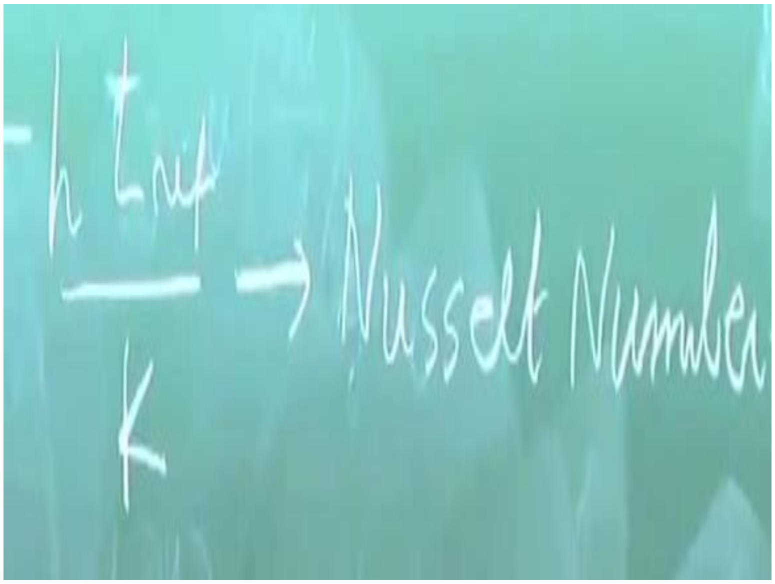

And it is known as, this h L reference by k is known as Nusselt number. So, here in the derivation of Nusselt number by expressing, derivation means to have an idea of Nusselt number, we see that Nusselt number is non-dimensional heat transfer coefficient based on a reference length and K like Reynolds number. Rho b d by Mu based on Rho b d means hydraulic diameter for internal fluid, for the external fluid, is the current coordinate or the length of the plate for a flat plate that means it is a characteristic reference dimension.

So, based on a reference geometrical dimension L reference divided by K and 8 is the local character. So, Nusselt number also has a local character and this Nusselt number is nothing but the non-dimensional temperature gradient that means if we can find out the non-dimensional temperature gradient, we can find out the Nusselt number.

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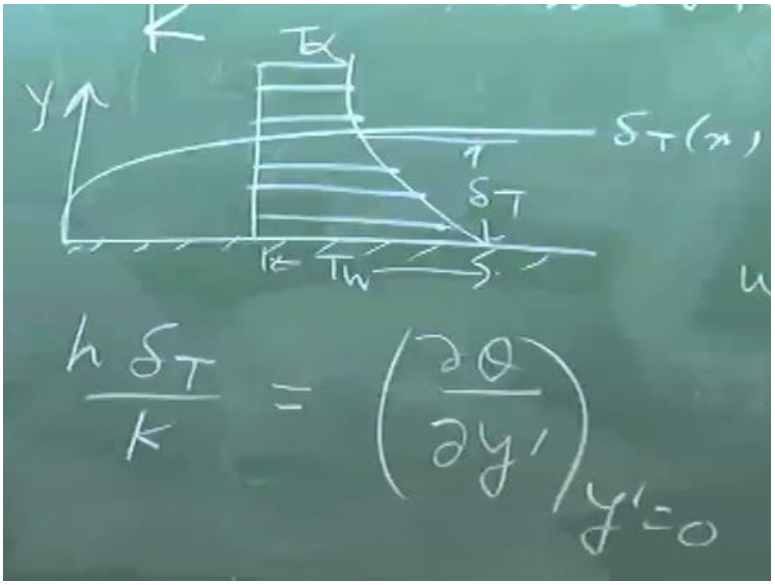

But here one information you must have that in case of flat plate, for example this one, if I draw the thermal boundary layer like this Delta t and let us have this thing T, this is Tw and this is T infinity and this is Delta t. Now, usually the reference dimension for normalizing the y coordinate is not taken anything along the plate.

The normalizing diameter is taken as the boundary layer thickness, Delta T, so L reference is taken as Delta T. So, therefore L reference is taken as Delta T in this case. So, therefore we write h Delta T by K is the Nusselt number which is nothing but Del teta Del y dash at y dash is equal to zero, where y dash is equal to y by Delta T. So, therefore where y dash is equals to y by Delta T. Usually it is scaled with Delta T and in that case h Delta T by K is this.

L reference becomes Delta T but at the same time, it is the convention that the dimensional less heat transfer coefficient h that is h L reference by K, which is known as the Nusselt number. And this local character comes from the local heat flux.

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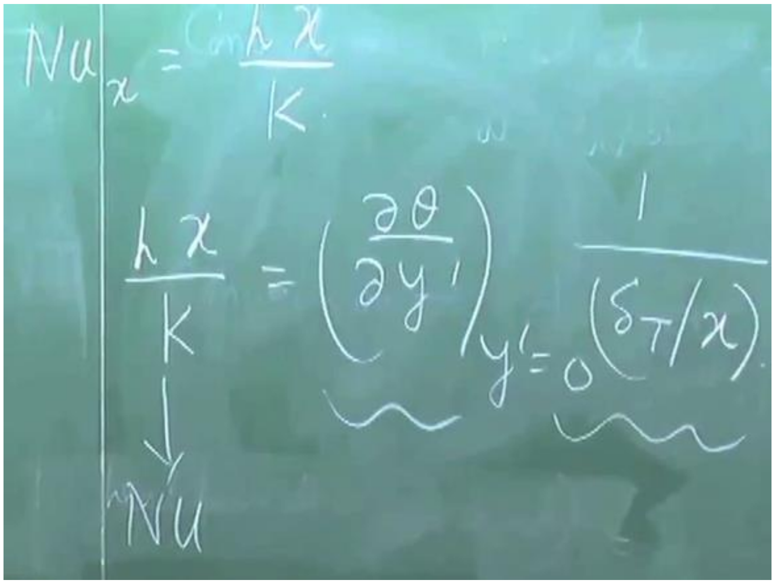

The local heat transfer coefficient is conventionally defined as Nu for example x, the local point h into x by K not Delta T by q. So, therefore if I rearrange it then h x by k is equal to Del teta Del y dash at y dash equals to zero into 1 by Delta T by x. Because I multiply x by Delta T, which goes this side.

So therefore, we see a Nusselt number, this is the Nusselt number in Nu equals to the non-dimensional temperature gradient times the ratio of thermal boundary layer thickness to x, which is also a non-dimensional form of the thermal boundary layer thickness, Delta T by x. And these 2 things are totally dependent on the fluid mechanics that is the flow field and the temperature field.

And this temperature field in turn depends upon the flow field. Temperature field in turn depends upon the flow field. Earlier we have assumed certain things that for a given Prandtl number if we have Delta as the hydrodynamic boundary layer, we have Delta T as the thermal boundary layer, which is connected by a property known as Prandtl number. But how is it connected? How do I get the temperature field?

Now the entire essence comes that if I have to find out the Nusselt number based on this heat transfer coefficient, this is the local Nusselt number similarly we can have average Nusselt number based on average heat transfer coefficient. We have to know the temperature gradient at the wall that means we have to know the temperature field and the growth of thermal boundary layer.

And these things are governed or guided by the flow field and the temperature field, so therefore, we have to know fluid mechanics and we have to know how the flow field affects the temperature field. For which we have to know the energy equation and we have to derive the energy equation. What is the difference between heat conduction equations? Energy equation is an equation for energy balance. In heat conduction equation, we have considered an element and made the energy balance by heat conducting.

Heat conduction coming in, going out, energy generation that is all or a convection heat transfer which we took through only a convection heat transfer coefficient. And we made a balance, but when there is a flow of fluid and we take an element along with the conduction, we have to consider the advection of energy, which is very important as I told, that advection of energy, total energy, which comprises internal energy that is intermolecular energy plus the flow arc, an internal energy not only the intermolecular energy, kinetic energy and the potential energy plus the flow arc.

All these things have to be taken together along with that generation of energy by the physical dissipation, because of the work done against the physical forces. All these things will be taken, will be considered, will be taken together to derive the energy equations, where the velocities will appear. And the velocity field will be given by the Navier Stokes equation. So, therefore you see to understand more of the convection heat transfer, we have to know the temperature field, which in turn governed by the velocity and the velocity fields.

So therefore, a review of the fluid mechanics, derivation of energy equations all will be followed after wards with this primary introduction to convection. So, this way we define a Nusselt number, which is h x by K, now I tell you that this thing depends upon the flow field and the temperature field which are guided by 2 very important parameters. This will again be taught in details that these are guided by very 2 important parameters.

If we consider the flow to be incompressible, that means whose velocity is relatively much smaller compared to sonic velocity and we consider the flow of fluid through a very narrow channel for example in a micro channel or micro tube, where the stranglers are very high we can discard the viscous dissipation, that means dissipation of mechanical energy in terms of thermal energy and we can tell in that case, the fluid mechanics along with the energy equation, that is the temperature field is guided by these 2 parameters.

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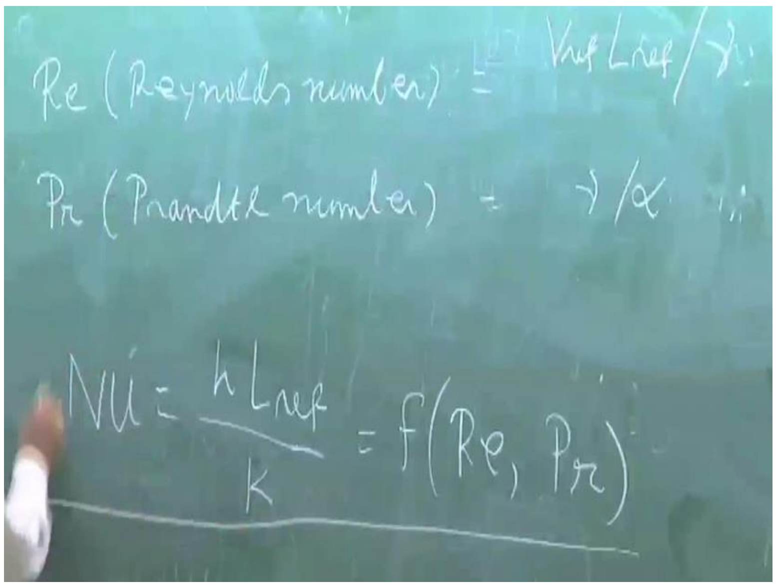

One is Reynolds number, now this is for your information, Reynolds number another is Prandtl number. And Reynolds number as you know is defined as reference velocity, reference length divided by the kinematic viscosity and Prandtl number is defined as kinematic viscosity by thermal diffusity, a momentum diffusity by thermal diffusity.

And this two non-dimensional parameters, we will see afterwards in our theoretical analysis govern both the temperature and the velocity field for usual external and internal flow, where we discuss the effect of compressibility in consideration of the flow to be relatively acting. Law and velocity is compared to the sonic one and also, we discuss high viscous flow through a very small micro channels, where the standards are high so that viscous distribution will be high.

So, if we accept this then we can tell that Nusselt number, which is the non-dimensional form, now I write in a most generic form L reference, which is 'x'. But in case of internal flow, this L reference is the hydraulic diameter. For an example, Reynolds number, this L reference is x for a flat plate, external flow for example over a flat plate. But, this becomes the hydraulic diameter in case of internal flow.

So, therefore I write in a generic fashion is a function of Reynolds number and Prandtl number and entire heat transfer analysis will be centered on this to find the relationship between Nusselt number with Reynolds number and Prandtl number for such flow and this relation depends upon the type of flows. So, there it is a flow, first flat plate, flow past external flows, flow past circular cylinder or internal flow, flow past cylindrical tube like that.

That means depending upon the flow configuration this relationship will be differing, but this Nusselt number will be function of Reynolds number and Prandtl number. But the question comes for free convection. In case of free convection, you can ask me that Sir, there is no Reynolds number in free convections because in a free convections there is no force flow, because flow is not induced and the flow is not imposed by an external agency. That means there is no velocity scale.

For example, in a force convection this V reference is the fisting velocity for external flow. And for internal flow, it is the volume average velocity. The average velocity based on volumetric fluid. This we have told in fluid mechanism class or the teachers might have told in fluid mechanism class. But in free convection flow, there is no reference velocity. This is because the flow is not imposed by the external agency, so we cannot identify the scale of velocity.

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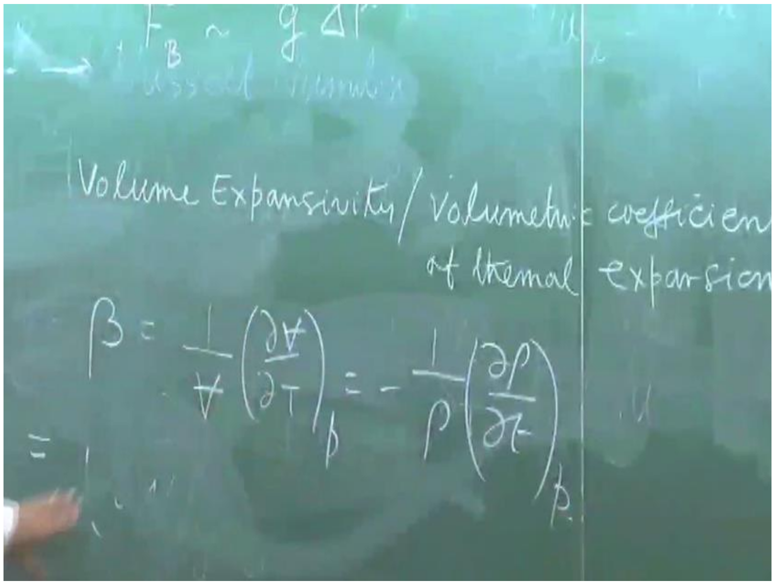

In that case what is done we consider the Buoyancy force by unit volume as the mathematical criteria, for that which is the cause of the flow because it is Buoyancy driven flow. And you know that Buoyancy force by unit volume is g times changing the density. Buoyancy means changing the density and that changing the density usually occurs because of the temperature gradient.

As you know, if you heat from the bottom the fluid would be lighter and go up and there will be a circulatory motion that the way I have explained. Therefore, it is the Buoyancy driven flow, g delta Rho. This change in the density may also take place because of another gradient, which is the concentration gradient for multicomponent that means if the medium consists of different components with different concentrations, then the concentration gradient may change the density, but we are not going into that complication.

Our heat transfer studies are far apart from that. We are not considering that, only a single component flow. So, therefore change in density only because of the temperature gradient. And the heat transfer scientist usually what they do is substitute this change in density terms of the volume expansivity, in terms of a parameters known as volume expansivity or volumetric coefficient of thermal expansion.

This is the usual style but not very universal, when the concentration gradient is there we don't do that. But usual style is that this is defined as volumetric coefficient of thermal expansion is 1 by V, V is the volume, I use to distinguish from the velocity * Del V by Del T at constant pressure. This can be written as -1 upon Rho in terms of density.

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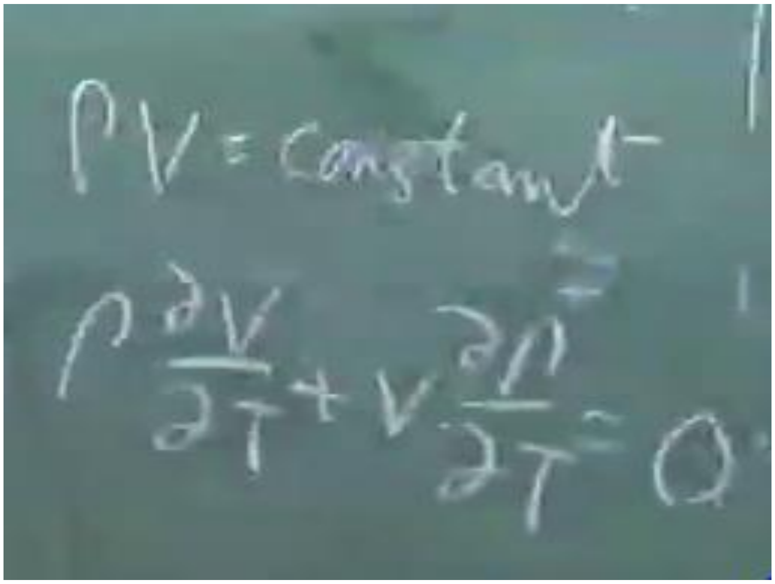

We should understand from the change of, because Rho V is constant, for a given mass Rho V is constant. So, therefore Rho * Del V / Del T + V * Del Rho / Del t is zero. So, it is very simple that Rho V constant either this or this. And this minus sign takes care of the sign of Del Rho / Del t, so that beta is a scalar coefficient is always positive.

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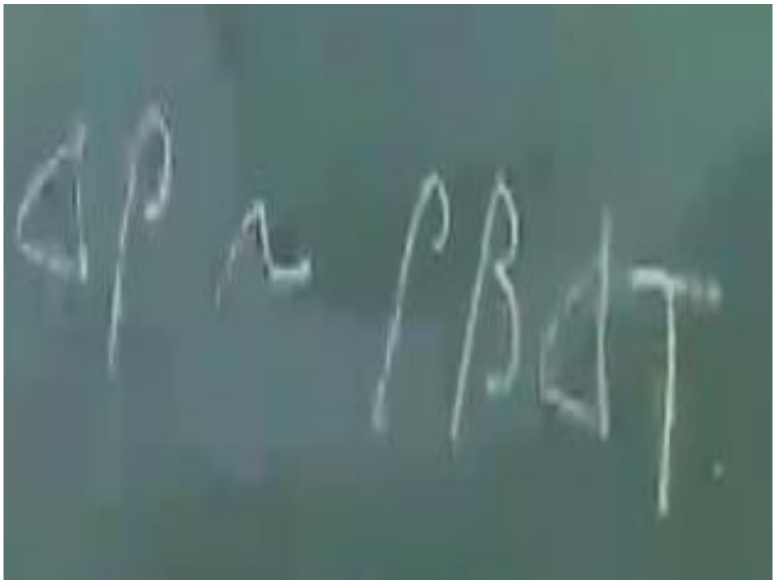

If you do that by a first order approximation, one can tell that Delta Rho that change in density is scale doing Rho beta * Delta T.

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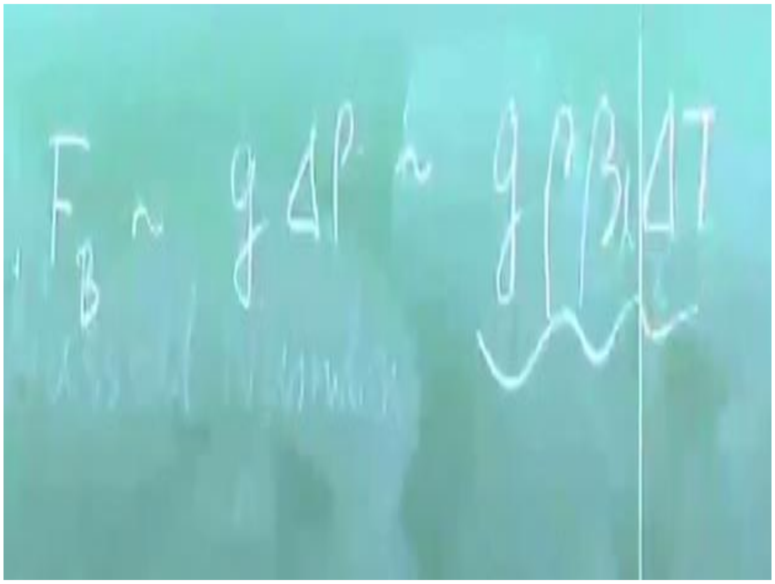

This can be written like that, the Delta Rho that change of density for a giving change of temperature, with the first order approximation can be scaled liked that, Rho beta * Delta T. So, therefore this becomes g * Rho beta * Delta T, that means instead of a velocity scale, we consider the Buoyancy force or the buoyancy force per unit volume, in place of the velocity we consider this in the theoretical equation, equation of motion, in the Navier Stokes equation, we will see afterwards or you will solve those category of problems.

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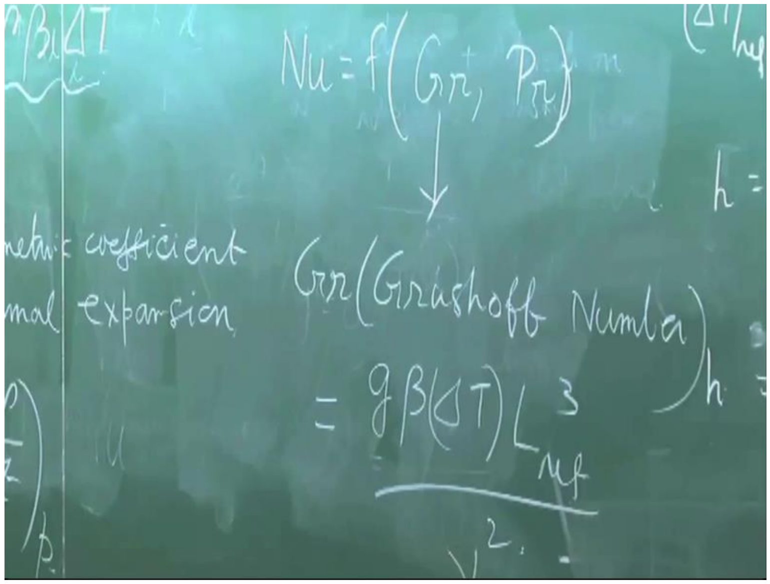

And even in the dimensional analysis, we replace the reference velocity with that and instead of Reynolds number we come up to another number, so that the Nusselt number becomes a function of Gr Grashoff number and Prandtl number. So, Reynolds number is replaced by Gr which is known as Grashoff. All these people are big scientists, great scientist. Grashoff number. And this is equal to G beta * Delta T * L reference length Q / Mu square.

So, physical significance of Reynolds number you know, it is proportional to the ratio of inertia to viscous force. So, Reynolds number implies the inertia to viscous force. And it represent the dynamic similarity of the force governed by inertia force, viscous force and pressure force. And the physical significance is the Prandtl number is the ratio of the two-diffusivity relative, that means Mu is responsible for taking the momentum transport that the presence of the plate in flow field. How much?

The presence of the flow goes to the flow field. What is the extend of hydrodynamic boundary layer? Similarly, alpha represents the presence of the hot surface or cold surface in the stream, which is at a different temperature. How much? The influence goes. Therefore, this represents the extent of the thermal boundary layer. Therefore, their ratio is proportional to the ratio of the two-boundary layer. More is the Mu, more is hydrodynamic boundary layer.

More the alpha is more the thermal boundary layer. Similarly, the Grashoff number, this is for your information today, this all will be done afterwards, so that you can enjoy this thing afterwards. This Grashoff number is a number, which physically signifies, is proportional to not directly to Buoyancy force to viscous force. It is exactly not buoyancy force to viscous force. It is buoyancy force into inertia force by viscous force square.

There is a term like that and where the inertia force is defined on a velocity represented as a velocity scale, which will be derived from the Buoyancy driven flow. That is the buoyancy per unit volume. That I am not doing at the present moment in the introductory class. So, this is physical implication of Grashoff number, which is ratio of Buoyancy force to viscous force, but not exactly so.

So, therefore the entire thing falls down to this fact to know this dependence, the Nusselt number with Reynolds and Prandtl, we have to solve for temperature field and velocity field and for the solution of temperature field, we have to know the velocity because the energy equation, which involves temperature as the unknown dependent variable contains the velocity as the known parameter, which will be fate from the solution of the Navier Stokes equation.

And similarly, to know the Nusselt number this relationship for free convection flow. We have to solve similarly the temperature and the velocity field where the flow is because of Buoyancy. With this, I like to end and I tell you that for this we have to know the fluid mechanics in detail and also the energy equation. The derivations of energy equation and all these things will come which will be followed afterwards. Before I end I just show you one application which will be very simple. It will not take more than 2 minutes.

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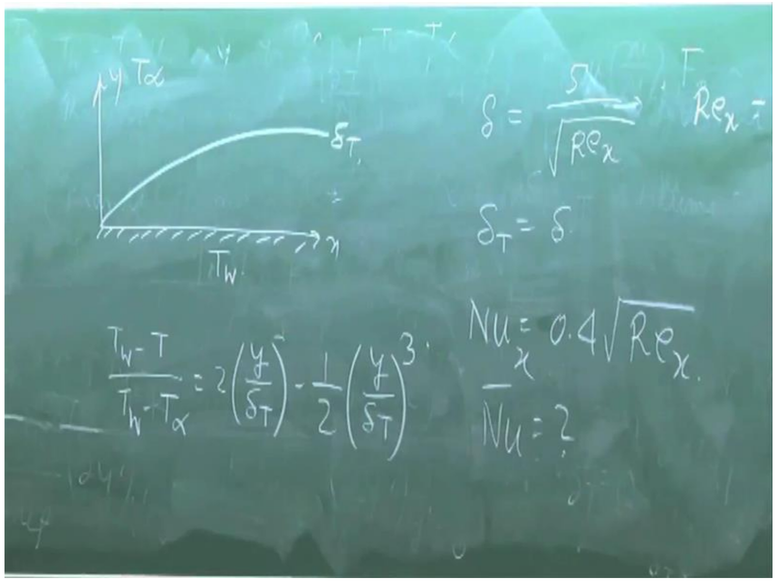

A problem like this. There is a flat plate at a temperature T w. And there is a cold steam at a temperature T infinity. Now the temperature distribution is given by this, Tw – T divided by Tw – T infinity is 2 * y by Delta T – half y by Delta T the whole cube. And it is given that hydrodynamic boundary layer is 5 by root over Re x where Re x is equal to U infinity * x by Nu. And also given that Delta T is Delta. That means Nu is equal to alpha.

So, prove that Nusselt number is 0.4 root over Re x. The local Nusselt number Nu x. And what is the value of average Nusselt number? How do you solve the problem? The temperature is the function of both x and y. This is x. This is y. But, y is this x is taken in Delta T. Similar to parameter y / Delta T. How to solve this problem? Very simple. Let us do this at a direct application of what we have discussed today. Nothing can be simpler than this. How to do this problem?

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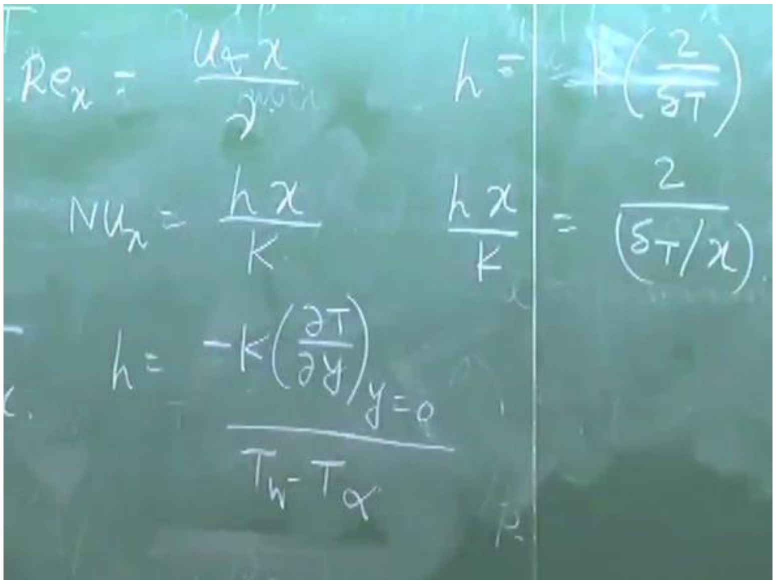

Now, we have to find out Nusselt number. Now, Nusselt number x is h, this is the convention x / K. How to find out h? h is –K * Del T / Del Y at y = 0 divided by Tw – T infinity. What is Del T, Del Y? Del T is – T infinity by T w into 2 by delta T. This will be 0. That means T w. So – will go down. That means – and – will cancel, equals to k * 2 by Delta T. Del T del Y at Y is equal to 0 is T infinity – T w into 2 by delta T and T w. So, hx/ K is 2 / Delta T / x. Now, what is delta T by X?

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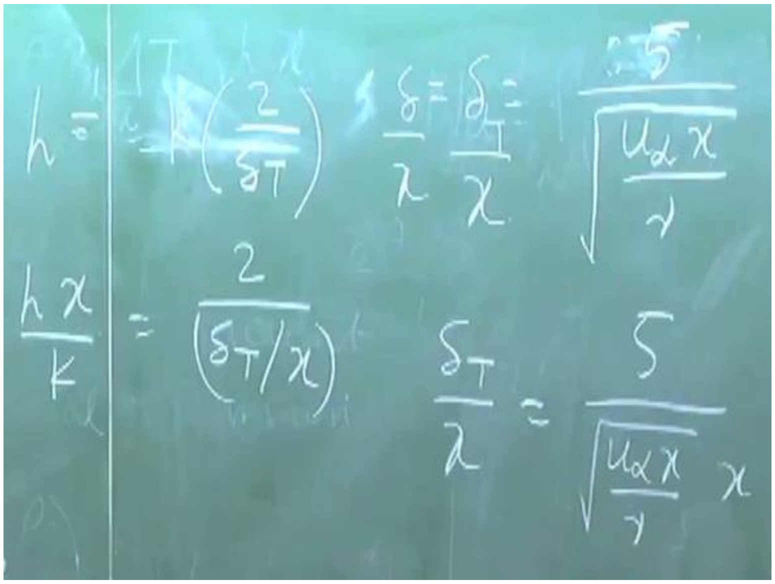

Delta is 5 / root of Re x. Delta T is root over this thing. So, what is this Delta T / x? Find out. Hx / K is 2 / Delta T / x. Delta equal to Delta T equal to 0.5 / u infinity x by Mu under root. So, what is Delta T / x? It is 5 / under root Mu infinity x / Mu * x. If you put delta T y by x. It will be. Is there any mistake? Which one? Delta by x. Sorry. I am sorry. It is non-dimensional delta. It is root over Re x. Very Good. It is delta by x.

So therefore, delta y, x is this. So, therefore there is no point of doing all those things. So, delta T by x. This cannot be. This is so simple. So, delta T by x. This is the expression of delta by x. Non-dimension. So, therefore, if you put that automatically you get Nusselt number x is 2 by 5. That means 0.4 into root over Re x. This is root over Re x. We no need to write in terms of u infinity x.

How to find out the average Nusselt number? You have to find the average heat transfer co-efficient. So, how to find average heat transfer co-efficient?

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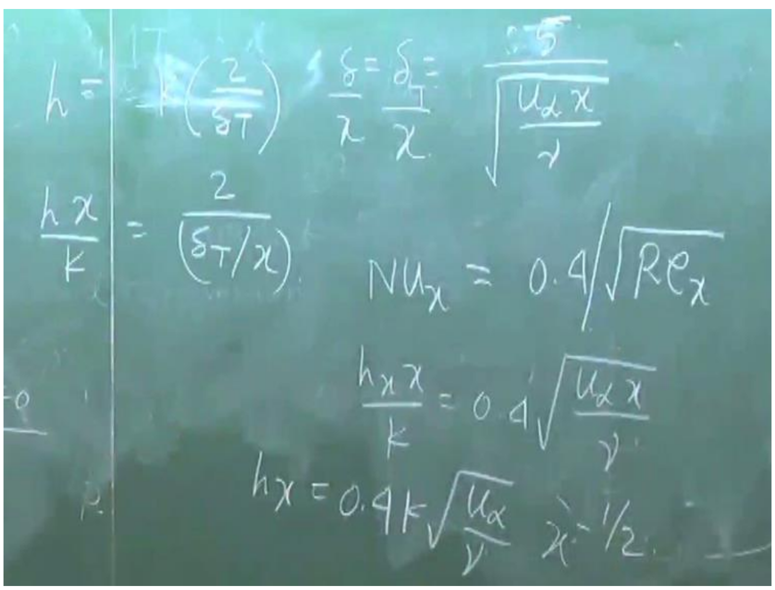

The average heat transfer co-efficient to find out in this case that I have already told, 1 by hdA h that means in this case the average heat transfer co-efficient as it is defined 1 area average hdA. In this case, it is 1 by L, over a length L. To find the average heat transfer co-efficient over a length L. It has to be prescribed over which area you are telling. That means about which area it is being prescribed. It is length L for a given value.

Therefore, it is nothing but 1 by L hdx 0 to L. And the beauty is that here, this is hxx by K is 0.4 root over u infinity ax by Nu. 0.4 by root over Re x. I am sorry. 0.4 * root over Re x. So, 0.4 into root over u infinity x by Nu. So therefore, you find out the heat transfer co-efficient. Hx = 0.4 k root over u infinity by Nu, x to the power minus half. That means it is decreasing x to the power minus half.

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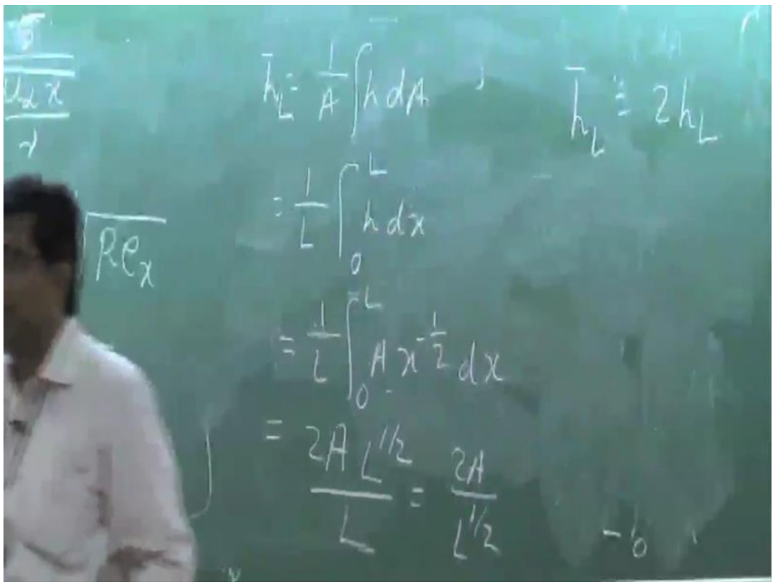

If you take this as constant then a beautiful thing you will see that is 1 by L, 0 to L, A x to the power minus of dx. What is the result? The result is 1 by L x to the power minus or 1x to the power minus half divided by the half. That means 2AL to the power half divided by L. That is 2A / L to the power half. That means this tells you all a very interesting thing that Nusselt number.

So, therefore HL average over a length L is equal to 2 times the hx. Because the same thing 2A L to the power half. Because here A is 0.4 K. That means since it is bearing with x to the power minus half. So, this is a very good conclusion that I will tell you it is a thumb rule, h L average over a length L is 2 times locally because local heat transfer co-efficient is some constant, which is 0.4 K root over u infinity Nu in this case. Because, this is the law which we have found out divided by x to the power half.

If you put x as L, that means it is the local heat transfer co-efficient at x = L. And twice the local heat transfer coefficient equals to the average heat transfer co-efficient up to that length. That means you can make a thumb rule, very preliminary thing that x to the power – 1 upon. So, it will be becoming N plus 1 times. The local heat transfer co-efficient, and while it is x to the power -1, then it is twice the local heat transfer co-efficient. That means the average Nusselt number over a length L will be 0.8.

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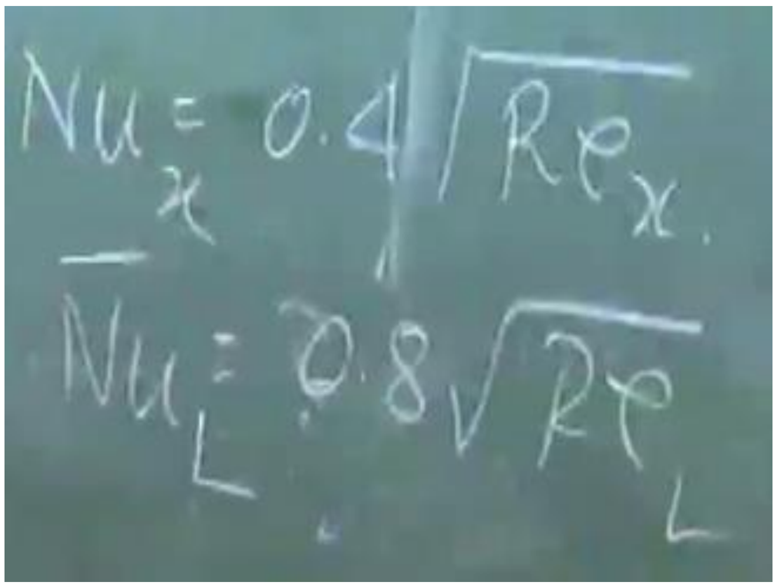

That means I will write 0.8 into root over Rebase L. Because this mathematics is very simple but doing in hurry there might be some mistakes. It is very easy doing this mathematics depends on exponent. So, the concept is area average heat transfer co-efficient is the average heat transfer co-efficient. And Nusselt number, average Nusselt number over a given area is the average heat transfer co-efficient times the reference geometrical dimension by L ref / K.

Therefore, we have appreciated one thing that to know details about the convection heat transfer and to develop this fundamental relation of Nusselt number as a function of Reynolds number and Prandtl number, we have to know the Prandtl number and Grashoff number, we have to know the flow field and the temperature field for which we have to recapsulate our fluid mechanics, Navier-Stokes equation, Bound layer equations and also the energy equation. In next class, the derivation of Navier-Stokes equations will be taken by Professor. Suman Chakraborty.