Conduction and Convection Heat Transfer
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Lecture-10
1D Steady State Heat Conduction in Spherical Geometry

Good morning and welcome you all to this session of conduction and convection heat transfer. Now today, we will solve 1 or 2 problem relating to 1-dimensional steady state heat conduction in cylindrical geometry, cylindrical wall, that is in a pipe of circular cross section and a very interesting related problem is the critical thickness of insulation. So, let us solve 1 or 2 problems just to habits applications what yesterday also we solved a problem that is at the last class.

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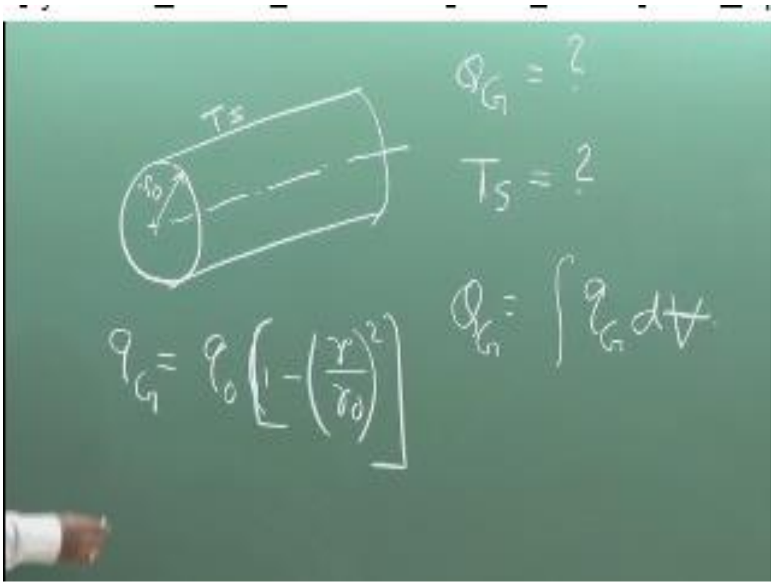

Now the problem states like this, I am telling if there is a cylindrical container, this is the axis; cylindrical container of radius r0 and the language of the problem is like that radioactive Os are packed in a long thin wall cylindrical container, that means the radioactive Os are packed that means ultimately works as a solid cylinder type of thing. The radioactive Os are packed in a long thin wall cylindrical container.

The waste generate thermal energy non uniformly according to the relation that the volumetric energy generation rate at a point is given by qG, q0*(1- r/r0) square, where q0 and r0 are constant. That means, it is varying with r, which means there is a cylinder solid, because this thin wall container is packed with the radioactive Os, quick generates in such a way.

So, what they want to obtain expression for the total rate at which the energy is generated, total rate of energy generation and use this result to obtain an expression for this surface temperature Ts. So, this is simplest again problem. First part is the total energy generation from the volumetric energy generation rate so it is nothing but qG is integral qG dV. I usually represent volume as a V with a cart to distinguish it from velocity.

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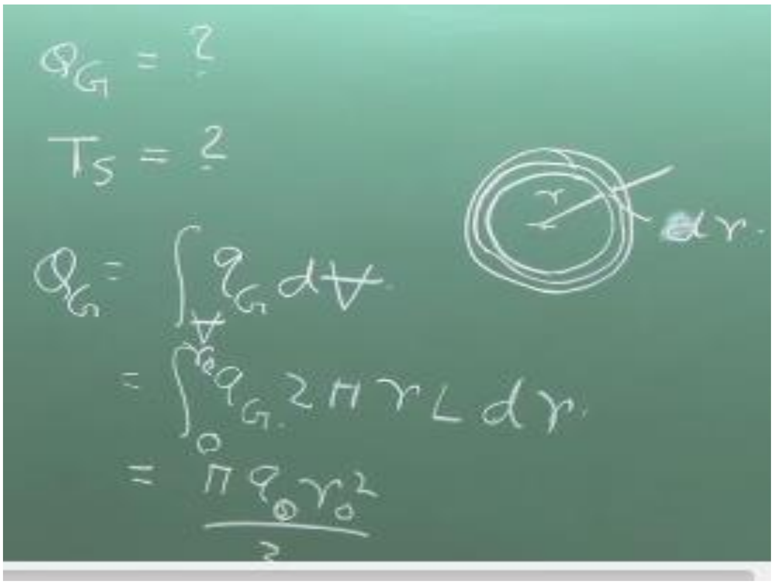

The convective heat transfer is associated with fluid mechanics fluid flow we use v as the velocity, that is why I use this big V, this V is the volume; dV is this small elemental volume, that means if we consider an elemental area like this in the container, thin cylindrical elemental area at a radius r and the thickness of this, is dr for example, then this will be nothing but qG. What will be the volume? 2 pi rL*dr.

If you do that, you get the expression this as, pi q0 r0 square/2, very simple. You put this 2 pi L will come out so r, dr r square/ 2 that is r0 square /2. Because this now will be; this is for the entire volume. Now when I put in terms of d r, it will be 0, 2r0. So, the first term will be dr, rdr that is r0 square/2 and second term will be r square /r0 square, rq/r0 square dr, that means r0,4 and denominator r0 square, it will be again r0 square/2.

So, ultimately you get pi q0 or 0 square/2. This thing, I am not doing in the class. So, this is the expression (()) (5:51) nothing, but what is the Ts? Now to find Ts, expression at the surface temperature, you have to apply certain intelligence that always we do not want a routine (()) (6:06), that we have to find out a temperature distribution, heat generation; is not required. Simply, the gross energy balance, the total heat which is being generated or total thermal energy which is being generated must go out from the surface.

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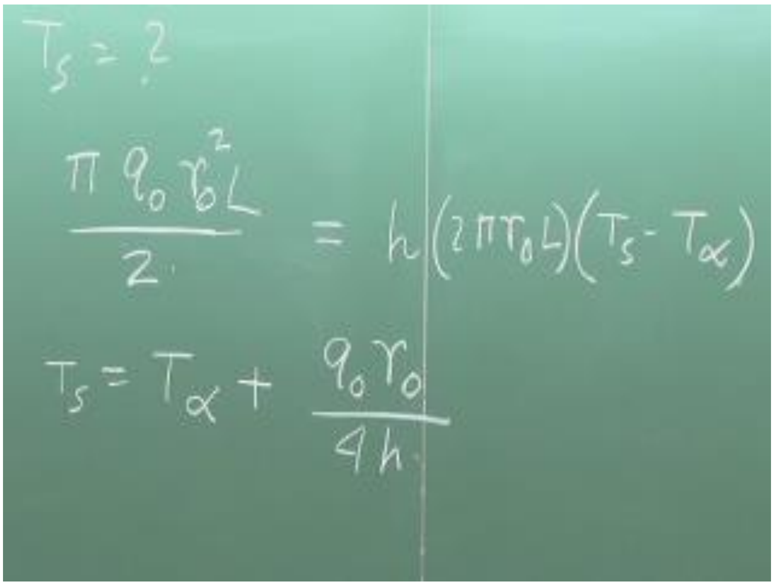

Because the problem is prescribed by a heat transfer coefficient h and the T infinity. This problem is prescribed that the container is immersed in a liquid or fluid that provides a uniform convection coefficient h and at a temperature T infinity and expressed Ts in terms of T infinity, h and q0. So, therefore what we can write? We can write pi q0 r square/2. This is per unit length. So, therefore, these as to be this. So, this is qG per unit length. I am sorry.

The result I have written since I have not worked it out here that is why this mistake is there that qG/L is pi q0r0 square/2. So, this L is there, so L is comes out, L will come out so qG/L. So, the total qG is this, this must be =h*outer surface area which will be twice by r0*L*Ts-T infinity. So, therefore from here we get Ts, Ts will be = T infinity that is the ambient temperature +, pi, pi will cancel, so 4 will be there that means q0r0/4h, pi r0 q0 r0 square, pi r0 q0 r0 /4h.

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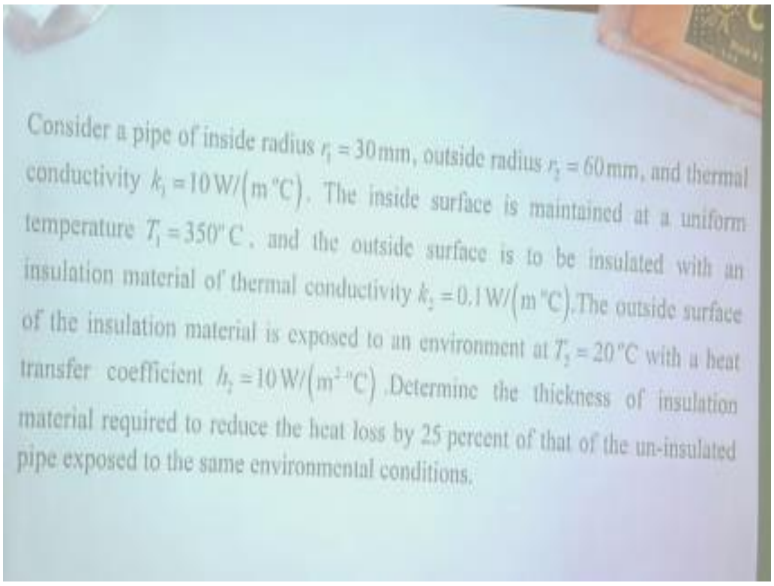

I think it is okay. This is the problem. Next problem, consider a pipe of inside radius r1, 30mm, outside radius r2, 60mm. This is the pipe of inside and outside radius and the thermal conductivity K1, that is the pipe material is 10watt/meter degree Celsius. The inside surface is maintained at a uniform temperature 350 degree Celsius but some (()) (9:32), there may be some hot fluid going inside but the problem is prescribed this way.

The inside surface temperature is 350 degree Celsius and the outside surface is to be insulated with an insulation material of thermal conductivity, that means from this outside surface temperature, heat loss will be more to the ambient. Because of that insulation is required, then the problem states that an insulation is made with a material, insulating material of thermal conductivity K2, 0.1watt/meter degree Celsius.

Now the outside surface of the insulation material is exposed to an environment. This is an environment temperature T2 which is a 20 degree Celsius. After the insulation, the insulating material is exposed with a heat transfer coefficient h2, that means the environment is at a temperature 20 degree Celsius and it has a heat transfer coefficient 10 watt/meter degree Celsius.

Now we have to determine the thickness of insulation material required to reduce the heat loss by 25% of that of an insulated pipe exposed to the same environmental conditions. This is the problem, problem is very simple. That means first we have to find out without the insulating material, what will be the heat loss from a pipe of 30mm inside radius and 60mm outside radius? where the inside surface temperature is 350 and the ambient temperature is 20 degree going through conduction resistance and the convection resistance.

Then we have to calculate the thickness of the insulation, we have to consider an another radius of insulation after putting the insulating material and then we have to find out again the rate of heat transfer considering that insulating material. That means, adding another thermal resistance, conduction thermal resistance and we will find out another convection resistance, which will be reduced, because of the increase in the surface area as I have told.

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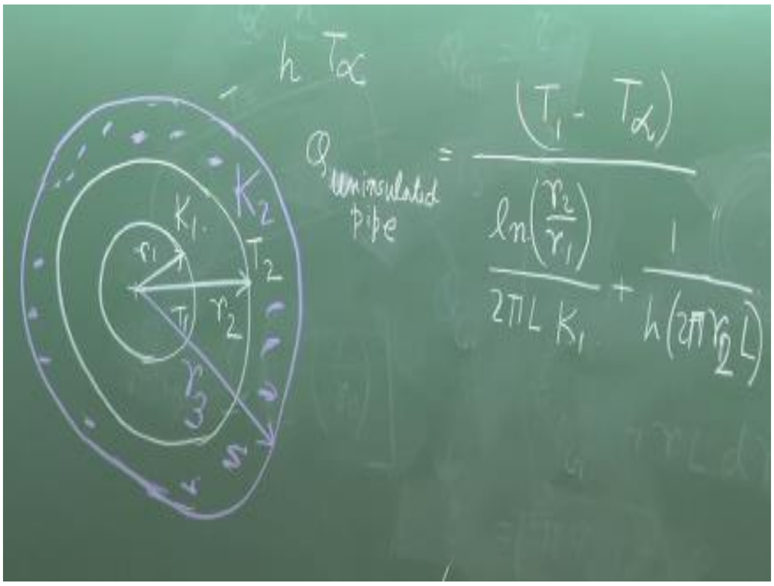

That will be 0.75 of the without insulation because it will be reduced by 25% so, this is the problem. So, there is heat transfer is 20% in this problem, so the rest part is the mathematics. Okay. So, let us solve this, so this is a problem, have a look, you can write in your own language not exactly all the watts have to be written. Okay. First, I find out the, this is the concentric cylinder this is r1, this is r2, this is T1, this is T2 which exposed to the same environment condition h T infinity.

Now if this be the cylinder the first now, heat transfer without the insulation Q without uninsulated pipe= we know that this is the potential difference which we have done, so we do not want to deduce it ln. If you want to deduce it, you can deduce it but if you want to remember it, you can write a straight forward ln. Sometime certain thing have to be remember, I tell you because the time is short.

We will try to read problems in the examination were very very less amount of thing you have to remember almost but sometimes what these problems again to generate this, then this is the conduction resistance twice pi L K1. I tell you K1, in Nomenclature is like that K1 is this, in the problem which is the thermal conductivity of the cylinder material + the convective resistance.

When the pipe, is the bare pipe without any insulation then it is h*twice pi r1, r2 very good, twice pi r2, tell loudly, twice pi r2*L. Very good. Okay. Now we have the insulating material, this is the insulating material with the thermal conductivity K2 and we consider which we have to find out the, we take this radius to be r2, that means insulating material is provided of to a radius of r3 and we have to find out this r2

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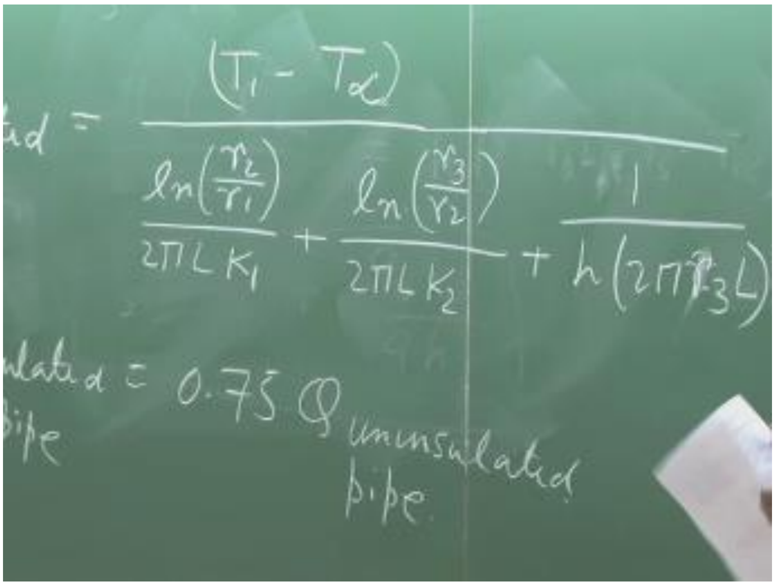

What is the Q insulated pipe = their subjective to same potential difference because environment remains same and the inside surface remains same, Q1-T infinity, very simple problem, heat transfer until and unless you come to convection, heat transfer things are very simple as far as this your under-graduate level material is concerned, how about conduction contents, high level mathematics after watts 3-dimensional conduction unsteady state.

T1-T infinity. Now in this case, there are 2 conduction raise them, one due to cylinder r2/r1, cylinder material twice pi L K1, another one ln due to, this is also a solid material conduction through this thick insulation which is extending from r2 to r3, so therefore it is ln r3/r2 and here it is twice pi L K2 and finally the convection resistance 1/h twice pi R3*L. Okay. 1/h twice pi R3*L, I am sorry, R3 small r. Okay.

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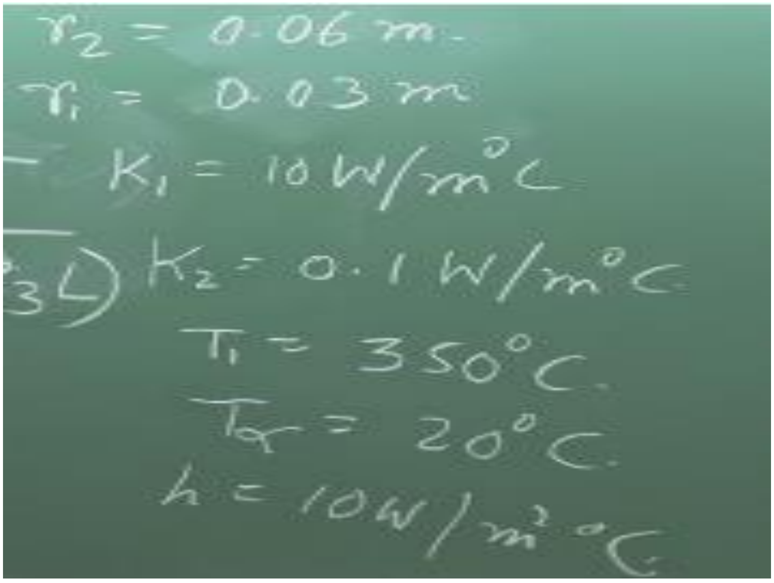

Now according to the problem Q insulated 0.75 Q uninsulated, (Q insulated pipe). So, if you make this, that this = to 3/4th of this Q1 uninsulated T1-T infinity will cancel then you can write this after putting, substituting the values, now I am writing the values here T1, what are value? r2 is; what is that 60mm that means 0.06m, r1 is 30mm, what is r1? 30mm, 60mm 0.6; I am sorry., 6m and this is 0.3m. 0.6 or 0.06, are you., 06.

It was all right, 0.06 and 0.03m, 0.06 as I am writing the value 0.06m, 60mm / 10 to the power of 3,0.38m, what is K1? K1 is 10watt/m degree Celsius and K2 is what?0.1watt/m degree Celsius, what is T1? T1 is 350 degree Celsius. What is T infinity? T infinity is 20 degree Celsius but incidentally this T1 and T infinity will not be require to solve the problem, but the problem gives as like that and h which is very very important which is 10watt/m square that is its units.

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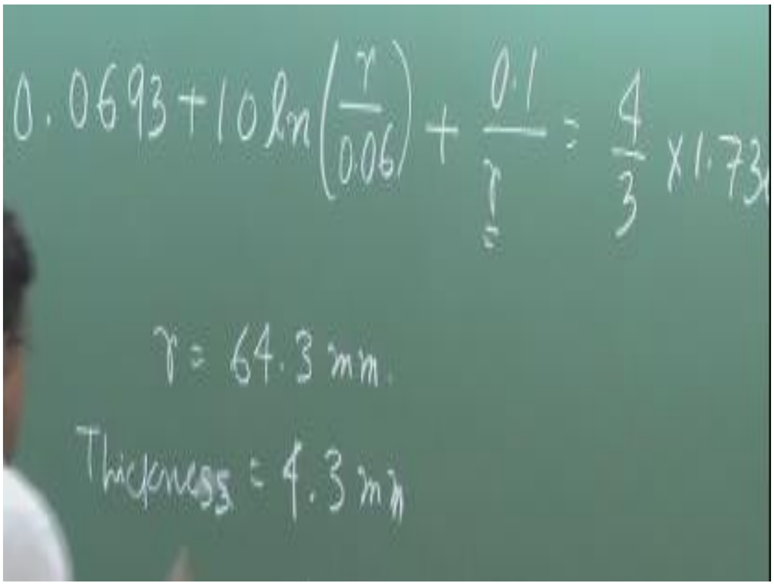

At the same time, this is given that Q insulated is 0.75. Now if you put these value, finally you have a relationship like this, this denominator is totally calculated, because r2 L K1 everything I know, so this becomes a finally a shape like this. I am telling the final step 0.0693+10 after some simplification, it is only a simple algebra ln(r/0.06) +0.1/r=4/3*1.736, actually this 1.736 is these value.

I can tell you that, 1/2 pi L *, that I have give you a hint, that these value if you calculate, actually 2 pi L will cancel from both the sides,1.76. So, these denominators, these resistance is 1/2 pi L*1.76, so this is a tedious calculation. So, after writing this, Q insulated 0.75 this and if you write this is = 0.75, then cancel it, 80% credit is there but do not take these advantage. This is because I am telling you, it is not only the marks that matters.

I can give you marks but you have to develop your ability of completing the work, finally when you will go to the shop or actual field, then you have to deliver things, deliver the final products so if you say sir these. Because at certain stage you cannot tell okay my assistance will do it. If you grow, go up then it will be possible but at some stage you have to work so definitely I am telling both the things.

I am not telling that final answer is be the only thing that you will get marks otherwise you will get 0, No. You may get 80% credit, if you just write this, even up to this, even you do not solve it but this type of problem may not come where the iterative solution is there. So, there is an iterative solution, because r this fashion. So, r comes out to be 64.3mm, that means the t insulation thickness=4.3mm.

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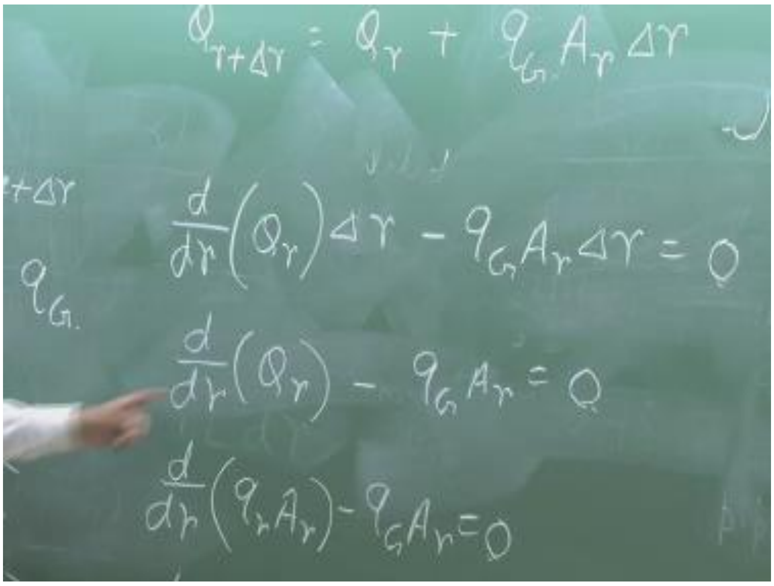

Now my intention to tell it that, up to this okay, you can get 80% credit but you have to complete the work, that will help you in developing this T2 complete a thing. Okay. So, this is the simple problem just application of this principle of critical thickness of insulation. Now I will discuss about this steady one-dimensional heat transfer through spherical geometry. Spherical wall.

Now discussing different types of geometry gives you additional information at the same time blushing up again and again the fundamental equations which we are using, so this will built be in your memory. Now spherical geometry, the most simple example is the simple spherical shell, just like this a reactor, spherical reactor, you consider a spherical reactor. All these deductions are also the problem.

Spherical reactor which is spherical in nature with radius r1 and r2. Now the same thing that is why I am telling that recapitulating; how to write the heat balance here again? you take the cell, spherical shell, only difference is that it is spherical in nature, in case of cylinder, it is cylindrical in nature, in case of plane wall, it is plane in nature, that is the only difference, but our problem, tackling the problem is same.

Let us consider at a radius r, we have this shell, which is having a thickness delta r and the same thing that the heat which is coming here Qr and the heat which is going out, this is, here I will not draw this will be overlapping here, for example, Qr this is going form the outer surface of the cell Qr + delta and again, small r + delta r and again, we consider a volumetric heat generation per unit volume defined at each and every point in the spherical cell.

This material, which extends from r1 to r2, this is the final r2. Okay. r1 to r2, this is r2 and we take an element at radius r of delta r and Qr is the heat, total heat coming into this elemental volume and Qr + delta r going out and qG is the volumetric internal thermal energy generation. The same equation again and again Qr+ delta r must be equal to Qr+qG* what is Ar? Ar is the area and delta r.

Always Ar is the area, which is perpendicular to the direction of heat flow, perpendicular to the r direction, that is all and now we expand in Taylor series, Qr+delta, then we get d/dr of, then this will be Qr+d/dr of Qr* delta neglecting the higher rod at tone, that we had done enough in plane and cylindrical geometry that means Qr delta r- qG Ar delta r is 0, again and again you are practice, so d/dr of Qr r direction –qGAr is 0.

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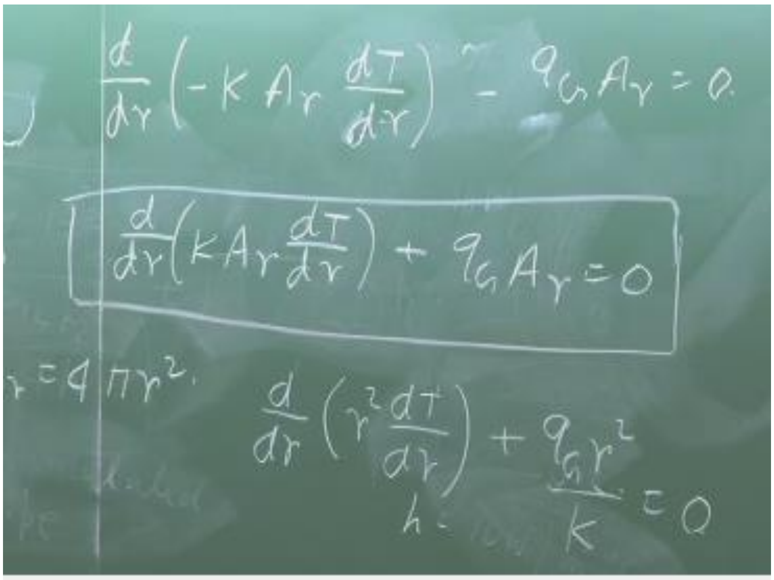

Now Fourier heat conduction equation d/dr of, that is heat flux, heat transfer at any direction is heat flux into the area –qG, Ar is 0, Again and again I am doing the same thing. Now this is not the Fourier heat conduction equation sorry, I am just splitting it heat flux into area, then Fourier heat conduction equation here, heat flux is –K Ar(dT/dr) because heat flux is –K dT/dr.-qG ar is 0, take this – sign out, so that d/dr of K Ar dT/dr + qG*Ar.

Why I am writing this again and again, Prof. Som is writing, this is because if a student can do this thing well, he cannot use problem in 1 dimensional steady state heat conduction, so far up to this equation, this is the most generalised equation, I have not imposed any geometric constants, this is the direction of heat flow. If it is a plane area, then it will be x, Ax, qG Ax, plane area may vary with, normal area may not be vary with normal area.

We have done it earlier. It may be cylindrical area, where r is the direction of heat flow, so therefore if you can remember this equation or you can immediately derive this equation from the conservation of energy, things are over. Now for any condition, if you consider K is independent of, now what or they are generalised, K is temperature dependent, may be Ar may be dependent on r, it is the 1 dimension, direction of heat flow.

qG may be constant, may be a function of r or may be 0, that means there may not be any internal energy generation. So, if you take the most simple case where K is not dependent on temperature, A does not depend on the path of the heat flow, this happens only in the plane area, okay, and there is no heat generation, then it give simple solution that d square dT r square is 0, that means dT/dr is constant.

So, if you can had hide this thing, everything is done, so spherical geometry now it becomes application, Ar is 4 pi r square, put that and take K constant, so what will be the result? 4 pi r square and K constant. This becomes d/dr of r square dT/dr +qG r square /K is 0. Simply, so from here I can go anywhere and in any problem, if everything varies with their independent variable, okay, then it is a problem of little complicated mathematics, nothing else.

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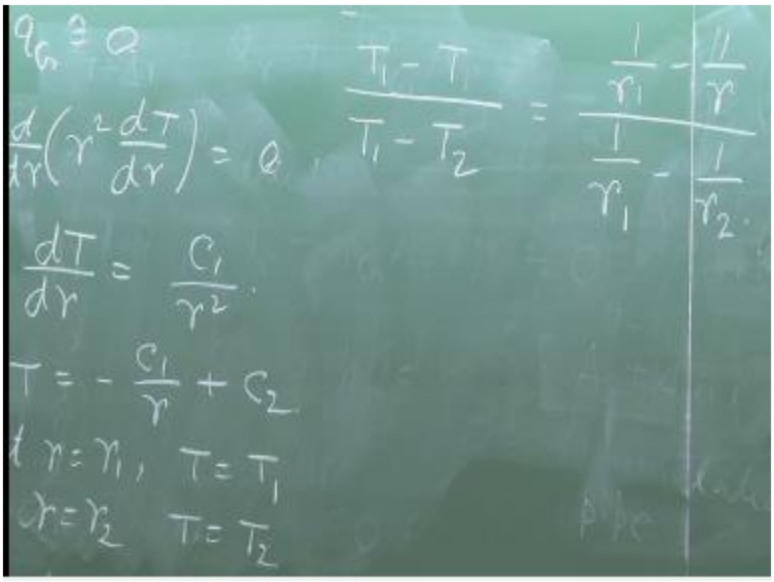

So, now if I solve this for a simple case without heat generation what do I get? Please tell me what do I get? Without the heat generation, do it, this is the problem, tutorial problem type of T. without the heat generation, when qG=0, then d/dr of r square dT/ dr, did that means dT, dr is some constant by r square, first integration. Second one is T is, so if you integrate this – c1r+c2, now it becomes routine and to some extent boring.

But as a student you have to go through this part also, so it is not challenging at this moment at boundary condition r=r1, T=T1 at r=r2, T=T2 and if you do it meticulously then what you get this. If you solve this with the boundary condition, you get the temperature distribution like this, T1-T/T1-T2, straight forward you get, 1/r1-1/r, r is the current radius where the temperature is T, 1/r1-1/r2, so this type of function will come.

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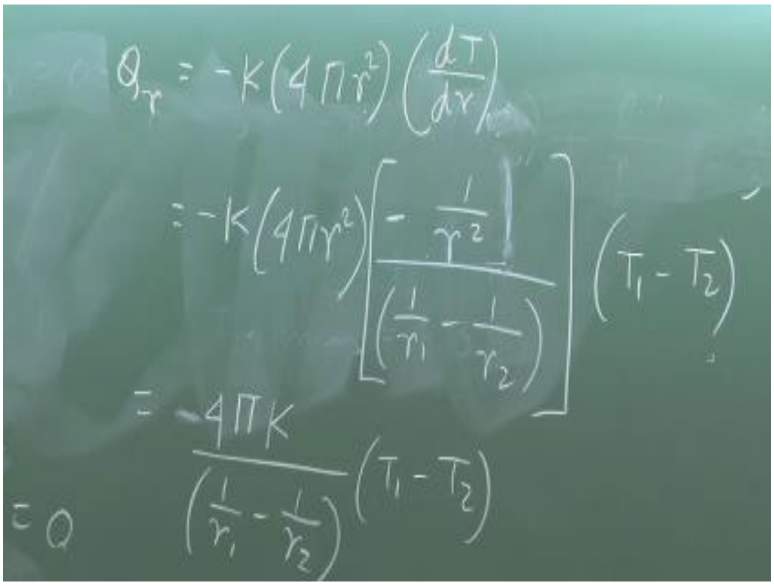

Next part is to find out the heat transfer rate at any radius r, it is =-K*Area, (4 pi r square) *dT/dr at r=, sorry not r, at any r, because I am finding out at any current radius which is Ar, 4 pi r square-K(dT/dr), Now what is dT/dr? 4 pi r square, now dT/dr is, there is a – sign, 1/r1-1/r2, so there will be a – sign, so -, dT/dr is 1/r, -1/r, that it 1/r square, - will be there, sorry, that means these r square and r square will cancel out, 4 pi r square.

dT/dr okay, -4 pi K/ and T1-T2, yes I have done a mistake, T1-T2, meticulously you do, dT/dr is -, this is the – sign, T1-T2*-1/r will be 1/r square, so therefore this -, -, will be, therefore, 4 pi K/ (1/r1-1/r2) * T1-T2, you can check whether you are doing a mistake or not, if you remember the final formula, that it will be again the potential difference, -, divided by a resistance, whose formula is this one r2-r1/4 pi K (r1 r2).

That means this is the conduction resistance, which is ln r2/r1 by twice pi L K, sometimes the memory helps this way, that you can check Oh, this is not coming, somewhere I have done some mistake. Another thing by your concept, when you are finding Q at any cross-sectional area, where the area is the function of r or function of x, that will be cancelled finally with the dT/dr expression, because Q total heat transfer has to be independent of the direction.

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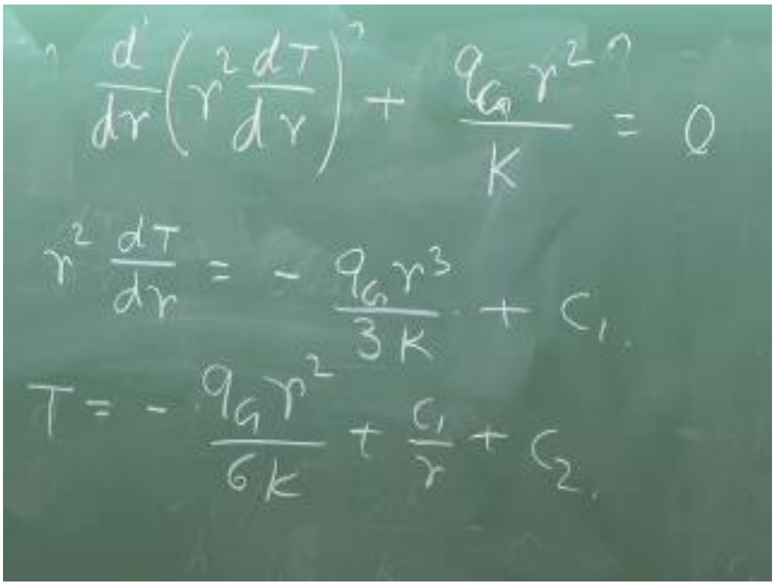

Because at all section same, clear. With heat generation, how to find out with heat generation? To find out the temperature distribution, you have to solve this equation, d/dr of r square (dT/dr) which we arrived earlier, we discarded this term considering qG to be 0, qG r square/K. Now with heat generation, but qG to be uniform, that means constant, volumetric heat generation at any location is independent of the coordinate not dependent on the direction along the heat flow.

That means, qG is constant, not be the function of r. In that case it will be again a very simple integration considering qG constant and you can find out, should I do it or you will be able to find it out? What is the expression you can find it out? that d/dr of r square (dT/dr), that means you take this that side, then you, first integration r square (dT/dr) is –qG r3/3K, Am I right? then dT/dr is qG (r/3K).

Next is + some integration constant will come. The next will be T=-qG, this will be again r, so this will be ultimately r square/6K+c1/r+c2, before writing the boundary condition, now this problem is not a spherical cell, yes, just I am solving as a mathematician, a differential equation I am provoke, tempted to solve it, but what is the practical problem? There is a spherical container packed with material, that means this is a solid spherical container, whose radius is r0.

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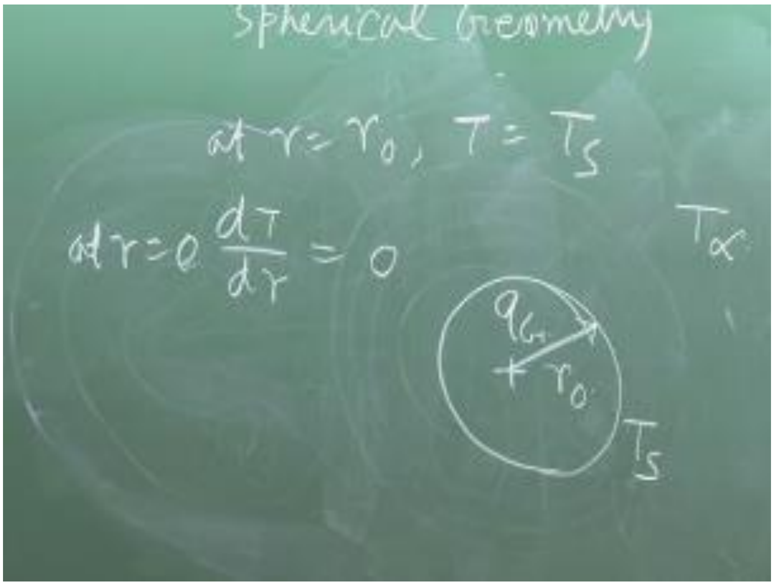

The problem is prescribed that thermal energy is generated at a rate of qG per unit volume which is constant throughout and I have to keep this surface temperature Ts constant, well there is an ambient T infinity which is not very important in determining the temperature distribution, so this is a solid sphere, so the boundary condition requires that at same thing, at r=r0, T=Ts.

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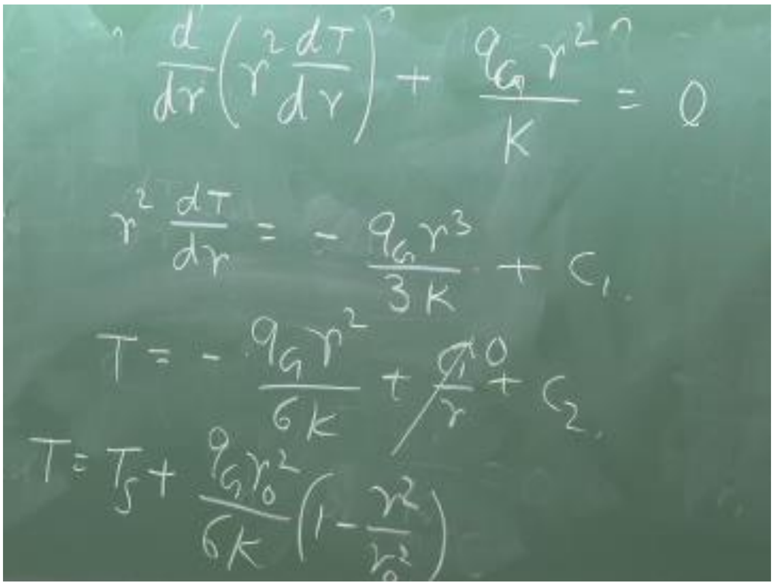

Because of the symmetry dT/dr is 0, at r=0, centre, which means that c1 has to be 0. c1 cannot exist because there should not be any term 1/r, which will give a discontinuity at the centre and moreover dT/dr is 0, at r=0 and first boundary condition Ts gives c2=Ts+qG r 0 square/6K, so therefore we can write the expression as + qG r0 square/6K, this is so simple that is the repetition.

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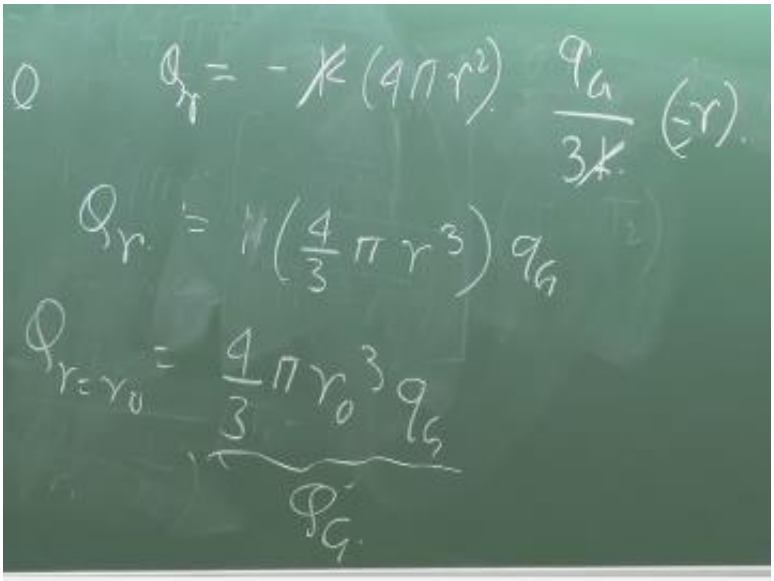

If you find out heat transfer at any section Q, what the expression is?Q= 4 pi r square- K A, again and again doing means, this will be memorised. Q= at any r, -K*4 pi r square, are into dT/dr, what is dT/dr?2 r or r0 square, r0 square will cancel, that means qG/3K, 2r, qG/3K,2r, that means 2, 2 will cancel 3k and this will be*r. that means and there will be a –sign, so –r. So, if you take care of this, it will be K*4/3 pi r cube*qG and this K, K will cancel, sorry.

This K will not come, So, why I have written that? I want to show you here with heat generation at any radial location, heat transfer rate is the function of r, because it is changing, because of the generation of local generation of thermal energy. It is not constant, without qG, it is constant, but that r=r0, this value must match with the total gross energy balance, total heat generation. This is total heat generation.

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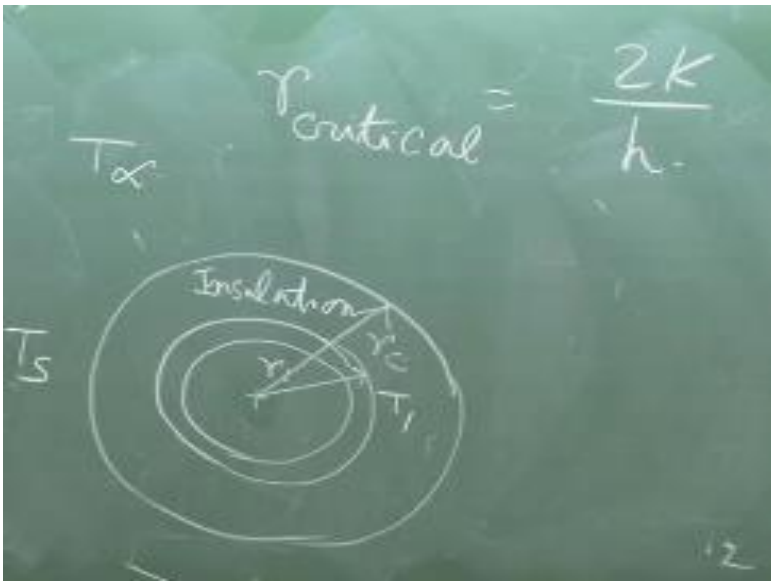

That is all, here also if you add insulating material in the spherical cell, the concept of critical radius of insulation will come and you derive it, it is your task that I will not do, the critical radius of insulation for spherical wall is r critical is 2 K/h, that means if I have a spherical cell or a spherical ball, whatever you call, spherical cell and the problem is prescribe like this, this is r1, the outer radius and heat at a temperature of T1.

Then if I put insulation, where this is the radius rc, the critical radius, the insulation. In a similar way, we derive for a cylindrical coordinate, we can derive the heat transfer rate will maximize, initially increase and then maximize where r critical is 2 K/h, this is the conduction resistance, you will have to consider the convection resistance, that means if this sphere, a spherical cell is subjected to convection.

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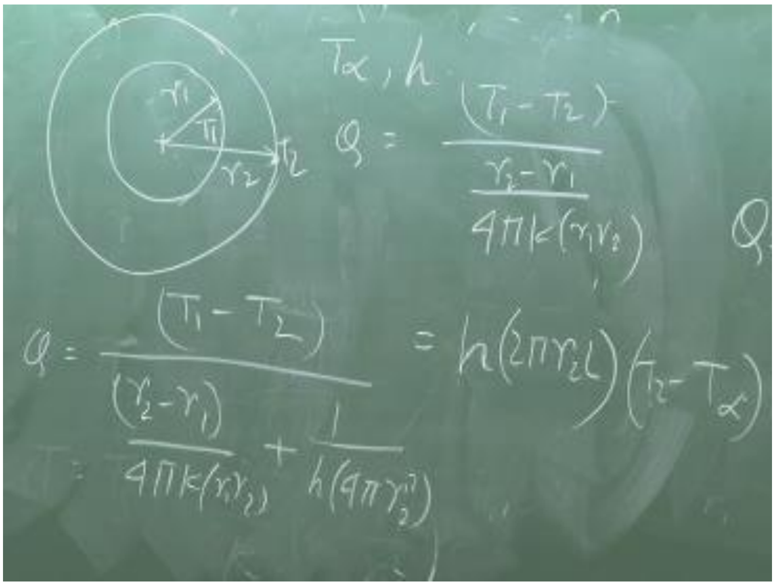

The temperature is given of the ambient, I am giving you hints that how to solve this problem, so this is the conduction resistance, but you have to give, in the convection resistance that if a, before solving this problem, you must have this thing in your mind that if a spherical that like the same cylindrical one. If this is r1, this is r2 and we are provided at a temperature at T1 and T infinity and h, then Q=(T1-T2)/the conduction resistance.

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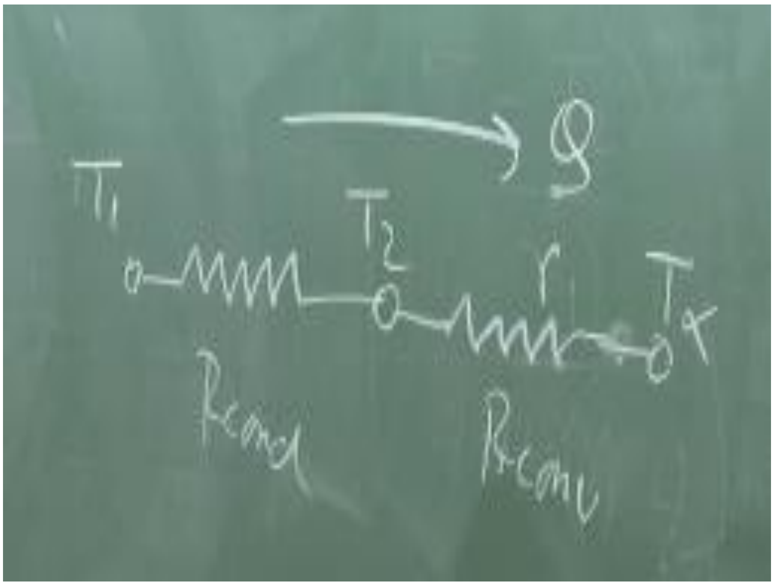

That is r2-r1 as we are generated that is 4 pi K(r1r2) + i/h (4 pi r2 square), this is because heat is, Q is T1-, if I consider this surface temperature is T2, which is not prescribed by, but I can write in these way r2-r1/4 pi K (r1r2) and the same heat is flowing in series under steady state with T2-T infinity1/h, rather it is better to write this fashion, it will be understandable by the definition of h, h* h*(2 pi r2 L) *T2-T infinity T2-T infinity.

That means again it is a series problem that means series network is like this, conduction resistance similar to that cylindrical and convection resistance T1, T2, T infinity, so this is R conduction, this is R convection this term, so therefore this is R, S this series is the similar to that of cylindrical geometry and then you have to take care of and then differentiate to find out these, critical radius, very good, T2-T infinity, correct anything okay, any question? any question? okay all right.