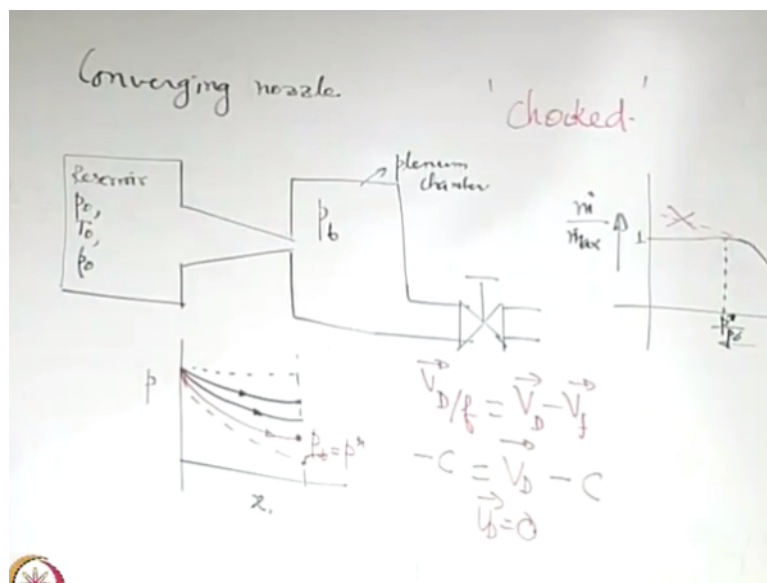


Introduction to Fluid Mechanics and Fluid Engineering
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Lecture – 57
Compressible Flows (Contd.)

Till now, we have discussed the examples of isentropic flows and normal shocks and we will try to utilize this understanding that we have till now to develop a feel of compressible flows through nozzles.

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So, we will first start with example of a converging nozzle, we have seen earlier that one of the important objectives that you have with our nozzle is to accelerate the flow and to do that let us say that you have a tank, you connect a converging nozzle with the tank, we have seen earlier that the nozzle shape always did not be converging and in fact, one could have accelerating flow with a diverging section also.

So, that is a non-intuitive manifestation of the compressible flow, which is not there in an incompressible flow because density variation also is very, very important, it is not just $A * V$ is constant, $\rho * A * V$ is a constant and ρ variation is important. Now, let us say that this is connected to a chamber; this is the reservoir where you have roughly the stagnation conditions prevailing.

Here, you have the nozzle; the converging nozzle this is known as a plenum chamber in which you have air maintained at the desired exit condition; the desired exit condition is not your choice but the desired pressure that is there in the chamber that you may control and there may be a discontinuity between the exit pressure and the pressure that you are having in the plenum chamber.

The pressure that you are having in the plenum chamber is known as a back pressure, so you have here the pressure as P_b which is known as a back pressure. How you control the pressure here is by regulating the flow through a valve, which is located in this pipeline, so if you want to reduce the pressure you may create vacuum by allowing more and more air from these 2 escape.

So, what happens in this kind of a situation, let us try to draw a graph of pressure may be we draw the graph according to the nozzle inlet and the exit. So, we draw a graph of pressure versus x , we will also try to draw a graph of mass flow rate versus the pressure; the back pressure. We are plotting \dot{m}/\dot{m}_{\max} ; \dot{m}_{\max} is the maximum mass flow rate, so we are normalizing the mass flow rate as a function of the P_b/P_0 .

So, we are varying the back pressure, so this is an experiment. Initially, the pressures at the 2 places are the same, now we are creating lower and lower pressure by allowing the air to escape from this pipeline, so that the back pressure falls. So, when the back pressure falls, what will happen? So, let us say that the back pressure you have initially the back pressure falls such that it comes to a level like this, so this is a pressure variation with x .

So, when you have a low back pressure, what will happen; that because of this pressure drop there will be an added mass flow rate, so initially there is; it is quite intuitive you have initially equal pressure, no mass flow rate, now you have a driving pressure gradient. So, you have a lower pressure here, you have a higher pressure here that will drive a mass flow rate. What do you expect?

You expect that as you reduce the back pressure, you will have higher and higher mass flow rate. So, if you make a plot like this, so if you reduce the back pressure, you will see that the mass flow rate is increasing. In this way, so you reduce the back pressure you have the mass

flow rate increasing and increasing till you come to a very important limit say, this is the limit where the back pressure is $= P^*$.

What is the star condition? The star condition is the sonic condition, so at the sonic condition you will have Mach number $= 1$, see you started with what Mach number; you started with 0 Mach number, so to say, so there was no flow initially then your Mach number was increasing. In this way, you are coming to a state where the Mach number is 1 and there you have a flow rate which is corresponding to a velocity which is same as the sonic velocity.

Now, if you reduce the back pressure further, so if you make a plot like this, you will interestingly see that when you have a situation where P_b is $= P^*$, you have P^*/P_0 then $\dot{m} = \dot{m}_{\max}$, so this will become $= 1$ and then if you reduce the pressure further, mass flow rate does not increase anymore. So, intuitive portion; intuitive variation of the graph could be something like this that is as you reduce the back pressure, there will be more and more mass flow rate but this is not correct.

And you come to a saturation state, which corresponds to Mach number $= 1$ beyond which further reduction of back pressure will not increase the mass flow rate. Physically, why it happens? So, remember that when you reduce the back pressure, what happens? It is a pressure perturbation; these pressure perturbation travels within the fluid medium at a speed of the sonic speed relative to the fluid medium, it travels with the sonic velocity.

So, if you write say, velocity of the disturbance relative to the fluid is $=$ velocity of the disturbance - the velocity of the fluid. What is the velocity of the disturbance relative to the fluid? It is in terms of magnitude, it is the sonic speed, in terms of the direction; what is the direction in which the disturbance is moving? So, disturbance is created in the downstream in terms of the back pressure it has to propagate and go to the desorber just in very simple terms.

If you have to explain it to say, a senior school student it is something like this, so you reduce the back pressure, when you reduce this pressure this reservoir should know that yes the pressure is reduced, so I have to compensate it by changing by increasing the mass flow rate. So, as if here, there is a decision maker, who is sitting, who will know the message that there is a reduction in back pressure and accordingly will increase the mass flow rate.

So, the message has to propagate, what makes the message propagate? The speed at which the disturbance propagates is governed by the sonic speed and because of the propagation of the disturbance in terms of a way through the media that is how this reservoir knows that yes, I now have to reduce, I now have to increase the mass flow rate to compensate for the reduction in pressure.

So, velocity of the disturbance relative to the fluid; if you just take the algebraic sign of it, it is $-C$ because let us say positive x direction is right; towards the right, so negative x direction is towards the left = velocity of the disturbance $-$; when the sonic condition is achieved P^* then velocity of flow is C and velocity of flow is towards the right with a velocity C and this shows that you have velocity of the disturbance $= 0$.

What it means is that; if you reduce the back pressure further, the message that at the such a disturbance is there is unable to propagate to the reservoir and make it actuated by allowing more mass flow rate, so it is sort of say to be coming to a saturated condition for the mass flow rate and in that case, the technical name is that the nozzle is choked. So, the nozzle is choked that means, it has come to a critical mass flow rate.

Beyond which, if you reduce the back pressure further, you will not be able to increase the mass flow rate and the corresponding Mach number at that condition is 1. If you reduce the back pressure further what will happen? Of course, you may be able to reduce the back pressure further because reduction of pressure is your task that is an exercise that you are experimentally doing.

So, what it will do is; if you reduce the pressure further there will be a sort of oblique expansion wave outside and such that at the end; so there will be something which is occurring in outside will not go into the details that what is occurring outside but there will be some phenomenon that is occurring just outside but important thing that we are getting out of this is that if you reduce the back pressure further at least in terms of getting the mass flow rate, it will not improve any further.

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Handwritten derivation of the critical mass flow rate equation. At the top, p_0 is written. The main equation is $\dot{m}_{max} = (\rho^* A^* u^*) \rightarrow \sqrt{\gamma R T^*}$. Below this, ρ^* is written as $\frac{p^*}{R T^*}$, and A^* is written as A^* . The final expression is $\dot{m}_{max} = \frac{p_0}{\sqrt{\gamma R T^*}} \frac{1}{\sqrt{T^*}} A^* = f(p_0, T_0, \gamma, A^*)$.

And you can find out what is that critical mass flow rate, what is the critical mass flow rate? \dot{m}_{max} is $= \rho^* A^* u^*$, right. What is A^* ? A^* is the area of the exit in the converging nozzle. What is ρ^* ? ρ^* you can write p^*/RT^* and u^* is; u is Mach number $\times C$, so $m \times \text{square root of } \gamma RT$, the start condition means Mach number = 1. So, u^* is square root of γRT^* .

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Handwritten derivation of the temperature and pressure ratios at the critical point. The first equation is $\frac{T_0}{T^*} = \left[1 + \frac{\gamma - 1}{2} M^2 \right]$ with an arrow pointing to $M^2 = 1$. The second equation is $\frac{p_0}{p} = \left(\frac{T_0}{T} \right)^{\frac{\gamma}{\gamma - 1}}$.

So, this is $p^* \text{ square root of } \gamma / R \times 1/\text{square root of } T^*$, right. You know the expression of p/p_0 , yes; into A^* . You know the expression of p/p_0 , you know the expression of T/T_0 as a function of Mach number for example, T/T_0 is $1 + \frac{\gamma - 1}{2} M^2$, p/p_0 is T/T_0 to the power, ah sorry, this is T_0/T , is the other way. So, this is T_0/T , this is p_0/p is T_0/T to the power of $\gamma / (\gamma - 1)$.

So, you can calculate T_0/T^* by substituting $m = 1$ and it will be a sole function of γ , so the expression here \dot{m}_{max} will be a function of what? P_0 , T_0 , γ and A^* , so whatever is the maximum flow rate that you will get is decided interestingly not by the pressure the back pressure; which is there but the pressure at the inlet section or the pressure in the reservoir because these are the stagnation pressures and the stagnation temperature.

So, this is about the converging nozzle, let us consider what happens for a converging diverging nozzle. We can see that what is the limitation of the converging nozzle? If you want to accelerate a flow says, from a subsonic state to a supersonic state that you cannot get in a converging nozzle because the maximum Mach number that you can get is 1 and that is at the exit below a critical back pressure.

So, once you have that and still if you have an intention of accelerating your flow further, you also have to add a diverging section and that is why the importance of a converging diverging nozzle. Before going into that maybe let us work out a problem on the converging nozzle itself.

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Handwritten notes on a slide showing a problem statement and its solution:

Prob. $T_0 \rightarrow 100^\circ\text{C}$, $P_0 \rightarrow 150\text{ kPa}$
 5 cm^2 throat area
 (1) 100 kPa
 (2) 60 kPa
 (3) 30 kPa

Equation for critical pressure ratio:

$$\frac{P_0}{P^*} = \left[1 + \frac{\gamma - 1}{2} \text{Ma}^2 \right]^{\frac{\gamma}{\gamma - 1}}$$

Result:

$$\rightarrow P^* = 79\text{ kPa}$$

So, the problem statement is like this; here in a large tank at 100 degree centigrade and 150 kilopascal of pressure exhaust to atmosphere through a converging nozzle with 5 centimeter square as throat area; throat area means the exit area for the converging nozzle. Compute the exit mass flow rate, if the atmospheric pressure is; number 1, the atmospheric pressure is like the back pressure is = 100 kilopascal.

So, here there is no plenum chamber, it is exiting 2 atmosphere directly, so one is 100 kilopascal, number 2 is 60 kilopascal, number 3 is 30 kilopascal, okay. So, clearly these corresponds to the stagnation temperature, this corresponds to the stagnation pressure, so this; when these pressures are given, what is your first decision making, what is the critical back pressure to get the sonic condition at the throat, okay?

So, that is the star's condition, so we know that $P_0/P = 1 + \gamma - 1/2 M^2$ to the power $\gamma/(\gamma - 1)$. So, when you want the star condition, when you want this P_{star} then this is M_{star} or this is $= 1$, so from here you can find out what is P_{star} , so P_{star} for this case is 79 kilopascal, okay. So, 79 kilopascal is your P_{star} and your given pressure is 100 kilopascal.

So, for the one it is some state like this, this one, say as an example which is $> P_{star}$, so you have not yet come up with the maximum mass flow rate. So, how do you decide that what is your \dot{m} ; that will be dictated by the ρ , A and u , which are in turn dictated by the Mach number. So, what is the Mach number?

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Handwritten notes and a diagram illustrating the relationship between stagnation pressure, Mach number, and mass flow rate in a nozzle.

Equation (1): $\frac{P_0}{P} = \left[1 + \frac{\gamma - 1}{2} M^2\right]^{\frac{\gamma}{\gamma - 1}}$

Given: $P_0 = 100 \text{ kPa} \rightarrow M_e = ?$

Mass flow rate equation: $\dot{m} = \rho_{exit} A_{exit} u_{exit} = \frac{P_{exit} A_{exit}}{R T_{exit}} M_e \sqrt{\gamma R T_{exit}}$

Temperature ratio: $\frac{T_0}{T} = \left[1 + \frac{\gamma - 1}{2} M^2\right]$

Diagram: A graph showing the relationship between pressure ratio P/P_0 and Mach number M . The curve starts at $M=1$ and $P/P_0 = 0.5$ (labeled P^*). The exit conditions are marked at M_e and P_e . The stagnation pressure is P_0 . The mass flow rate is given as $\dot{m} = 0.1514 \text{ kg/s}$.

So, now you have $P_0/P = 1 + \gamma - 1/2 M^2$ to the power of $\gamma / (\gamma - 1)$. What is for case 1; what is the value of P ? 100 kilopascal, so from here you can find out what is the Mach number at the exit, so when you know the Mach number basically all the expression derived in such a way you know all the flow parameters, so \dot{m} is $= \rho_{exit} * A_{exit} * u_{exit}$. So, ρ_{exit} is $P_{exit} / RT_{exit} * A_{exit} * u_{exit}$ is $Mach_{exit} * \text{square root of } \gamma RT_{exit}$, right.

And T_{exit}/T_0 also you can; T_0/T using the expression of T_0/T is $1 + \frac{\gamma - 1}{2} M^2$ from that you can find out what is T_{exit} , if you use m_{exit} , T_0 is given, 293, sorry; 273 + 100, 373 kelvin. So, from here when you get T_{exit} you may substitute all the values to get \dot{m} and this \dot{m} is going to be 0.15 kg per second. Now, consider the cases 2 and 3, when you consider the case 2, you see that your decision making now says that the pressure is < the critical pressure that is P^* .

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Handwritten equations from a slide:

$$\begin{aligned} \dot{m} &= \dot{m}_{max} & (1) \\ &= \rho^* A^* u^* & (2) \\ &= 0.157 \text{ kg/s} & (3) \\ &= 0.15 \text{ kg/s} \end{aligned}$$

Below these, the pressure ratio is shown:

$$\frac{P_0}{P^*} = 1$$

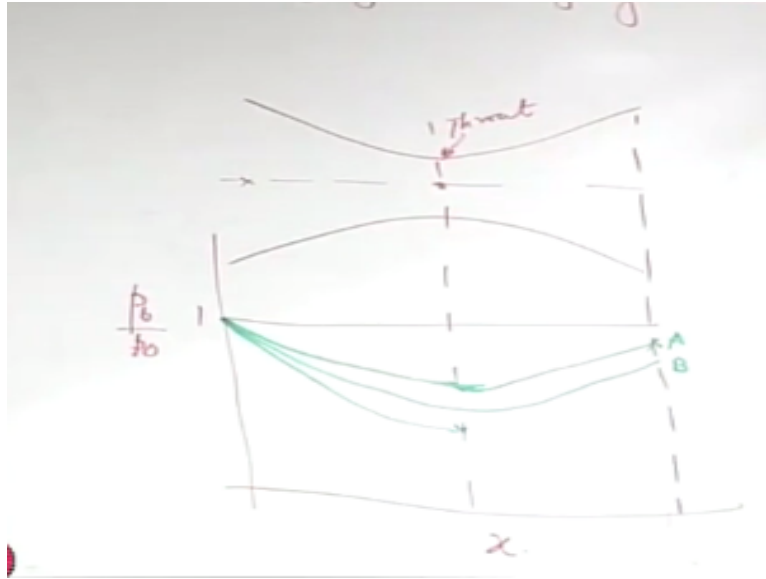
And finally, the critical pressure is determined:

$$\Rightarrow P^* = 7$$

That means, the saturation in the mass flow rate has already been achieved and therefore, for calculating the mass flow rate you should not go through this route but it should go through the \dot{m}_{max} route, so for both 2 and 3, you have $\dot{m} = \dot{m}_{max}$, this is $\rho^* A^* u^*$ which we have just derived. So, this you can write exclusively in terms of the stagnation properties in terms of P_0 and T_0

And A^* is the area of the exit or the throat area; throat is the exit for the converging nozzle. So, if you calculate this, this will come out to be 0.157 kg per second, so you can see that this is > the previous mass flow rate and is the maximum mass flow rate that you can get from this nozzle, so with this background now we will go to the converging diverging nozzle.

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So, for the converging diverging nozzle, let us say that you have a nozzle like this which has a converging part and a diverging part, so if you refer to the expression for dm/m , you can clearly see that for $m > 1$, dm/m is > 0 , if dA/A is > 0 , that means it is possible to have an increase of Mach number from the critical case, which you may come here as $m = 1$, so this part is just like a converging nozzle, so the critical condition that you may achieve here is $m = 1$.

In the converging part, you cannot achieve a Mach number $> m = 1$ but from $m = 1$, if you want to further increase it to greater and greater and greater values then using the expression for dA/A in terms of dm/m , we can see for $m > 1$, you can have $dm/m > 0$, if you have; only if you have $dA/A > 0$ that does not mean that you will always be able to achieve that because that expression was derived with an isentropic flow in mind.

And we are not sure that in the diverging section always it will be isentropic flow, why? Because that in the diverging section, if you want to accelerate the flow you go from a subsonic to supersonic condition and when you have a supersonic condition, the flow is prone to shocks and therefore, it is not trivially possible that you maintain an isentropic flow in the diverging portion of the channel that makes the diverging portion somewhat more interesting and complicated than the converging portion.

So, if you want to make a plot of; again let us do a similar experiment, where we make a plot of P_b/P_0 and $m \dot{m}/m \dot{m}_{max}$ and here we plot, P_b/P_0 as function of x , this is the throat, so throat for a converging diverging nozzle is the minimum area that is called as a throat, this is

just a technical term, this is the throat. Initial case; when no flow is like you have same uniform pressure, which is $P_b = P_0 = 1$, the ratio is 1.

Now, you reduce the back pressure, so when you reduce the back pressure, see there are 2 possibilities; think of one possibility that here the Mach number is say, here the Mach number is < 1 , so the Mach number here is; I mean whatever is the inlet flow, from the inlet to this one the Mach number at the most may be 1 but it will in general, be < 1 . So, if it is < 1 , then what will happen?

So, if the back pressure is such that it could not bring it to the limit of a choking condition at the throat, then what will happen? So, let us consider such cases first. So, when you have such a case, you have; at the throat you have what condition; at the throat you have the maximum Mach number but still that Mach number is $\neq 1$, it is somewhat < 1 . So, when it is < 1 and you go along the diverging section, what will happen?

It will not be able to accelerate the flow, so it will be a sort of recovery your pressure from this, okay, so that means if you keep these are the back pressure, you may have a flow where it is accelerating in the converging section and decelerating in the diverging section, just like an incompressible flow and never you have; know where you have reached a sonic condition, your Mach number everywhere is < 1 and it is an isentropic flow throughout, okay.

So, let us say now you reduce the back pressure and similar things will occur till you come to a critical limit, so let us say that we will consider that critical limit separately, so first we consider the exit plane and consider the non-criticalities. Let us say that this is a state A, this is a state B, now we come to a case, where it comes to a critical state, where the throat condition is Mach number = 1 that means the sonic or the start condition is achieved at the throat.

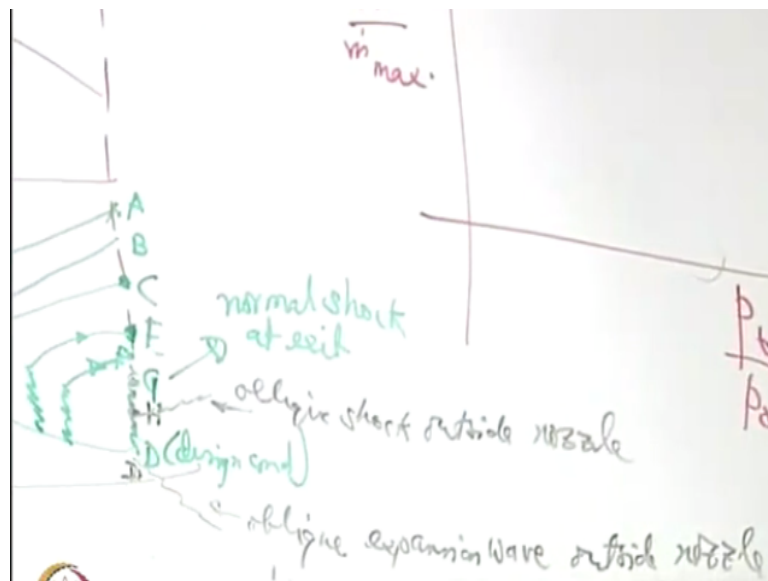
When you have a Mach number = 1 here, what will happen in the diverging section; we will have 2 possibilities, why it will have 2 possibilities? Still, we are considering isentropic flow, it will have 2 possibilities with isentropic flow, it may have multiple possibilities if it is not isentropic flow but with isentropic flow, it will have 2 possibilities, why?

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$$\frac{A_e}{A^*} \rightarrow f(M_e)$$

Think about the exit area, so you have A_e/A^* , when the sonic condition is achieved at the throat, then A^* is = A_{throat} , so A/A^* is a function of the Mach number at the exit where multiple values of Mach number at the exit will satisfy a unique A/A^* that is the equation that we have derived in the previous; one of the previous lectures that A/A^* is a function of; it is a nonlinear function of Mach number at the section that we are considering.

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If you are considering the exit section then the Mach number is the exit Mach number, there are 2 isentropic solutions for this; one isentropic solution is a subsonic solution, another isentropic solution is a supersonic solution. So, if you want to have an isentropic flow at the exit with the throat Mach number = 1, then you have 2 possibilities; one is a subsonic flow, another is a supersonic flow.

So, you have these as let us say C, let us say this is D, so if your design is so perfect that you are keeping your back pressure same as the pressure at D what you get out of this exercise, it is easy to get what is the pressure at D because you can from A/A^* , the supersonic solution will give what is the Mach number at the exit and corresponding to that Mach number P/P_0 , if you calculate that P will be the P at D, which is known as the design condition.

Why it is the design condition? It gives you the highest Mach number, so it is making the flow utilize its entire potential of acceleration, so it is accelerating to the highest Mach number with an isentropic condition throughout, so this is known as the design condition that means of course, if you keep at C, still you get an isentropic flow throughout but it is a subsonic flow throughout, it is not a supersonic flow.

Now, so if you want to utilize the diverging section with a continuously increasing velocity or the accelerating flow, then the design condition is the correct condition that one may maintain but in reality, it may not be possible to maintain the design condition, so we have to see interestingly that what are the off design conditions. So, some of the off design conditions are what; so let us say that you have a back pressure say somewhere like here say, E.

So, when the back pressure is maintained at E, then what will happen? Say, there are 2 cases; the back pressure is maintained at E and the back pressure is maintained at F. So, when the back pressure is maintained at E and F, so let us consider F as a very limiting case but let us consider first E. So, let us say that E is such that at the end C whatever is the pressure, it has to match with the back pressure at the exit.

So, how it may match? See, the pressure was going to reduce, it has to increase to match it, how it may be increased; by a shock, so inside the nozzle, there may be an; there may be a normal shock, which will make it, so the normal shock will give symbolically like this, which will make a pressure rise and match the condition with the exit condition. **“Professor – student conversation starts”** It depends on the back pressure at E.

There were limiting condition as you reduce the back pressure that the shock occurs exactly at the exit plane, so that is given by this F, say as an example. So, here there will be a normal shock, so if you have say or let us put up point F maybe it is difficult to put too many points in

the diagram but let us just try, so we put a point F maybe that is still a case where the shock occurs at further downstream.

So, in this way as you reduce the back pressure, the normal shock within the diverging portion goes closer and closer to the exit section and if you come to a condition say, G as a limiting condition, there the normal shock will occur at the exit plane. So, G corresponds to normal shock at exit. If your back pressure is between G and D that also may be a case, so let us say that the back pressure is at H, which is between G and D.

Then this is accompanied by; because see this discontinuity has somehow to be adjusted, so that is adjusted by a shock not within the nozzle but just outside the nozzle and that is an oblique shock. So, this is oblique shock outside the nozzle, if the back pressure is $< D$, some greedy designer may think that well I want to reduce the back pressure below D and C because it might be technologically possible.

So, let us say, I; yes, which? Yes, see this oblique shock will occur just outside the; so this is a symbol, this is not a pressure curve; this is just a symbol of oblique shock that is all, just a symbolic way of representing it. The whole idea is that whatever may be the pressure here, the pressure has to eventually match with the back pressure. See, so when it comes with the pressure at this point say, you think about this; think about this point.

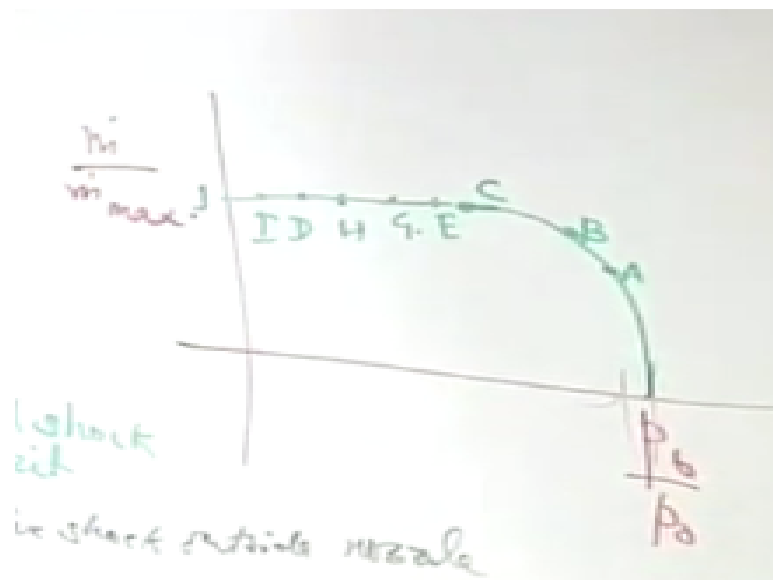
So, when you have the pressure same as the back pressure as G, so till the exit it has come with an isentropic condition, it has come to a point very close to the point D, it is not technically same as the point D but infinitesimally away from the point D just at this one but you have kept your back pressure same as the point G, so at the exit, there should be a discontinuity to match with the exit pressure from; so the exit pressure should go through a discontinuity to match with the back pressure and that is through a normal shock at the exit.

Similarly, if the back pressures are lower and say here that discontinuity is not in the form of a normal shock but in a form of oblique shock just outside the exit plane. **“Professor – student conversation ends”**. When you come to the point I, which is $< D$; where the back pressure is $< D$, so let us say that you want your back pressure at I, when you have your; so up to D, very close to the D, it could come to what state?

It could come to a state, where it is an isentropic flow almost throughout very close to the point D but back pressure has to match with pressure at I, so there should be some sort of discontinuity between D to I that is in the form of, what? See, D to I is what; pressure increase or pressure decrease? Pressure decrease, so it is an expansion, shockwave is the; till now whatever we have considered as a shock wave is the compression wave.

So, here it is an expansion, so it is an oblique expansion wave that is there at the exit, so this is an oblique expansion wave occurring outside the nozzle. So, if you consider from a design perspective, D is that is; back pressure at D is the design condition, if you have your back pressure between G and D then still you get almost entirely isentropic flow within the nozzle. If you consider your back pressure $< D$, then also you get almost entirely isentropic flow within the nozzle.

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But the discontinuity to match up the back pressure and the exit pressure is through different forms of waves; shock wave or expansion wave, in terms of \dot{m} / \dot{m}_{\max} . Now, let us see this graph. So, when you reduce your back pressure just as the previous case, the mass flow rate will increase. So, when the mass flow rate increases, it will come to a saturation, out of these all these points in the diagram, which point will be the point corresponding to the saturation mass flow rate, beyond which mass flow rate will not increase further, C right.

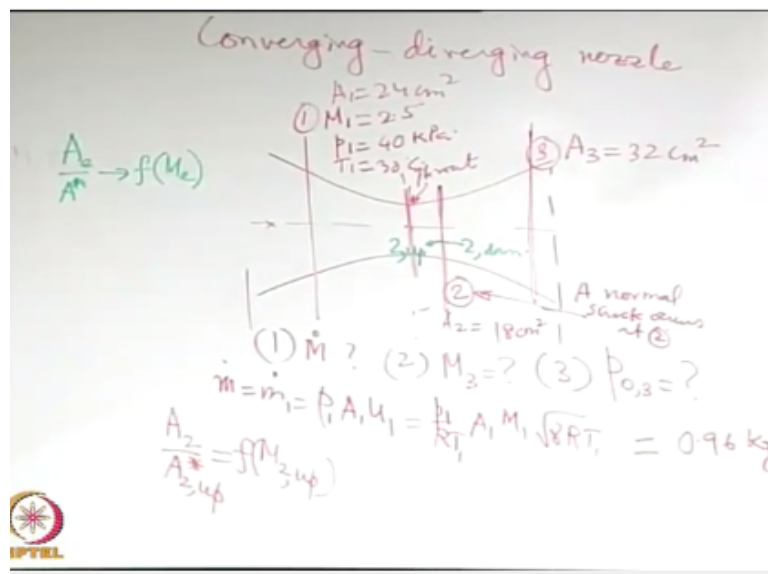
So, C; because it corresponds to the critical; the sonic condition gives you the maximum mass flow rate, see for the mass flow rate converging portion is important because whatever mass flow rate the converging section is delivering that is going through the diverging section, so you

come to the point C maybe, A is these and B is this, then if you come to this one, this is $\dot{m}/\dot{m}_{\max} = 1$.

All your points like E, G, H, D, I will give the same maximum \dot{m} , which is given by the sonic condition at the throat, so it means that if you want to have a supersonic flow in the diverging section, it must pass through the sonic point at the throat in a converging diverging that is a very important understanding, so we have seen that if you have such a geometry where it is a converging section or a diverging section, it is not always necessary to have achieve a sonic condition and the throat, right.

So, just in the previous class, we have walked out an example where the geometry was like this and at the throat, we got a condition where the Mach number is < 1 , so it is not a must that at the throat, the Mach number should be $= 1$ but if it at all has to be 1, it has to be at the throat. So, these are the conditions for which the Mach number is 1 and that is at the throat these are the conditions, where the Mach number is < 1 , right.

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So, with this understanding, now let us try to work out a couple of problems on the converging diverging nozzle. So, the problem is like this; air flows through a duct as shown in the figure, the duct is like whatever is drawn in the sketch with there are 3 sections; section one with given $M_1 = 2.5$, $P_1 = 40$ kilopascal and T_1 is 40 degree centigrade, the throat name is section 2, no sorry, throat name is not identified as any section.

It is told that in some section 2, a normal shock occurs, wherever it is a normal shock occurs here and there is a section 3 also, which is marked, section 2 has $A_2 = 18$ centimeter square, section 3 has $A_3 = 32$ centimeter square and $A_1 = 24$ centimeter square, these are given. So, with this given data, you have to find out the mass flow rate, number 2; the Mach number at section 3 and the stagnation pressure at section 3, okay.

See, at the section 1 everything is given to calculate the mass flow rate because at the section 1, you have P_1 , T_1 and therefore you can find out ρ_1 and you have the Mach number that means, you know, what is the corresponding u_1 , so \dot{m} is same as \dot{m}_1 that is $\rho_1 A_1 u_1$, so ρ_1 is P_1 / RT_1 , A_1 is given, u_1 is M_1 square root of γT_1 , so all these data are given.

If you substitute that you will get this as 0.96 kg per second, now you move on to section 2; see, section 2 is such a section, where you have a discontinuity, so you have a 2 upstream and you have a 2 downstream, the properties are different there because there is a shock at section 2, so at the section 2, how do you find out what is the Mach number in the upstream of the section 2, you can write A_2/A_2^* as a function of Mach number at 2 upstream, when you consider A_2^* as upstream A_2^* .

Remember A^* changes across the shock, so when we say A_2^* , it has no meaning, is it in the upstream side or is it in the downstream side, accordingly the Mach number is upstream side or downstream side, okay. So, how do you calculate A_2^* in the upstream? It is same as A_1^* , so because from 1 to 2 upstream is isentropic flow.

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$$\frac{A_1}{A_1^*} = f(M_1) \rightarrow A_1^* = ?$$

$$= A_{2,up}^*$$

9/2

So, A_1/A_1^* star is what? A_1/A_1^* star is function of Mach number at 1, so from here you can get what is A_1^* star, which is same as A_2^* star upstream, so you can; from that you can get what is A_2/A_2^* star upstream. Once you get what is A_2/A_2^* star upstream this; if you look into the table of the compressible flows, it will have 2 solutions; one is supersonic, another is subsonic, which one would you have accepted as M_2 upstream? Supersonic.

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$$\dot{m} = \rho_1 A_1 u_1 = \frac{p_1}{RT_1} A_1 M_1 \sqrt{\gamma RT_1} = 0.96 \text{ kg/s}$$

$\dot{m} = \rho_2 A_2 u_2 \rightarrow \text{get supersonic } M_2 \text{ up}$

$M_{2,up} = 2.18$

Because the shock is from supersonic to subsonic that is what the second law of thermodynamics says. So, from here you get the supersonic solution, for M_2 and that M_2 upstream are 2.18. Once you know M_2 upstream, you can calculate M_2 downstream because you have the formula for M_2/M_1 for a shock.

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0.96 kg/s \rightarrow by $\frac{M_{\text{down}}}{M_{\text{up}}}$ formula for shock
 $= 2.18 \rightarrow M_{2, \text{downstream}} = 0.54977$

So, M_2 upstream; this will give you what is M_2 downstream by M downstream/ M upstream formula for shock that is M_2 square in terms of M_1 square that formula. So, that will give you M_2 downstream = 0.54977, when you have got an M_2 downstream, you can calculate A_2/A_2^* star downstream.

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$\frac{A_2}{A_{2, \text{down}}^*} = f(M_{2, \text{down}}) \rightarrow A_{2, \text{down}}^* = A_3^*$
 \Downarrow
 $\frac{A_3}{A_3^*} = \dots$
 - diverging nozzle
 24 cm^2

So, A_2/A_2^* star downstream is a function of M_2 downstream, so from here, A_2 still remains the same, so from here you can get what is A_2^* star downstream and that is same as A_3^* star. So, then from this, you therefore have what is A_3/A_3^* star, this is a function of M_3 . So, out of the possible M_3 is again, it has 2 solutions which M_3 we will take; subsonic or supersonic? Subsonic because it is downstream of the shock.

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$$\frac{A_3}{A_3^*} = \psi = f(M_3)$$

$M_3 \downarrow$ (subsonic)
 $= 0.27$

$$\frac{A_1}{A_1^*} = f(M_1) \rightarrow A_1^* = ?$$

$$\frac{P_0}{P} = \left[1 + \frac{\gamma-1}{2} M^2 \right]^{\frac{\gamma}{\gamma-1}} = A_2^*, \text{ up}$$

So, this you get what is M_3 as the subsonic solution that is $= 0.27$ that is the part 2 of the solution. What is the part 3 of the solution, what is P_{03} , so how do you find out P_{03} that is; yes, how do you calculate P_3 , how do you calculate P_{03} ? This you tell, how do you calculate P_{03} ? See, you have P^* okay, then so you have P_0/P as a function of what; $1 + \frac{\gamma-1}{2} M^2$ square to the power $\frac{\gamma}{\gamma-1}$.

So, when you consider P^* , so what is the; how do you get the corresponding P^* ? Remember, P^* is a reference state, which is not any physical state out of 1, 2 or 3, so in principle obviously, if you know say P you could calculate, what is P_0 but how do you calculate P_0 ? You need to refer it with a star state, how do you calculate that star state? Let me give you the answer at least.

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$$\frac{P_0}{P_1} = f(M_1) \text{ Converging-diverging}$$

$A_2^*, \text{ down}$

$P_{0, \text{up}} \rightarrow P_{2, \text{up}} \xrightarrow{\text{shock}} P_{2, \text{down}}$

$A_1 = 24 \text{ cm}^2$

① $M_1 = 2.5$

See, if you want to calculate this in this way, the trouble is; you are required to know what is P_0 ; but if you forget about that; you forget about that just go to the upstream of the shock, can you tell what is P_0 at the upstream of the shock? Yes, P_0/P_1 , you know, right. So, in the upstream of the shock, you know P_0/P_1 as a function of M_1 , right, so this is P_0 at upstream, right. Can you relate P_0 at upstream with P_0 at downstream?

“Professor – student conversation starts” That is P_2/P_1 ; that is P_2 ratio, what about P_0 ratio? Once you have that the situation is from here you can find out what is; what is P_1 , sorry what is P_0 upstream, so you can calculate P_2 upstream, once you calculate P_2 upstream, you can calculate P_2 downstream, right. So, these will give you the possibility of calculating P_2 upstream because P_2/P_0 is a function of Mach number at the upstream.

P_2 upstream by the shock relationship; normal shock relationship will enable you to calculate P_2 downstream, right. When you calculate P_2 downstream, you get P_0 downstream because P_2 downstream/ P_0 downstream is a function of Mach number at 2 downstream. **“Professor – student conversation ends”**

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Handwritten notes on a whiteboard showing calculations for a normal shock. The notes include:

- $P_{2,down} = f(M_{2,down})$
- $P_{0,down} = P_{03} = 435 \text{ kPa}$
- $A_3 = 32 \text{ cm}^2$
- $\frac{A_3}{A_3^*} = f(M_3) = 0.27$
- $\frac{A_1}{A_1^*} = f(M_1) \rightarrow A_1^*$

So, P_2 downstream/ P_0 downstream is a function of Mach number at 2 downstream, which is already calculated. So, from here you can find out what is P_0 downstream that is same as P_{03} , okay but if you use the table of shocks, you need not do all these, it is simply given a P_0 downstream/ P_0 upstream because it goes through all these calculations and eventually tabulates in the form of that.

So, if you do that this will be 435 kilopascal that is the answer to the third part, so that is the P_{03} downstream, okay, so we can see that given certain conditions, we need to keep in mind that A^* is a reference quantity, the A^* may not be; the any physical A^* , which we are getting for these sections, it is just a hypothetical state, where in an isentropic condition, the system is brought to a sonic state.

So, it is different for the isentropic flow in the upstream and isentropic flow in the downstream, so what we are assuming is that upstream of the normal shock is one isentropic flow, downstream of the normal shock is an isentropic flow, so in the upstream of the normal shock P_0 , T_0 , A^* the remaining same, downstream of the normal shock P_0 , T_0 and A^* , they are remaining another set of same values.

And there is a discontinuity of those as you go across the shock because of the non-isentropic nature of the flow across the shock, okay. We stop with this lecture now and we will continue with the next lecture. Thank you.