Introduction to Fluid Mechanics and Fluid Engineering
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Lecture – 58
Compressible Flows (Contd.)

In our previous lecture, we were discussing about the converging-diverging nozzles and let us workout an illustrative example on that.

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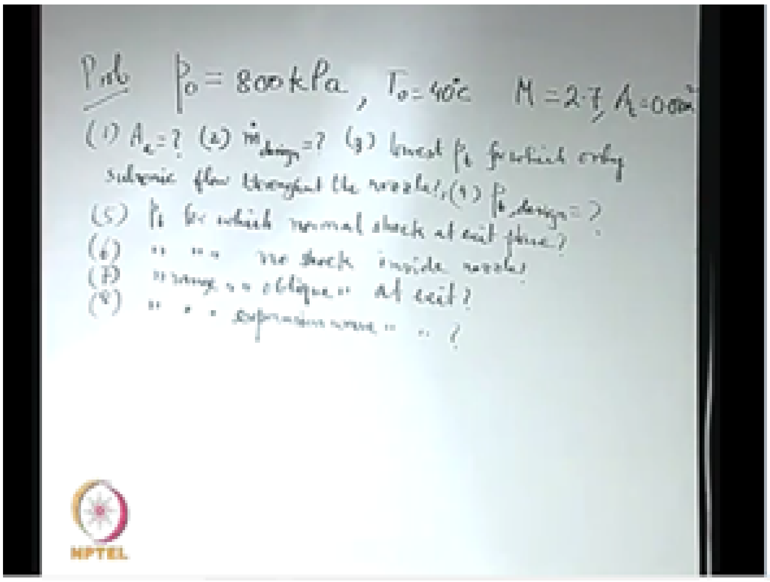

The problem is as follows; a convergent-divergent nozzle is designed to expand air from a chamber in which P0=800 kPa which is the stagnation pressure. T0=40 degree centigrade to give the Mach number equal to 2.7 at the exit. The area of the throat is 0.08-meter square this is the given data.

So the following questions are to be answered:

1. What is the area of the exit?
2. What is the designed mass flow rate?
3. What is the lowest back pressure for which there is only subsonic flow throughout the nozzle?
4. What is the designed back pressure?

5. What is the back pressure for which there is normal shock at the exit plane?
6. What is the back pressure for which there is no shock inside the nozzle?

7. What is the back pressure range for which there is oblique shock at the exit plane, oblique shock at exit?
8. What is the back pressure range for which there is expansion wave at the exit?
So this is the set of question that we would like to answer. So let us look into this one by one.
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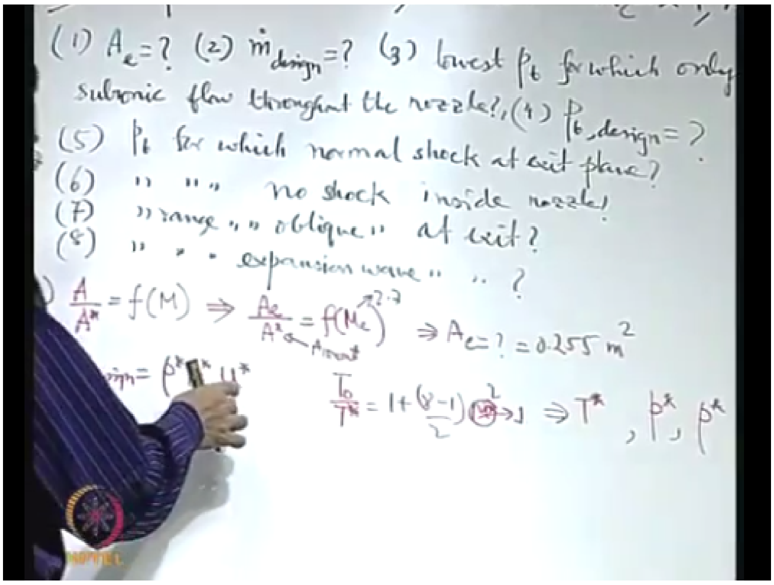

First of all, what is the area of the exit. So to know that if we refer to A/A star, we know that A/A star is the function of Mach number only. This we have derived earlier, so based on that we can find out what is Ae because the Mach number at the exit is given which is 2.7. A star is nothing but area of the throat because the sonic condition here is reached at the throat so from here we can get what is the value of Ae which is 0.255-meter square for this problem.

Second point, what is the designed mass flow rate? Remember the designed mass flow rate corresponds to the star condition that is M dot design is nothing but Rho star A star * u star. So individually we have to calculate Rho star, A star, u star, so how do we calculate Rho star? May be we may calculate T star or P star, so to calculate T star let us use any of the isentropic relationships that is T/T star or says T0/T = 1+ gamma-1/2 * M2, that we know.

So we can find out what is T star by nothing that when the star condition is there it is M star and M star = 1, so putting that value we may find out what is T star. Similarly, we may find out P star, Rho star, et cetera., by using the isentropic equation of change. Now how do you calculate u star? remember that u = M * C which is square root of gamma T, so u star = M star square root of gamma T star with M star = 1.

So if you know T star you can get u star and A star is nothing but equal to A throat. So from all these considerations by substituting the values you can calculate what is the M dot design which for these case will come out to be 146 kg per second. Third part, what is the lowest back pressure for which there is only subsonic flow throughout the nozzle?

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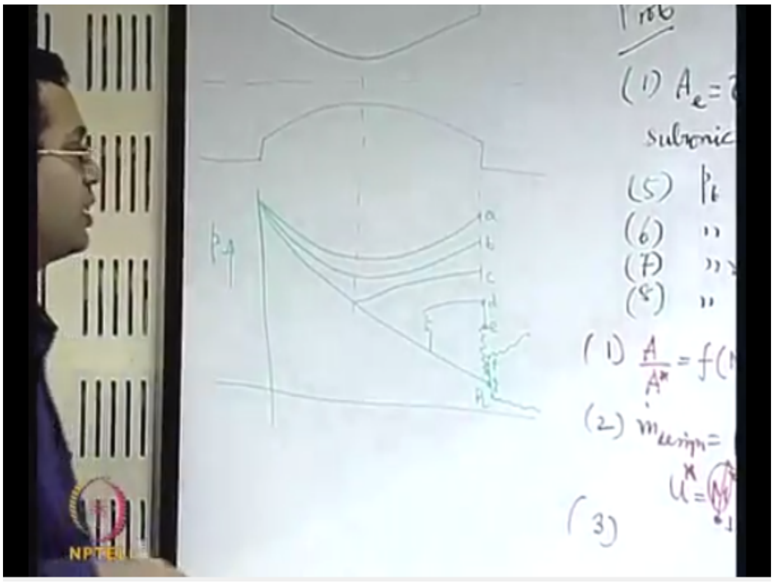

So if you look at this pressure profile for different back pressures we can see that the back pressure corresponding to the point C shown in the diagram is the lowest back pressure for which there is only subsonic flow throughout the nozzle. So how do you calculate that one? You know, P/P0 is the function of the Mach number and you know that you may also write it as A/A star as some another function of Mach number, say if 1 Mach number.

So when you know that what is the area of the exit Ae/A star, Ae we have already evaluated, it is the function of Mach number at the exit, this has 2 isentropic solutions. One is corresponding to the point c; another is corresponding to the point g shown in the figure. Out of these 2 one is the subsonic solution which corresponds to c, another supersonic solution corresponding to g.

So we get the subsonic solution from this say you can get Me subsonic. Remember that 2.7 is the supersonic solution, so we can get a subsonic solution also from the A/A star by referring to the isentropic tables, so once you get that one you may plug it back here to get P as a function of P0 so that P will be the lowest back pressure for which the subsonic flow exists throughout the nozzle.

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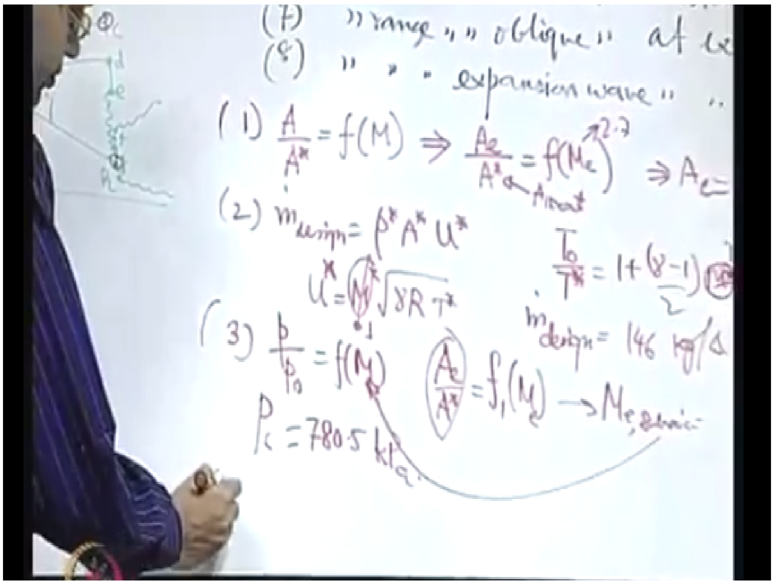

So we can find out this one to be equal to that is the pressure at this point C as 780.5 kilo pascal. Part 4, what is the design back pressure? The design back pressure you may calculate exactly in the same way as part 3, but you replace the subsonic solution with the supersonic solution, so here M corresponds to the supersonic solution that is 2.7 that is the back pressure that is pressure at the point g in the figure by P0.

So from that you can get what is the pressure at the point g which is the design pressure and which may be evaluated as 34.4 kilopascal. Next let us refer to the part 5, what is the back pressure for which there is normal shock at the exit plane? So normal shock at the exit plane means we are referring to a condition which is like the back pressure corresponding to E. So just upstream of the shock, it was expanding like an isentropic flow so the Mach number was 2.7.

What is the Mach number at the downstream of the shock? You know what the relationship between M2/M1 for the shock. So from the value of M1 you can get what is the value of M2 which is just at the downstream of the shock. So once you know that what is the value of M2 you also know what is the value of P2/P1 shock as a function of M2 and M1.

So if you know the value of M1 and M2 which you know from the previous step you can find out the ratio of the pressure downstream and upstream of the shock and from there you can find out the pressure at e because you know what is the pressure at g. So P2/P1 is as good as Pe/Pg in the figure. So from here you can find out what is Pe. So that value of P for this problem the answer will be 286.5 kilo pascal.

So we know the pressures at points c, e and g. These we have already evaluated and the remaining 3 parts, the answers to the remaining 3 parts of the problem may be given on the basis of these. Part number 6, what is the back pressure for which there is no shock inside the nozzle? So we can see that the shock occurs outside the nozzle when the back pressure falls below the pressure of the point e. So when P is < Pe then there is no shock inside the nozzle.

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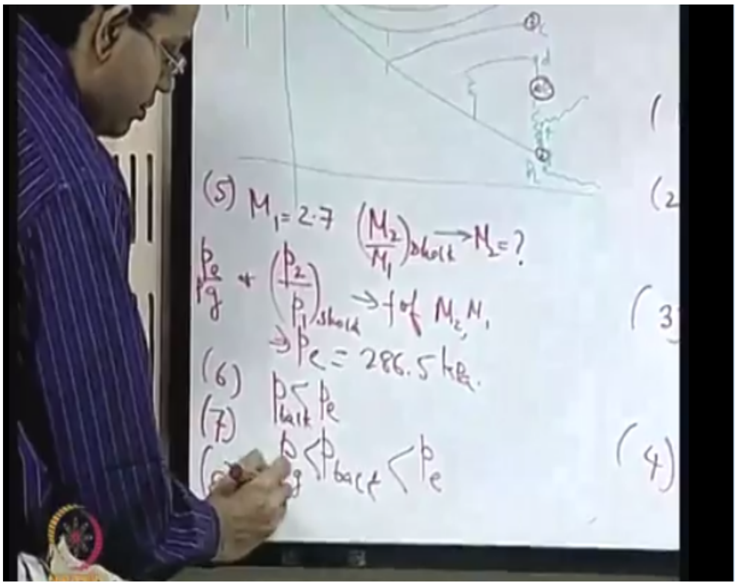

Part 7, what is the back pressure range for oblique shock at the exit, so you can see that if the back pressure is between the points e and g marked in the figure, there will be oblique shock inside the nozzle. So the back pressure has to be between Pg and Pe. And part 8, the back pressure range for expansion wave at the exit when the back pressure falls below the design pressure that is when P back is < Pg, for example the point h as shown in the figure you will get expansion wave at the exit.

So depending on the back pressure which is there you may have different interesting phenomenon at the exit and within the nozzle itself. Now till this time whatever we have discussed on the compressible flows through ducts or nozzles we have assumed that the flow is isentropic. In other words, we have assumed that the flow is reversible and adiabatic.

However, it is not always true that the flow will be reversible and adiabatic, in reality hardly there is a case when the flow is reversible and adiabatic. Therefore, we need to consider situations when more general types of flow occur. So the generalization may be there with sudden possibilities, one is instead of the flow being adiabatic it may be a flow with heat transfer at the wall or and because of friction the flow becomes irreversible and that means that you could consider a case when the flow is irreversible and non-adiabatic.

In this particular course we do not have the possibility or scope of discussing all the detailed aspects of irreversible and non-adiabatic flows together so we will consider a specific example where we have a deviation from the isentropic flow in the sense that the flow is adiabatic, but irreversible because of friction in the flow. So we will consider the adiabatic flow with friction.

And for simplicity we will consider the flow in a constant area duct. So schematically the situation is like this, you have a duct of say area A and some compressible flow is occurring through this duct. The flow direction is along x and let us write the basic governing equations by assuming a one dimensional flow for such a case.

So the first basic equation is the continuity equation so what we get from the continuity equation Rho * A * u that is equal to constant, so Rho*A*u = constant which means remember here A is also a constant because it is a constant area duct so it boils down to Rho * u = constant. So we may take log of the expression and then differentiate to get d Rho/Rho + du/u = 0, let us say it is equation number 1.

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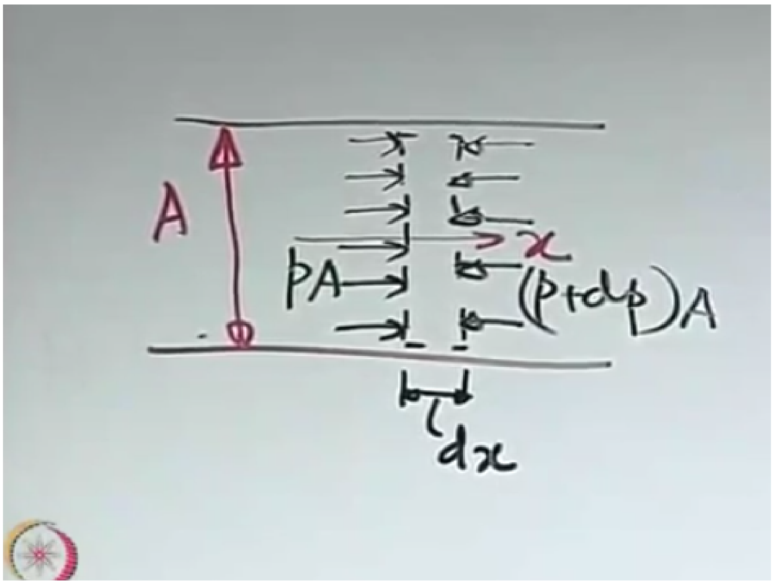

The next fundamental equation that we may think of is momentum conservation. So let us try to write the momentum conservation equation, to do that let us consider a control volume of width dx. The pressure acting on one side of the control volume is P so the force is P * A, pressure acting on the other side of the control volume is P + dp so the force is P + dp * A.

The velocities on the 2 sides of the control volume, so on one side the velocity is U, let us say on the other side the velocity is u + du. The other forces that act on the control volume are the frictional forces because of the wall shear stress. So if Tau wall is the wall shear stress, it is the Tau wall times the area on which it acts as the total shear force. What is the area on which it acts? let us say it is a circular duct as an example.

So if you consider this as a circular duct, the area on which it acts is 2 Pi r * dx. What is 2 Pi r? 2 Pi r is the perimeter of cross section, so in general we can say that it is Tau wall * P * dx. Where P is the perimeter of cross section of the duct. So we can write a momentum conservation principle by using the momentum QM as a resultant force as P * A – P + dp * A – Tau wall * P * dx, this is nothing but equal to M dot * u + du – u. So M dot * u + du –u and M dot is Rho * A x u, so Rho A u du.

So from this it follows that if you divide all the terms by A you have dp + Tau wall * P/Adx + Rho u du = 0. To write it in a more compact form we may divide all the terms by Rho u2 so it becomes dp/Rho u2 + Tau wall/Rho u2 * P/Adx + du /u = 0. Let us say this is equation number 2. The next equation we may write let us consider the energy conservation, so for that we are basically writing the first law of thermodynamics for a steady flow process.

There is no heat transfer because it is an adiabatic flow given so we have h + u2/2 = constant. So we can write dh + u du =0 remember that for an ideal gas dh = CpdT + udu = 0. Again we may write it in a compact form by dividing by u2 so you have CpdT. Let us call this as equation number 3.

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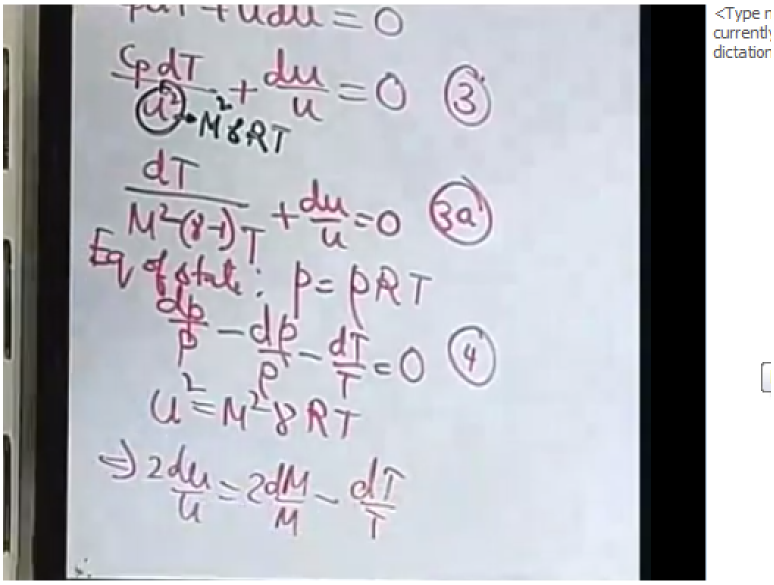

We will have a further simplification of equation number 3 or maybe you will call the next step as equation number 3 so CpdT/u2 +du/u = 0 by dividing by u2. Remember that u2 is nothing but M2 x gamma RT and accordingly we may write in the next step that using CP/R is gamma by gamma -1 so this becomes dT/m2 * gamma -1 in T + du/u = 0, let us say this is equation number 3a.

So this is the energy equation, what other basic equations we have, we have equation of state, P = Rho RT so we may again take log of both sides and differentiate to get dp/p – d Rho/Rho - dT/T = 0, let us say it is equation number 4. And the 5th equation that we may get is the relationship between the sonic speed and the Mach number, so u2 = m2 gamma RT. This is the property relationship, so from here we may again take log of both sides and differentiate to get 2 du/u = 2 dM/M + dT/T, this is equation number 5.

Now we have from equation 1 through equation 5, five independent equations and these equations have their (()) (23:25) you have dp, d Rho, dT, dM and du, so by eliminating variables it is possible to obtain expressions of each of these variables in terms of the other and when we say in terms of the other one of the objectives will be to express these in terms of the Mach number.

To do that let us just eliminate certain variables, for example we may eliminate d Rho from equation number 1 which is the continuity equation and equation number 4 which is the equation of state. So from equation 1 and 4 we can write that dp/p – d Rho/Rho is + du/u - dT/T = 0, say this is equation number 6.

So this is one equation also you may write du/u in terms of dM/M and dT/T that is also possible and you may write du/u in terms of dT/T so you may use equation number 3 and 5 to express du/u in terms of the other variables. So let us use that 3 and 5. So in the previous step we eliminated d Rho/Rho, in this step we will eliminate du/u, so we can write here CpdT/M2 * gamma RT + du/u, dU/U is dM/M + 1/2 dT/T = 0, this is from equations 3 and 5 by eliminating du/u.

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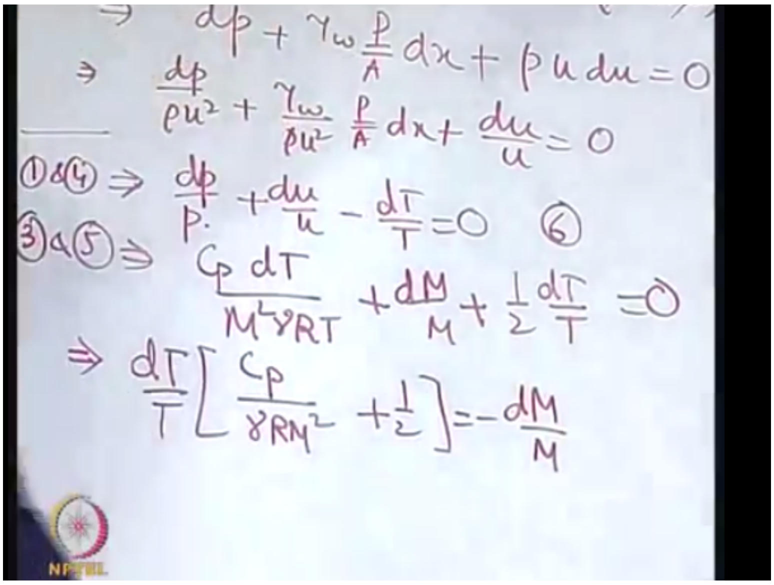

So from this expression it is possible to express dM/M as a sole function of dT/T involving the Mach number or dT/T as a sole function of dM/M as a function of Mach number. So if you take dT/T common we have Cp/gamma RM2 then + 1/2 = - dM/M, so this mean that we have dT/T = - dM/M/Cp/gamma RM2 + 1/2. Let us give it an equation number 7.

Without going too much into the algebra we may have a very important observation which we will note. See, the denominator of this expression is always positive because M2, R, gamma, Cp all these are positive, so from these we may conclude that if dM/M is positive then dT/T is negative that means if M increases then T will decrease and if M decreases then T will increase.

This is a very important observation that we get from here and remember this observation till now whatever we have made is independent of whether it is having a friction or whether it is having no friction because till now we have not yet utilized the fact that it is a flow with friction and if you observe very carefully we will see that the only place where it has been utilized that it is a flow with friction is the turn in the box mentioned in the equation number 2 that is there.

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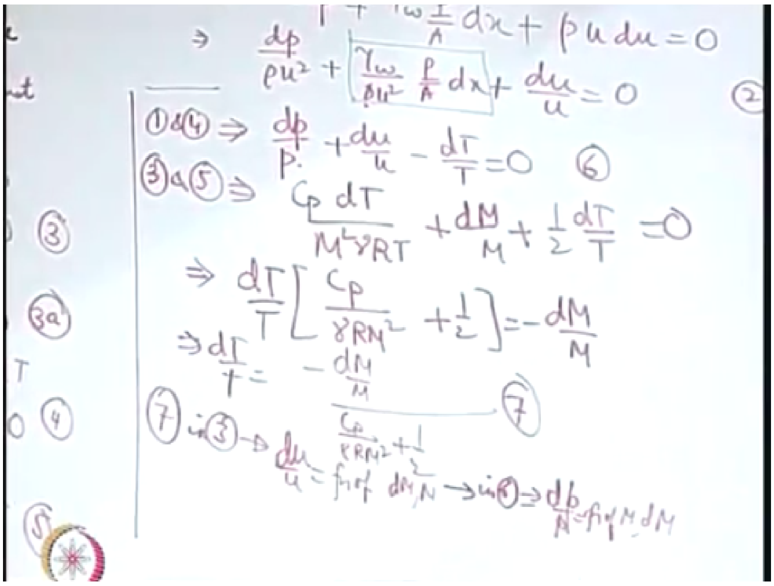

Now when you have this dT/T from here you may get other expressions also in terms of dM/M. For example, if you refer to equation 3 now, so equations 3 and 7, using equation 7 in equation 3 you may get du/u as a function of dM and N. Once you get dU/U as a function of dM/M it is possible to get other parameters for example dP/P as a function of dM/M by referring to equation 6, so now if you use this in 6 we will get dP/P as function of M and dM.

Similarly, if you now substitute that in equation number 4 which is the equation of state you can get d Rho/Rho as a function of d M/N so that you substitute in 4 to get d Rho/Rho as function of M and dM, that means we have been able to express in principle dT/T dP/P du/u and d Rho/Rho as a function of dM/M, but that does not allow us to calculate the change in pressure, change in temperature, change in velocity along the length of the duct because we still do not know how M varies with X.

So how will we know how M varies with X for that we refer to the equation number 2 which is the momentum equation so if we refer to the equation number 2 what we will get, so you may substitute that dP/Rho both in terms of the Mach number then du/u as a function of Mach number and Tau wall by Rho u2 in terms of the friction factor which depends on the Reynolds number of flow.

So we know that how friction factor is related to the Reynolds number and the surface roughness by the considerations of viscous flow as we discussed for flow through pipes and ducts in one of our previous chapter. So when the variations of all the variables expressed in terms of the Mach number and it is differential is substituted in equation number 2, it gives a governing equation for the variation of Mach number and from that we may obtain a Mach number as a function of x.

Therefore, the relative dependences of the variables temperature, velocity, density, pressure, all depend on the Mach number and that nature of the dependence is not dependent on whether the flow has friction or not, but how Mach number varies with X very much depends on the friction in the flow by virtue of equation number 2 and therefore that Mach number as the function of X when it is substituted in different relationships.

There will be differences in results for flow with frictional flow without friction because the Mach number for a given X will be different for flow with friction and flow without friction, but in terms of the Mach number the dependences appear to be independent of friction, but it is dependent on friction implicitly because Mach number as the function of x is dependent on extent of friction in the flow.

So this is about that how we can get an estimate of the variation of the temperature with Mach number, the pressure with Mach number, the density with Mach number and so on. Now next thing is about the directionality of the process in this types of ducts. So if you have such a duct the processes which take place inside these duct, these processes are in general adiabatic but these processes are processes with friction.

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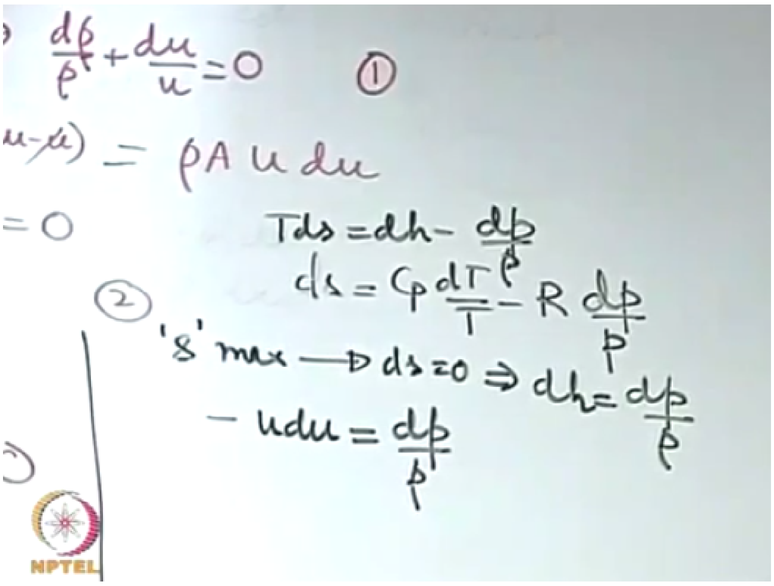

So if you recall that if you want to calculate or if you want to identify the direction of a process we have to see that what is the direction in which the net change in entropy of the system plus surrounding is positive. So if we calculate the change in entropy of the system so $ds = Cp\, dT/T – R\, dp/p$ for an ideal gas or if we consider its form in the previous step we have $Tds = dh - dp/Rho$ where substitution of the ideal gas equation has given rise to the simplified step.

Now if we want to identify a state where S is maximum, say we want to get a maximum entropy state. So S is maximum that will imply the $dS = 0$ that means $dh = dp/Rho$. Also from the energy equation we have $dh + udu = 0$. Therefore, in place of dh we can write $–udu = dp/Rho$. This is why substituting the energy equation. Next, we may substitute the d Rho/Rho in terms of du/u or du/u in terms of d Rho/Rho from equation number 1 by using the continuity equation.

So in place of du we may write – u d Rho/Rho, so this is – u in place of du – u d Rho/Rho = dp/Rho this is from equation number 1. So from this what follows is that the Rho gets cancelled out so you get u2 = dp d Rho. Remember that dp/d Rho is also equal to C2, the square of the sonic speed and therefore at the maximum entropic condition u2 = C2 that means M2 = 1 or M = 1, so this is a very important observation that at the maximum entropic condition you have the Mach number = 1.

You could express the maximum entropic condition in different ways so if you want to evaluate actually what is the change in entropy as a function of temperature you have to basically integrate the expression of ds, you have one dT/T, you have another dP/P so it is important that you eliminate dp/p or write express dp/p in terms of the other parameters so if you refer to equation number 6.

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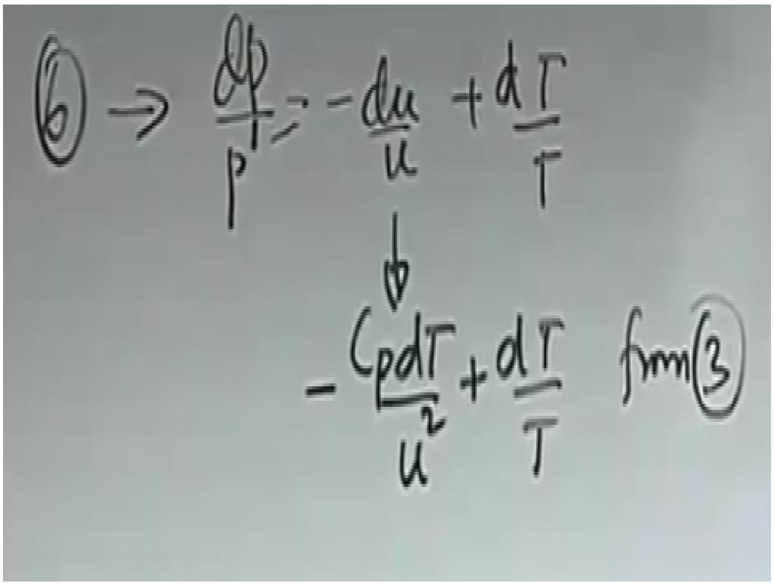

Equation number 6 will give you dp/p = - du/u + dT/T and du/u in terms of dT/T you can get from equation number 3, so in place of du/u we will be writing minus of CpdT/u2 + dT/T this is from equation 3. Now it is possible to write again u2 as M2 * gamma RT but again M as an unknown will appear so to keep it explicitly as a function of T only, the temperature as a variable without bringing Mach number in the manipulation what we will do is, we will just refer to the definition of the stagnation temperature.

So remember that if you have h0 as the stagnation enthalpy then you will have h + u2/2 = h0 in terms of Cp it becomes CpT + u2/2 = CpT0 so in place of u2 you can write 2 * Cp * T0-T so that we may substitute here to get dP/P so CP/u2 will become 1/2 dT/T-T0 so if you look at this equation Cp/u2 will be one half * T0 - T that minus sign adjusted it will become T - T0 + dT/T.

So let us call it as maybe equation number 8, so if we substitute this equation number 8 into the change in entropy equation, let us see what we get out of that. So we get from the change in entropy equation that ds = CpdT/T - R/2 dT/T - T0 –R dT/T. Noting that Cp-R = cb so this become cb * dT/T – R/2 dT/T - T0 this is ds. So we may integrate it from a state say 1 to the given state so we can write S - S1= CV ln T/T1 - R/2 ln T - T0/T1 - T0 so this means that we are able to write S - S1 as a sole function of T T0 and T1.

So with this background let us try to make up sketch of the temperature versus entropy diagram for this kind of a place. So if you make a plot you will see that first of all we have already seen that there is a maximum in the S so there will be maximum in the S and the curve looks like this so if you identify these point at which you have the maximum in the S we know that the value of the Mach number here is equal to 1 that is what we have shown.

Now clearly if you see we need to find a directionality of the processes for the curve which is in the upper part of M = 1 and for the curve which is the lower part of M = 1 remember when we say that total change in entropy it is dS system plus dS surrounding, here dS surrounding is 0 because it is an adiabatic process. Because it is an adiabatic process you do not have any change in entropy because of heat transfer with the surroundings.

So the change in entropy is only due to the change in entropy of the system. So when the change in entropy is taking place, it is taking place in a direction such that the entropy is increasing so when the entropy is increasing if you consider the upper curve it is moving towards M = 1 when you consider the lower curve it is also moving towards M = 1.

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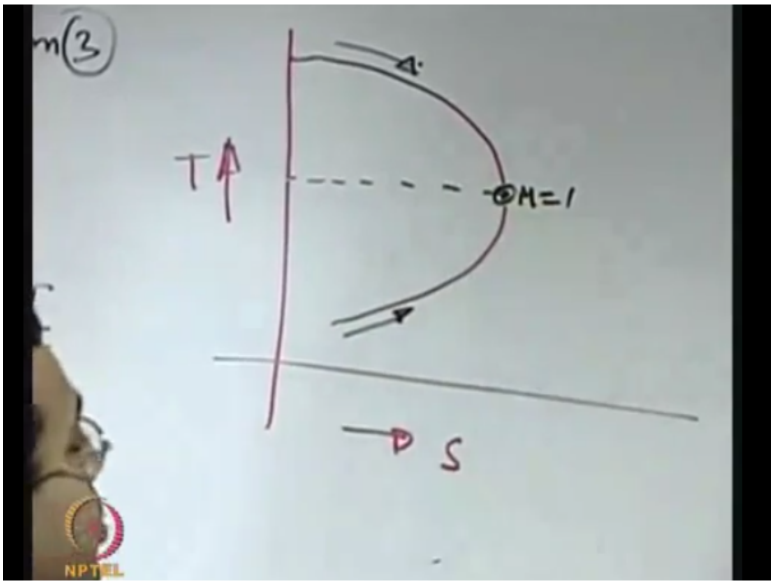

Now what are the differences when you are having the process which is going towards the maximum entropy which is the natural spontaneous way by which a process may thermodynamically take place, but there are 2 different parts by which one way reach that process for this case. For the part above if you look into it very carefully we will see that as it is moving towards M = 1 it is temperature is decreasing so for the upper part of the curve since the temperature is decreasing the Mach number should be increasing.

Now refer to this conclusion that we had from equation number 7. The relationship between the change in Mach number and the change in temperature which is this equation number 7. So from this equation number 7 which is identified here we have earlier concluded that if you have a positive dM/M we will have a negative dT/T so if you have a negative dT/T it must be a positive dM/M that means along this path the Mach number is increasing because the temperature is decreasing.

So that means you have M < 1 here so if the Mach number has to go to 1 and if it has to increase along that path that means along it is path Mach number must be < 1 so it is going increasing and becoming = 1. If you consider the lower part of the curve if you have the temperature increasing from the same logic, we can say that the Mach number is decreasing.

So since the Mach number is decreasing it must be M > 1 here. So the upper part is M < 1, it is temperature is decreasing that means the density is decreasing, the pressure is decreasing whereas the lower part M is > 1, the pressure is increasing, the density is increasing and temperature is increasing so on

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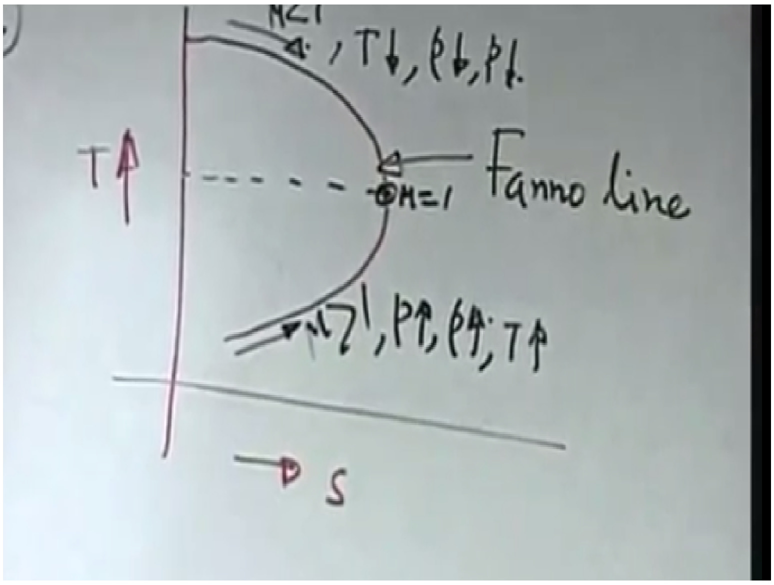

So the upper part corresponds to a subsonic flow and the lower part corresponds to a supersonic flow and these parts go through a point of maximum entropy where M=1, so this line in the TS diagram is known as Fanno line. Which is a very interesting line because it shows the locus of all points in the TS diagram corresponding to $M > 1$ and $M < 1$.

And you can see that if the initial state is subsonic that is $M < 1$, the process tends to go towards the state where $M = 1$ which is the maximum entropic condition where as if M is > 1 from that also the process tends to go towards $M=1$ which corresponds to the maximum entropic condition. So if you have a duct of constant area and if you have friction in the duct then there are 2 possibilities of the inlet flow, say one is the subsonic flow, another is the supersonic flow.

So if you have a subsonic flow at the inlet then the Mach number will be going on increasing till it covers the length that is necessary to achieve a Mach number equal to 1 and that depends on the friction in the flow because we have seen that the variation of Mach number depends on the wall shear stress because if we substitute all the variables, pressure, density, velocity, all these variables in terms of Mach number in the momentum equation then you can see that the momentum equation gives the Mach number variation in terms of the wall shear stress.

Therefore, the wall shear stress dictates strongly that how the Mach number will vary with X so it requires a threshold X for the Mach number to become 1 from an initially subsonic state. If the length of the duct is less than that threshold length, then obviously Mach number equal to 1 or the sonic condition will not be achieved.

On the other hand, if the length of the duct is greater than that threshold length for achieving Mach number equal to 1 for the given frictional condition then obviously what will happen then the flow will be choked at critical length and that will give the maximum flow rate. On the other hand, if the Mach number is > 1 at the inlet then if it has the threshold length for which the Mach number may become 1 then at certain length it will have a Mach number equal to 1.

And from a supersonic to that critical or the sonic state it may come if the length is short enough then there may be shock close to the exit of the channel or exit of the duct if the length is very large then it may be seen that relative to the length of the duct the location of the shock is moving closer and closer to the inlet for an initially supersonic flow.

So it depends on whether the flow is initially subsonic or supersonic to figure out that what will be the physical change in the flow as the compressible fluid is moving along the duct. For all this cases the variation in the Mach number is important and the variation in the Mach number depends on the variation in the temperature, density, pressure and velocity.

These dependences inform are sort of independent of whether the flow has friction or not, but implicitly depends on friction because how the Mach number itself varies is strongly dependent on the friction which is manifested through the wall shear stress. So from this discussion we may conclude certain interesting things, one is that this flow maybe considered as a more general case of the isentropic flow.

Here if you substitute the wall shear stress equal to 0 all the conclusions that you get should be corresponding to isentropic flow, that means if you substitute the wall shear stress equal to 0 all the expression that you get here will be the expressions corresponding to the isentropic flow and the change in entropy in that case will turn out to be 0, so the non0 change in entropy is the sole consequence of friction in this particular flow.

If you have heat transfer obviously the situation will be algebraically more complicated how algebraically will it be different? Let us figure out that in which equation it will be different. So if you recall our first equation was the continuity equation, it does not sense whether there is heat transfer or not so that equation will remain unchanged. Our second equation was the momentum equation, there friction itself appeared and it is again insensitive to whether there is heat transfer or not.

Our third equation was the energy equation and this is very much dependent on whether there is heat transfer or not. So if there is heat transfer what will happen, if there is heat transfer then in place of H + u2/2 = constant this will be replaced by an energy equation with heat transfers. So this will be like Hi + Ui2/2 + q = He + U2/2, where i and e are the inlet and exit sections and this is the rate of heat transfer per unit mass flow rate.

So this heat transfer term will be the only new term that will feature in the energy equation. For other equations like the equation of state that will also not be altered and the property relationship that give the sonic speed that also will remain unaltered, so the similar analysis will be valid but it will be more complicated because of appearance of a new parameter in terms of the heat transfer.

And therefore one has to separately consider this heat transfer and it is possible to write another differential form of energy equation considering this heat transfer by taking a small control volume and by making an energy balance over the control volume just like the momentum balance of the control volume gives the momentum equation.

Similarly, the energy balance over the control volume with the possibility of heat transfer at the wall will give a new differential form of the energy equation which is different from the case with no heat transfer at the wall, so that will be the only change in terms of in principle, so we will still have these 5 equations with 5 unknowns.

You will have a heat transfer at the wall which you may prescribe and therefore your energy equation will be different because it will now involve the heat transfer so you may manipulate all the equations to come up with another temperature entropy diagram which will involve not only the frictional characteristics but also the heat transfers in the flow and that is the more general case than the case that we have considered here where there is no heat transfer.

And then you can get such lines in the TS diagram known as the Rayleigh line just like the Fanno line that you get here. So more and more general cases, they may be treated in a more and more general way, remember that the treatment that we have discussed in this particular course that is only a one dimensional treatment to give you the essential physics of the problem.

In reality none of these problems are one dimensional, these are all 2 dimensional or in the more general way 3 dimensional problems and one needs to solve the proper governing differential equations in 2 and 3 dimensions to get the flow feel and the density feel and the pressure feel in the compressible fluid flow medium. In the medium where the compressible flow is occurring.

But here as the part of the scope of this particular chapter in this particular course we have only got restricted to one dimensional flow with an understating that how to write the basic equations in terms of approximate one dimensional analysis and how to express the dependent variables in terms of the independent variables and how to figure out that what will be the permissible direction in which different processes may take place under those conditions typically for more general case with friction being present and with heat transfer being present.

With this kind of a background what we believe is that in high level courses if you come across situation where you require the analysis of compressible flows in (()) (55:36) and greater details these type of physical understanding will be of good help for you to begin with typically when you consider the more general partial differential equations by which you may have your analysis and get the results not by such a simple one dimensional form, but through more (()) (56:00) and complicated exercise of solving partial differential equations.

Even in such cases these type of one dimensional treatment will give you a significant amount of physical insight on the variations of different parameters in a compressible flow. With this we will conclude this lecture. Thank you very much.