

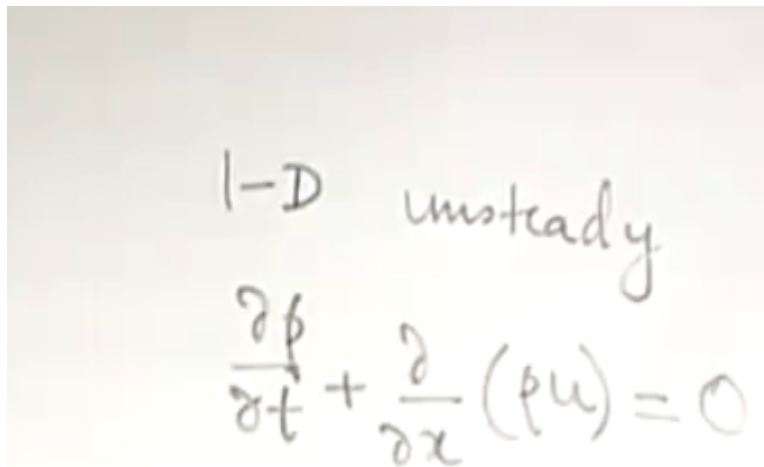
Introduction to Fluid Mechanics and Fluid Engineering
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Lecture – 54
Compressible Flows

So far in this particular course, we have emphasized more strongly on incompressible flows, but compressible flows are also very important and the scope of this particular course does not contain too much of an opportunity of discussing the details of the compressive flows but we will try to identify some of the important conceptual paradises which are associated with the compressive flows and we will see that how they may be strikingly different from that of incompressible flows.

So first of all let us revisit the requirement of compressibility or incompressibility to do that let us consider that we have one dimensional unsteady flow. So that if you write the continuity equation, you are having these particular terms.

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1-D unsteady

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) = 0$$

Now let us try to have a sort of a feel of what would be the corresponding form, if it is an incompressible flow. So to do that let us say that we divide this term into 2 parts. One is, if it were an incompressible flow then obviously that only term that would have mattered for that case is these term.

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$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x} (\rho u) = 0$$
$$\downarrow$$
$$u \frac{\partial \rho}{\partial x} + \rho \frac{\partial u}{\partial x}$$

Whereas if it is a compressible flow we expect that this particular term would also turn out to be important. So if this term does not turn out to be important then it is as good as treating the flow like an incompressible one. That means we are interested to investigate a situation when you may have a sort of effect which give rise to some kind of compressibility but the compressibility is not substantial. That compressibility will be substantial, when you have these term < this particular term.

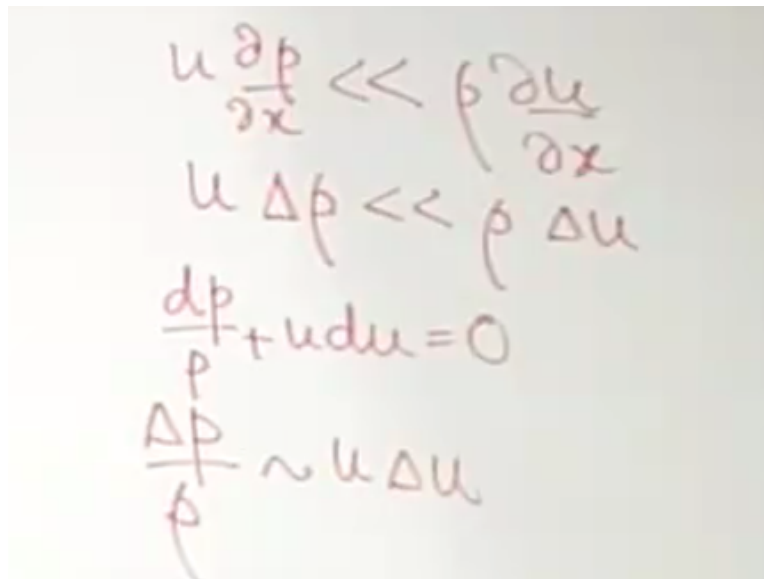
So that means if you just write in term of order of magnitude. $u \cdot \Delta \rho \ll \rho \cdot \Delta u$.

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$$u \frac{\partial \rho}{\partial x} \ll \rho \frac{\partial u}{\partial x}$$
$$u \Delta \rho \ll \rho \Delta u$$

Also just to get a feel of the relationship between the Δu , ρ and Δp . We may refer to the Euler equation. That is $\frac{dp}{\rho} + u du = 0$. Assuming negligible change in potential energy, so this we are treating it in an inviscid limit and try to figure out that what could be the important parameter that may dictate the extent of compressibility in the fluid. So this is as good as writing the $\Delta p/\rho$ is of the order of $u \Delta u$. So our requirement is $\Delta \rho/\rho$ is much $<$ $\Delta u/u$.

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Handwritten mathematical derivations on a whiteboard:

$$u \frac{\partial p}{\partial x} \ll \rho \frac{\partial u}{\partial x}$$

$$u \Delta p \ll \rho \Delta u$$

$$\frac{dp}{\rho} + u du = 0$$

$$\frac{\Delta p}{\rho} \sim u \Delta u$$

And in place of Δu you can write these as $\Delta p/\rho u^2$ is $\Delta u/u$. so from here what we can say is that $\Delta \rho/\rho$ is \ll or we can simply rewrite it, by writing in term of $\Delta p/\Delta \rho \gg u^2$. Right? So what is $\Delta p/\Delta \rho$? We will see, we soon that these $\Delta p/\Delta \rho$ is an indicator of the sonic speed within the medium. So let us say that c is the sonic speed within the medium these in fact may be written as c^2 .

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$$\frac{\Delta \rho}{\rho} \ll \frac{\Delta u}{u} \quad \frac{\Delta \rho}{\rho} \ll \frac{\rho u^2}{\rho u^2}$$

Where c is the sonic speed in that is velocity, the speed at which a disturbance propagates through a medium. So since u/c that is the fluid speed by sonic speed is known the Mach number. So we can say that this is as good as mach number square is $\ll 1$ in term of the order of magnitude.

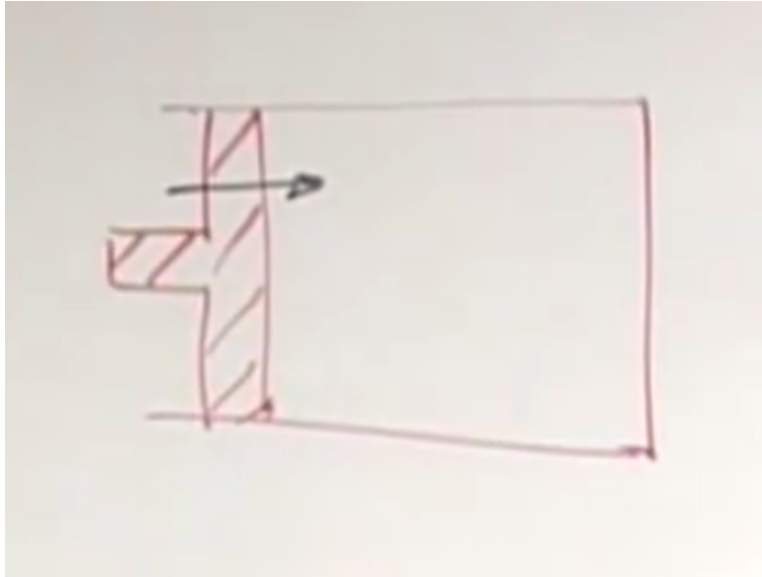
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$$\frac{\Delta \rho}{\Delta \rho} \gg u^2 \quad \frac{u}{c} = Ma$$

So from these what follows is that, if it is very low mach number case then the situation may be treated as it is like incompressible fluid. So from this very simple understanding what we can conclude is that one of the very important roles that is being played by the sonic speed and therefore it is important to know that what are the factor and what are the parameters on which the sonic speed depends.

To do that we will consider a very simple experiment. Let us say that you have piston cylinder arrangement and there is a container and there is the piston.

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So the container is like a cylinder. What is being done is that, let us say that this confinement. This piston says is moving towards the right with the particular speed. So when the piston is moving towards the right with the particular speed, it is forcing the molecules which are adhering to that we are disturbed and that means it creates a disturbance in the fluid medium that is there space between the piston and the cylinder.

Question is that what is this medium let us say that this medium is the gas. And a let us try to see what it could be a difference if replaceable by gas it would have been some liquid. Now what may be say this that if it is that a sort of a liquid or highly compact system then if you create a disturbance? Now if the question is if you create a disturbance and if it is liquid with high compactness.

The disturbance whatever it is, you may not be able to compress this further, because of the very poor compressibility of the medium. But what you may get assured, when try to impose the disturbance. This disturbance gets propagated from one point to another in the medium at the

infinitely large speed. So it is like all the molecules or all the fluid particles, if they are very compact they respond almost instantaneously towards this disturbance.

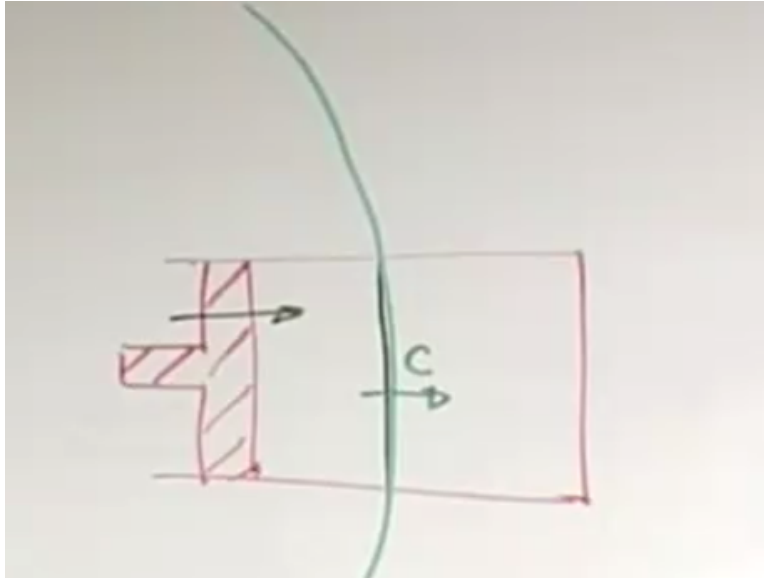
On the other hand, if the medium is not that compact then what happens? Then if they are certain molecules which are contact with these particular pistons then those molecules are first replaced from their mean position. So it is the sort of the pressure pulsation. So that pressure pulsation then it is propagated to some other molecules in the surrounding and from that to another set of molecules in the surrounding to maintain the continuity.

But all the molecules in the medium are not simultaneously disturbed by that pressure perturbation. So what happen is that there is sort of disturbance, The disturbance propagates through the medium at the finest speed and if it is sort of the weak pressure wave that propagates through the medium at a speed which is the function of some of the properties of the medium and one of their objectives will be to identify that how it depends on some of the properties of the medium.

But before that we may appreciate that their effect of this pressure perturbation they may reach at the certain location and beyond that location the effect is not failed at the given time. As the time progresses may be this location up to which the location of pressure perturbation is failed propagates further and further. So we may call these as a sort of a wave front remembers that is not exactly like a flat line that we shown in the diagram.

So the wave front may be typically a part of a very large sphere with the particular radius, but we are considering only the small part of it that is almost like a flat surface and these is propagating relative to the medium at a certain speed says c which we call as sonic speed.

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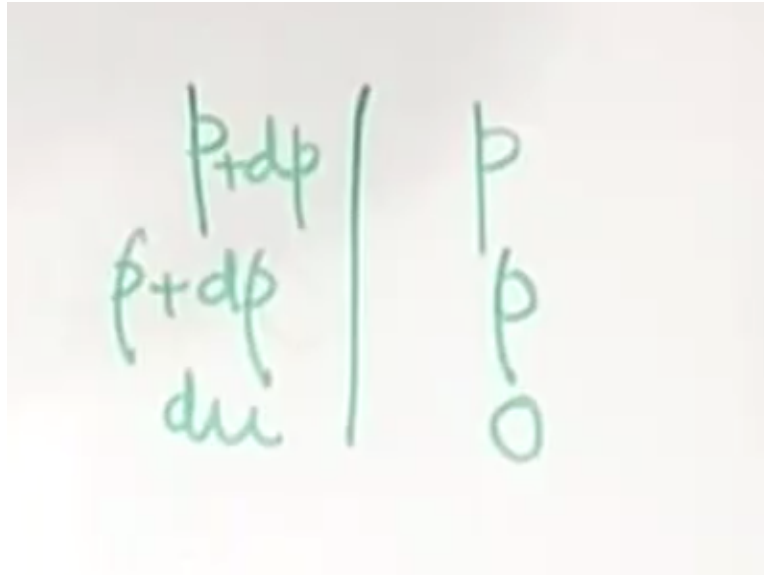


That is the speed at which the disturbance in terms of the pressure perturbation travels relative to the medium in form of a weak pressure wave. Now we need to identify what happens upstream and downstream of these. So if we just consider this particular line which is propagating towards the right at a particular speed c and because the line is very thin almost all properties are continuous across c .

We will see that not in all cases it may be possible that even if you have a limitingly thin interface across which properties are continuous. But here we will assume that the properties are continuous across this. Because this is just a normal wave front but if it is a short wave front then that may not be possible. So let us say that the pressure on this side is p or let us say pressure on the right side is p then the pressure $p + dp$.

In the right side the pressure is the density is ρ . Here the density is $\rho + d\rho$. In the right side the velocity is 0 and in the left side the velocity is said to be u .

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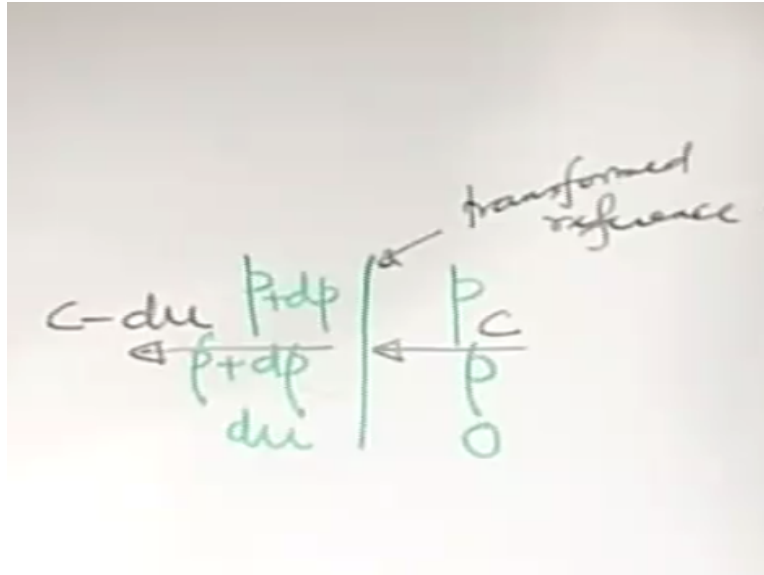


So this is undisturbed and this is disturbed. Now remember this front is propagating towards the right with the velocity u . To make the analysis it may be important to transform our reference to a front which is stationary. To do that what we will do as if we will add the speed of c but opposite to the towards the positive x direction. so that the speed of the front relative to the reference is 0.

And then all other quantities may that wit respect to that so team the situation will be that here we made a transformation as if you are sitting on the reference frame that moves with the velocity same as this one. So with respect to that moving reference frame moving with the velocity c . What would be the velocity of flow here? Remember what we are doing we are adding as if to make it stationary,

We are adding a velocity of $-c$. so this is in this direction c . And here $+du$ is along the positive x and $-c$ superimposed on that, so here it will be $c-du$ in the negative x direction.

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To maintain it this is the transformed reference. Now what will you do we will write equation of mass balance and momentum balance for a control volume that surrounds this particular front. First of all mass balance so conservation of mass, Now one of the important assumption that we are going to make here and possibly for most of the important fundamental curative derivation for compressive flows at least within a scope of this particular course is that

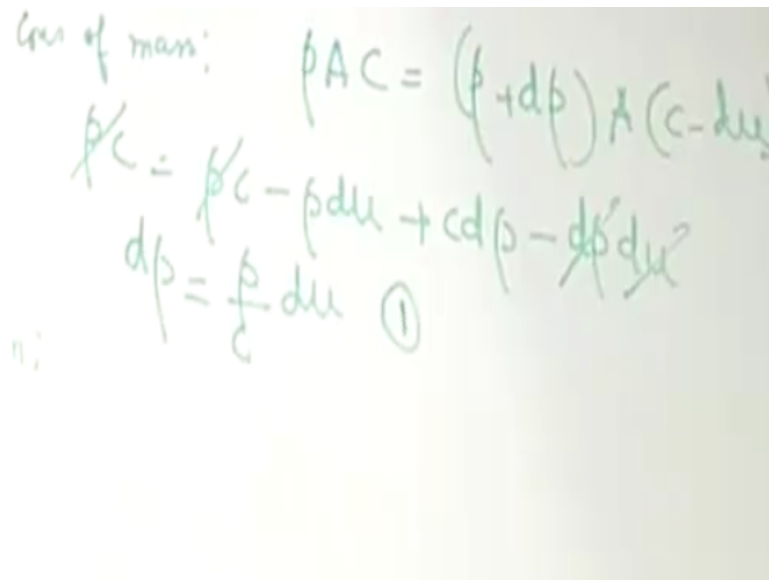
Flow is like a 1 dimensional flow. That means u is not a pairing with y so then conservation of mass what we can write that $\rho A C = \rho + d\rho A c - du$. Okay?

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$$\rho A C = (\rho + d\rho) A (c - du)$$

Where A is the area of cross section of control volume that is drawn. If the velocity is where non uniform on would have taken the average velocity in the place of this. Now from here what we can write is that $\rho \cdot c = \rho \cdot c - \rho \, du + c \, d\rho - d\rho \, du$. $\rho \cdot c$ get cancelled out. $d \cdot u$ is the product of differentially small quantities we may be neglected in comparison to the other terms. So we can write that $d\rho = \rho/c \, du$. This is what we get from conservation of mass c .

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Cons of mass: $\rho A c = (\rho + d\rho) A (c - du)$
 $\rho c = \rho c - \rho du + c d\rho - d\rho du$
 $d\rho = \frac{\rho}{c} du$ ①

Let us say this is the equation number 1. Next conversation of linear momentum, so the conversation of linear momentum what we get the resultant force that act on a center volume we just using the Reynolds Transport Theorem. The resultant force that acts on a counter volume is what? From the left side it is $p + dp \cdot f$ and from the right side it is opposite reaction $p \cdot A$. So $dp \cdot A = m \dot{v}_2 - v_1$.

So since all the flow shown in the negative x direction may be later write it conservation of linear momentum along negative x direction, so may be next the algebraic sign symbol. So $m \dot{c} - du - c$. right? In place of $m \dot{c}$ we can write $\rho \cdot c$. So these becomes $-\rho \cdot A c \cdot du$. So dp you can write $\rho \cdot c \cdot du$. okay?

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$$\begin{aligned} \therefore dp &= \rho du \quad (1) \\ -dp A &= \dot{m} [c(-du) - c] = -\rho A c du \\ dp &= \rho c du \quad (2) \end{aligned}$$

So now if you divide 1/2, what it gives $dp/d\rho = c^2$. Right?

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$$\begin{aligned} (1) \div (2) &\rightarrow \\ \frac{dp}{d\rho} &= c^2 \\ c &= \sqrt{\frac{dp}{d\rho}} \end{aligned}$$

So from here we may get an expression for the sonic speed which is the square root of $dp/d\rho$. Okay? So we get that an important property which sort of this related to the elasticity of the medium because what it shows, it shows the effect of compressibility directly that is in another words it is impressively related to what is the rate of change of density with respect to change in pressure.

So if you pressurize the system whether how strongly the density will change or not. So if you require large pressure to create a small change of density then c is very large, that means if when

you require a large pressure to get change in density when it is not a very compact system. When it is not a very compact system you require a very small pressure to create whatever change in density you want.

But more and more compact system is you require more and more pressure that means more and more pressure compact the system had and have will be the sonic speed. So the fact that this kind of sonic speed finites sonic speed existing it shows that it is not a perfectly compact system that we are talking about. So analogy here with mechanics are basic or classical mechanic is like between rigid bodies and deformable bodies, the rigid bodies and informal bodies.

Rigid bodies say you have disturbance all points equally feel the disturbance. If you have an elastic body there is an elastic response to it. So all points do not respond equally to the disturbance so it is compressible medium is like elastic body and in fact this is related to the elasticity properties of the medium. The next important question is that when you write $dp/d\rho$

It depends on the thermodynamic process by which it is occurring. So have a further simplification of that we must refer to certain processes. So let us try to see the feel of what kind of thermodynamic process talking about. In most of the example, we will be discussing the particular things in this chapter will be regarding to the special type of flow known as isentropic flow. So the process thermo dynamically is an isentropic process.

Isentropic process is just equivalent to something known as reversible and adiabatic process.

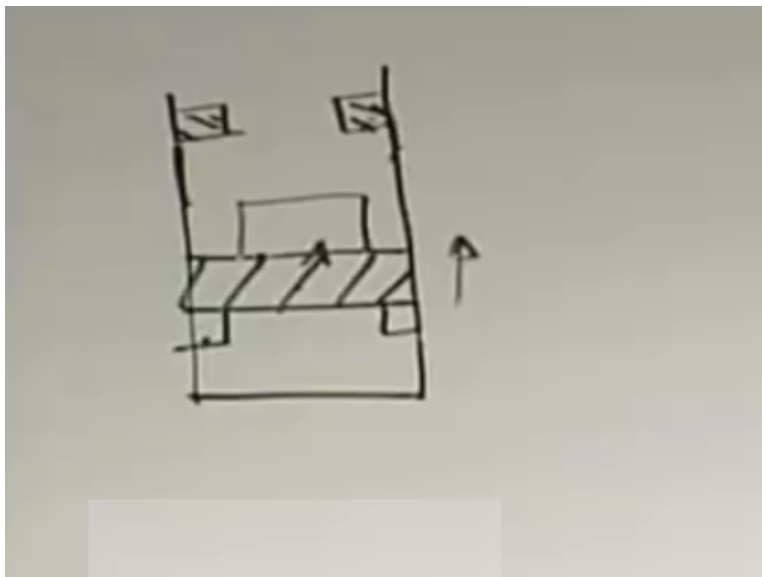
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Isentropic
 \equiv reversible
+ adiabatic

Of course this is not a course of thermodynamics. So we will not be going to details what is the reversible process and what is adiabatic process is straight forward. You know that the process where heat transfer is 0 is adiabatic process that is relatively clear but what is reversible process? So the reversible process is defined as such a process. So which it has taken place it may be reversed and why it is been reversed and then it leaves no change in a system or surrounding.

So how it is possible? so it may be possible only in a hypothetical environment that does not exist in reality because the reversible process has to be a very slow process. Example if you have a piston and cylinder like this?

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Now say that you are moving the piston in the cylinder upwards. So how you may move it upwards? One of the possibilities is that let say that we want to move it from initial level to the final level dictated by the stop. Where you want to stop, so one of the possibilities is initially it was restrained by a huge weight.

Which was pressing it with the bottom stop? Now take this weight away, you take this weight away it suddenly jumps from this location and reach the top, it is a very fast process. Now if you want to revert it back to its original state what will happen? Say it has come to this 1, now you put the load back and it will come back here, when it comes back here that it does not mean that it will come back to the thermodynamic state.

Its temperature, pressure all the state would be different from what it was initially and only way it can be made sure that you also return it back to the thermodynamic state is by making arrangement in this way, making this as the summation of very small slices, very small weights. Remove weights one after the other so that it will move only slightly from its bottom position to the top position.

Once all the loads are removed, so their loads are decided, it just reach the top most position. If you want to revert it back then put the loads back one by one slowly then it will come back to its original position. So if that is done then what will happen is this will be back to its original state spontaneously, on other hand if it moves very fast when it had its forward motion and then a reversed motion.

After that it would be found that inside its temperature is higher say as an example then what it started to bring it back to its original state some transfer of heat to the surrounding will be required and then in the process the system will come back to its original state. But the surrounding are no more at this stage because of the making transfer not a reversible process. Hence to have a reversible process one must have a slow process.

So in reality such a slow process is known as a quasi equilibrium or quasi static process. So in reality such a slow process may not be achievable. And therefore a reversible process in a reality

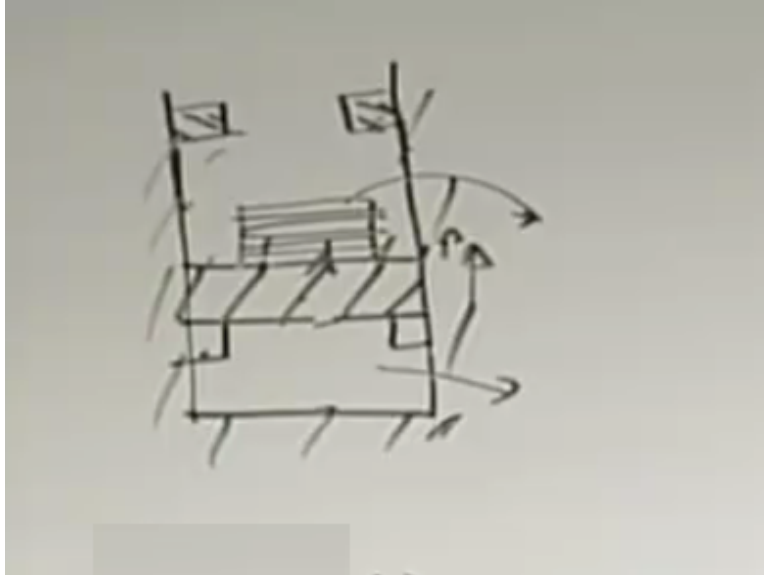
is something which is sort of an imagination. It may be only achieve by thought experiment but real processes may only approach that limit but it will not be really reversible, because if it really reversible it will take infinite time to complete the process.

Now if you are thinking of as reversible process they are many factors actually which are against it in a reality which makes real process reversible. One of the important factors is friction. Friction is such a factor will make real process reversible in a way we will not have enough scope here to discuss but keep one important thing in a mind. If you want to have a reversible flow or a reversible thermodynamics process it should be divides a friction.

Because friction is one thing that makes a real process deviated from a reversible one. When we say a reversible and adiabatic process we mean basically a sort of friction less adiabatic process and a very slow process. It will be striking to some of you who first learnt the adiabatic process is the fast process. Because that is how the adiabatic process are introduced in a school physics that you have piston cylinder arrangement and you make a process

So fast that insufficient time for heat transfer to take place and then you call it adiabatic. Yes? That is one of the ways of achieving the adiabatic process approximately but this is not reversible adiabatic process. To have reversible adiabatic process it has to be adiabatic, it has to be slow for example in this particular system a thought experiment you could have a reversible and adiabatic process by putting the insulation around the cylinder everywhere.

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So there is no scope of heat transfer to take place and insulating material you put and then make a slow change. So that makes a sort of reversible and adiabatic process. So we are thinking of to get a fair idea first that this process by which the pressure at the density change and related to each other a reversible adiabatic process. It may be shown that if it is reversible and adiabatic process of an ideal gas then you have $p/\rho^\gamma = \text{constant}$. Where γ is c_p/c_v ?

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$$\frac{p}{\rho^\gamma} = \text{const}$$

$$\gamma = \frac{c_p}{c_v}$$

Remember its misconception that it is valid from any adiabatic process of the ideal gas, it is only reversible adiabatic process of an ideal gas. If it is adiabatic but reversible this is not valid. The other important thing is that when you that this is the constant of what are the factor of c_p and c_v

depend. For the ideal gases it may be shown that c_p and c_v are function of temperature only. Such that $c_p - c_v = R$, but it does not mean c_p and c_v are constant.

This may be function of temperature but when you have the difference, the function of temperature gets cancelled off. But certain special type of ideal gases we will have constant c_p and c_v and those are known as calorically perfect gases. This is the difference between perfect gas and ideal gases. So for calorically perfect gas, you have c_p and c_v as constant not function of temperature. Ideal gas in general to c_p and c_v may be function of temperature.

So here we are discussing about perfect gases. So we will not explicitly tell all the time but in this particular chapter whatever substance we are considering is calorically perfect gas. Ideal gas with constant c_p and c_v . So for particular substance we are having this p/ρ to the power γ as constant as a process. So far let us then find out what is dp , $d\rho$ so if we say this reversible and adiabatic process why it is called as an adiabatic isentropic process.

Because if it is reversible and adiabatic process it may be shown that there is no change in entropy during the process. That is why it is called as isentropic process. So if we want to find out that what is the value C corresponding to isentropic flow then it better to write the in more formal way, that is partial derivative of c with the respect to ρ by fixing the entropy, then shows that it is reversible adiabatic process?

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$$C = \sqrt{\left(\frac{\partial p}{\partial \rho}\right)_s}$$

Because you could have many different types of thermodynamics processes by which you would have change in pressure and change in density. So if you have p/ρ to the gamma as constant you may take log of both sides and then differentiate so you get $dp/p - \gamma d\rho/\rho = 0$.

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$$\begin{aligned} \frac{p}{\rho^\gamma} &= \text{const} \\ \gamma &= \frac{C_p}{C_v} \\ C_p - C_v &= R \\ \Rightarrow \ln p - \gamma \ln \rho &= \ln \text{const} \\ \frac{dp}{p} - \gamma \frac{d\rho}{\rho} &= 0 \end{aligned}$$

So from here you can write $dp/d\rho = \gamma p/\rho$, so this is root over gamma p/ρ for a reversible adiabatic process. And we know from the equation of state that $p/\rho = R T$. This R keep in mind this is not a universal constant. Universal constant divided by the molecular weight of the substance so this is gas specific constant.

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$$\frac{p}{\rho} = R T$$

$$\bar{R} = 8.314 \text{ J/mole K}$$

$$R = \frac{\bar{R}}{\text{Mol wt}}$$

So the universal constant usually given as symbol of r by common literature that is 8.4 joule/mole kelvin. Kilo joule/mole kelvin or joule/mole kelvin. And these are this divided by the molecular weight. So it is for air, this turn out to be roughly 287 joule/kg kelvin. Of course molecular weight of air is not a very simple calculation. but you may be roughly considered it as some proposition of nitrogen and oxygen

And a like 79 is to 21 roughly and then calculate equivalent average molecular weight. So if you do that these will be value of R . so keeping that in mind we may replace p/ρ as RT this becomes root over gamma t . okay? So we can see that what are factor on which this sonic speed depends it of course depend on a gas component constant it depends on a cp/cv and the absolute temperature but we have to keep in mind that this not a general expression.

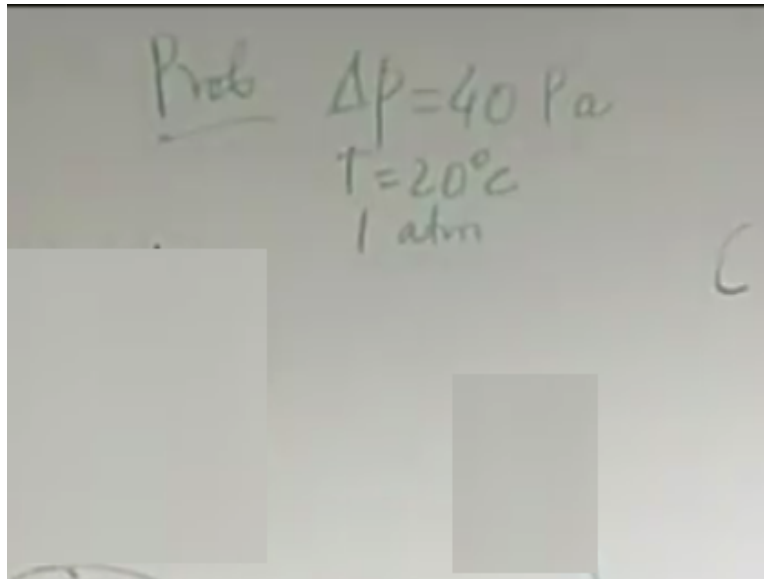
The most general expression will be this one because then depending on a nature of a process dp and $d\rho$ will be different expression this for an isometric process. So for a normal temperature condition this will be closed to 340 meter per second and since sound waves also propagates from a weak pressure wave in a medium, so it is natural coincidence that this is also same as the speed of the sound in a medium.

Sonic speed is not a speed in a medium by a fundamental definition. Because sound also propagates in a form of a weak pressure wave it coincidental and scientifically coincidental that

they are the same. Now let us work out the simple small problem to just illustrate some of the facts that we have discussed till now. So that problem is like is as weak pressure wave which is the sound wave

With the weak pressure wave with the pressure change of $\Delta p = 45$ Pascal propagates through still air at 20 degree centigrade and one atmosphere.

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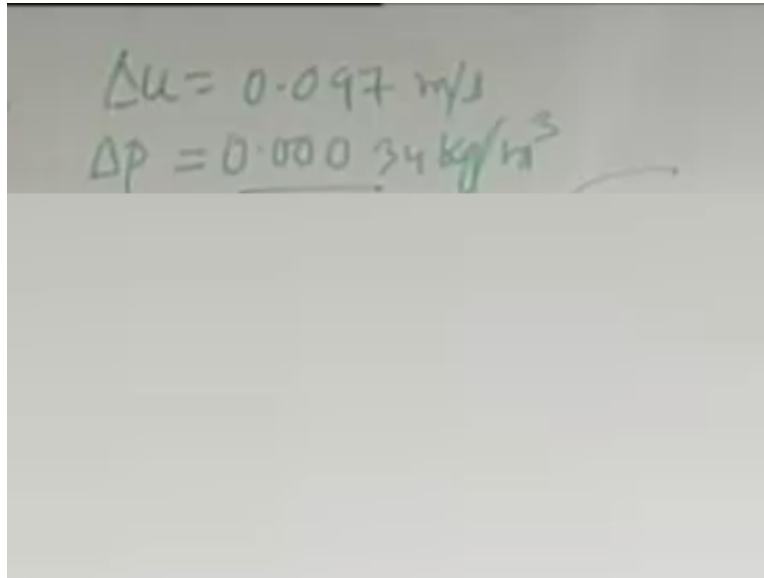


Estimates number 1 the density change and number 2 the velocity change across wave okay. The velocity change and density change across wave. It is very straight forward problem. It is the use of the formulas expression 1 and 2 for calculating the density change and the pressure change. So if you calculate the density change and pressure change first of all. you see here the $dp = \rho c du$.

Just look into equation number 2. So you have dp that is given that is Δp for T Pascal. From here you have to find out what is Δu ? The ρ of the medium you can find out given the temperature and pressure as one atmosphere. You can use the ideal gas equation to find out the density. Okay? So let me give you the answer so may verify this. But it is a straight forward use of this one.

So the answer are $\Delta u = 0.097$ meter per second and $\Delta \rho$ is $= 0.00034$ kg /meter cubic.
Okay?

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The image shows a handwritten note on a slide. The first line reads $\Delta u = 0.097 \text{ m/s}$. The second line reads $\Delta \rho = 0.00034 \text{ kg/m}^3$. The handwriting is in green ink on a light gray background.

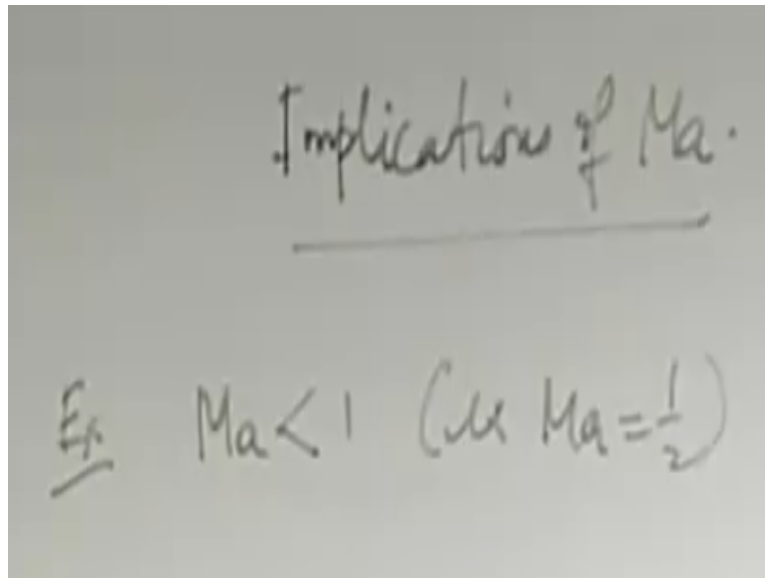
So for this expression you assume that R as 287 temperature remember. the value of the temperature could be absolute temperature so not 20 . $20+273$. Because all the calculation here where second law of thermodynamics is involved, will require absolute temperature. that is the only place where we have to be careful otherwise it will be a straight forward use of the formula that we have. Okay?

Now the next thing we will try to see is that still now we see what is the sonic speed? What is it physical simplification and what are the parameter on the sonic speed depends? The next question will be that we have seen in that there is the important non dimensional number + mach number which is the ratio of the fluid speed to the sonic speed. And therefore it might have it is important consequence so far as the extend of compressibility.

Concerned that also we have seen, so how that mach number influences the nature of compressibility of nature of propagation of disturbance within a particular medium. Let us try to look into that bit more carefully. So we consider some of important limiting cases. So this is just to have a physical understanding on the implication of mach number. So as the first example let

us take say mach number $\ll 1$. Just let us say the mach number $= 1/2$. Let us say that source of disturbance which we are approximating on the point source.

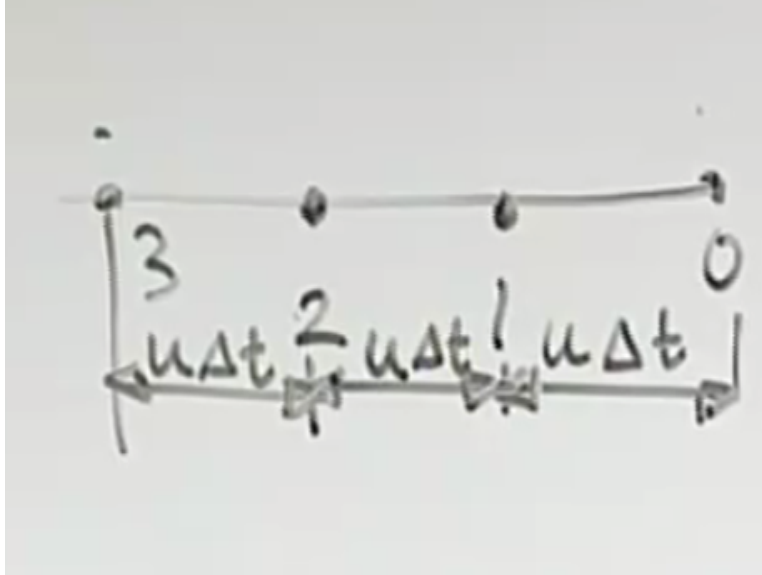
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The source is moving in the medium with the certain speed. And let us say that speed is u . So the medium is stationary and the source is moving with the speed u as good as relative to the source the fluid is flowing with the speed of u but in opposite direction. So there is the source which is at a time $= 0$ here.

Then it moving along this direction say at time $= \Delta t$. It is there at a position 1, Which is given by $u \cdot \Delta t$. Say it is moving towards the left that is relative to the source flows moving towards the right. Then at time $2 \Delta t$ it is there at point 2 and so on.

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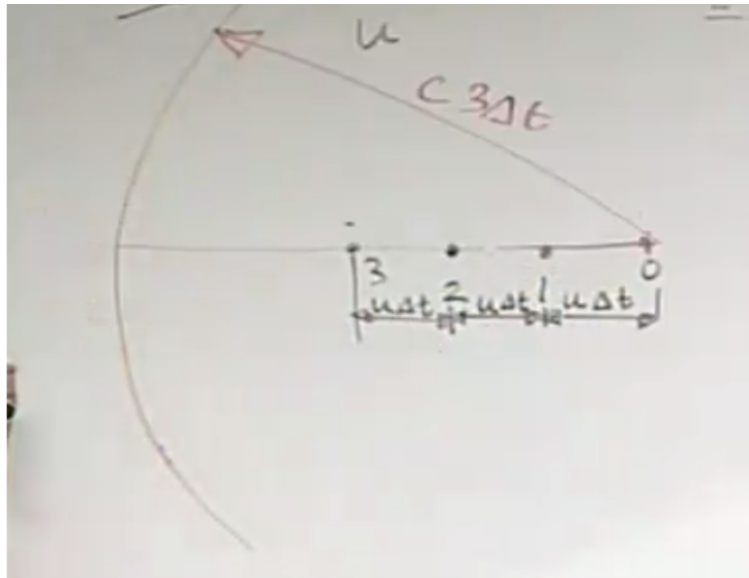


So we consider another time at $3\Delta t$ it is at a point 3, which is again another distance $u\Delta t$. In this way the source is moving and from each of this point through which, the source passed waves are emitted and when the waves are emitted see from the point. Let us say that we are now considering the current incident of time. So at the current incident of time, how much time elapsed $3 * \Delta t$.

So the time $3 * \Delta t$ if there was a wave that was initiated propagated from the point O then how much the wave would travel in a time $3 * \Delta t$. C is the disturbance of propagation of the wave. So $C * 3\Delta t$ would be the distance at which the wave would travel. So you have mach number = $1/2$ that means $u / c = 1/2$. That means in time $3\Delta t$ the wave will propagate by a distance of c will be $2u$.

so that means double of this one. So by taking these are the centre and $6u\Delta t$ as a radius, if you draw a circle in all side it will be very big circle. Then that will be a wave front in the plane of the figure. The radius of the circle is $C * 3\Delta t$ where c is $2u$. okay?

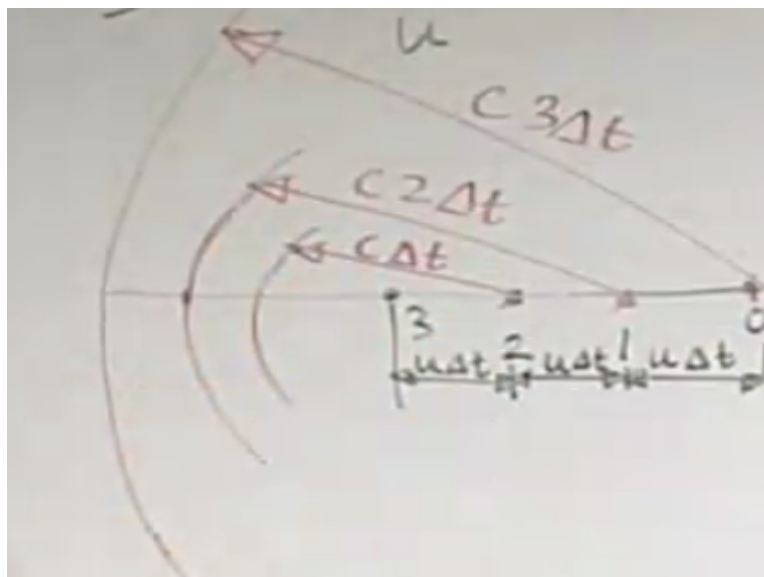
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Thus from the point 1, when source was a point one after that how much time as elapsed. $2\Delta t$. so by this time now. This as emitted something which will go by a distance of $c * 2\Delta t$. so by taking these as the center and $c * 2\Delta t$ as radius this is the wave front according to the wave that was emitted from one. Similarly, when the source was a 2 after that Δt time t is fast so it is emitting a wave. That is now having a front like this, where c is $2u$. okay?

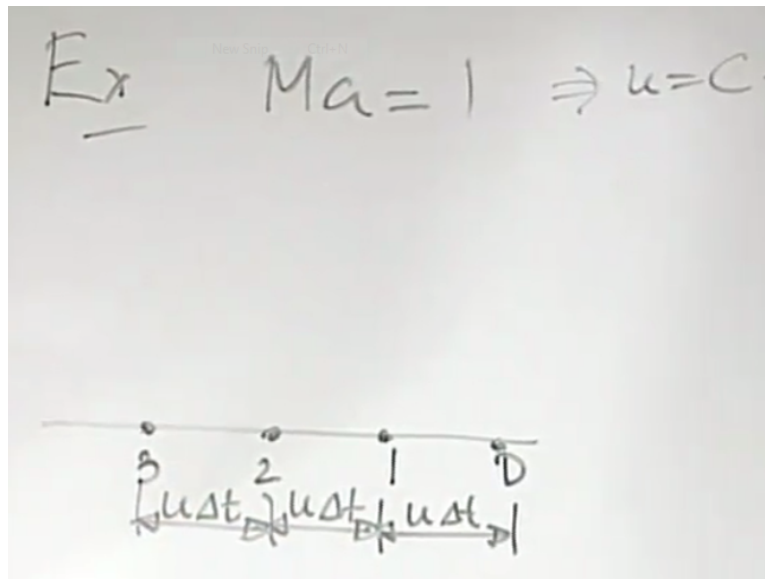
So you can see that the disturbance c is always ahead of the source. And the disturbance propagates in all direction just because the figure will be clumpy have not drawn the other part of the figure. It is too big to draw here.

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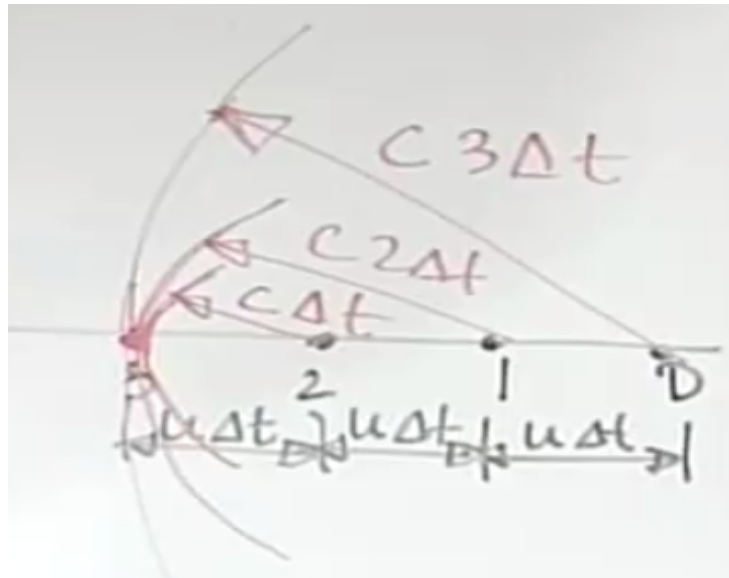
But imagine this full circle so that the disturbance is propagated in all the direction. And the heat moves ahead of the source. That mean if you think of this point these point knows the message that there is the disturbance before the source of disturbance comes there. This is very important. Now let us consider the second limiting case say mach num = 1. So when you consider mach num = 1, again let us take some examples but that the different mach num. number so 0, 1, 2, 3 like that. So here mach num = 1 that means $u = c$. okay?

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So if you construct similar things for just as a wave front we have drawn for the previous figure. C and u are the same. We will see that these fronts are still circles. But having a sort of common tangent at the point 3 right?

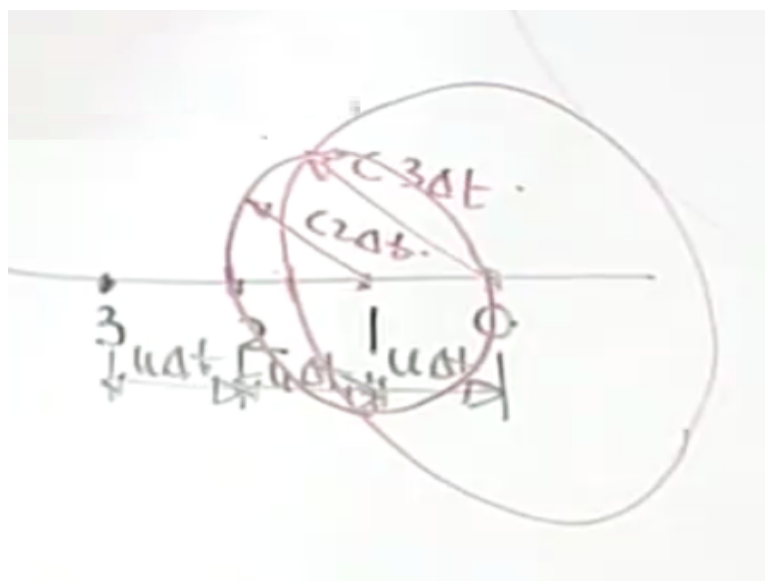
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So this is the limiting case. let us take the third example mach number > 1 . Again we draw the similar figures. All the intervals at $u \Delta t$. so when the wave is emitted from 0 in time let us take the example of mach number = 2. That means $u/c = 2$. So when a wave is emitted from 0 in a time of $3 \Delta t$ it will have it is zone of influence as $c \cdot 3 \Delta t$, then $c = u/2$ that means it will be circle with $\frac{1}{2}$ of a radius of the distance has between 0 to 3.

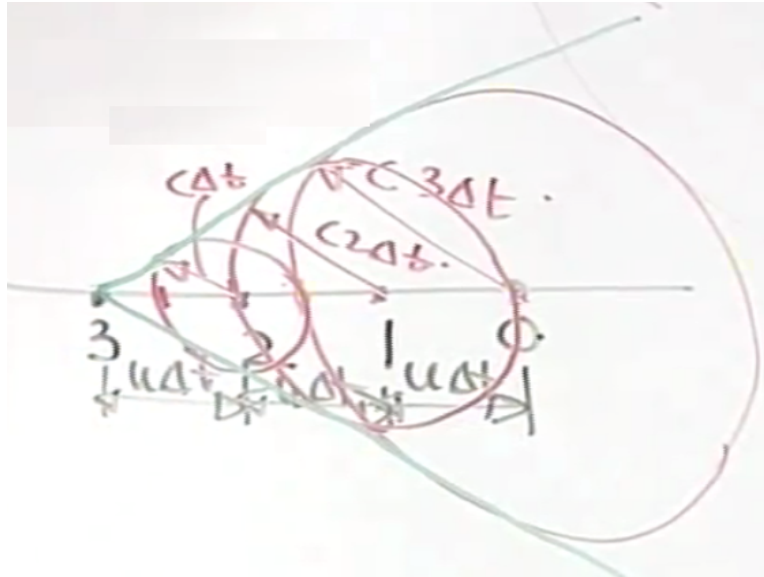
So these will be $c \cdot 3 \Delta t$. when it is at one by the time $2 \Delta t$ it will emit a wave with radius $c \cdot 2 \Delta t$. where it same as $u \cdot \Delta t$. okay?

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When it was a 2 similarly this is $c \cdot \Delta t$. So in this way it is possible to draw envelopes of this particular figure so it gives something which is very intrinsic. So if you draw an envelope of as a common tangent to all this circles. What does this line represents see all the wave confined within this zone? this like a cone in a 3 dimensional space.

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So these green color lines are bounding lines edge view of that bounding line of a cone. So all the waves are confined within that and the disturbance can be understood only if the wave is propagated at a particular location. So outside these, at any point the disturbance is not failed. And within this cone the disturbance is failed. So within this zone is known as zone of action and this is known as zone of silence.

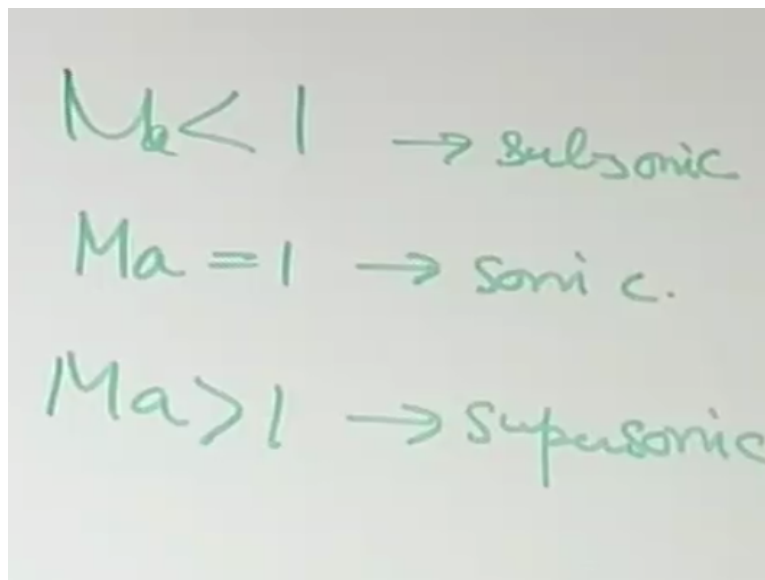
And the cone which is generated is known as mach cone and the semi particle angle number of this cone which is obtained very easily. Let us say that μ is the semi particle angle of cone. So you can write.

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Then therefore there is strong discontinuity across this boundary lines. And as if the distance is propagating following this bounding lines. So on one side disturbance is failed and on other side disturbance is not failed. And there is sharp discontinuity across the wave. These types of lines are known as characteristics of the physical system. Here if we think the corresponding partial differential equation these are known as hyperbolic partial differential equation.

And these lines are known as characteristics lines of the corresponding hyperbolic partial differential equation. Will not going to the mathematical theory of these thing the interesting observation, if you see that say if a aircraft is moving with such a speed so roughly you see mach number < 1 is relatively at low speed flow and these are known as subsonic flow. So mach number $= 1$ is sonic. Mach number > 1 is called supersonic.

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Handwritten notes on a green background:

- $Ma < 1 \rightarrow \text{subsonic}$
- $Ma = 1 \rightarrow \text{sonic}$
- $Ma > 1 \rightarrow \text{supersonic}$

Thus some important names is it revolves around the sonic which is mach number $= 1$. Why sonic? There is sonic speed at a same at that particular condition. Close to mach number $= 1$ is there are certain names it is known as stream sonic. So it is like close to mach number $= 1$, but one may have a slight height variation that we some mach number say within the range point 8 to one point.

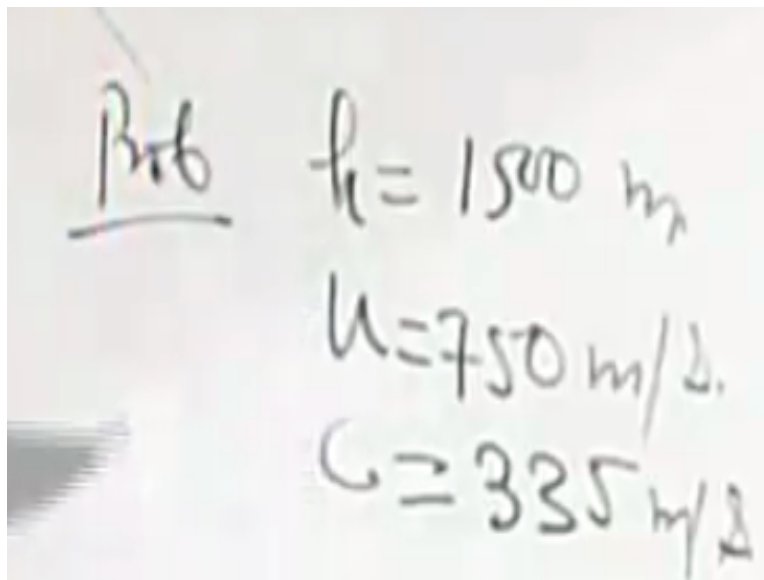
2 or whatever like near sonic region. Is like a transonic region. And very high mach number are known as hypersonic flows. So very high, anything > 1 is supersonic. So very high value of

mach number typically > 3 is known as hypersonic flows. So if you have a supersonic flow, then what is the important thing observe say observe a jet plane moving in a sky. So we will see that once jet plane as moved.

After some part of the movement then only you realize that its sound you are getting. And you will see that smoke emitted from that it forms that cone. If you observe jet plane moving in the sky, you will see it will form this type of environment of disturbance. Once you are with envelope of disturbance then only you will feel that that is why there at some distance such that you are within that cone. Then only you will feel that you have heard the sound of that otherwise.

If you are outside the mach cone you are not hearing that let us work out the problem to illustrate these. So the problem statement is like this a supersonic aircraft fly's horizontally at 15000-meter altitude, so there is supersonic aircraft which is moving at a altitude of $h = 15000$ meter with the constant speed of 750 meter per second.

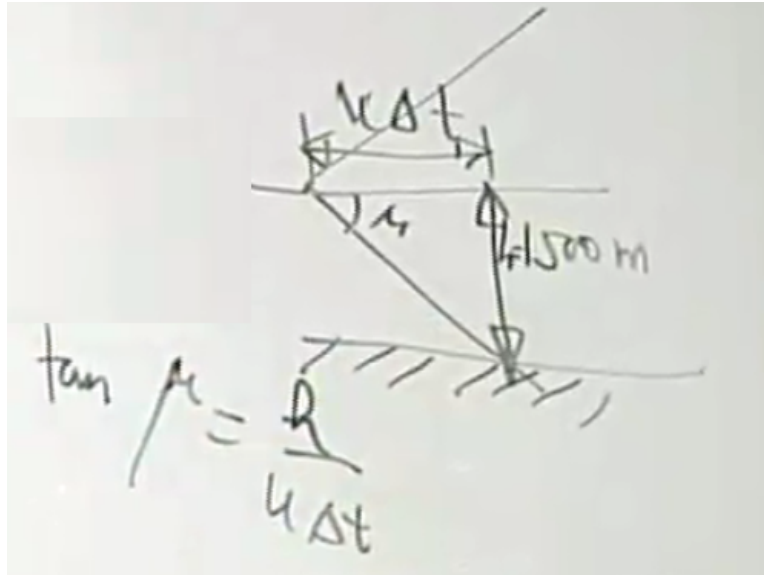
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Prob $h = 1500 \text{ m}$
 $u = 750 \text{ m/s}$
 $c = 335 \text{ m/s}$

The aircraft passes directly over the stationary ground observer. The aircraft passes over the stationary ground observer how much time elapses after it as passed over the observer before the observer hears the aircraft? Assume the sonic speed as 335 sound wave meter per second. And the aircraft creates the small disturbance that may be treated sound wave.

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So if you schematically consider it there is mach cone like this and consider that this is the ground. Observer standing here and it is 15000 meter, height at which the aircraft is moving .in the time interval of Δt the aircraft / $v \cdot \Delta t$. where v is the origin $u \cdot \Delta t$ where u is the speed of the aircraft. We find now what is the Δt ?

So that is the now comes within the mach cone and this is the limiting configuration when the observer comes within the mach cone. So these angle is given by μ , $\mu = h / u \cdot \Delta t$ we know what $\mu = \sin^{-1}$ of $1/\text{Mach number}$ from that periodic diagram and mach number is u / c both are given, therefore μ may be obtained h is given, u is given so from here you can find what is Δt . and answer to this is 4.0068 okay?

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$$\tan \mu = \frac{R}{4 \Delta t} \Rightarrow \Delta t = 4.86 \text{ s}$$

So we have seen that what is the important application of different ranges of mach number. Sand in terms of propagation of disturbance and how that be related to the sonic speed and in the next lecture we will go ahead with these and we will move on into some more interesting and important of compressive flows that we will take up in the next lecture. Thank you.