Conduction and Convection Heat Transfer
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Lecture - 11
Steady State Heat Conduction Fins

Good morning and welcome you all to this session on Conduction and Convection Heat Transfer. Now, in this session we will start a new topic Extended Surface and Fins. You have seen in several essays practical scenario we use an extended surface. For example, in a wall, hot wall, if you have to enhance the heat transfer we add some extended surfaces you will see in many places the radiators of a car.

In heat exchange, as if you see the two fluids exchange heat between each other, one is hot another is cold and they pass through different passages one is for example in a shell and tube heat exchanger they are on the inside tube another is the outside annular area. And you often see that to enhance heat transfer there are extended surfaces attach to the inside tube to enhance the heat transfer there.

So, several applications that they are in buildings you will see, the fins at there to enhance the rate of heat transfer at the outside wall. Now the basic principle of using extended surfaces which are known as fins to enhance the heat transfer depends upon again the Fourier Law, whole life time, the entire steady of heat transfer I tell you those who of you who may make career heat transfer will remember this depend, that the pivotal point is the Fourier heat conduction law which is used in convection heat transfer.

Also, that is heat fluke, total heat transfer rate is K-thermal conductivity, times the area, times the temperature gradient. So, we can increase the heat transfer rate by increasing the thermal conductivity of the fluid or dissolute that is a property. We can increase the heat transfer rate again by increasing the area that is the geometry, more is the area more is the heat transfer, you know that when you want to cool or tea sometimes.

We pour it on a flat plate where it gets more surface area, it is cold being rate of heat transfer is enhanced? The third one is the dT/dX or dT/dR at temperature gradient. Now one is the property another is the geometry and dT/dR is the temperature gradient that depends upon the boundary condition and geometry for conduction and for convection heat transfer these DT/DR depends upon the flow condition which will be dealt in more details in the convection class.

And the example of that is again, first we pour that tea on the plate but still we are not satisfied than we try to blow air over it or put it in the bottom of a fan. That means in that case we create a higher temperature gradient, we do not touch area, we do not touch the property. What do we touch? Change the flow field as I explained earlier temperature gradient.

So, therefore, in heat transfer this has to be remembered for whole life that these three parameters scientists are searching that these three parameters had to be dealt to it to change the heat transfer rate. Now in extended surface or fins are used to increase the heat transfer rate by increasing more surface area. How it is done?

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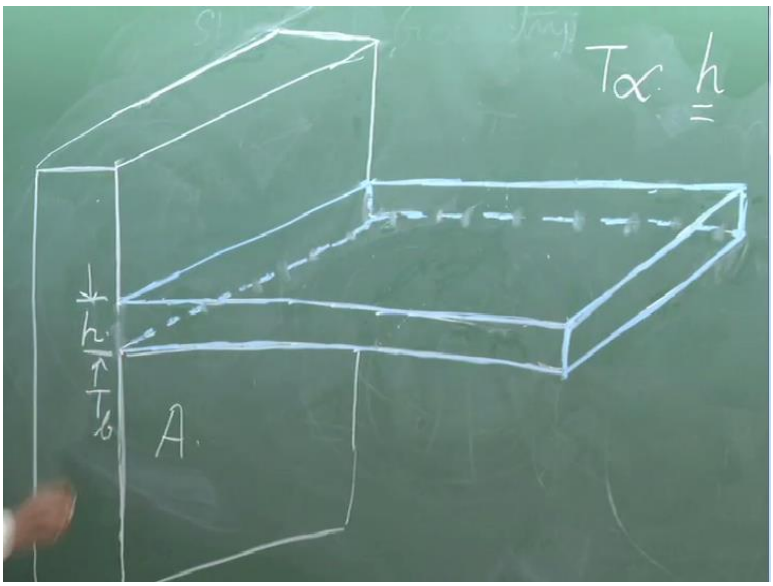

Let us consider a wall, this surface of wall, we are not interested what happens this side of the wall; just I have drawn the figure for your understanding. The problem is pour that this is the outer surface of the wall this surface which is kept at a temperature Tb in nomenclature b is used that is base, base temperature that means from this wall. And this wall is exposed to an ambient T infinitive it is hot wall this may be consider that something is generated yet in terms of energy.

It may be the outer wall of a building where too much energy is generated because of some action, huge people are there making noise, making some functions and all these things where this becomes hot, or it may be the wall of a farness where this energy is generated. These are the practical example that we get a hot exposed wall, this surface of the wall at some height in temperature Tb which is greater than the T infinitive.

Now immediately one person will tell, okay, then you have a heat transfer Q I tell you, it is h if you prescribe h as heat transfer coefficient which is again a very complicated thing, you will see can how complicated it is but it is the fun when you will deal with convection. But at present we are happy that h is given as a parameter which is constant. Then people will tell if A is your surface area of this wall then fine you have this heat transfer. Are you happy? No.

I am not satisfied with this heat transfer rate. But h is fixed I cannot change. The flow field is such of the outside there, no I cannot do. So, what you can do? I cannot change the T infinitive the ambient temperature. Then I have to set another refrigerator or air conditioning plant to reduce the temperature. So, this we have to accept. Okay, I will enhance the heat transfer rate by enhancing my area. How?

Wall is fixed, how you will increase the area? Or can I increase the outer wall area? So, what I do, I add additional surfaces like this in a plane geometry for simple understanding. Now let us consider this part of the wall I attach a rectangular surface like this a three-dimensional, my drawing is not that good, I think you can understand it, so I at this surface. Quiet a high thermal conductivity.

It is not insulating material it is a thermal conductivity high. K, let K is the thermal conductivity which is very important of this. Now what happens actually by adding this, if I tell that it provides additional area for heat transfer it is understood. How? Now when heat is flowing from this area – now this unexposed area remains safe they behave as the same thing h*A (Tb – T infinitive) area plus this area.

So, this area, let us now consider that h– it is very important I am telling you, few books I do not know explain this way, they simply write by providing more surface area, but how? You have understand surface area is being provided at the cost of what we are providing the surface area? “Professor - student conversation starts” -Hey, you please come to the first bench, you, hello, you yes. Yours next to you, you yes, hello you, yes come and sit here – not you in front of you, yes come and sit here please. “Professor - student conversation ends”

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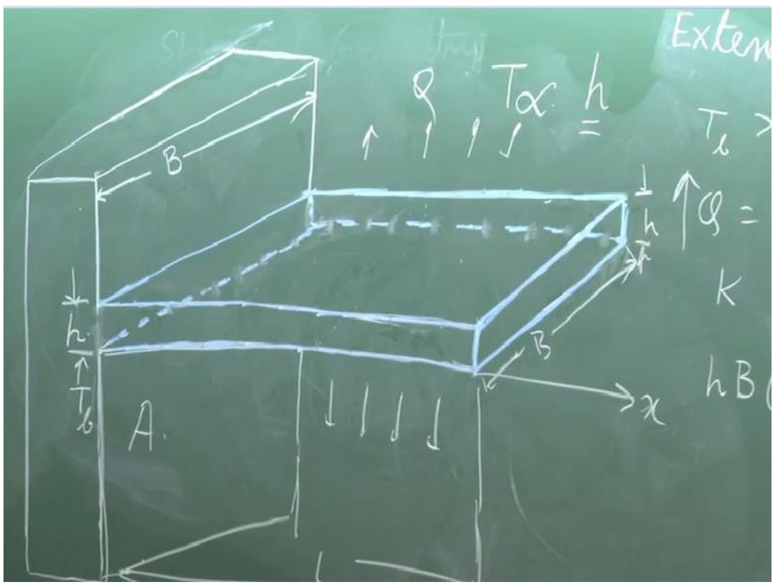

So, now, a thermal conductivity K. Now what happens if this is the height h and if I consider that this with understanding is our B, then from this part we have a heat transfer Tb minus T infinitive. H*Bb (Tb minus T infinitive), try to understand it this is very important I tell you, but the first part of deduction is may not be important but to understand the feel efficient it is very important. Now you can tell sir, I am generating the same area h*B here, so why you are bothered so much?

I am generating the same area, but you are adding materials that means the thermal resistance, what does it do? That this exposed area hB is not at a temperature Tb because there is a temperature dropped, that is why we tell this is the thermal resistance, that if you add more material and take the surface area somewhere here in the direction of heat flow means the temperature drop is there until and unless thermal conductivity is infinitive.

Less is the thermal conductivity more is the temperature, this way we realize that there is a reduction in the heat transfer, that means adding this material reduces heat loss from this part of the surface where it is attached. But, does not matter each and every portion that heat is being transmitted in this direction in this direction. Why? This is because we have a temperature gradient from Tb to T infinitive.

So, each portion at each length if we consider this as a distance x and let this be the length of this from here actually this length this is in isometric view this length L. So, along the length you have a temperature drop from Tb to some temperature here. But this is relatively higher but lower then Tb. So, at each and every point even if the temperature is lower than the base temperature but it exchanges heat to the surrounding because of the huge surface area.

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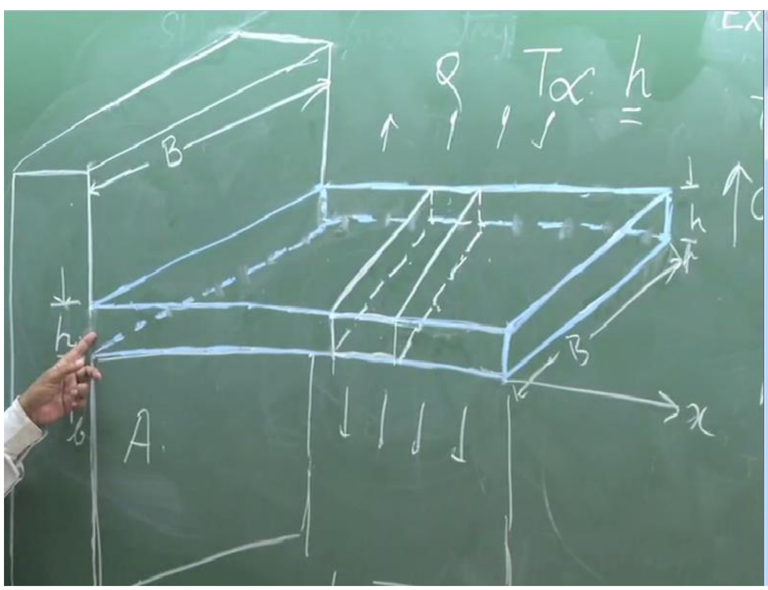

Let us consider a section here at a distance x. I draw a simple two-dimensional surface like this then it will be easier for us to understand that let us consider at a distance x, we take an element of length del x and heat is conducted from the base which is at a temperature Tb. From here it is conducted at position, let Qx is the heat coming in then there is heat transfer through the surfaces of this element.

And rest part of the heat is being conducted, that means this heat transfer through this lateral surfaces is huge. Because of this it draws more heat here. Finally, it is connected to here that means from the base it draws more heat that means what are those lateral surfaces, that means this one. I draw it here that is del x you can understand this, like this. That means, I think you can see it.

That means this lateral surfaces one is the top surface another is the bottom of surface that means del x into b, top and bottom then this surface that del x * 8, del x * 8. So, you will see there are four surfaces from which it transfers heat by convection. Here, if you cannot guess from this three-dimensional drawing that this surface, this surface top and the bottom that means one is this top surface perpendicular to this direction that means this top surface, this one that is B times delta x.

Similarly, the bottom on B time delta x from the bottom surface delta x and quiet in this direction perpendicular to the plane of the boat. And also, this surface that is this one whose 8 into delta x this area and the rare one that means it had this open or exposed surface to be ambience. Quite transfers heat and it has some temperature T at distance x which is less then Tb because of the thermal conductivity of this material that I accept it is not at Tb.

But even if that less temperature it has huge area to transfer a huge amount of heat and this why it is added up for the entire fin so that it helps to draw more heat from the base then that was being transferred without the attachment of this extended surface which is known as fin, this is a rectangular fin. Clear to everybody that how then the fin works that it pours an additional thermal resistance in conduction along the direction of flow.

But it gives huge heat transfer by convection by allowing a lateral, more lateral surface which are even being even less being even less then the base temperature but transfer the huge amount of it because of a surface area. So, now if this be the problem which we have understood then next one becomes so simple that why not we then find out the conservation of energy principle, no thermal energy generation.

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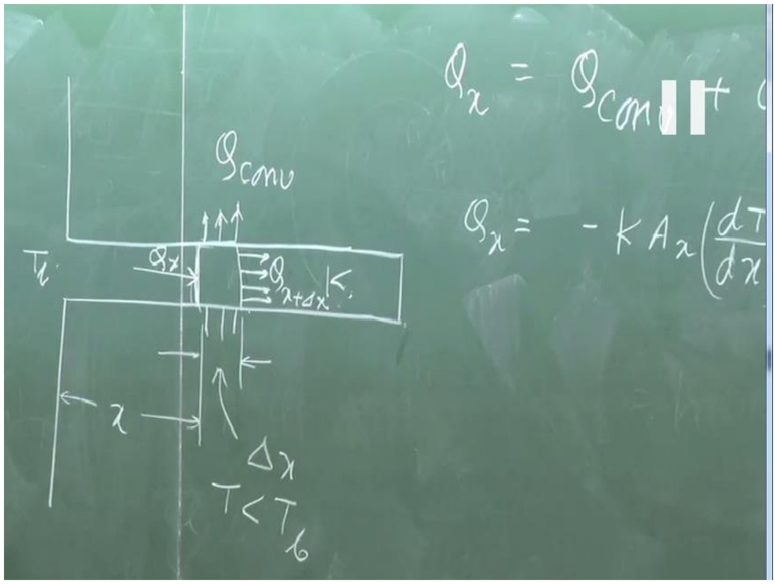

One rectangular fin if you consider let this is Q convection from the lateral surfaces. Then Qx which is coming is distributed as Q convection plus Qx plus delta x. Clear? Now Qx is –KAx dT/dX. What is Ax? Ax is the surface area here. Here you can say that it is uniform but I take a variable area. Let us consider a variable area.

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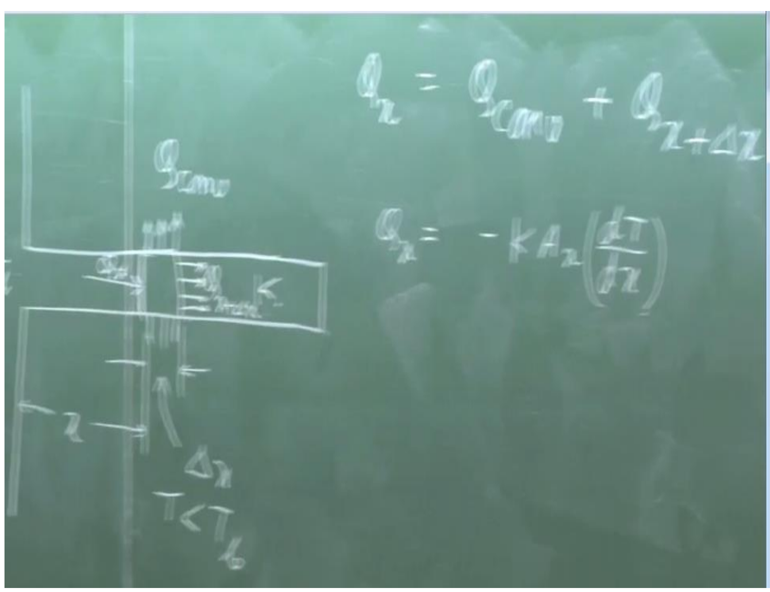

Let us consider a variable area. I have shown you here area which is straight but we can consider a variable area now for general purpose. But ultimately you will be solving this, this is variable area, this is a variable area, okay. So, this is Qx, this is Ax at a distance x then this is Qx plus del x and here this is del x, very simple, here Ax is a function it, I am not writing Ax plus del x not necessary.

Then Q convection that is the heat rate transfer. So, that is why minus Kh (dT/dX). Now if you solve this by expanding Taylor's series and take it here then Qx plus del x is Qx plus dTX of Qx dX, that means minus K, take all sorts of Qx plus Q d/dX of Q is del x plus Q convection. What is Q convection?

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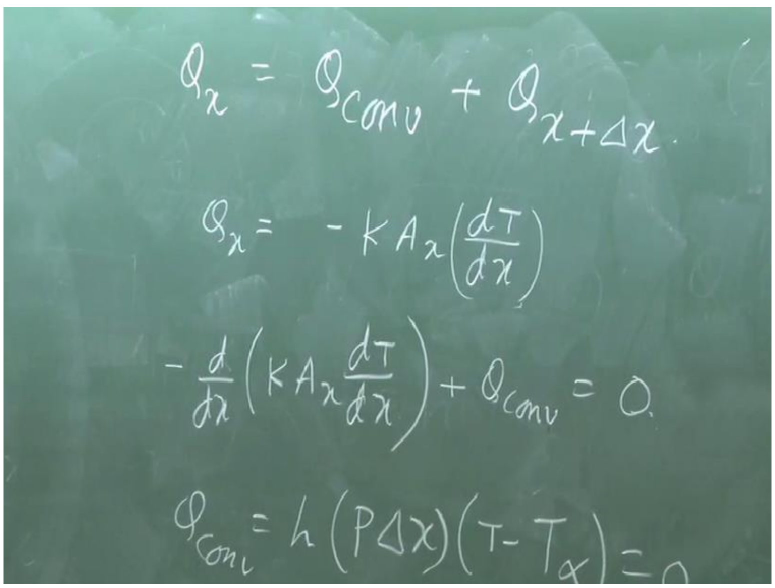

Now Q convection can be written like this, we can now rub this one, we will go by that. Now Q convection can be expressed as this. If you define a perimeter P, for example here you can understand better the perimeter in this case is the perimeter of this surface, that means here P plus 8 plus B plus 8 2b plus 8. So, therefore, if you consider P as the perimeter of this lateral surface which is perpendicular to the plane of the board then Q convection can be written as area 8 times the area, area become the perimeter into delta x.

In terms of perimeter, it is perimeter into delta into T minus T is the temperature there at that location x T is the temperature. So, in terms of the nomenclature perimeter I can write this equation as hP delta x T minus T infinitive.

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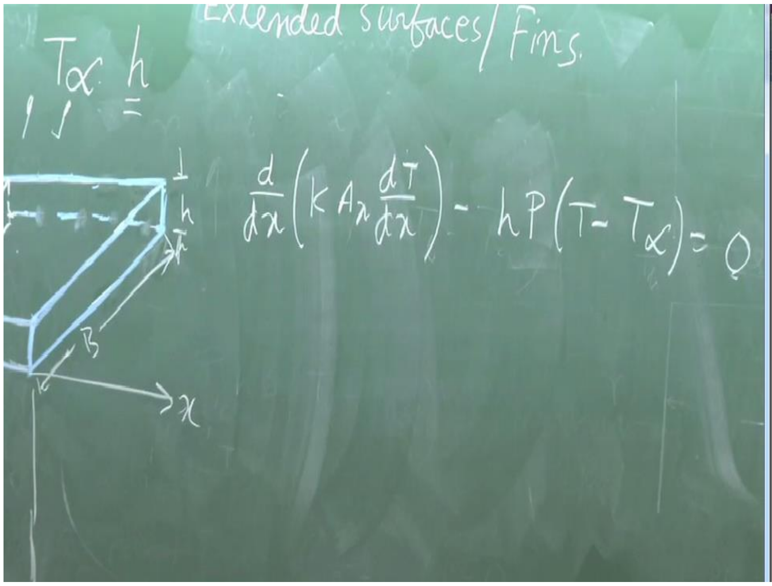

So, therefore I can write the equations in this fashion that d/dX of KAx, dT/dX minus hP, delta x you are very, very good delta x, I am sorry there will be delta x, so hP into T minus T infinitive very good there should be delta x, yes because this is Qx plus del x is Qx plus d/dX of Qx into delta x neglecting the higher order terms, very good. So, that delta x gets canceled. So, now this is the temperature distribution equation.

Difference is there, here heat is transferred simultaneously for each and every element both by conduction and conduction through lateral surfaces which is so far, we did not consider so that means it precisely boils down a conduction problem where lateral surfaces at each and every section of the conducting material along the length of the heat flow shares in the convective transfer. Okay that is the only the balance.

So, therefore, if you we know physics then we can find out the basic governing differential equation for the temperature distribution, let us consider a very simple case. Cost and thermal conductivity, no temperature dependences and this type of rectangular fin, that means A is constant. Then we get a very simple equation d square T/dX square minus hP/ KA (T minus T infinitive), okay.

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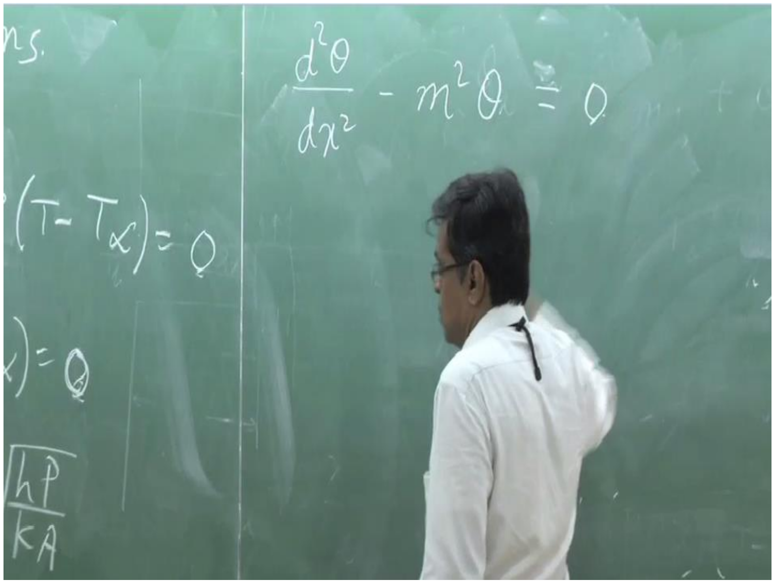

Now if we write by transforming the variable dependent variable temperature in the form of x temperature that means I change the temperature variable T transformation T minus T infinitive as the variable, that means it is the temperature over the ambient temperature. Then dT/dX is the theta dX, then dx square dX square is theta dX square but the advantage is there here only we get theta that means this looks like a simple school level problem that this square.

By defining this and by defining m as root over hP by KA, m is a perimeter which is just defined by root over hP/KA, hp/KA is the m. Okay, if you define m as root over root over hP/KA then we get an expression, d square theta and dX square minus m square theta is equal to 0. If we define m as a dimensional parameter then we get an expression d square theta and dX square minus m square is equal to 0 whose solution is very simple. Now we define this as square root or rather m square is hP/KA this because hP K all are positive.

Okay so m is a real quantity that means hP/K can be expressed as a square of a real number m square, m is a real number because hP/KA cannot become negative, that is the logic otherwise we cannot define just square root of something as a real number, we have to investigate whether this is positive or not.

So, if we do so rest part the solution of this complimentary function you find out, there is no particular integral, that side is 0 this is a second there you differential equation it is an exponential solution e power mx plus C2 minus mx hyperbolic functions. C1 exponential function whatever you call e power mx. That is all.

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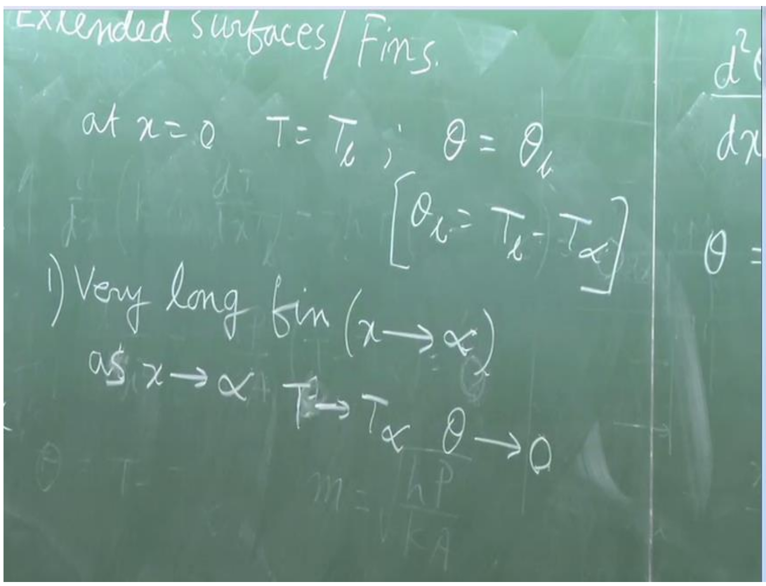

Now the boundary condition. Okay. Now the boundary conditions. What are the boundary conditions? Tell me, now boundary conditions then you ask me sir, how do I know? The governing differential equation comes from the principle of conservation of energy or anywhere principles of conservation principles. And the boundary conditions comes from the physical problem define to it boundary and other conditions that means the boundary conditions have to be physically defined.

One boundary condition that is defined or prescribed by the physics is that, base temperature is Tb where from x is measured. So, therefore that x=0, T=Tb which means theta is equal to theta b. Theta b means Tb minus T infinitive that mean Theta in terms of Theta it is Theta b. What is another boundary condition? One another boundary condition, you have to search for another boundary condition with respect to x that means you have to find out what is the boundary condition at x is equal to in.

If L is the length of the fin, so you have to know whether it is insulated or something else. So, one very simple case, there are various cases, one case very long fin, sometimes in problem we tell that fin is very long, very long fin, engineers are always smart, they always take an approximation, they do not like mathematician. Very long mean consider x tens to infinitive that means the very long fin.

And if x is very long tens to infinitive then eventually the trailing surface the excrement of the fin will attend the environmental temperature, very good. So, this boundary condition is that at – as it is written as x tens to infinitive T tens to, obviously this is mathematics T infinitive and Theta tens to 0. This is the simple one. So, first boundary condition gives you Theta b C1plusC2. Now in this case if you consider a very long fin so automatically you will see C1 is 0.

Because if you make this x tens to infinitive this term vanishes and if Theta is 0 means C1 is 0, that means the solution is that Theta by Theta b is e power minus mx that means there is an exponential of temperature to what 0. Very simple, that means if you draw this graph exponential, sorry I will show this thing afterward.

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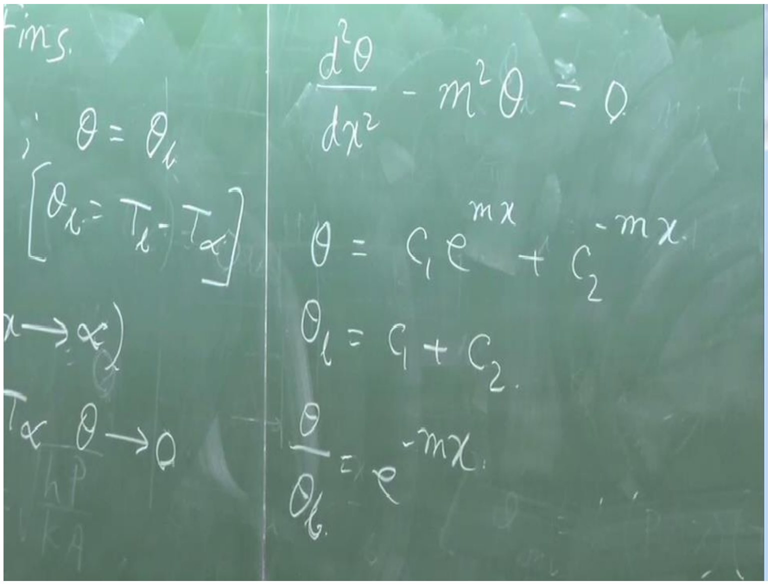

Theta by Theta b is e power minus mx at x at x tens to infinitive, Theta tens to 0. So, what is the value of Q? Now Q means what? What is Q? Try to understand.

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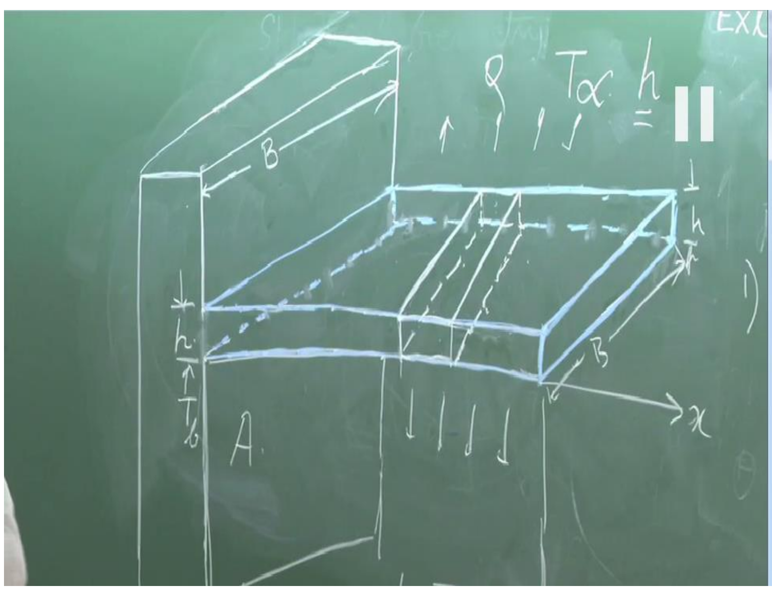

Q at every section is changing. What happens again try to understand this, these surfaces can transfer heat that means that any section that heat which is coming in is getting transferred by convection from the lateral surface and the rest part is being conducted. That means the heat conduction is getting through this is reduced that means the heat which is taking from that base extracting from the base by the fin is almost giving to the atmosphere why is lateral surface due to convection and a very less amount is being convicted from this surface.

And in a long fin, the entire heat which is takes from the base is being convicted by the lateral surface, because when it reaches this surface expose the atmosphere at the excrement, it has reached almost T infinitive, because T tens to T if you know, heat transfer, heat transfer is 0, delta T, delta T is 0. However, this is the concept, we will go by mathematics, we are interested at Qx is equal to 0. What is the heat transfer from the base?

Engineers are interested, heat transfer how much one-minute base I have attached a fin what is my heat transfer rate from the base that means how much heat it is extracting from the base.

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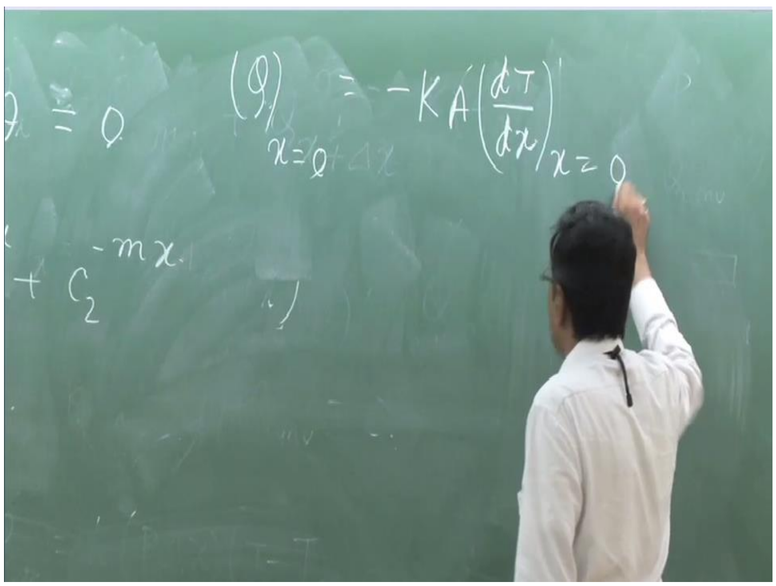

Okay I will tell you very simple minus K area dT/dX, this area now I am using as the area of the fin, earlier when I talk to you to introduce the problem I told this is the area of the wall A but now whenever I am using area A this is for the cross-sectional area of the fin. In the equations also I derived that, do not get confuse with this area. So, this is the cross-sectional area of the fine and in this problem, we are considering a rectangular area having constant cross-sectional area.

So, therefore this is the cross-sectional area A, so therefore –KA and dT/dX at x=0.

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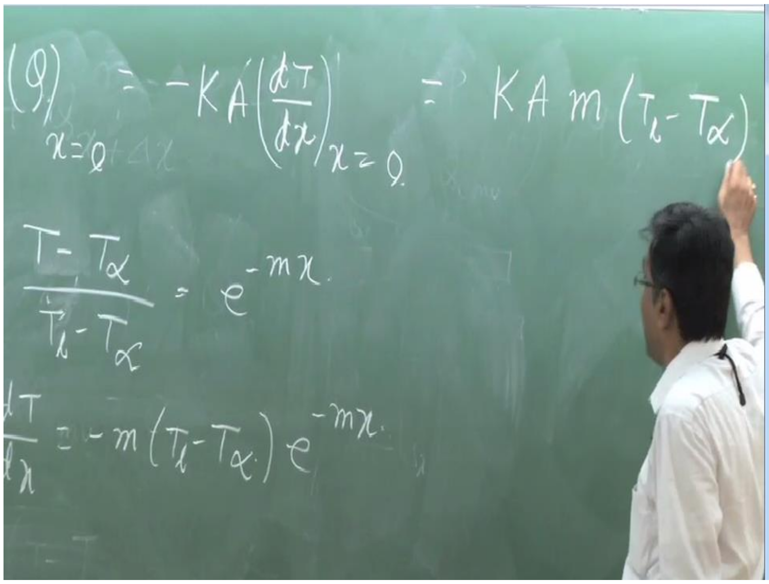

So, what is dT/dX at x=0? Theta by Theta b means I write yet T minus T infinitive divided by Tb in terms of temperature decoding the variable transform, variable into the actual temperature is e power minus mx, so therefore it is very simple that dT/dX is equal to – m (Tb minus T infinitive) e power minus mx and that x=0 the exponential function will be 1 so it will be –m Tb minus, minus will plus that is KAm into Tb minus T infinitive.

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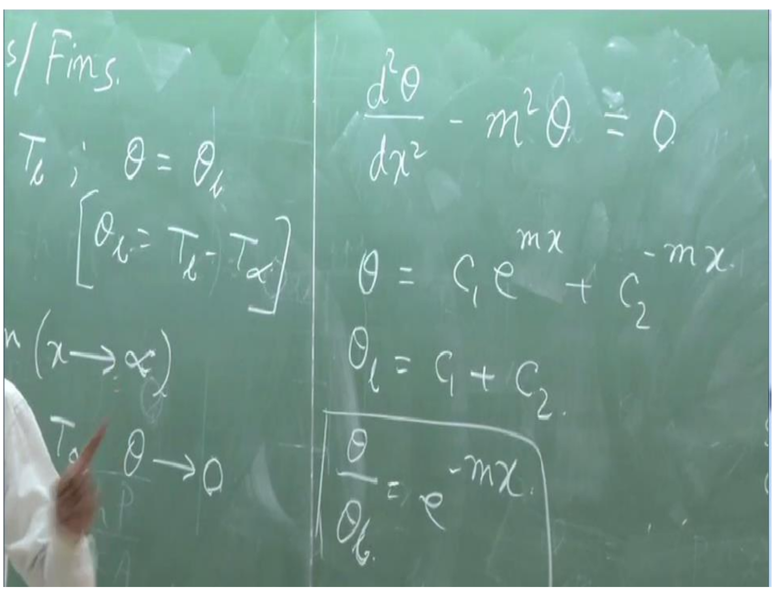

Now m is root over hP/KA I defined earlier, again I write m was defined as root over hP/KA. And if we put that then Q at x=0 becomes root over hPKA * Tb minus T infinitive which can be written as Theta. So, this is the heat transfer from the base and this is the temperature distribution. You do not have to remember any formula but you have to know how it has been derived.

Again, and again I am telling you, you have to be capable of generating the governing differential equation by taking an element with the understanding, physical understanding of the conservation of energy value, what is happening, something is coming, it will go whether generation inside, whether there is a lateral convection altogether you have to develop the basic equation.

And then slowly you have to think that which are constants given in the problem or everything is varying then it becomes a problem of mathematics, how complicated it will be. And if you do it meticulously you will arrive at any equation. In the examination, also if any problem is there we have to derive the equations. So, do not mug up that Theta by Theta b to the power of minus mx, Q is equal to root over Hp K, Theta b for a long fin, not required.

But sequentially, how the deduction is made understand a problem both physics and the mathematics.

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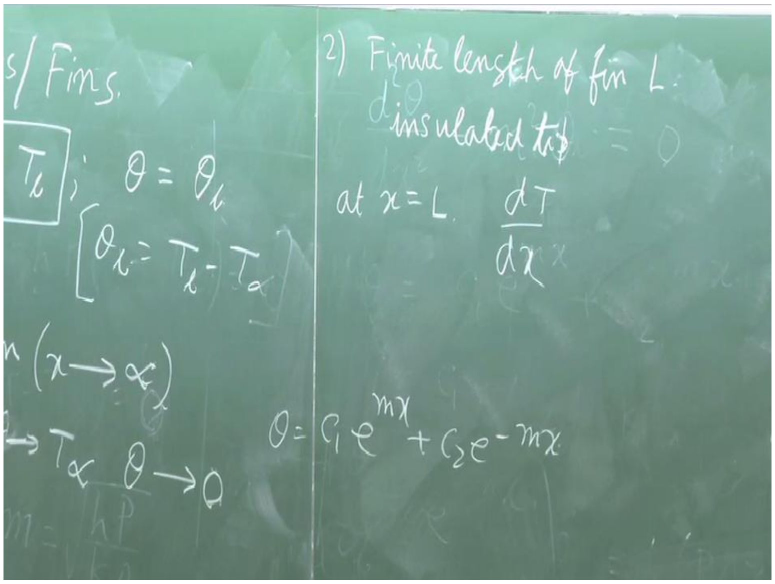

Now the second group, second boundary condition is another type of fins, secondary boundary condition means another type of fin. We may consider. Note long fin, Finite length of fin, let it be L. But, insulated tip that means tip is insulated; insulating material is pasted on it that means no heat will be transfer. So, this boundary condition is common to all, this temperature Tb. So, what is the second boundary condition, in this case will please tell at x = L.

What is the boundary condition if it is insulated? In which form of boundary, it will come if it is insulated means no heat transfer, please anybody flux 0 means I have to solve the temperature equation. dT/dX 0 who has told it? dT dX is 0 you tell that because I have to solve the, I have to solve which equation? Theta is C1 e power mx plus C2 e power minus mx. I have to solve this equation, let me write this here.

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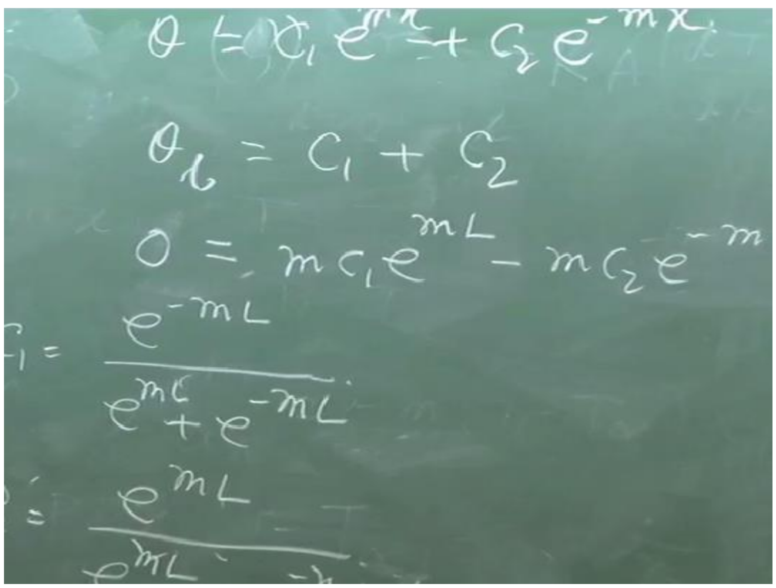

So, I have to generate from – you are correct, you are also correct but ultimately you have to translate in this form because I have to solve this C1 C2, Theta is C1 e power mx plus C2 e power minus mx. And one condition is that Theta b at x=0 that means Theta b is C1plusC2 another condition is use, heat transfer 0, heat fluke 0 means –K dT/dX is 0 that mean dT/dX is 0 that means dT/dX at x= L is 0, what is that?

That means 0= dT/dX 0 means, the Theta dX is 0 same thing because Theta is T minus T infinitive. 0 is mC1e power mL- mC2e power -mL. If you solve it, you will get C1 is equal to e power -mL/e power mL plus e power -ml and you will get C2 is e power mL/e power mL plus e power -mL and if you substitute this from the second case you get the expression Theta by Theta b in terms of hyperbolic function cos m (L- x) this is the argument divided by cos hyperbolic mL.

You know the hyperbolic function that cos x is equal to exponential x plus exponential plus exponential -x by 2, okay. So, this is the final expression. Now it becomes a routine job. Only thing is that you have write the boundary condition correctly. Then things are done.

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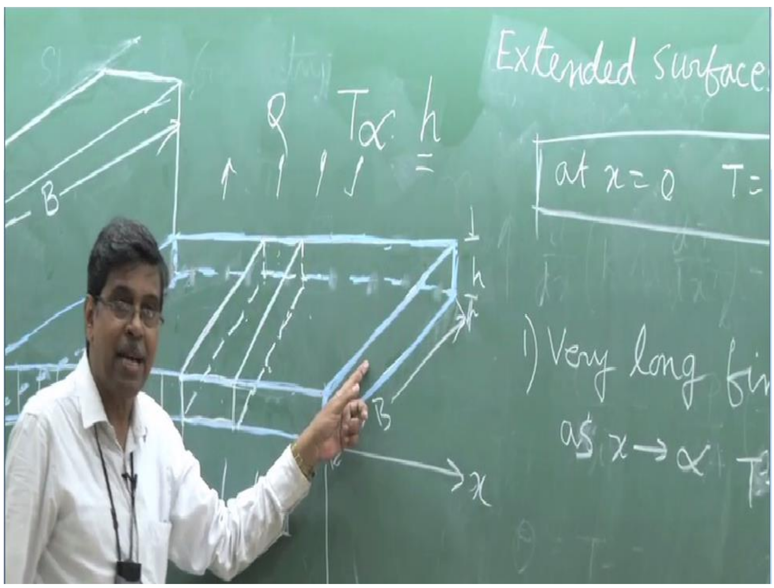

Now third category of problem is fine of finite length L but no insulation. It is the most practical problem. If there are long fin means what? What how much long? How do you take that T is T infinitive, T maybe very low, if there is a very huge drop from Tb depends upon thermal conductivity also. Now insulated sometimes the fin surfaces maybe not be insulated and even with insulation there maybe heat loss so the third one is the most practical condition.

That Fin of finite length L but with convective heat loss at the tip. What is meant by that? That means at x = L what do you have? This is a conjugate heat transfer problem, K (dT/dX) L, for unit area I am writing or you can write area –K into area that means the heat which is coming to this surface by conduction -K dT/KA dT/dX which we did earlier combined conduction convection problem in series the same it is being transferred to the h*A (T- T) T at L minus that means T L is the temperature at L.

So, if you can understand this and correctly write this then things are okay that means -K dT/DX at x=L must be equal to h into TL - T finite. TL means T at this is clear to everybody? That means this is the conduction heat at the tip which is being conducted that is what. I told you that ultimately what is happening if you go on dividing into number of elements each element this takes heat some is going by lateral surface then some is going by lateral surface then it is conducted next.

So, this will be conduction heat transfer is getting reduced when it come here most of the heat which has been taken from the base has been convicted by the lateral surface. So, the rest part of the heat which is conducted finally to the extreme expose surface is being lost to the surrounding ambience my convection from this part of the expose surface, which is the same area because we are considering constant area rectangular fin.

So, with this thing, this concept in mind one can write the differential – sorry boundary conditions this. Clear?

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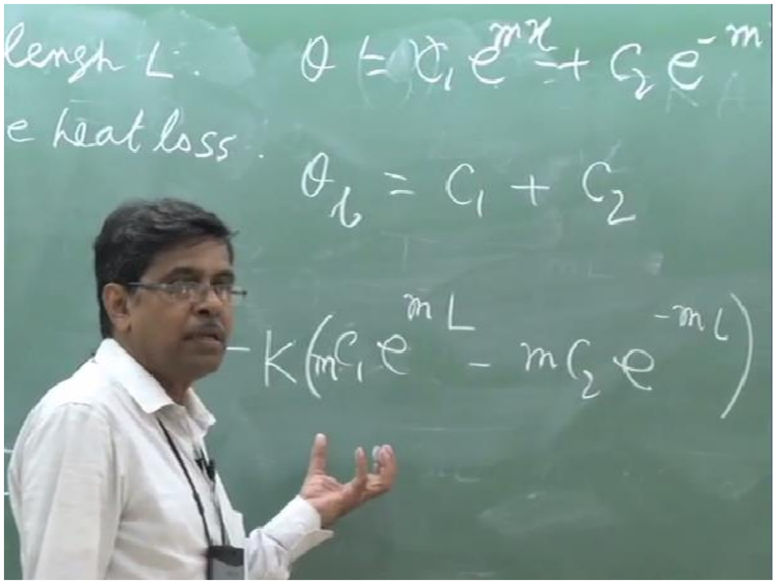

Step two is, now is convicted boundary condition in terms of our Theta and all these things. How to do it? Now dT/dX the Theta dX that means –K dT/dX means the Theta dX that means I can write now this as the Theta dX and this has Theta L in my nomenclature because my equation is in terms of the transform variable Theta that is excess temperature Theta L. Now what is -K d Theta dX, what is d Theta dX?

C1 e power mL - mC2e power -mL. h into Theta L that is C1 e power mL plus C2 e power -mL. No, this side C1 mC1, this side is simply C1. Sorry mC1 d Theta dX is mC1 e power mL plus mC2 e power -mL. So, this is another equation. You can rearrange it some coefficient into c1 plus some another coefficient into c2 equals to something that means these are two equations for c1, c2 in terms of m and L. And if you substitute this then you get a solution like this.

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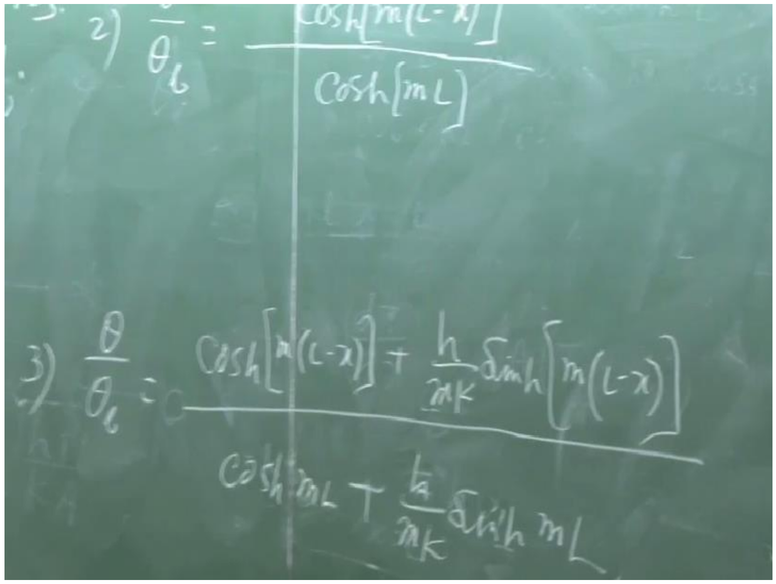

Now you get a solution like this Theta by Theta b. Now earlier case we got a solution Theta by Theta b is Cosh hyperbolic m(L-X) that is the earlier case insulated tip divided by cos hyperbolic [mL]. Now I get a solution Theta by Theta b for this third case, this is the second case this is the third case, Theta by Theta b is Cosh hyperbolic mL- x plus h/mK sine mL-x divided by Cosh hyperbolic mL plus h/mK sine mL.

This is a routine matter but tedious job to find out the especially the rearrangement things that means you have to find out c1, c2 and then you can find it. Other two things I have forgotten to tell you that even in the earlier case this is insulating tip, this one of finite length L, difference is that this is non-insulating tip of finite length L which transfers heat with the ambient in terms of convection where h is the convection coefficient, okay. Now what is the heat transfer?

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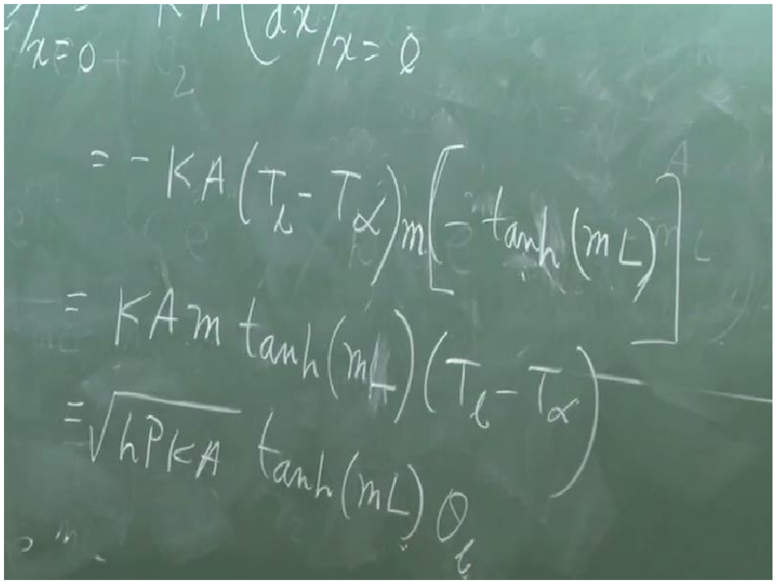

Earlier I did it for long fin, (Q)x=0 = -KA (dT/dX) at x=0. Now if you do it from here what is dT/dX? This Theta by Theta b means T minus, Theta is T minus T infinitive, Theta b is Tb - T infinitive. So, if you do it then you get it is equal to -KA, T - T infinitive, so therefore Tb - T infinitive * Cosh hyperbolic mL - x that is sine mL-x with a minus sign and Cosh ml at x=0 Tanh mL.

That means minus –tan(hmL). That means this becomes is equal to KA, sorry m, m will be there m. m Tanh mL Tb - T infinitive and we recall that m is root over hP/KA, so it is root over hPKA Tanh mL * Theta b. This is an expression. Now these become all routine task.

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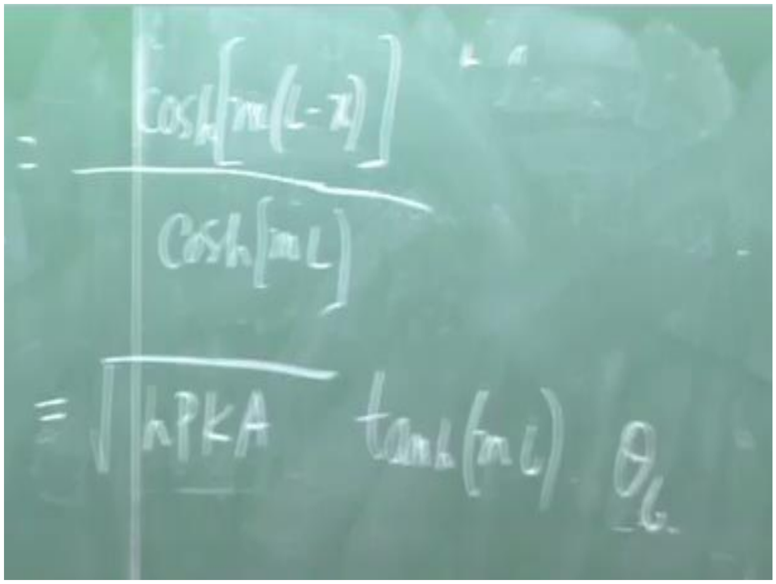

It is root over hPKA tanh mL into Theta. This is the heat flux at x=0. Similar you can find out the heat flux expression for this. It is a lengthy expression that I can tell you, you can see in the book also.

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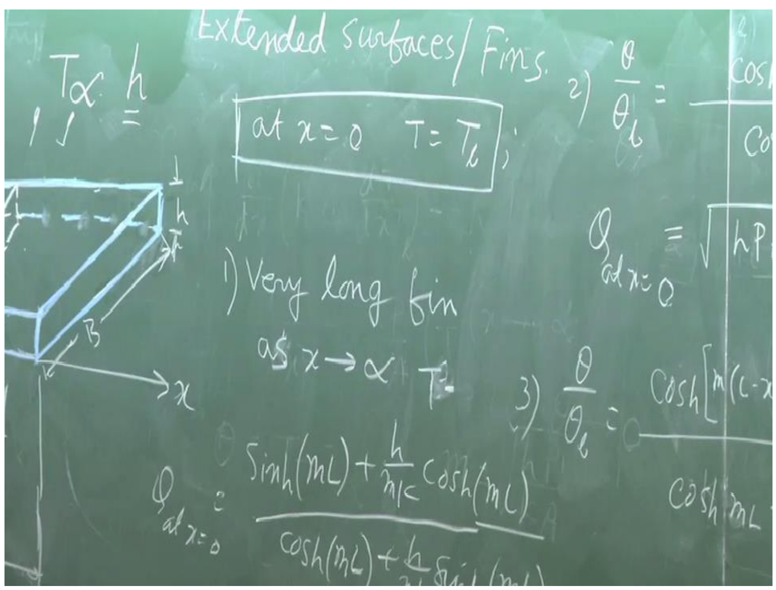

For this case, number three, the heat flux at x=0 will be let me see that it is difficult that which function will come. With insulated fin tip sine mL plus h/ mK cos mL that means it will sine (mL)plus h/mK cos (mL)/ cos (mL) plus h/mK sine (mL) this is very tedious I know also bore you but you have to afford to do this because without this it is not complete. Concept is very interesting that how the fin enhances the heat transfer, then by setting up the governing differential equation.

You have to solve it with the boundary condition. And for more practical cases the boundary conditions are such things are little complicated. So, complicated means tedious in equation in solving the things. So, that finally we solved for temperature distribution in terms of the excess temperature and the heat transfer from the base which is enhanced and temperature distribution heat transfer from the base for different cases.

So, these three cases are most important boundary conditions. Another boundary condition is there that fin is a finite length and its end surface is kept at a temperature T_L that means T_L is specified. So, everything has to be found in terms of T_L that is given as a task for you to do it. Then we will be solving few problems on this fins extended surfaces so that we know that how we can apply our knowledge that means these equations which we have derived along with our knowledge to those practical problems in the next class.

And I will finally give you a generalized approach mathematical approach for any one-dimensional steady heat conduction problem which will act as a fin; which will act as a simple geometry without extended surface, lateral surface all those things combined. So, therefore next class, we will be most probably the concluding class for the one-dimensional steady state heat conduction.

I will be solving few problems and then we will start the two-dimensional heat conduction steady state and then we will go for unsteady heat conduction, okay. So, next class means tomorrow. Okay thank you.