**Conduction and Convection Heat Transfer**
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**Lecture - 15**
**Unsteady State Heat Conduction - II**

So, let us continue with what we are discussing in the previous lecture. We were trying to solve a problem of unsteady heat transfer, unsteady heat conduction using the lumped parameter analysis and what we are trying to do is we are trying to figure out the temperature as a function of time assuming that there is no temperature variation within the body as a function of position.

Now, when is that assumption valid? So, we will try to answer the first question.

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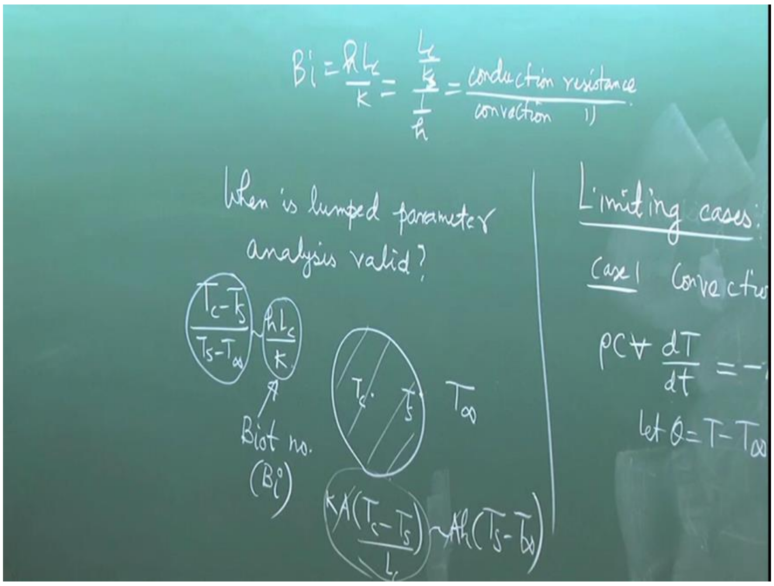


So, let us take this example. Let us say that you have a spherical ball with T c as the centile temperature, T s as the surface temperature and T infinity as the temperature of the surrounding fluid. This is a solid ball. So, you can write not exactly equal because it is not a steady state problem, so the heat transfer due to conduction is not exactly minus K a into delta T by L.

But we can write order of magnitude wise whatever is the rate at which heat is being transferred here due to conduction at the same rate heat is being dissipated by convection and this is an approximate estimation of the rate of conduction heat transfer. Had it been one dimensional steady state heat conduction that would have been exactly equal to this then instead of this order sign we could have put the equality sign.

But here it is just the order because it is not a 1-dimensional steady state problem. So, you can write there will be a here right, A into h into T minus T infinity. So, this A will get canceled from both sides and you will get T c minus T s divided by T s minus T infinity is of the order of h L c by k. What is this L c? We have not discussed what is this L c, this is called as characteristic length scale of the body.

So, typically the characteristic length scale is a length that describes the physical change or that change of temperature within the body. So, you can see that in this expression that characteristic length comes out to be volume by area. So, you can take it as a volume by area as the characteristic length. So, here just to symbolize we have retreat it as L c without mentioning what is this L c. So, this is a non-dimensional number, right?

Therefore, this is also a non-dimensional number. What is the name of this non-dimensional number? The name of this non-dimensional number is called as Biot number, ok. So, what does it physically represent? So, the Biot number which is h L c by k right. What is the numerator? It is the conduction resistance and denominator is the convection resistance. So, it is a ratio of conduction resistance to convection resistance.

Remember this k very very important. This k is k of solid, not k of fluid. We will later on see a non-dimensional number in the context of convection where it looks very similar, but the k is k of fluid and not k of solid. So, now when can we think of that the entire solid is at a uniform temperature? When can we think of like that? We can think of the entire solid to be at a uniform temperature when T c minus T s is much much less than T s minus T infinity.

That is temperature variation within the body is much much less as compared to temperature variation outside the body.

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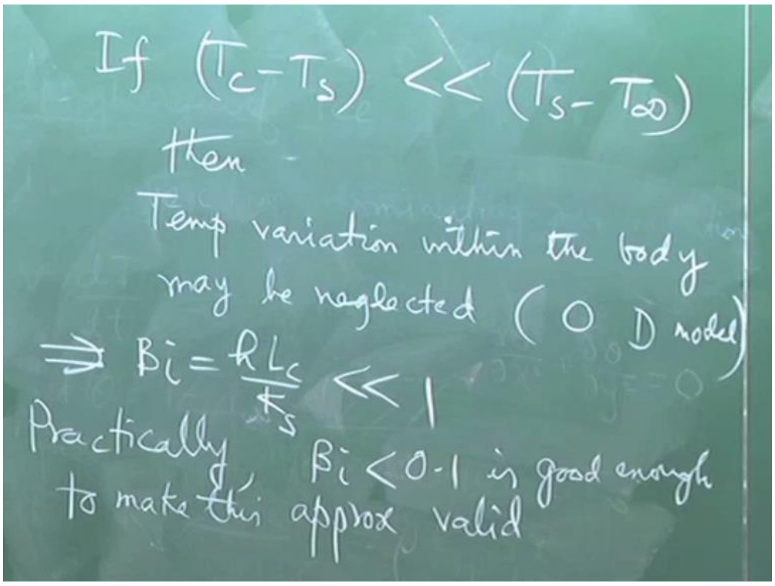


So, we can write if T c minus T s is much much less than T s minus T infinity then temperature variation within the body may be neglected. This is zero-dimensional model. So, the model is valid see you cannot adjust this (()) (7:47) because you do not know what are the temperature, so you cannot put this values and check. So, what you can check? You can check this parameter.

So, this parameter implies that the Biot number which is h L c by k of the solid much much less than 1. So, please do keep in mind that for the lumped parameter analysis to be valid the Biot number must be very very small, much much less than 1. Now, question is, what is much much less than 1? For all practical engineering purposes, see for mathematical purposes it is in the limit as it tends to zero, but for all practical purposes the Biot number may be considered to be small if it is 1 order less than 1 that is less than 0.1.

So, practically Biot number less than 0.1 is good enough to make this approximation valid. So, let us say that condition is true.

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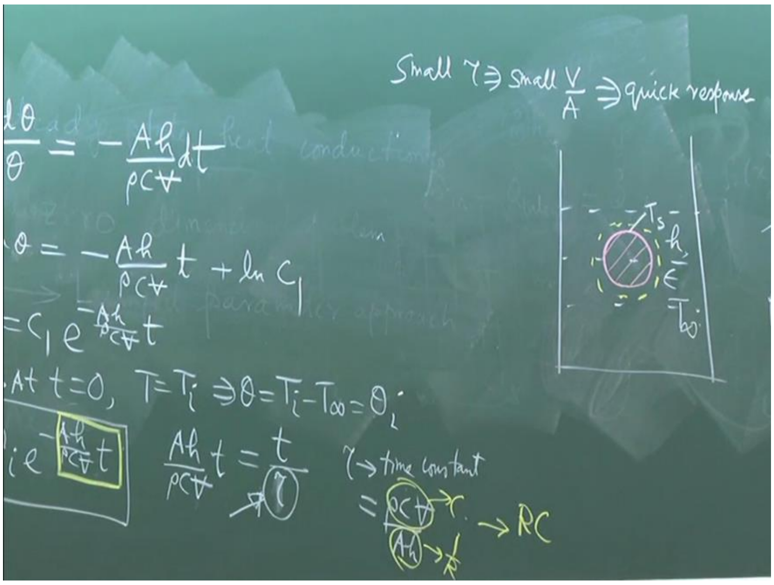


Now, what is the physical interpretation of the time constant that is the first question that would like to answer. Second question is based on that physical interpretation can we write it in terms of the Biot number, ok. So, now the first question? Now look at it, let us say that tau is small, if tau is small then what will happen? See let us say this is a lump body. You create a change and this body is adjusting to that change in temperature.

So, question is how fast it is adjusting or how slow it is adjusting. If tau is small will it adjust fast or will it adjust slow? If tau is small then for even a small value of T, T by tau is large right. T by tau is large means e to the power minus large will be tending to zero that means theta will be tending to zero. Theta is T minus T infinity. That means in a very short time T will come to T infinity.

That means the solid body will very quickly come to the ambient temperature. That means the response of the solid body is very fast. So, a short time constant means it responds to the temperature change very fast and short time constant means what? Short time constant means small volume by area. Small volume by area means short small tau. So, small tau means small V by A. This means quick response.

Why is it so? because volume is what, volume is the storage of thermal energy and what is area, area is the surface over which heat gets transferred. So, if the area is much much larger as compared to volume then there will be rapid transfer of heat as compared to thermal storage. So, V by A small means actually area much much large as compared to volume. So, then the body will quickly adjust to its change.

So, in other words in this quenching example it will cool fast if V by A is small, right. So, if V by A is small it will cool fast. On the other hand, if tau is large then what will happen? If tau is large then for a short time it will be what, e to the power zero, right. If tau is large then for a short time divided by a large time, so that means it will tend to zero. So, e to the power zero will tend to 1.

That means theta will tend to theta i. That means the temperature of the solid will remain same as the initial temperature even for a substantial time. The change will be slow, clear. So, we can say that tau is an indicator of the rate at which the time rate at which the body adopts to a thermal change. If it is small it adopts quickly. If it is large it adopts slowly. So, now how can this be applied?

This can be applied to many engineering design problems including say the design of a riser in a casting system. So, in a casting process you know that there are shrinkages at various levels and when there is a shrinkage during solidification what will happen, so there will be a volume that needs to be replenished and the riser is there as the supply of the molten metal which will supply to the mould till the entire mould solidifies that is the requirement of a riser in a casting problem.

So, when you are having such a requirement the riser should solidify at the last because it must supply the molten material to the mould till the final mould solidifies. So, riser must solidify last. So that means riser must lose the heat most slowly.

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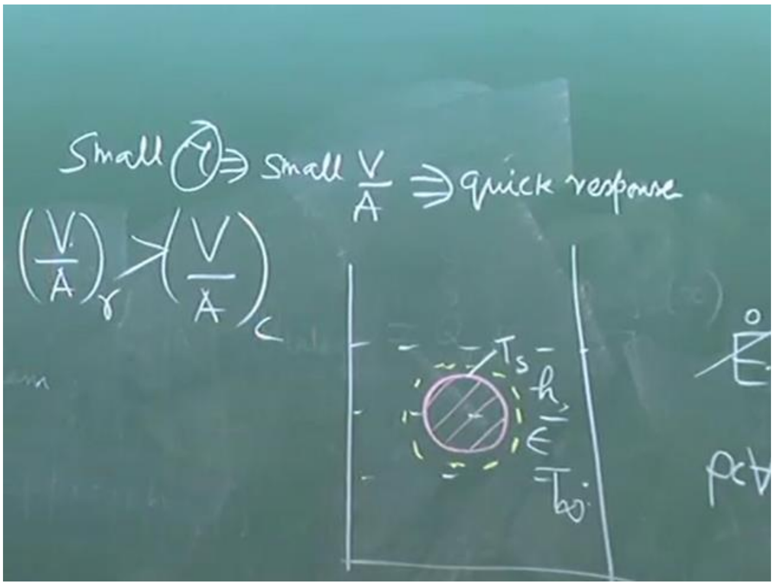


So, if the riser must lose heat most slowly then the time constant of the riser must be large, right. That means volume by area of the riser must be greater than volume by area of the casting because its time constant should be large so that it should respond slowly to the thermal change so that it will not immediately freeze. This in riser design problem is known as (()) (15:00) rule.

So, there are see classically manufacturing process or studies in manufacturing in many cases have been experienced based so before the science of this was properly known people from experienced found that this should be the case for the riser design. So, now you can see that the rule basis which are taught in manufacturing processes can be fundamentally derived if you think of a mathematical analysis following heat transfer, ok.

Now, how can you apply it for a practical problem? Let us say you are measuring the temperature of a body with the aid of a thermometer or a thermocouple. What kind of thermometer or thermocouple will you want? Something with high time constant or with low time constant. In that case you want a low time constant because you want the thermometer to quickly adjust to the temperature of the body with which it is in contact, right.

Otherwise, it will take a long time for it to read the actual temperature.

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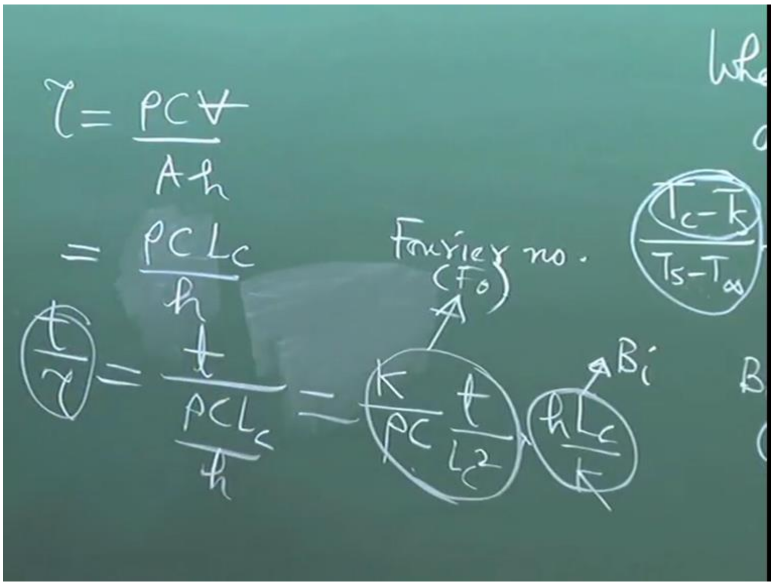


So, when you go for purchasing of a temperature measuring instrument you should look into the time constant and make sure that the time constant is small. Then how can we write this A h by rho C v into T, so you can just manipulate the algebra and you can write T by tau in this way. So, T by tau is T by rho C into L c by h, L c is viva A, ok. So, now you can multiply by K. Why you multiply by K? because you want to get the Biot number.

H L c by k, so this is a non-dimensional number. We have discussed about its physical significance earlier. This is also non-dimensional that means this is also non-dimensional. This is called as Fourier number.

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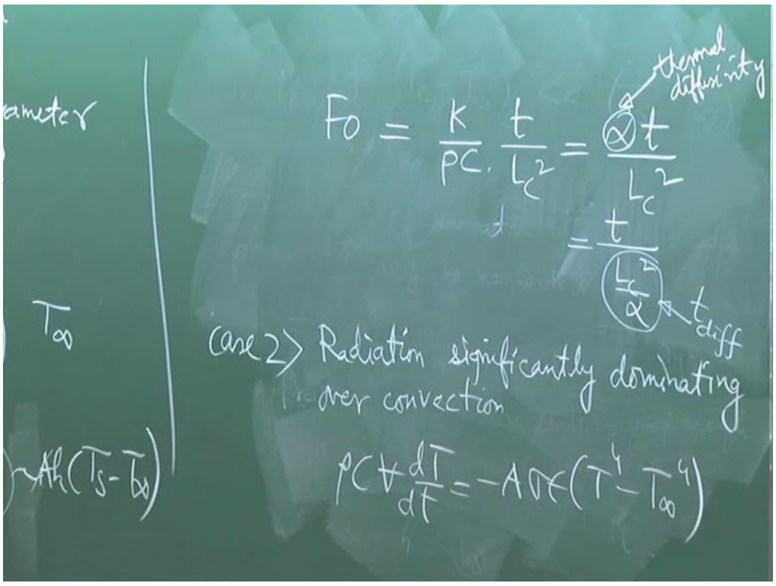


So, how do you define Fourier number? Fourier number where alpha is the thermal diffusivity. Alpha is K by rho C that is the thermal diffusivity. So, this can be written as T by

L c square by alpha. What is this? If this is unit of time, this is a non-dimensional number. This is also unit of time, this is called as diffusion time. Diffusion time is the time over which heat diffuses over a given length.

So, the time over which heat diffuses by a length L c is L c square by alpha, ok. So, we have studied the lumped parameter analysis for a limiting case when conduction is dominating. We will consider the other limiting case briefly before working out a couple of problems. The other limiting case is when radiation is significantly dominating over convection. So, let us look into the governing equation.

This equation which we derived in the previous lecture. Now, this term will not be important as compared to this term. So, you can write rho C V d T by d T is equal to minus A sigma epsilon T to the power 4 minus T infinity to the power 4, ok.

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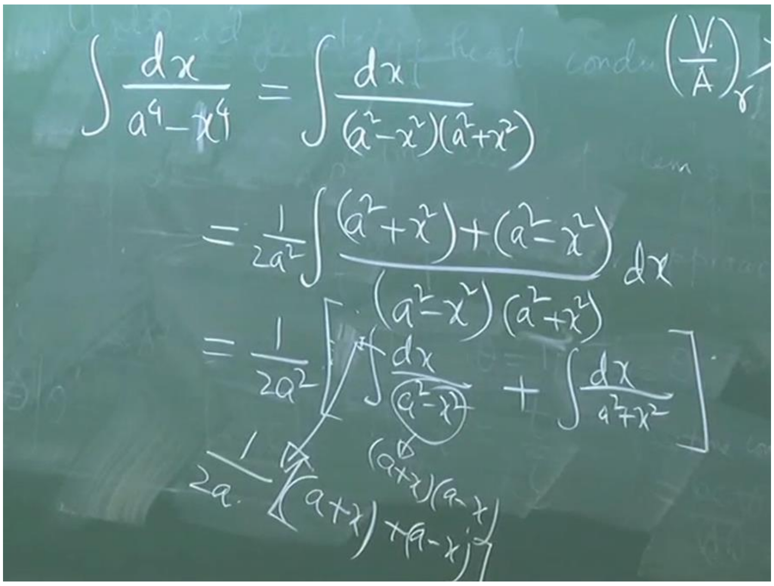


So, this is also a very straight forward problem. In the previous case the integration was d x by x. now the integration is of the form d x by say A to the power 4 minus x to the power 4 right. So, I will not go through that, but I leave it on you as an exercise but very simply you can do that. This you have done in school level. So, integral of this is the form of the integration a is T infinity this is T.

So, you can write this as integral of d x by a square minus x square into a square plus x square. So, this you can write integral of 1 by 2 a square, a square plus x square, plus a square minus x square. Again, for this term you can write a plus x into a minus x and numerator you can write 1 by 2 a, right. So, one of the integrals will be L n of a plus x, another will be L n of a minus x and this if you substitute x equal to a tan theta it will be a straight forward integration.

So, the integration is quite easy even if you do not remember any formula you can reproduce it quickly. The remaining part of the problem is just purely algebraic simplification. Concept wise we do not learn anything new. So, we will work out a couple of problems related to the use of the lumped parameter method and we will start with working out the problems.

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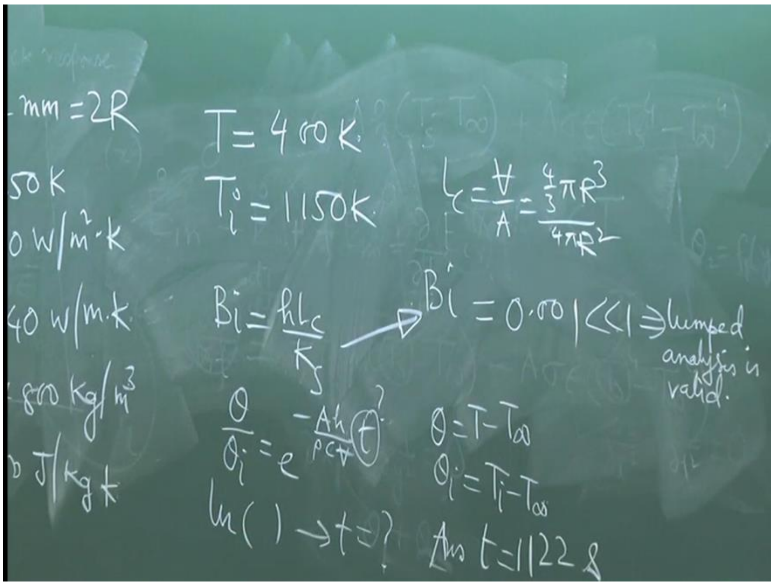


So, the first problem, please note down the problem. This is a problem from incropera book. Steel balls of 12-millimeter diameter, so D is equal to 12 millimeter, this is 2 R or annealed (annealed means annealing is a heat treatment process) by heating to 1150 Kelvin and then slowly cooling to 400 Kelvin. I am repeating steel balls of 12-millimeter diameter or annealed by heating to 1150 Kelvin and then slowly cooling to 400 Kelvin.

In an air environment for which T infinity is equal to 350 Kelvin, so T infinity is equal to 350 Kelvin. H is equal to 20 watt per meter square Kelvin. Assume k of the steel is equal to 40 watt per meter Kelvin, rho of steel is equal to 7800 kg per meter cube. C of steel is equal to 600 joules per kg Kelvin. Calculate the time required for the cooling process, how much time it will require to cool. So, what is the final temperature?

Final temperature is 400 Kelvin right. What is the initial temperature? 1150 Kelvin. So, we can straight away use the lumped analysis provided we justify that the lumped analysis is valid. So, for solving any problem using lumped analysis, first you have to calculate the Biot number and then tell. So, what is the Biot number? H L c by k. what is L c here? volume by area. So, L c is volume by area is equal to 4 by 3 pi R cube by 4 pi R square, ok.

So, if you calculate these, this will come out to be if you substitute the values this will be 0.001. Sorry not the L c the Biot number, ok. So, we can assume this to be much much less than 1 that means lumped analysis is valid. So, then we can write the governing equation and solve the governing equation, I have already done it. So, I am not going to write it again, but I will just the final solution.

So, theta by theta I is equal to e to the power minus A h by rho C v t, right. What is theta. T minus T infinity and theta, I is equal T I minus T infinity. So, in this expression you have to find out what is the time. So, you can take log of both sides and find out what is the time. So, log of both sides and what is the time, so the answer I am giving T is equal to 1122 second. This is the straight forward use of the theory that we have discussed in the class.

Now, let me ask you a question? For the lumped analysis to be valid is it necessary that the Biot number has to always be small. See why did we impose the condition of Biot number small, we wanted to make sure that the entire body is at a uniform temperature right. Let us think of a situation.

Let us say that you have one glass of hot milk and the milk is slowly stirred by a spoon because of the stirring with the help of the spoon the entire milk will come to a uniform temperature close to practical purposes. So, then whether the Biot number based on the volume and area is small or not is not important because we have ensured by stirring that the milk is at a uniform temperature.

So, the requirement of Biot number much much less than one is not a fundamental requirement. The fundamental requirement is the uniformity of temperature. So, we can say that Biot number much much less than 1 is a sufficient condition for lumped parameter analysis to hold but not a necessary condition, right. If it is true then the lumped analysis will be valid.

But even if it is not true but the substance is homogeneous to a uniform temperature then you can use the lumped analysis. Now can you tell that why I was telling that the milk is slowly stirred? If the milk is stirred very fast then convection within the system is important and here, we are assuming that the convection is taking place outside the system. Within the system it is pure conduction, ok.

So, conduction analysis will not work for a system where because of very vigorous stirring convection within the system is itself important. Then you have to solve the fluid flow equations, ok. So, finally we will work out another problem and call it a day today.

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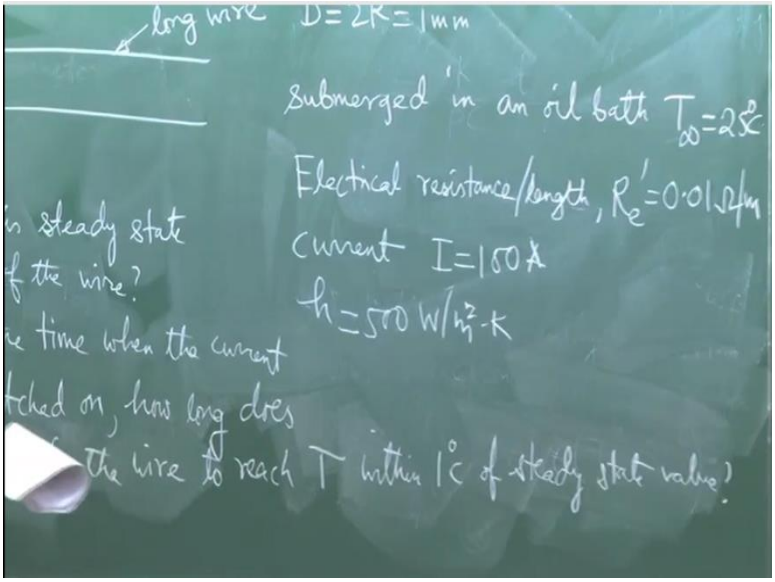


So, that is the last problem that we are solving today. So, the problem statement is like this: this is also a problem from the exercise of incropera textbook. So, you have a long wire of diameter is equal to 1 millimeter. It is submerged in an oil bath of T infinity is equal to 25 degree centigrade. Electrical resistance per unit length is 0.01 ohm per meter. The current that flows through the wire is 100 amp here.

And the convection heat transfer coefficient is 500 watt per meter square Kelvin. The first question is what is the steady state temperature of the wire? This is the first question. Second question is from the time when the current is switched on how long does it take for the wire to reach a temperature within 1 degree centigrade of steady state value. Other properties are given rho is equal to 8000 kg per meter cube, C is equal to 500 Joules per kg Kelvin and K is 20 watt per meter Kelvin, ok.

So, the physical problem is that there is a current which flows through a resistance so there is a Joules heating because of Joules effect there is a heat generation and that heat is dissipated to the surroundings. At steady state the rate of heat generation is same as rate of heat dissipation to the surroundings but during unsteady state, what will happen? The temperature within the wire will change with time before achieving steady state.

So, the first part of the problem deals with the steady state, second part is the unsteady state. So, we will write first develop the general formulation and then we will apply it to both steady state and unsteady state.

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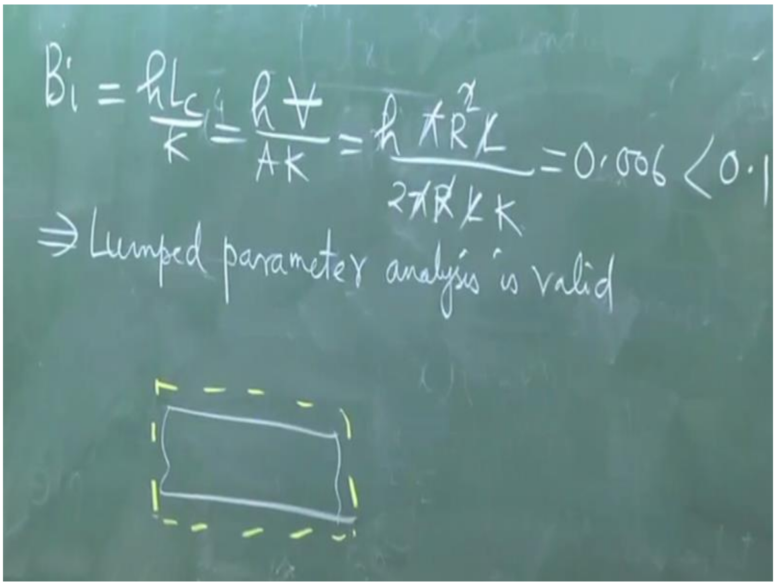


First of all, before applying the unsteady state situation can we use the lumped model here? So, let us say that length of the wire is L. So, the Biot number h R by 2 K, ok. So, if you substitute these properties this will be 0.006. This is less than 0.1. So, it a sufficient condition for the lumped parameter analysis to be valid. So, now let us try to make a control volume analysis of this electrical wire.

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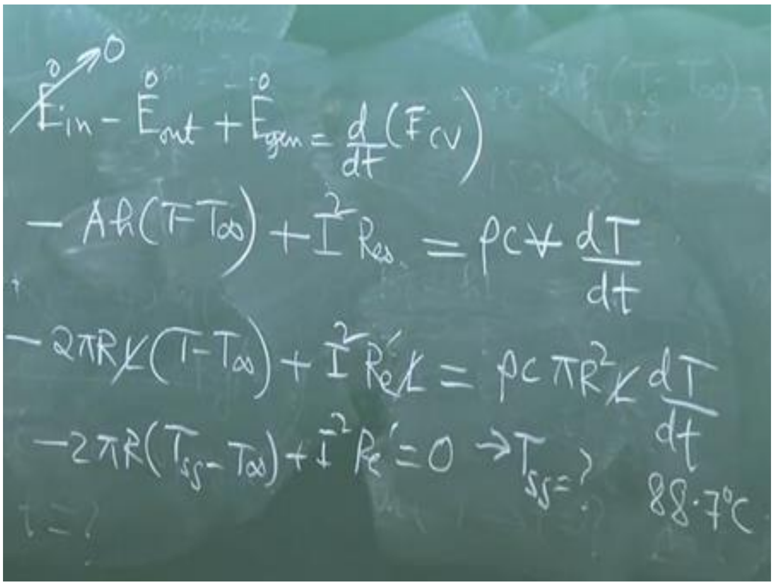

So, energy in minus energy out plus energy generated because it is a lumped body we can write d t instead of the partial derivative, ok. So, what is the rate of energy in? There is no rate of energy in. What is the rate of energy out? So, there are 2 modes: convection and radiation. We assume that convection is the dominant mode as compared to radiation. So, minus A h into T minus T infinity plus what is energy generation?

I square R because R I have used already for radius, so I am using a different symbol Res. So, we can write what is A? A is 2 pi R L into T minus T infinity plus I square. Now resistance is given see Re dash ohm per meter. This multiplied by the length will be the total resistance. So, I square Re dash into L is equal to rho C Pi R square L into d t by d t, ok. So, L gets cancelled from both sides. It is given that it is a long wire.

Otherwise, it will not be a 1-dimensional problem. In fact, 1 dimensional here becomes zero dimensional. It is not even 1 dimensional, it is a zero-dimensional problem. Now, all the values are known so what is the first part? What is the steady state temperature of the wire? So steady state temperature of the wire is obtained when this temperature stops changing with time then its steady state. So, when this term is zero, then generation balances heat loss.

So left hand side equal to zero will give the temperature, the steady state temperature. So, minus 2 pi R into T steady state minus T infinity plus I square Re dash equal to zero. This will give you what is T steady state. So, if you calculate this. So, T steady state will be 88.7 degree centigrade.

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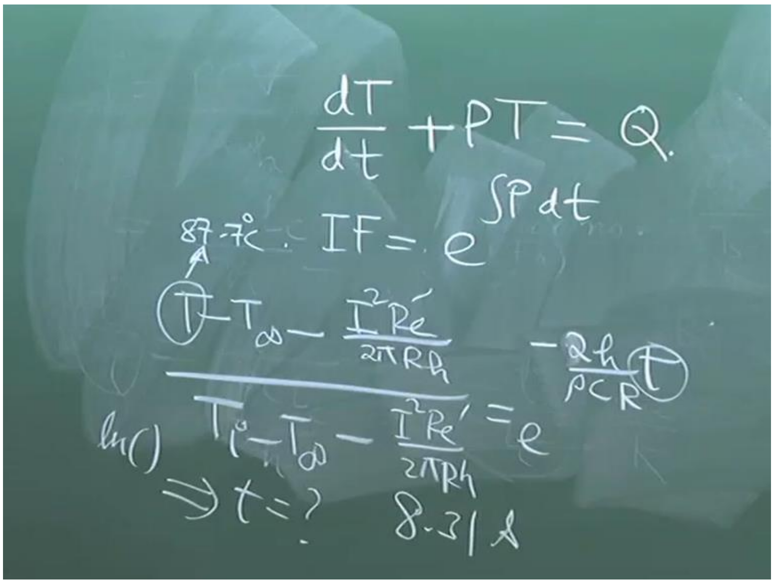


Now what will be the unsteady state variation because for the second problem basically we require to find out temperature as a function of time, right. So, if you look into the differential equation. The differential equation is of this form right, where P and Q are some coefficients. This is the form of the differential equation. So, for solving this differential equation, we have to multiply with an integrating factor e to the power integral P d t.

That is the integrating factor by which you have to integrate. So, if you do that it is a straight forward solution of the differential equation, ok. So, let me give you the solution to the problem T minus T infinity. So, this is the solution to the equation. Now, you have to find out the time at which this temperature becomes 1 degree less than the steady state temperature, right. So, the steady state temperature is 88.7.

So that will be 87.7 and then you take log of both sides and find out what is the time. The answer is 8.31 second ok. So, you can complete the calculations, but you have understood the method of how to solve this problem. So, to summarize we have mainly discussed today about first some aspects of lumped parameter analysis and then what are the conditions under which the lumped parameter analysis works.

And we have used the lumped parameter analysis to solve a couple of problems of practical interest, ok. Thank you very much. Let us stop here today.