Introduction to Fluid Mechanics and Fluid Engineering
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Lecture – 55
Compressible Flows (Contd.)

We will continue with our discussion on compressible flows. We will now discuss on something know as stagnation properties.

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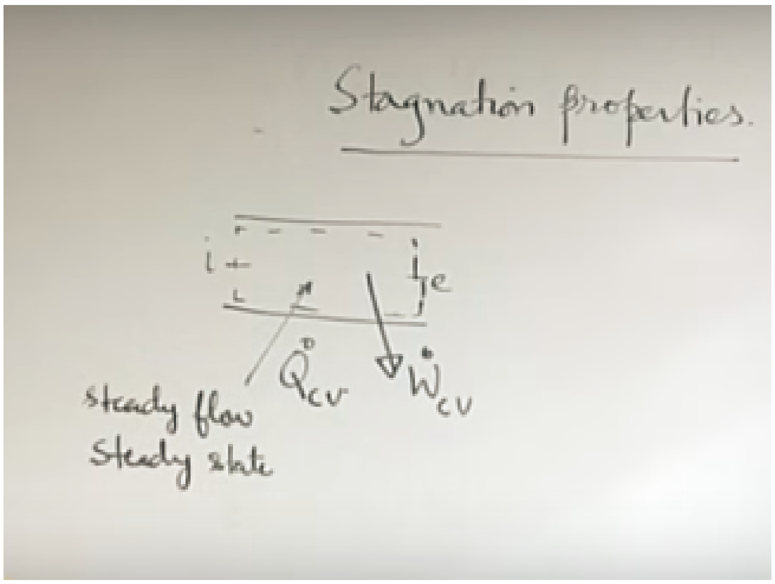

This terms stagnation we have come across during our earlier topic that we have covered in fluid mechanics and one of the literal meaning is that the fluid is brought to rest starter point, so that the velocity is 0. So stagnation point is a point where the velocity of the fluid is 0. But that does not mean that its suppress the requirement of description of stagnation properties. So, we will now look into more deeply about some of the important properties which dictate the nature of the process by which the stagnation state is touching.

To that we will refer to first an important thermodynamic property and when we are referring to the thermodynamic property, we would be basically referring to the first law of thermodynamics to specify that. So, if we say that you have control volume, say some control volume which has some inlet i and some exit e. Let us say, that there is some rate of heat transfer to the control volume Q dot cv and some of the work done by the control volume as W dot cv.

And let us say that the flow is steady and the state or the properties of fluid within control volume is steady that means the property within the control volume do not change with time. So, if these 2 conditions are simultaneously achieved then the corresponding form of the first law of the thermodynamics is like this. Where h is the property in enthalpy, which is the internal energy plus pressure by the density.

Now, this particular form is also known as steady flow energy equation. Just for common understanding. So this is nothing, but first law of thermodynamics expressed for a flow process across the control volume. When, the flow is steady and the state within the control volumes also steady.

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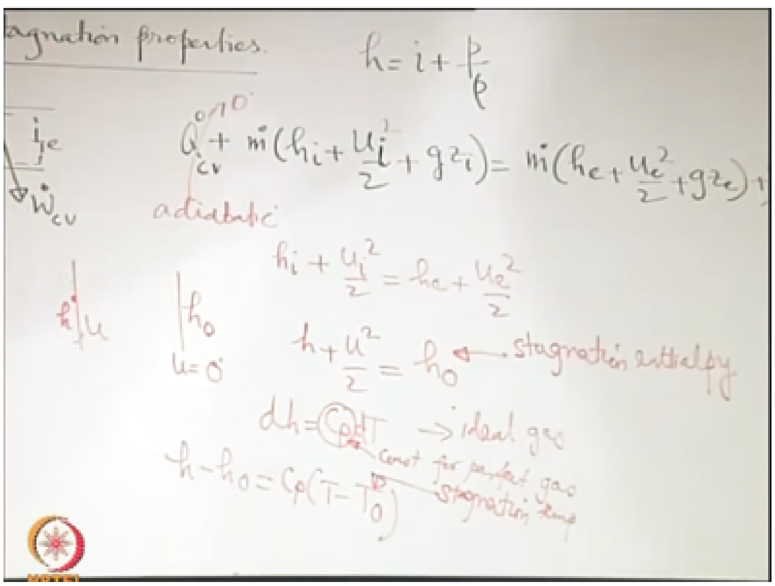

Not only that there is certain more important assumption, what are the important assumption? All the properties at the inlet and exit are uniform that is the velocities profiles are at uniform, the thermodynamics properties like enthalpy those are uniform. So it is like approximately 1-dimension representation where uniform properties across the cross section for the inlet and outlet. Now we are interested to apply this equation for compressible flows.

So, when we apply this equation for compressible flow first of all if you have system were you are not having mechanism of extracting work from that, so then this rate of work done will be 0 by the control volume. Then, if what kind of process we are considering. We are considering adiabatic process. If it is an adiabatic process, so if it is an adiabatic process that means the heat transfer across the control volume is 0.

We are not commuting, whether it is reversible or irreversible, so reversible to irreversibility features when we talk about second law of thermodynamics. When we discuss about that, but in the first law of thermodynamics it is just good enough to say whether it is adiabatic or not, it is reversible or irreversible does not feature here. Now in most of the high speed gas flows the effects of thermal effects and kinetic energy effects are more important than changes in potential energy that is negligible and therefore your left with h i+u2 i=h e+u2 e. Okay?

Now, let us say that we are thinking about 2 sections instead of i and e. Some genetic section where the enthalpy this is specific enthalpy that is enthalpy per unit mass is h, the velocity u this is at some section and when you go to some another section that section is special section when the velocity is brought to 0. So this is section of stagnation. So we are interested to see that what a corresponding enthalpy there is and let us say that the name enthalpy at that stagnation section is h0.

Since it is a one dimensional treatment section and point are same. That it is basically uniform, so we can say from here that h+u 2/2=h0, because at the stagnation u=0. So this h0 know as stagnation enthalpy. For an ideal gas it is know that dh=C pdt this is for an ideal gas where Cp is functional of temperature in general what when you will talk about the perfect gas Cp is a constant. So if you say a perfect gas it is constant for perfect gas.

Therefore, we may say h-h0. For perfect gas is Cp(T-T0), where T0 is the temperature corresponding to the stagnation enthalpy this is known as stagnation temperature, while we refer to the temperature because temperature is a direct measureable quantity from experiments. So it is important that we refer to that. So this known as stagnation temperature. So we can write Cp (T-T0) +u square/2=0. Okay?

Now we may write Cp in terms of R and gamma, because just we call that Cp-C0=R and Cp/Cv = gamma. Okay? So we can write Cp1-1/gamma=R that means Cp = gamma R/gamma-1. So we can write T o - T in place of Cp we write gamma R/gamma-1=U square/2. U square/2 is what you can write U/C=Mac number say M. So U square=M square*C square.

Now what will be expression for C will depend on the nature of the process, if it is an isentropic flow C square is gamma at T. So this is=M square*gamma at T for isentropic flow. See till now where ever we define the stagnation temperature so the stagnation temperature how it is defined. The Stagnation temperature is defined from this equation as T0=T+U square/2Cp, Right?

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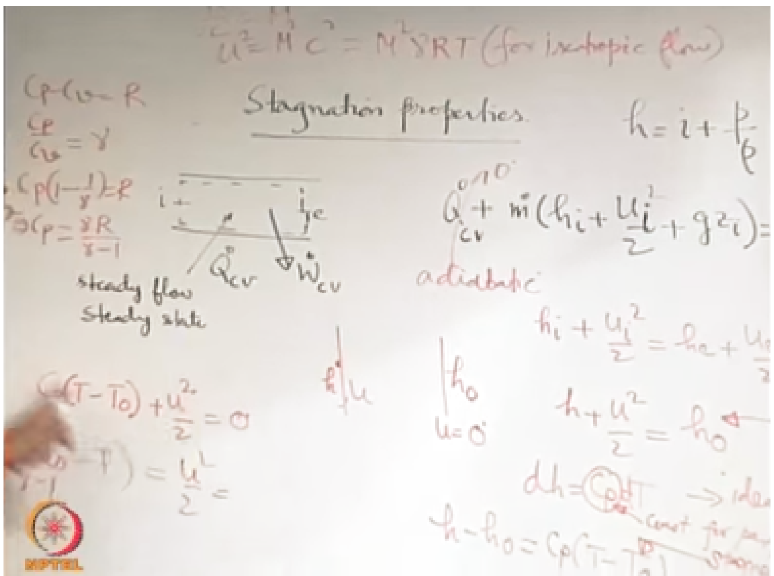

This definition does not require to process to be reversible. So if you want to find out what is stagnation temperature, infect this definition does not require any process because it is just an expression. So if you know the temperature and velocity at a point and specific heat at a point you know the stagnation temperature at a point. So Stagnation temperature is defined at a point irrespective of what is. I mean what kind of state is there at the point.

If you want to physical achieve stagnation temperature, you have to bring the process to rest at a point in an adiabatic manner. It need not be reversible. That is not necessary; because the definition followed from the first law thermodynamics by setting heat transfer = 0 without imposing any condition of reversible process. Okay. So keep one thing in mind stagnation temperature just like any stagnation property is a property.

So it does not mean that at one point. If it is not a stagnation point it will not have stagnation temperature it will have stagnation temperature, because it is just dependent on the local temperature, velocity and Cp. So it is just like a combination of properties, therefore it is property. If you say that, I want to physically achieve that property then you have to follow this kind of process adiabatic process.

Now if you want to use this expression U square=M square * gamma at T that means now you are imposing additional constrain that it is an isentropic process that means it is a reversible process for an isentropic process this will be M square gamma RT/2. So from this what follows is gamma, so into R will cancel from both sides, so we will get T0/T=1+gamma-1/2M square, Right?

This is the relationship between the stagnation temperature and the temperature at a point. What are the assumptions under which this is valid it is an isentropic process that you have to keep in mind, otherwise the more general expression is this one. Now you may relate the stagnation just like stagnation temperature you may have a stagnation pressure and stagnation density.

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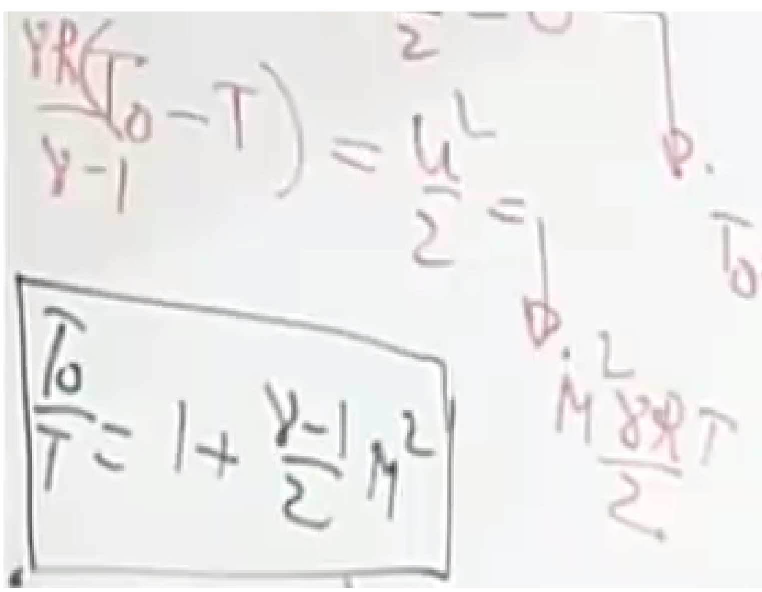

So those properties also you may find out, so for an isentropic flow. For isentropic process you have T p to the power gamma=constant, that means basically in terms of T and P you can write T2/T1=P2/P1 to the power gamma-1/gamma. So just like you have related the stagnation temperatures, similarly you can relate stagnation pressure P0/P=T0/T to the power gamma/gamma-1 that means 1+gamma-1/2M square to the power gamma/gamma-1. Okay?

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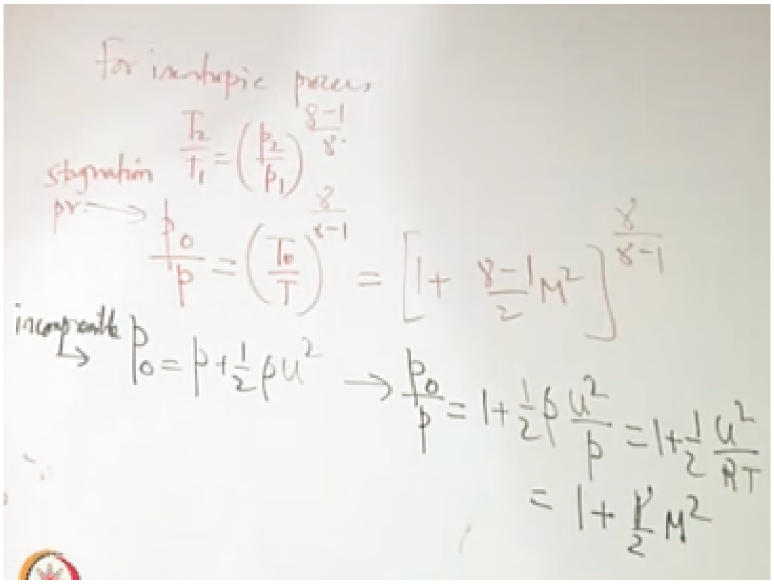

This is this P0 stagnation pressure remember assumption again is isentropic flow. Now so this considers compressibility, we have earlier seen a case. Where we used Bernoulli equation to define a stagnation pressure and that was without consideration of any compressibility. So we use just a Bernoulli equation. Bernoulli equation means it is a incompressible flow assumption and the stagnation pressure.

So if you consider incompressible limit P0=P+1/2Pu square, neglecting the potential energy effect. What are the assumptions? it is an incompressible flow, we use the Bernoulli equations that means we implicitly assume that, it is a frictionless flow. So if you want to achieve physically this P0 you have to bring the fluid to rest in a reversible adiabatic process that will mean it is a frictionless process.

If you want to achieve P0 physically you have to bring the fluid to rest in a adiabatic process, it need not to be reversible. Where as to achieve P0 physically? You to bring to rest by ensuring both that it is reversible and as always as adiabatic. This is very important distinction between like how you achieve physically stagnation pressure and stagnation temperature.

So, now when you come to this incompressible flow here you can write P0/P. For an ideal gas P/rho1/2T. So this is 1+1/2u square/1/2T. Remember that, if it is an isentropic flow C square=gamma1/2T. So this you can write 1+gamma/2U square/C square=M square. Okay? So, this is an expression which is valid for incompressible flow this is an expression that is valid for compressible flow.

It may be interesting to see that in a certain approximately limit this to equations in certain approximation they agree with each other to do that, what we may do this is more general expression. So we may expand this binomial theorem, just like 1+X to the power of n. So, if we expand that in a binomial theorem. So P0/P, compression so 1+X to codeine is 1+nX+n*n-1/factorial 2*X square+.

Let us just write another term n*n-1*n-2/factorial 3*X cube like that. So this is 1+gamma/2M square, so you can see that up to the first term it is just like an incompressible flow expression that the remaining term and the correction because of the compressibility effect. So, what are the corrections? You can write may be the first 2 corrections, so one is gamma-, so this is gamma/8M 4, Right?

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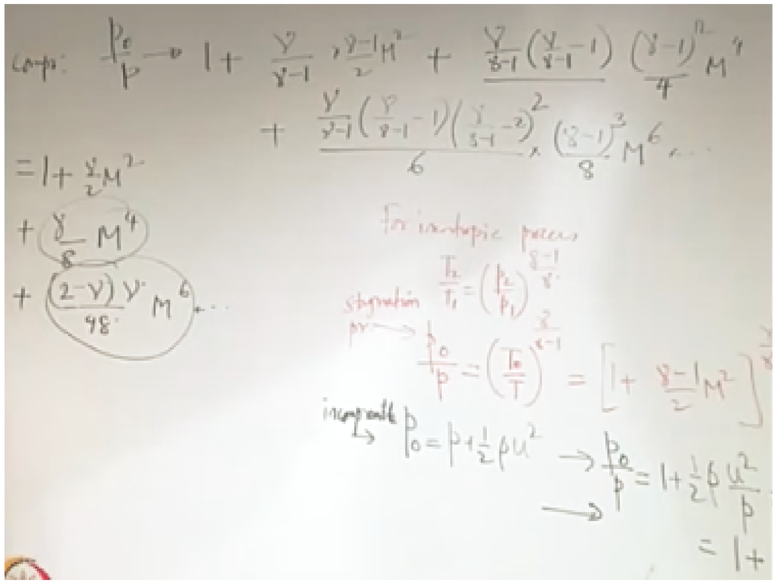

Then + this is 2 + gamma * gamma/48 * the power 6 like that. Sorry, 2-gamma, right? So what it is important implication say, you are using a P dot U to measure flow velocity at a point and your negligent of the compressibility effect and your using this expression for P0/P. Because in a P dot U, you may get the difference between stagnation and static pressure by connecting a manometer between the stagnation point and another point which is point locator upstream to that.

So when you get that expression that expression will require a correction and leading on term that will dictate correction. The first 2 leading on term are like this. So, if it as to have the compressibility effect this extra term call correction needs to be invoke and we can see this. These terms are higher powers of M. So as M become smaller and smaller this become more and more irrelevant. That means for no values of Mac number the compressibility effects are smaller and smaller that is quiet intuitive.

Next what we will consider? Next, we will consider the similar thing that we have discussed till now may be isentropic begin with, but through a variable area ducts**.** Till now we are not taken area as variable parameter, but now let us generalize the pervious discussion somewhat and consider isentropic flow through a variable area ducts. So when we say variable area ducts may be it is something this, maybe it is something like this we will not specify what it is but the area of cross section will vary.

So what are basic equations that we may use here? One is of course continuity equation and in all cases we are assuming it is a one dimensional flow. So rho*A*U=constant this is from the continuity equation. Then the fluid flow equations here remember, we are talking about isentropic flow. So reversible and adiabatic, so it is having no friction that means it is a inviscid flow.

So what will be governing equation for that Euler equation 1-dimension form of the Euler equation? So Euler equation, so that is what dp/rho + UdU=0?, that is a differential form of the Euler equation. Remember that in all cases of compressible flow that we are discussing we neglecting the changes in potential energy. Gravity effects are negligible compared to the other effects, so this sort of like the fluid flow equations and the thermodynamic constrain is the energy equation.

So energy equation is like h+U square/2=constant. But, first of all we will not consider this energy equation as an important consideration. But, we will concentrate on these 2 equations and see what we will get out of that. So continuity equation rho*A*U = constant. So what we will do is we will take log on both sides so we will ln rho+ ln A+ln U=ln constant and then differentiate that means d rho/rho + dA/A + dU/U=0, say this is equation number 1 and say this is equation number 2.

Now, if want to relate this with sonic speed see equation number 1 there is d rho in Equation 2 there is dp, so somehow we find out dp d rho, that will give C square. So we can relate this behavior with sonic speed, so we can we write from 2 that dp=-rho U dU and or if you want to relate with it U square. So dp/U square=-rho U d U/U square. So, - rho d U/U. Why we are written in this form?

Is because you have another d U/U in equation 1, so let us say that this is 3 and from equation 1 you have d rho=-rho*dA/A + dU/U say this is equation 4. So, if you divide equation 2/equation sorry equation 3/equation 4. You get dp d rho*1/U square=d U/U/dA + dU /U. If consider as an isentropic flow dP d rho=C square. So, for an isentropic flow the left and side is 1/M square because M=UYC.

So, from you can write dA/A+dU/U=M square dU/U. Which means dA/A=1- or =M square-1*dU/U, this is relationship is that govern the change in area which change in velocity and it is one of the very important physical relationship which we have to carefully study. So if you write d A by A so when you write d A/A, we think about 2 possibilities.

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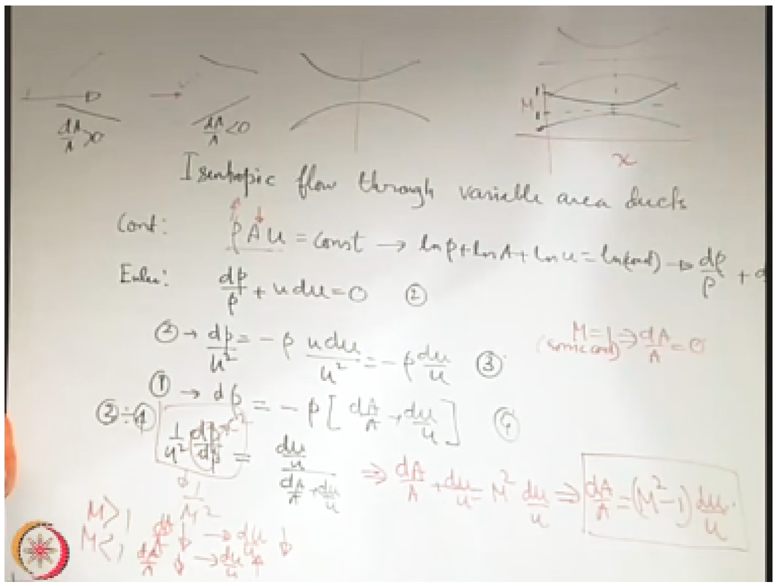

One of the possibilities is along the deduction of the flow, the area increasing or area is decreasing this is an example with dA/A>0, this is an example with dA/A<0. Assuming, that to be the deduction of flow. Okay? Now if you consider M=1 as case so M=1, if you consider that will imply what dA/A=0, right? That means if you want to achieve M=1. M=1 is called as a sonic condition. In an isentropic flow it must confirm to dA/A=0. dA/A=0, if you look into mathematically it may be either maximum area or minimum area, Right?

Physically, we will see it is confirmed to the minimum area, so if you consider possibility of this 2 types say you consider as this are the case. Let us say that you have the inlet flow with particular Mac number whatever the Mac number at the inlet may be > 1 or may be < 1. Now, we interest to see the consequence. If you consider this particular geometry, if you see that no matter where ever M=1 is achieved we will see later on with the physical example that how M=1 may be achieved.

See just think this geometry, so this particular geometry even, if it is an incompressible flow you what is your target. The target could be to increase the velocity that is, why? You reduced the area of cross section, so this type of thing where you have variable cross section area with the intension of increasing the velocity and, therefore consequence reduction of pressure that is now as a nozzle.

So, if you have a nozzle your target is to have an increase in velocity. But just this geometry does not ensure that will be like that for a compressible flow for an incompressible flow area reduction will imply velocity will increase. For compressible flow, you think it like that it may be wrong because rho is also another factor its rho*A*V, not just A*V. So rho may vary in such way it might that area is increasing the velocity also increasing, so it all depends on the Mac number at which the fluid is there.

So we will see later on that. What is physical implication of the minimum area, when we discuss more about nozzle? Converging nozzle this is called as converging nozzle or converging diverging nozzle. But look it in the other way if M is != 1 then can you have dA/A=0 definitely M is != 1. Then you may have dA by A =0 then dU by U = 0 that means U is either are minimum or a maximum.

So, from this we come to a very important conclusion that if for the time being we take that at the minimum area M=1 is achieved that does not mean that at minimum area M=1 is always achieved it means that M=1 is achieved. Somewhere it has to be minimum area location, but the minimum area location will not always have M=1, if it does not have M=1 it will have either a minimum name or maximum name.

And we will that see how it is possible to understand that what we will do his we will consider to different cases one is M>1 and another one is M<1. If you consider M>1, you can see that if you have a reduction in A that means dA/A reducing. Then dU/U that is also reducing, Right? That means, if area decreases the velocity also decreases, if the flow supersonic this nonentity. Incompressible flow consideration, we say, if area increases the velocity will decrease so why it happens roughly if you see that what you are keeping fixed rho * A * U.

So your reducing A, your expecting U to increase but with reduction in A maybe there is, such a high increase in density that there is actually significant reduction in U to keep the product as constant. Okay? That what is happening here physically? So what we get of this is M >1 that means you have dA/A. That is area is reducing then the velocity is reducing, therefore it is not acting like a nozzle.

It looks like a nozzle, but it is acting like a diffuser. Diffuser is a something, where do want decrease in velocity and an increase in pressure. So here it is acting like a diffuser, so physically looking like nozzle does not mean it is a nozzle one as to see the Mac number range which it is operating. On the other hand, if M<1 you have dA/A will be, if it decreases that will mean that dU/U is positive that means U increases, Right?

So, if so this situation we may intuitively think as physically analogous to the incompressible flow behavior in a nozzle. Only if Mac number<1, only if it is a subsonic flow, if it is a supersonic flow that is not the case. So, if it is a subsonic flow what happens, if it is a subsonic flow. So, now think about such an arrangement, so this is arrangement where the area is reducing it comes to the minimum and then increases. So this kind of an arrangement, if you say you plot -- So, let us say that the arrangement is like this we plot the Mac number verses X for 2 cases.

Assuming isentropic flow for 2 cases one is entry Mac number>1, another entry Mac number<1. So, if inlet Mac number<1.So this Mac number=1 inlet Mac number is 1/2 like this. So as you go through the converging section the velocity will increase, the Mac number will increase. So the Mac number will increase and it will and it come to a maximum when dA/A=0. Okay?

So it may not achieve one but it will come to a maximum and then, it will fall because the area of variation is difference. On the other hand, if the inlet Mac number >1, then the velocity will reduce in the converging section. Therefore, the Mac number will reduce it will come to a minimum, therefore here and then, will increase again. So if the increase is supersonic the throat may have the minimum Mac number.

If the increase subsonic the throat may have the maximum Mac number, but this are limited by Mac number=1. So, if it is supersonic it remaining supersonic for all the locations. It is coming only to minimum, here that is >1. On other if it is subsonic, it remaining subsonic everywhere at the throat, it is coming to the maximum. But still greater less then Mac number 1. So, from this, we conclude the very important thing.

It is not necessary that at the throat this location of minimum area called as throat. That at the location of the minimum area, it is not necessary that we always have the Mac number=1. But the converse is true, if we have at some location the Mac number=1, that must be at the throat. Okay? Again remember, there are many important assumptions associated with it. Assumption is isentropic flow, if it is a flow with friction and flow with heat transfer.

Then the location of this sonic point Mac number=1, shifts it is no more at the throat. It is at either the upstream or downstream depending on the heat transfer in the flow and friction in the flow. Okay? Now it is also possible to write the expression of dA/A, exclusively in term of dM/M. So, let us try to do that because it will give as exclusive variation of or exclusive relationship between Mac number and the area.

So, let us write say d. So, we know that the Mac number=CU. We are interest to express dU/U in some way in terms of dM/M. So, that we write dA/A explicitly in terms of dM/M. Sorry? U=CM right now, what you may do is? You may take log on both side and then differentiate. See, if for all this cases by this time you have released, that there purpose in taking the log and differentiating, because you are getting this type of expression dA/A dU/U by like this.

Taking log and differentiating, there is other ways of getting it this is the easiest way of getting such forms. So, logU=logC + logM and then you differentiate. So you will get dU/U=dC/C+dM/M. So, in an effects of expressing dU/U in terms dM/M. We are come up with new unknown dC/C. But C may be expressed in terms of the temperatures. So, let us give the equations some numbers, let say this is equation number 5 and say this is equation number 6. Now we know that for an isentropic flow.

See we writing all this equation for an isentropic flow C=root over gamma at T for an isentropic flow. Again, we take log. logC=1/2log gamma+1/2log R+1/2log T, that means dC/C=1/2dT/T, so dC/C=1/2dT/T. Therefore, you can write from equation 6. That dU/U=1/2dT/T+dM/M. Again, another new unknown dT/T as appear. But, till now we are not used the energy equation. That we may use so energy equation, what it was h+U square/2=constant? That was an energy equation.

So, you have dh + UdU=0, and for a perfect gas this is C pdT + dU=0, in place of C p, we can write gamma R/gamma-1. If you want it write in terms of dU/U, just divide both terms by U square. At U square=M square*gamma at T. So this is M Square gamma at T, so gamma R gets cancelled. So 1/gamma-1 M square dT/T + dU/U=0. So, we have been successful in writing dT/T in terms dU/U, so from this, we get dT/T = -Gamma-M square *dU/U, so this is 9.

So, if you combine 8 and 9 this is dU/U dT/T is written in terms of this 1/2 of this is 1/2 of the righted side and so that will become 1+gamma-1/2 M square dU/U=dM/M, Right? So that. We may substitute in equation number 5. Which is there to get dA/A=M square-1*in place of dU/U, we may write dM/M. So, from this equation, we may get explicit relationship between the area and the Mac number.

So you know, how area vary with X? You can easily get how the Mac number varies with X. So a given geometry A as the function of X, you will get output a Mac number as function of X and from, here you can clearly verify the statement that, we had made so M>1. See that the denominator always positive, because gamma is Cp/Cv, Cp>Cv always. So this is positive M square is positive denominator is positive numerator is dictating the sign.

If M >1, dA/A is positive means dM/M is positive and if M<1 dA/A is negative means dM/M is positive. Okay? So, whatever statement in terms of dU/U, similar logic holds for dM/M, hence this plots you verify the nature of this plots. Okay? Now other consideration regarding this types of flows is the achievement of sonic condition. So we have given some emphasis, on that we have and then, we have seen sonic condition at all as to be achieved that is achieved at the location dA/A=0.

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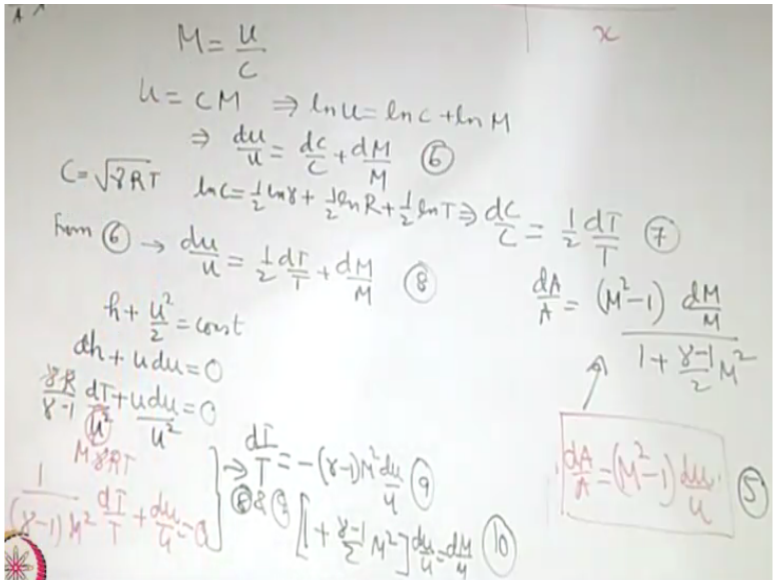

Although at dA/A=0, sonic condition need not necessary to be achieved. If is achieved at all achieved there. Now when the sonic condition is achieved the sonic states this usually given by quantity '*' in the description of the flow this is just a symbolic way of representing. So if at the state sonic condition is achieved. Let us say that a corresponding velocity is U we call it U *, corresponding pressure P v we call it as P *, corresponding density as rho *, corresponding area as A * like that.

The star quantities are important because, these are some important reference quantities. Remember that there may be cases when no sonic conditions is achieved at all look at this example, either the top curve or the bottom curve sonic condition achieved no were in the flow. So still U*, rho*, P*, A*. These quantities exist because these are hypothetical reference condition for example A* is, what A*? Is at a given condition, what could have been an area at which sonic condition would have been achieved not that the area is physically there in the flow.

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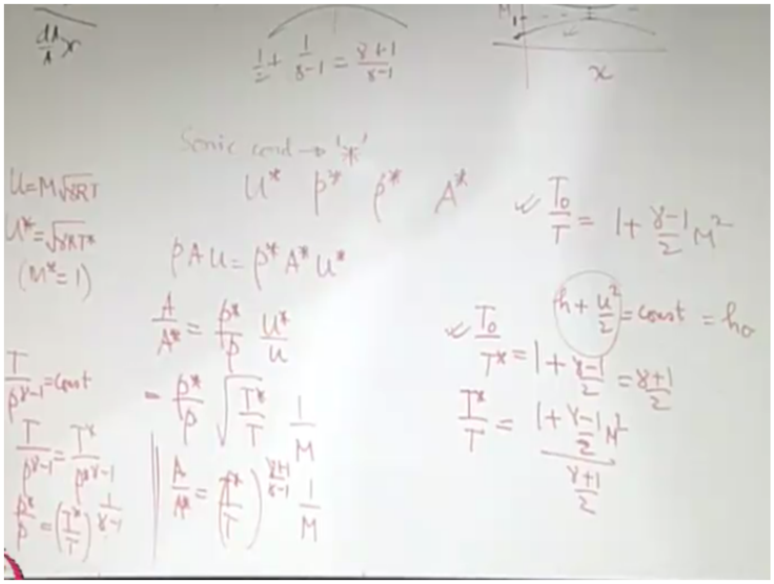

So corresponding to each state there is corresponding '*' quantity and, if you see tables of properties of compressible flow this '*' properties are there. So this are very important referral quantities not that this values have to exist in the particular flow condition but this is referral quantity with respect to which other quantities may be expressed. For example, if you have say a condition where the sonic state is achieved then you may write rho*A*U=rho *, A *, U *. Okay.

So, this is hypothetical condition, if the sonic state is achieved, it may be achieved in reality**.** It may not achieve in reality**.** Then, still it should satisfy the mass conservation. Okay? So you may write has A/A * as rho */rho*U */U. Remember that U = M * square root of gamma RT, Right? For an isentropic flow, when it is U *, it is square root of gamma RT *, because M *=1. '*' condition is sonic condition.

So, when you write star that means the Mac number at that state is 1. Okay. So this may be written as rho*U */U=root over T*/T*1/M, Right? How you can write T* / T? We may use the stagnation properties; we may use the reference with respect to stagnation properties. We had T0/T=1 + gamma-1/2M square. We just now derived this if it is a stagnation properties remember that, when it is an isentropic flow the stagnation does not change.

Why? For example, stagnation temperature does not change for an isentropic flow, why it does not change for isentropic flow? See consider the energy equation h + U square/2, this is a constant and this is=h0. So, what where may be, its h varies U also varies in such way that some this=constant. Therefore, h0 it is a constant h=C p=T, for a perfect gas that it is, why? To remains the constant infect it is not necessary that is has to be isentropic just for adiabatic flow T0=constant, because revisable condition never used here.

But, when you use this relationship it uses isentropic condition also because it uses T/rho to the power gamma=constant, for sonic speed derivation from which this expression comes. So T0=constant, that means you can write T0/T*, as what? What is T0/T*1+gamma-1/2? that is gamma+1/2, so you can write T*/T/dividing this 2 expression T*/T=1+gamma-1/2M square/gamma+1/2, Right?

Also, we know that T/rho to the power gamma-1=constant, for an isentropic flow, so you can write T/Rho to the power gamma-1=T */rho * to the power gamma-1. So rho */rho=T */T to the power 1/gamma-1. So A/A *, so you have T */T to the power 1/gamma-1*T */T to the power 1/2+gamma-1. So 1/2+, Sorry? 1/gamma-1, so gamma+1/gamma-1, Right? So this T */T to the power of gamma+1/gamma-1/M.

That means you have A/A*, sorry, one 1/2 is there to the power gamma+1/2*gamma-1*1/M. Right? So, what it tells is that given value of M you can get one A/A *, looking from the other angel given a one A/A*, you could have some value of M, but it is not a unique value of M you could multiple value of M that is very clear from this that is, because it is an equation having multiple roots.

So it will basically 2 acceptable value of M and if look into the tables of properties compressible flows usually these quantities are referenced for each Mac number certain important are given so you look isentropic flow table which in the next class we will look more carefully. So, if you have the isentropic flow important properties what are the important properties of for each Mac number? You can P/P0 T/T0 rho/rho 0 and A/A *.

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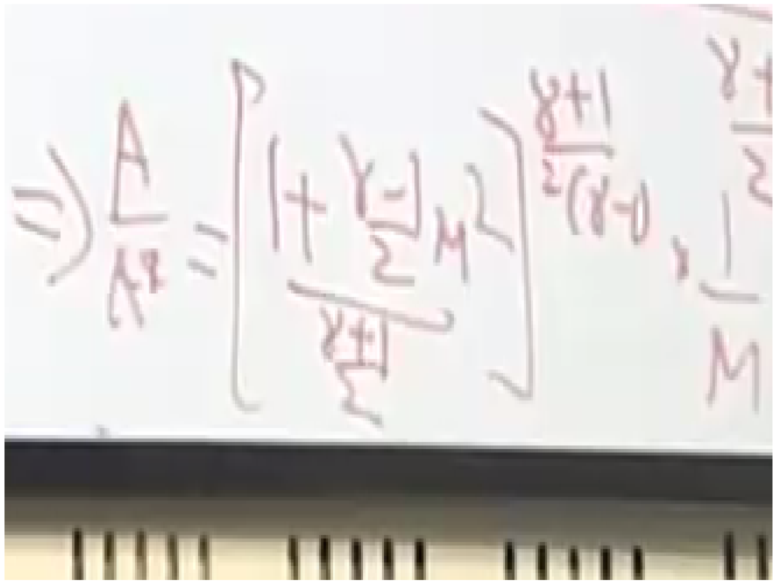

It is important to tabulate this because, if you know the Mac number, you can calculate all others, but if it is reverse problem, that is, if you know A/A *. Calculate what is Mac number all those multiple roots? You may have to solve nonlinear equations, but if you have a tabulated set of data you can just read from the table and that is, why? Compressible flow tables are there with every book in the chapters of compressible flow corresponding there will be an appendix where these properties are there.

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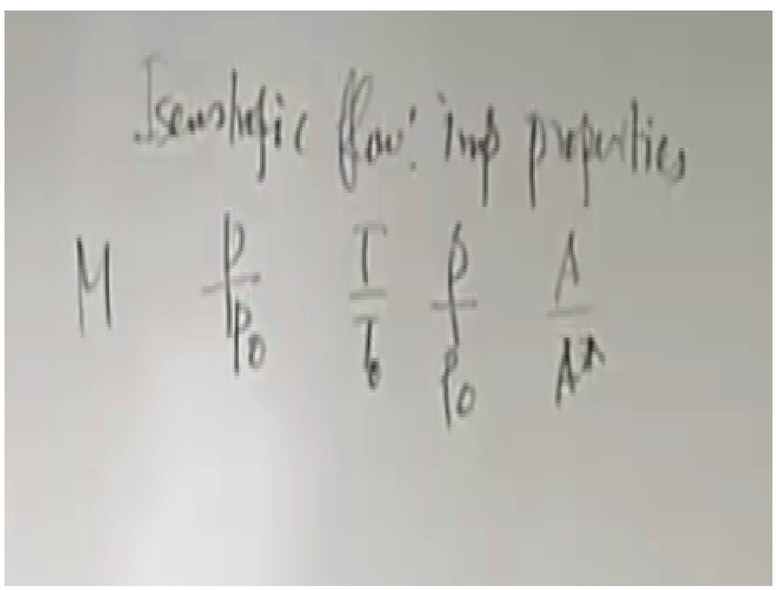

So, these properties will relate everything with reference '*' the reference either the stagnation state or the sonic important thing is that stagnation state is the same so long as it is isentropic flow if the isentropic nature of the flow is disturbed then the stagnation properties changes that means T0 is same so long as it is an isentropic, but if a isentropic nature is changed.

If is no more isentropic then the stagnation properties changed and one of the important mechanism that can create some abrupt change stagnation properties that is a shock this is presence of a shock wave. So in our next class we will see that how these properties get changed when you have abrupt discontinuity in a compressible medium represent in the form of the shock waves o that will take in a next class.