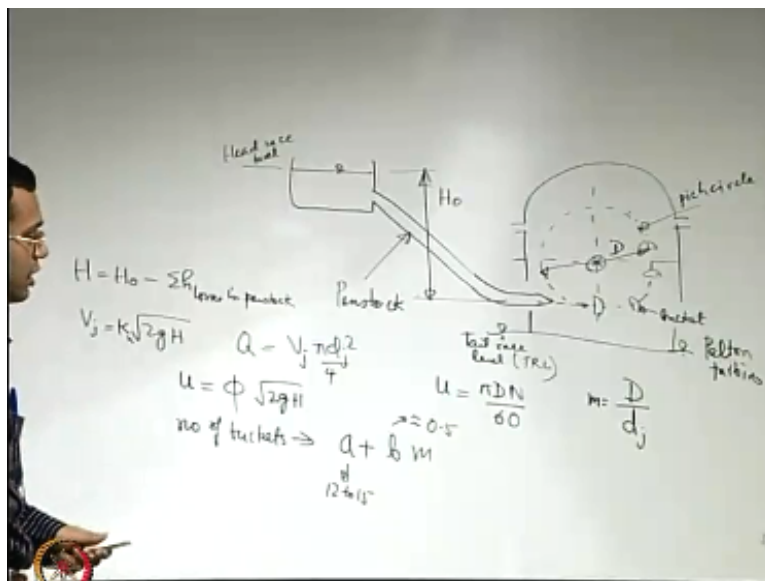


Introduction to Fluid Mechanics and Fluid Engineering
Prof. Suman Chakraborty
Department of Mechanical Engineering
Indian Institute of Technology – Kharagpur

Lecture - 53
Introduction to Fluid Machines

We continue with our discussion on the hydraulic turbines and we now try to develop a bit of a greater insight into the Pelton turbine working principle.

(Refer Slide Time: 00:32)



So if you have a nozzle here which is sort of diverting a fluid jet. What is the head that is available with the nozzle that is H_0 - summation of all the head losses in the penstock. And ideally the nozzle should develop a velocity or the velocity of the jet as what $\sqrt{2gH}$ that is the entire head could be converted into the kinetic energy of the jet, but in reality that does not happen because there is some loss in the nozzle.

So it is multiplied by some fraction K_n which is sort of the nozzle efficiency so to say and it is not very, very < 1 . It is quite efficient, but it may be something of the order of 0.95 like that. And you can find out what is the flow rate that the nozzle is supplying it is $V_j * \pi d_j^2 / 4$. Now the next thing is when you consider that what is the speed with which the jet is following on the bucket.

Next is that what is the speed with which, what is the rotational speed with which this wheel will rotate and if you have a tangential speed u then the tangential speed u is nothing, but that

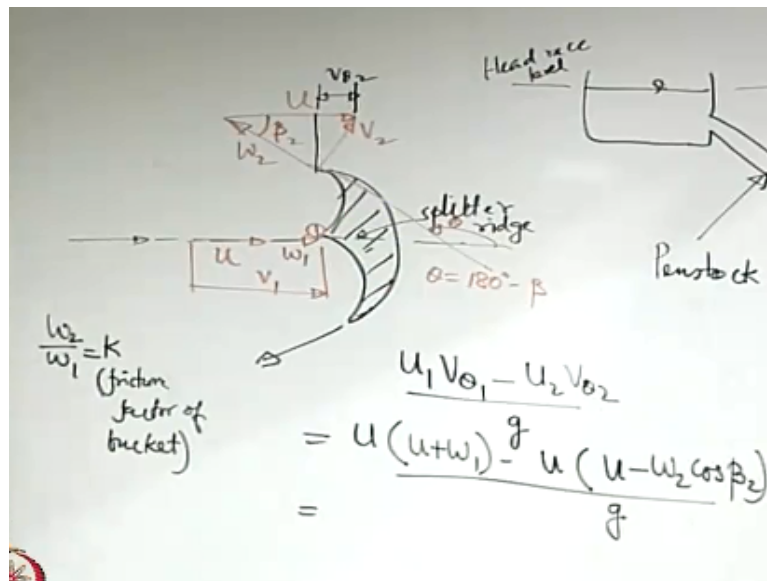
is $\omega * r$ so $\pi DN/60$. This is the tangential speed at the pitch circle so that you have to understand. This is related to the head with a particular efficiency because there is a relationship between say an optimum value of this u and the optimal value of the v at which the fluid is falling on the bucket.

So typically this is some parameter $\phi * \sqrt{2gH}$. We will see that theoretically an ideal value of this parameter ϕ 0.5 we will prove that theoretically, but the practical value is a bit less than that because of the losses in the nozzle and so on. So the practical value is like 0.46, 0.47 like that, but theoretical value of this is 0.5 that we will show. Now there are many other important parameters. One important parameter is the ratio of the diameter of the wheel is to so the diameter of the wheel is like this one that is capital D is to the diameter of the jet.

And this is a very important ratio and the number of buckets on a wheel is depended on this ratio. So what do you expect if this ratio is higher the number of buckets on the wheel will be more or less? It has to be more because for a given jet you have high diameter π or high diameter wheel so that should be if the pitch between the buckets is sort of something which you consider as something standard then you should have more number of buckets.

So typically the number of buckets will be of the form of say $a + b m$. A maybe like something between say 12 to 15 this is roughly like 0.5. These are rough figure and this come from some design considerations we do not have enough scope to discuss of all these design considerations. Now what we will do here is we will consider some of the basic theoretical aspects of this performance of the buckets.

(Refer Slide Time: 04:40)



So if you consider if you draw the bucket in the way in which it is facing the fluid it is something like this. So this is the bucket the water is coming from this side. It is interacting with the bucket and the bucket is deflecting it. So actually the water jet which is coming is getting split into 2 parts and there is something there is a particular structure which allows it to split it into 2 parts. This is known as a splitter ridge.

The flow rate may be controlled by something known as spear which is there in the nozzle. What it basically does? You can allow it to move axially in and out. So if you take it axially in then what it does is it restricts the flow passage. If you take it axially out then what happens it allows more flow passage. So there is something which can move along the axis of the nozzle and depending on its axial locations it may allow a particular area of flow and therefore a particular flow rate.

So this spear is something which is because of its movement it can control the rate of flow. So with that control rate of flow say there is some fluid which is coming and falling on this bucket. So we are just drawing one bucket and let us consider concentrate on what happens in one side of the bucket. So if you want to draw the velocity triangles. So first of all you have a velocity of the wheel as u you have the velocity of the water relative to the wheel.

And that is w . We are using the same symbol that we use for the pumps and here all those are in the same direction and the absolute velocity v_1 is the sum of the here just the scalar sum because they are in a same direction fundamentally the vector sum. Now when the fluid leaves the blade passage it has a change in direction. So it has a relative velocity with respect

to the bucket.

So that is w_2 then u_2 and U_1 are the same it is the same u because it is the same radius no matter what the inlet or the outlet this is the same radius and therefore the resultant velocity is the vector sum of these 2. So u_2 and u_1 are the same and this angle let us say this we call that as β_2 just by following the same nomenclature as we used for the pumps. And this when we consider that means there is a deflection of the jet by an angle θ where θ is $180^\circ - \beta_2$.

So let us find out that what is the power developed. So what is the power developed here? So you should have $u_1 v_{\theta 1} - u_2 v_{\theta 2} / g$ as the head developed. So the fluid had a particular head with which it enters. It sacrifices some head and its head came down. So this is a difference that is a head that is being imparted to the rotor. So $u_1 v_{\theta 1} - u_2 v_{\theta 2} / g$ the same formula just remember it is the reverse for the pump because the direction of energy fluid is reverse.

For the pump it was $u_2 v_{\theta 2} - u_1 v_{\theta 1} / g$. Here is the other way. So 1 and 2 points are just exchanged. So you have U_1 what is $v_{\theta 1}$? $v_{\theta 1}$ is v_1 that is let us write U_1 and u_2 same as u because they are the same. So in place of U_1 we are writing u in place of $v_{\theta 1}$, $v_{\theta 1}$ is same as v_1 that is $u + w_1$ from the velocity triangle. What is u ? U_2 is $u v_{\theta 2}$.

What is $v_{\theta 2}$ so if you consider this triangle this is $v_{\theta 2}$. So $v_{\theta 2}$ means $v_{\theta 2}$ is $u - W_2 \cos \beta_2$. Now typically there is a ratio between W_2 and w_1 which is called as a friction factor of the blade or here the bucket friction factor of the bucket. So you have $W_2 = k w_1$ because the physical reasoning is straight forward because of some friction in the flow passage there is some reduction in the relative velocity. If it is a frictionless flow, then $k=1$.

(Refer Slide Time: 10:12)

Handwritten notes on a whiteboard showing the derivation of wheel efficiency for an impulse turbine. The notes include a velocity triangle diagram on the left, a schematic of a Pelton turbine on the right, and a series of equations in the center. The equations start with efficiency $\eta_h = \frac{\dot{m} g H}{\frac{1}{2} \dot{m} V_1^2}$ and lead to $\eta_h = 2 \frac{u}{v_1} (1 - \frac{u}{v_1}) (1 + k \cos \beta_2)$. The final result for maximum efficiency is $\frac{u}{v_1} = 0.5$.

So let us consider $w_2 = k w_1$ so this here if you take u common this is $w_1 + k w_1 \cos \beta_2 / g$. So $u * w_1 * 1 + k \cos \beta_2 / g$. You can write w_1 as again $v_1 - u$. So $u * v_1 - u * 1 + k \cos \beta_2 / g$. For impulse turbines there is a particular terminology called as wheel efficiency. Usually hydraulic efficiency is used wherever you have pressure change as well. So here you just have some energy change based on the input form of energy which is the kinetic energy form.

So the wheel efficiency it is again indicator of the energy conversion efficiency of the wheel. It is defined as this like whatever is the energy whatever is the power output because of that. So $\dot{m} * g * H$ if this is H that is the power output. The power input is nothing, but $\frac{1}{2} \dot{m} v_1^2$ that is the kinetic energy input where \dot{m} is $\rho * Q$. So you can write this one as $2 u * v_1 - u * 1 + k \cos \beta_2 / v_1^2$.

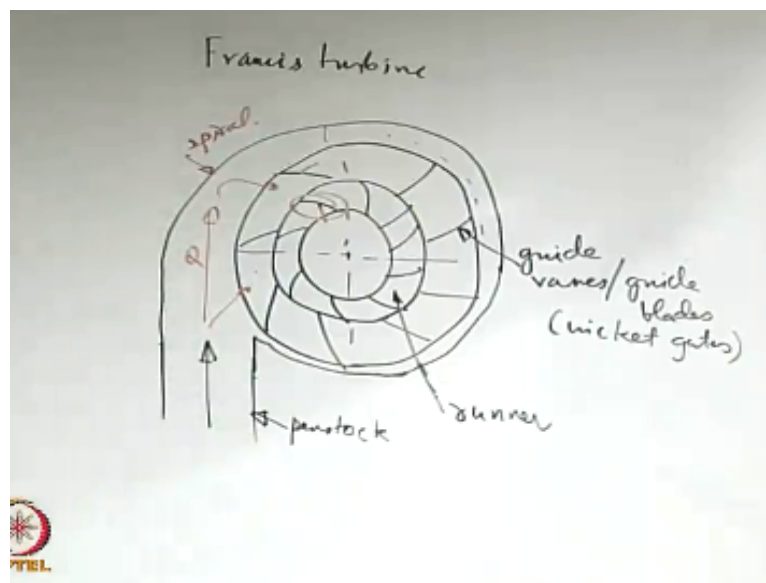
So it may also be written as $2 * u/v_1 * 1 - u/v_1 * 1 + k \cos \beta_2$. So you can find out so what it says. It says that what is the efficiency in conversion from the kinetic energy form to the energy that is the output from the turbine. So if you want to have a maximum value of this then that means most efficiency or most effectively the kinetic energy of the jet is converted into useful work.

So for maximum value of this you must have the derivative of the wheel efficiency with respect to u/v_1 . u/v_1 is the parameter here others things are geometrical parameters that is $\beta_2 = 0$. So from here you will get that $u/v_1 = 0.5$. So this is called as an optimum speed ratio. So speed ratio is the ratio between the speed of the wheel to the speed of the water that is falling on the wheel and an optimum value of that is 0.5.

As we mentioned that this optimum value may not be the reality because in practice there are losses and so on in the nozzle so this value maybe a bit deviated bit < 0.5 maybe 0.45, 0.46 like that. Now the next type of turbine that we will discuss is the Francis turbine. So when we discuss about the Francis turbine just for the sake of convenience we may keep in mind that as if we are inverting the operational direction of the centrifugal pump.

It is not exactly like that it is an over simplification, but somehow it helps in initial understanding.

(Refer Slide Time: 14:40)



So what you have in a Francis turbine. So we will first consider blade passage and then some of the more details. So you have a runner we have mentioned that the rotor of the Francis turbine is called as a runner. So you have a runner with certain blades. These are rotating blades, but there should be something which would be giving a fixed direction to the water falling on these blades.

So for that you should have certain blades which guide the flow and those are called as guide vanes or guide blades. In some cases, those are given alternative names as wicket gates. So these are rotating blades we will now on the top of this have some guide blades that we are drawing it schematically. So these are known as guide vanes or guide blades or sometimes wicket gates.

Now let us say that penstock. So penstock is the pipe through which you are having supply of

the water to the turbine runner. So this is the runner. So now what is happening is you are having a supply of flow in a way such that the area available for flow is continuously reducing. If you consider the flow passage or flow path that will be quite clear. So you have some water entering like this is the penstock.

So when the water is entering like this see first of all it goes through this. So this is in the shape of something like a spiral very much analogous so the (()) (17:22) of a centrifugal pump. So this is a spiral casing and you will see what is the role of this one. So continuously the fluid enters the blade passage. Say first some fluid enters the blade passage here. So when the fluid enters the blade passage you have the same rate still you want to maintain the same rate of flow.

Because you want to have the entire runner receiving the same rate of flow and that is possible if you continuously reduce the area. So what is basically happening is some flow is entering here so there is a reduced rate of flow so what I meant is the same rate of flow is not literally the same rate of flow, but flow velocity. So if you have a particular Q here because of the entrance of some water in this passage.

The Q has reduced so the area will also continuously reduce so that the velocity which is Q/A that does not get change as the fluid is entering different places in the different locations of the circumferential part of the runner. Now what this blade do These blades are sort of these are flexible blades. So these blades they may change their orientation and accordingly this may give certain directions to the water which is now falling on the actual blades of the turbine.

So these are the guides blades and these are the blades of the rotor. So the guide blade will or the guide vanes will give a guidance or direction to the water and based on their relative orientation they are flexible. They are actually pivoted and they might have different orientation around that pivot. So sometimes if these are fully closed then you may have very little flow of water that is entering into the runner.

And sometimes that is used for controlling the power output say there is a variation in the load. So when the load is high load means the demand of power so when the load is high you will have more power that needs to be produced by the turbine, but when the load is low you

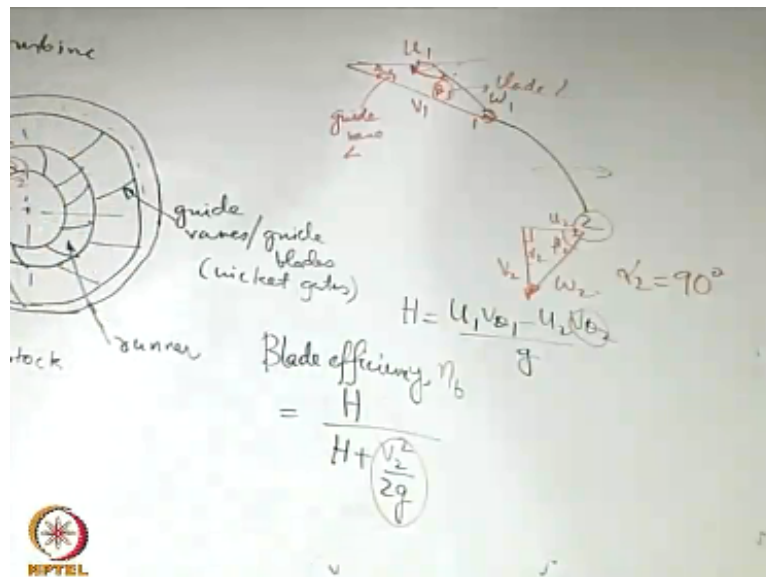
do not require that much of power to be produced, but the head developed by the runner that is given by that momentum principle.

So only way you can change or reduce the power is by playing with the flow rate Q . So you can play with the flow rate by having a closure of the partial closure of the inlet to the moving blades and that is also one of the important purposes fulfilled by the guide blades. They not only they give the orientation of the flow, but they may also have a control of the flow.

Not only that when the fluid passes through this passage between the guide blades it sorts of it is as if it is like a artificial nozzle. So it tries to impart some energy or as if there is a jet issued from the nozzle that is falling on the blades. There is no real nozzle, but the passage between the guide blades act like an artificial nozzle that is why we say that even though it is a reaction turbine it still has some impulse component.

Because at the end there is an impulse of a jet that is falling on the rotating blades.

(Refer Slide Time: 21:00)



When the jet falls on the rotating blades let us try to construct velocity triangles and see that what happens. So you have this as say 1 and this as 2. So the points 1 and 2 just very much analogous to centrifugal pumps only the reverse flow direction. So you have say this is as U_1 and this is w_1 and this is v_1 . Let us say that this is β_1 and let us say this is α_1 . What is the physical importance of the angle α_1 ?

See this is the angle at which the water jet falls on the blade who decides that the guide vanes decide that. So this is also known as the guide vane angle or guide blade angle. So there is a difference between the guide vane angle and the blade angle that you have to understand this is the blade angle. Again there is a matter of convention because sometimes 180 degree- this one is also considered as a blade angle depending on conventions.

So either this one or may be the other one, but physically it implies that the angle between u and w either at the acute one or the obtuse one depending on the convention. So here we will be following this convention where we will consider this angle as β_1 because that will make our analysis consistent with what we did for the centrifugal pump. Then for the outlet the water is coming say like this with w_2 then there is on the top of that there is u_2 .

And the resultant is v_2 . So what is the head developed by this one? $U_1 v \theta_1 - u_2 v \theta_2 / g$. The same formula we are not changing anything only application of the formula changes. So remember this is β_2 this is α_2 . So we can see that this will be the maximum 1 is $\alpha_2 = 90$ degree then β_2 is 0 there is no θ component of v_2 . So that is what we design here. Let us say we consider $\alpha_2 = 90$ degree.

If we consider $\alpha_2 = 90$ degree then it is just $U_1 v \theta_1 / g$. Now just like all rotor have certain efficiency of energy conversion the runner of a Francis turbine also has an efficiency of energy conversion. What is that? That is given by a name of blade efficiency. So blade efficiency is given as this ratio. Why this ratio see efficiency is what. Efficiency is always from output by the input where the difference in output or input represents some loss.

That means you can intuitively understand that this must represent some loss. Why this represent a loss? See when the fluid leaves the blade passage still it has some kinetic energy here $v^2 \text{ square} / 2g$. That kinetic energy head has not been utilized to develop power and therefore in terms of converting that energy of the fluid into what that is a loss and that is why this is the expression for the blade efficiency.

Now one may manipulate with this expression a bit more in terms of the geometrical features. This may be written solely in terms of the geometrical features. So you can see that you have v^2 .

(Refer Slide Time: 25:32)

$$1 - \frac{U_1 V_{\theta 1}}{g V_{r1}^2}$$

$$\frac{V_{\theta 1}}{V_{r1}} = \cot \alpha_1$$

$$U_1 = V_{\theta 1} - (-V_{r1} \cot \beta_1)$$

$$= V_{r1} \cot \alpha_1 + V_{r1} \cot \beta_1$$

$$\Rightarrow U_1 = V_{r1} (\cot \alpha_1 + \cot \beta_1)$$

$$\eta_b = \frac{\frac{U_1 V_{\theta 1}}{g}}{\frac{U_1 V_{\theta 1}}{g} + \frac{V_{r1}^2}{2g}} - 1 + 1$$

Since alpha 2 is 90 degree you have $v_2 = v_{r2}$. The total component of the velocity (v_2) (25:42) radial there is no theta component. Then usually these turbines are so designed that the blade will turn the radius they are adjusted in a way that you have $v_{r2} = v_{r1}$. You see this depend on what? V_r is Q by the area. So the area contains 1 diameter and 1 breadth at that particular diameter. If the diameter is reduced the breadth is increased in such a way that Q / that ratio that ratio that remains almost the same so that you have $v_{r2} = v_{r1}$.

This is the practical design consideration. So that means you can write this as well as if you consider $v_{\theta 0}$ as 0 as $u_1 v_{\theta 1} / g$ divided by $U_1 v_{\theta 1} / g + v_{r1}^2 / 2g$. In place of v_2 we have substituted v_{r1} we may just manipulate it a little bit by adding and subtracting 1 the whole purpose is in the numerator you will have now the $v_{r1}^2 / 2g$ then $u_1 v_{\theta 1} / g + v_{r1}^2 / 2g - 1 + 1$.

Now we just manipulate it a couple of more steps so the blade efficiency therefore maybe written as $1 - \frac{1}{1 + U_1 v_{\theta 1} / g * v_{r1}^2 / 2g}$. So the whole idea is that this ratio we may write in terms of the geometrical features of the triangle. So let us try to write this ratio this is $2 u_1 v_{\theta 1} / v_{r1}^2$. So first of all can you write U_1 and $v_{\theta 1}$ in terms of v_{r1} ? So look into the velocity triangle.

This is what? This is v_{r1} what is $v_{\theta 1}$ this is $v_{\theta 1}$. So you can write that $v_{\theta 1} / v_{r1} = \cot \alpha_1$ and how you can relate u_1 with that. You can write $u_1 = v_{\theta 1} -$ this part. What is this part? This part is $v_{r1} \cot 180^\circ - \beta_1$. So $- v_{r1} \cot \beta_1$. So $-$ of $- v_{r1} \cot \beta_1$. So from here you can write expressions so you can first of all you have $v_{\theta 1} =$

$v_{r1} \cot \alpha_1$ then you have $+ v_{r1} \cot \beta_1$. So you can write u_1 as $v_{r1} \cot \alpha_1 + v_{r1} \cot \beta_1$.

(Refer Slide Time: 30:12)

$v_{r1} \cot \alpha_1$
 $v_{r1} \cot \beta_1$
 $+ v_{r2} \cot \beta_1$
 $v_{r1} \cot \beta_1$
 $= \text{Reaction head}$
 $= \frac{\text{Reaction head}}{\text{Total head}}$
 $= \frac{u_1 v_{\theta 1}}{g} - \left(\frac{v_1^2 - v_2^2}{2g} \right)$
 $\frac{u_1 v_{\theta 1}}{g}$

$\frac{2u_1 v_{\theta 1}}{v_{r1}^2} = 2 \frac{v_{r1} (\cot \alpha_1 + \cot \beta_1) v_{r1} \cot \alpha_1}{v_{r1}^2}$
 $H_1 - \frac{u_1^2}{2g} = H_2 - \frac{u_2^2}{2g}$
 $H_1 = \left(\frac{p_1}{\rho g} + \frac{v_1^2}{2g} \right) - \left(\frac{p_2}{\rho g} + \frac{v_2^2}{2g} \right)$
 $\frac{u_1 v_{\theta 1}}{g} = \left(\frac{p_1}{\rho g} - \frac{p_2}{\rho g} \right) + \frac{v_1^2 - v_2^2}{2g}$

Similarly, you can write $v_{\theta 1}$ as a function of v_{r1} . So you can write this expression $2 u_1 v_{\theta 1} / v_{r1}^2 = 2$ in place of u_1 we write $v_{r1} \cot \alpha_1 + v_{r1} \cot \beta_1$. In place of $v_{\theta 1}$ you have $v_{r1} \cot \alpha_1 / v_{r1}$. So the v_{r1} get cancelled out and you can see that the blade efficiency may solely be expressed as a function of the guide vane angle and the blade angle. So it is a sole function of the geometrical features of the blade because any other parameters get cancelled out.

Next we will define another related quantity which is known as degree of reaction. We have already discussed that when you have a turbine when you have a reaction turbine it is not totally a reaction turbine because there is a part of the energy transfer which is by the impulsive effect. So the degree of reaction therefore it concerns the fractional head because of the reaction divided by the total head.

So reaction head by the total head. So when you consider the total head what you can say? See grossly you have the head at 1 as H_1 if you consider the head at 2 as H_2 then what is the difference between these 2? Head at 1 as H_1 some head is taken from that to do useful work that is $U_1 v_{\theta 1} / g$ neglecting the $v_{\theta 2}$ and that is nothing, but the H_2 . So fluid out at 2 with a lower head.

Now the head at 1 is what $p_1 / \rho g + v_1^2 / 2g$ neglecting the change in potential energy head that is considering at 1 and 2 do not have sufficient change in elevation. You can

therefore write $H_1 - H_2 = \text{this} - \frac{p_2}{\rho} + \frac{v_2^2}{2g}$. So what is the corresponding reaction head? The reaction head is if you isolate the pressure part of it $\frac{p_1}{\rho} - \frac{p_2}{\rho} + \frac{v_1^2}{2g} - \frac{v_2^2}{2g}$.

You can see that if $H_1 - H_2$ that is nothing, but $U_1 v_{\theta 1} / g$ that is the contribution from the 2 heads the first part is the reaction head that is the pressure head change and the other part is the kinetic energy head change. So when you consider this one you can write therefore that this is $U_1 v_{\theta 1} / g - \frac{v_1^2}{2g} - \frac{v_2^2}{2g}$ and this one as $v_1 v_{\theta 1} / g$ from that head consideration.

(Refer Slide Time: 34:24)

Handwritten derivations on a slide:

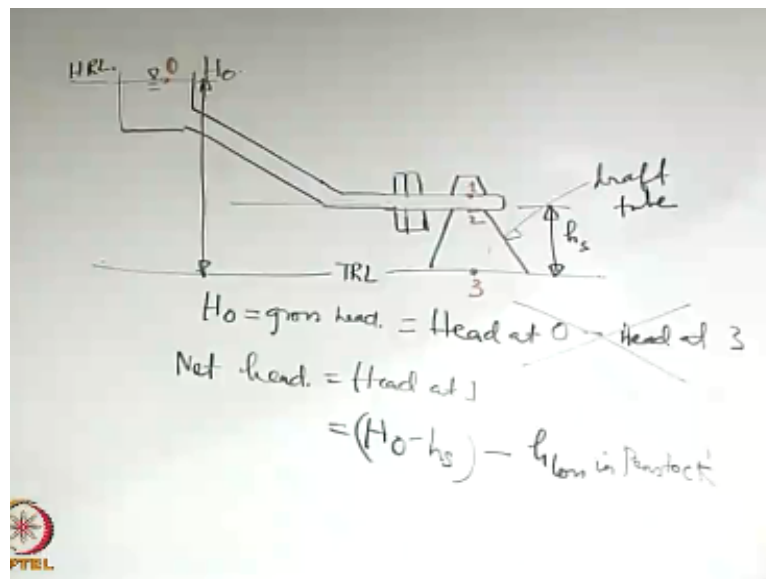
- $U_1 v_{\theta 1} / g$ (circled)
- $V_2 = V_{r2} = V_{r1}$
- $V_1^2 = V_{\theta 1}^2 + V_{r1}^2$
- $= V_{r1}^2 (\cot^2 \alpha_1 + 1)$
- $= V_{r1}^2 \operatorname{cosec}^2 \alpha_1$
- $\frac{V_{\theta 1}}{V_{r1}} = \cot \alpha_1$
- $U_1 = V_{\theta 1} - (-V_{r1} \cot \beta_1)$
- $= V_{r1} \cot \alpha_1 + V_{r1} \cot \beta_1$
- $U_1 = V_{r1} (\cot \alpha_1 + \cot \beta_1)$
- $\frac{2U_1 V_{r1}}{V_{r1}^2}$

And $\frac{v_1^2}{2g} - \frac{v_2^2}{2g}$ you can write v_2 as V_{r2} that is same as v_{r1} and what is v_1 you can write as $v_{\theta 1}^2 + V_{r1}^2$ and $v_{\theta 1}^2$ is $V_{r1}^2 \cot^2 \alpha_1$. So $\cot^2 \alpha_1 + 1$ so that is $V_{r1}^2 \operatorname{cosec}^2 \alpha_1$. So the whole idea is just you could write $U_1 v_{\theta 1}$ as in terms of v_r this also you can write in terms of v_r .

So the effect of v_r can get cancelled out here also and here also you may be able to come up with a final expression involving just α_1 and β_1 . So you can see that you will never have $v_1 = v_2$ and therefore you will never have a situation when the total head is the reaction head. So the degree of reaction is always < 1 therefore no reaction turbine is fully a reaction turbine.

Now till now we have considered the turbine runners as an isolated case, but just as we have discussed for the pump let us see that what happens if the turbine is put in a system.

(Refer Slide Time: 35:56)



So you have supply reservoir, you have the penstock basically same as for the impulse and the reaction turbine. Then we will draw it very, very schematically to get the picture. This is the head race level this is the tail race level and this is a schematic diagram of the turbine. So you may consider certain points. Let us say that this is section 1 this is section or may be let us call it 0 because it is just the reference.

Let us say this is section 1, this is a section 2 which is the outlet and this is the section 3 which is the discharge. We will try to see that what is this diverging portion and why there is a requirement for that. Let us give us a name of the height say h_s . So the head at section 0 is H_0 . So when you have this H_0 it is totally the potential energy head so that is this one. What is important so this H_0 is known as the gross head.

So this is basically Head at 0- Head at 3. In reality, it is not exactly correct as the Head at 0- Head at 3 why? Because this presumes that the only form of head is the potential energy head, but actually there is some kinetic energy head. So we will not write it as Head at 0- Head at 3. The expectation would have been like that if the velocity at 3 was 0, but in reality the velocity of 3 is not 0. So it is just considered I mean it is a convention to say that it is just the gross head is the height elevation difference between the head race and tail race level.

Tail race is where the water is rejected. Now important is not this one, but important is the net head. Net head is the head at 1. Again the head at 1 will be basically $H_0 - h_s$ that is the static thing -the losses -head loss in penstock. So these are 2 important terminologies the net head

and gross head across a reaction turbine. The next thing is that when you have the net head and the gross head.

First of all, for 1 and 2 are this 1 and 2 that we have mentioned that enters to the blade and exit from the blade. Now why we require such a divergent portion. See we could figure out that one of the reasons for a low efficiency is a high kinetic energy at the outlet because the blade efficiency was divided $U_1 v \theta \frac{1}{g}$ divided by $U_1 v \theta \frac{1}{g} + \frac{v^2}{2g}$. So higher v^2 is a loss.

And it is true that it is a loss in many ways because that kinetic energy is not utilizable. So is it possible to recover that kinetic energy somehow. One intuitive way of doing it is that you allow it to pass through a divergent tube because then the area of cross section increases and the velocity decreases. So it is expected that the velocity at which the water is eventually rejected to this tail race level is very, very small.

It is almost tending to 0. So that is possible if this area is a divergent one and this tube is therefore considered to be an integral part of the turbine and this is known as the draft tube. So one of the objectives of the draft tube is to recover the kinetic energy head at the outlet that is one of the basic objectives. So when you do that let us try to see that what happens to the other forms of the head.

So the draft tube is a tube it is divergent tube and it may have certain head losses. Let us neglect the head losses for the time being and let us see that what are the consequences of it.

(Refer Slide Time: 41:12)

$$\frac{p_2}{\rho g} + \frac{v_2^2}{2g} + h_s = \frac{p_3}{\rho g} + \frac{v_3^2}{2g} + h_{sf}$$

$$\frac{p_2}{\rho g} = - \left[h_s + \frac{v_2^2 - v_3^2}{2g} - h_{sf} \right]$$

$$0 = \frac{p_{atm}}{\rho g} - \frac{p_{min}}{\rho g} - h_s - h_{sf}$$

$$H = \frac{p_{atm} - p_{min}}{\rho g} - h_s - h_{sf}$$

So let us apply the energy equation between 2 and 3. So you have $\frac{p_2}{\rho g} + \frac{v_2^2}{2g} + h_s = \frac{p_3}{\rho g} + \frac{v_3^2}{2g} + h_{sf}$. If we neglect any losses neglect losses in draft tube. In reality that is not the case. So p_3 is the atmospheric pressure. So if we consider it as a 0 (()) (41:52) pressure. So you can write $\frac{p_2}{\rho g}$ as $- [h_s + \frac{v_2^2 - v_3^2}{2g} - h_{sf}]$. Now satisfying the purpose of the draft tube you must have $v_2 > v_3$ right that is for your whole purpose.

So you have $v_2 > v_3$ so this is positive. H_s is positive in this configuration that means p_2 is negative and this in fact is the location of minimum pressure in a reaction turbine. And when this pressure falls below the vapor pressure the cavitation phenomenon may occur exactly in a similar way in which we discussed about the cavitation in a centrifugal pump. There is no big difference.

So we have to first understand that the point 2 or the section 2 rather is a location which is prone to the minimum pressure in a reaction turbine system and that pressure if it falls below the vapor pressure it may give rise to a cavitation. Like the cavitation parameter for a pump a very similar cavitation parameter is defined for a turbine. So it is defined in the same way the NPSH available by H .

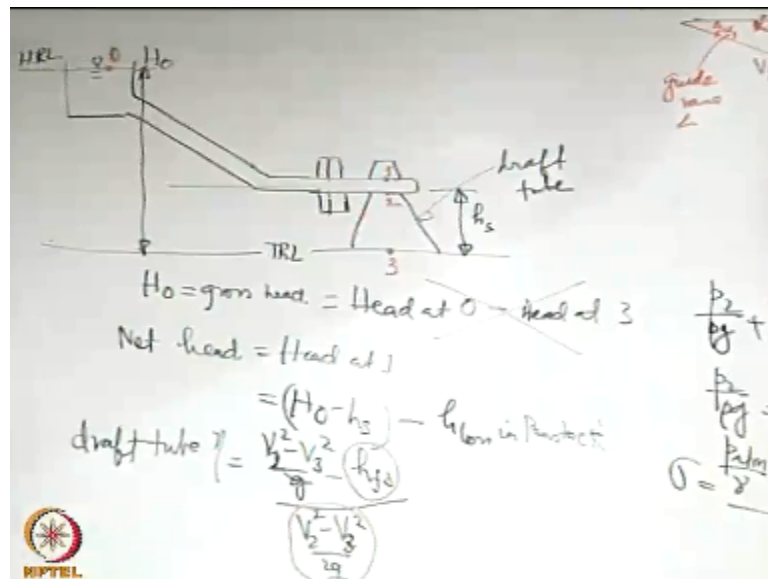
So $\frac{p_{atm}}{\rho g} - \frac{p_{vapor}}{\rho g}$ is $\frac{H_s}{H}$, but there is no minus h_{sf} . This is not considered why? So this is what this is the head loss in the section which is being considered for making this analysis. So head loss is there in the draft tube it is very much there in the draft tube, but it is not considered in this expression because the draft tube is considered to be an integral part of the turbine.

So head loss is considered to be in a part which is in a system outside the fluid machine. Like when we consider the cavitation in a pump. Head loss in the suction pipe and the delivery pipe those are parts outside the pump those are part belonging to the system. The draft tube is considered to be an integral part of the turbine itself. So it is not in the system and therefore the head loss within the draft tube is not considered for defining the cavitation parameter for the turbine.

That is a big difference between the turbine cavitation parameter definition and the pump cavitation parameter definition, but we have to keep in mind that if you want to write this equation realistically yes there is some head loss. So you have h_{sf} and accordingly here you have a minus h_{sf} . So the head loss is very much there, but that head loss is not considered while defining the cavitation parameter.

Now the draft tube because of the head loss in the draft tube there is some terminology which is defined as efficiency of the draft tube.

(Refer Slide Time: 45:06)



So how it is defined it is defined as draft tube efficiency which is $\frac{V_2^2 - V_3^2}{2g}$ - the head loss in the draft tube by this one. Again you see that here what is the purpose? The purpose is in the ideal case the draft tube could recover this amount of energy, but in reality it cannot recover this amount of energy because there is a head loss in the draft tube and that is why it is a efficiency of the draft tube.

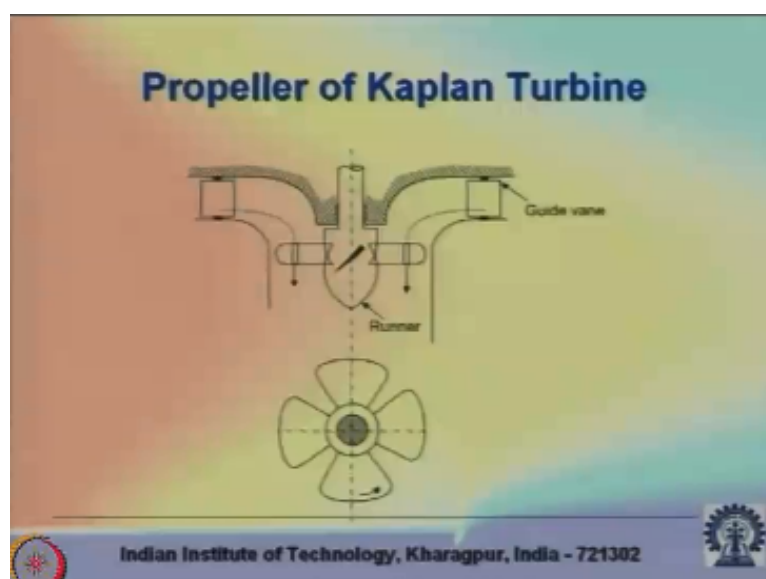
If there is no head loss in the draft tube it is as good as a draft tube with 100% efficiency. The big question is when you have a draft tube. I mean do you have a draft tube for an impulse turbine say would you have a draft tube for a Pelton turbine. See one of the important objectives of the draft tube that the draft tube fulfills first of all there is in the reaction turbine you can see that this there is a partitioning between the pressure change and the velocity change and that is important.

In an impulse turbine that is no more important because everywhere you have the pressure as the same. There is no change in pressure as the fluid is passing through the turbine and therefore you do not require something like a draft tube. So the draft tube also affects the change in pressure that is one of the very important thing and that is an integral part of the turbine.

By basic principle of an impulse turbine you should not have a change in pressure across that then it is no more an impulse turbine. Therefore, impulse turbine will never have a draft tube. The other thing is that will the impulse turbine have a cavitation prone behavior again the answer is no because cavitation is because of the local variation of pressure within the turbine.

In an impulse turbine there is no local variation. Therefore, cavitation is an irrelevant phenomenon for an impulse turbine that we have to keep in mind. So we have looked into 2 different types of turbines.

(Refer Slide Time: 47:22)



Let us just look into the Kaplan Turbine a bit more clearly than what we did earlier. So let us consider the axial flow turbine of the Kaplan Turbine. So if you consider the axial flow turbine and the Kaplan turbine just look into the figure. You can see that if you look into the runner you see that the blade is like sort of a twisted one. So when the fluid enters before encountering the runner it is almost like a free vortex flow.

And you can see that it basically enters sort of axially the guide vanes you can see these guide vanes may swivel with respect to the pivots which are shown in the figure and accordingly they will allow some fluid to enter. The fluid changes its direction and so long as it enters the runner it is just like a free vortex with $v_{\theta} * r = \text{constant}$.

(Refer Slide Time: 48:20)

Handwritten notes on a whiteboard:

- Top right: $\rightarrow V_{\theta} r = C \rightarrow V_{\theta} \propto \frac{1}{r}$
- Below that: $V_{\theta} \propto r$
- Left side: $N_s = \frac{N \sqrt{P}}{(H)^{5/4}}$
- Right side: 10-30, 300, 1500

So you have the first requirement is when it is entering the flow passage you have $v_{\theta} * r = \text{constant}$. So you have this $v_{\theta} * r = \text{constant}$ that is when it is entering the flow passage. Now on the top of that you have to keep in mind that when it is entering the runner you have v_{θ} is proportional to r it is like $v = \omega r$ type. So when it enters the runner you should have v_{θ} proportional to r here v_{θ} is proportional to $1/r$ before just entering the runner.

So when the blade is designed it has to meet these divergent requirements. One is the entering flow is like v_{θ} is proportional to $1/r$ and at the blade you should have the tangential velocity proportional to r . So that requirements combined together do not give rise to a very straight forward blade design, but a twisted type of blade design. So if you consider the runner you can see here that basically across the runner it just flows axially just in the same way as it does for a axial flow pump.

And therefore you may draw velocity triangles exactly in the same way as you could do for an axial flow pump only the direction of the flow is something that is reversed. So we have come across the different turbines and here again we have to distinguish the turbine in terms of their specific speed. So the specific speeds you have to recall that for a turbine it is $N \sqrt{P/H}$ to the power $5/4$ that is a specific speed for a turbine.

So it will strongly depend on the head. If the head is high, then the specific speed will be low. So therefore it is important to compare the heads of the different turbines that we have discussed. First the pelton turbine. So in the pelton turbine what happens the fluid comes from a very high level reservoir and it is now being converted as a jet with a high kinetic energy.

So the difference in level between the reservoir and the nozzle is something what is dictating the input form of energy because that is converted into the kinetic energy by the nozzle. What it means is that it should have a high head because that is its only input. So the Pelton turbine has a very high head. On the other hand, look into the Kaplan Turbine the Kaplan turbine is an axial flow machine.

And we have seen that it may operate on the basis of a high discharge, but not on a high head because the high head may give rise to flow separation and stalling type of situation. So it will be operating on a low head and the Francis Turbine is something which will be a sort of midway between these 2 behaviors. So we may conclude that the lowest specific speed out of this 3 will be Pelton turbine and the highest will be the Kaplan Turbine.

So typically the specific speed for the Pelton turbine maybe say around 10 to 30 like that for the Francis turbine may be say up to 300 or even close to 500 and for the Kaplan turbine it may be up to 1000 of the order of that. So these are just representative numbers and keep in mind that while calculating these numbers N was put as rpm. P was metric horsepower and H as meter. So let us now work out a problem on Francis turbine.

(Refer Slide Time: 52:18)

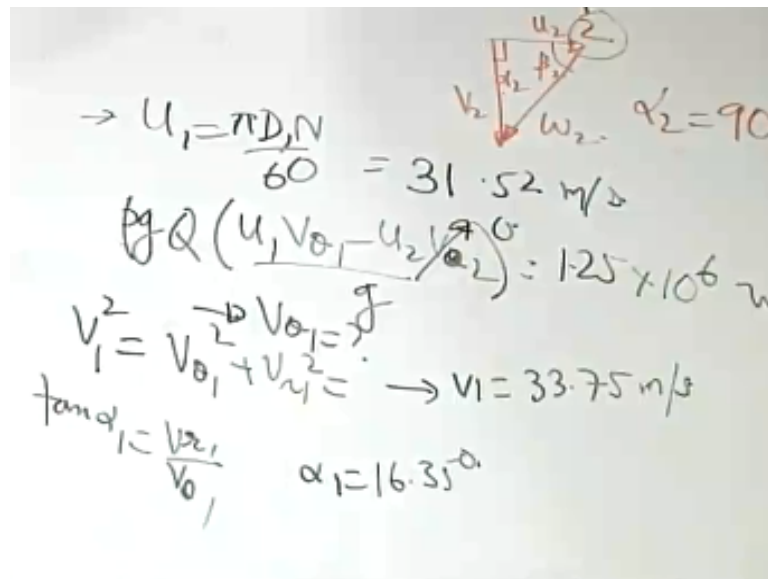
$$\begin{aligned}
 D &= 1.4 \text{ m} \\
 N &= 430 \text{ rpm} \\
 V_{r1} &= 9.5 \text{ m/s} \\
 \text{leaves without whirl, } V_2 &= 7 \text{ m/s} \\
 \left(\frac{p_1}{\rho g} + z_1 \right) - \left(\frac{p_2}{\rho g} + z_2 \right) &= 62 \text{ m} \\
 P_{\text{net}} &= 12.25 \text{ MW} \\
 Q &= 12 \text{ m}^3/\text{s} \\
 H_{\text{net}} &= 115 \text{ m}
 \end{aligned}$$

So the problem statement is like this a Francis turbine has a diameter of 1.4 meter and rotates at 430 rpm. Water enters the runner without shock with a flow velocity of 9.5 meter per second. When it says that is a flow velocity it means it is a relative velocity V_r because that is related to the flow and leaves the runner without whirl. With an absolute velocity of 7 meter per second.

The difference between the static and the potential heads at the entrance and exit of the runner that is $p_1/\rho g + z_1$ static and potential head $p_2/\rho g + z_2$ this difference is 62 meter. The turbine develops 12.25 megawatt of power flow rate through the turbine is 12-meter cube per second for a net head of 115 meter. Find the following number 1 the absolute velocity of water at the entry of the runner and the inlet and the angle of the guide vane then the inlet angle of the runner blade and the head loss in the runner.

So let us try to quickly work out this problem.

(Refer Slide Time: 54:30)



Handwritten calculations and a velocity triangle diagram for a turbine stage. The diagram shows a velocity triangle at the inlet (1) and outlet (2). At inlet 1, the absolute velocity U_1 is horizontal, the blade velocity U_2 is at an angle $\alpha_2 = 90^\circ$, and the relative velocity V_1 is at an angle α_1 . At outlet 2, the absolute velocity U_2 is horizontal, the blade velocity U_1 is at an angle α_1 , and the relative velocity V_2 is at an angle α_2 .

$$\rightarrow U_1 = \frac{\pi D_1 N}{60} = 31.52 \text{ m/s}$$

$$\rho g Q (U_1 V_{\theta 1} - U_2 V_{\theta 2}) = 12.25 \times 10^6 \text{ W}$$

$$V_1^2 = V_{\theta 1}^2 + V_{r1}^2 \rightarrow V_1 = 33.75 \text{ m/s}$$

$$\tan \alpha_1 = \frac{V_{r1}}{V_{\theta 1}} \quad \alpha_1 = 16.35^\circ$$

So you have first of all you can calculate U_1 that is $\pi D_1 N/60$. So I am just giving you the numerical answer so that you can check 31.52 meter per second. Then the power developed is $\rho g Q \cdot h$. h is $U_1 v_{\theta 1} - U_2 v_{\theta 2}/g$ that is given as 12.25 megawatt 12.25×10^6 watt. It is given that the flow leaves without whirl that means $v_{\theta 2}$ is 0. So this is one information that is given.

So from here you know what is U_1 you know ρg and you know Q . So from here you can find out what is $v_{\theta 1}$ you know what is V_{r1} that is already given to you. So v_1 the resultant velocity v_1^2 is $v_{\theta 1}^2 + V_{r1}^2$. So v_1 is 33.75 meter per second. So from here you can find out the guide vane angle. How do you find out the guide vein angle?

So $\tan \alpha_1$ is what $V_{r1}/v_{\theta 1}$. So from here you can find out the guide vane angle. So the α_1 is 16.35 degree. Then how do you find out the β_1 the blade angle?

(Refer Slide Time: 56:16)

$$V_{\theta 1} = u_1 + V_{r1} \cot(180^\circ - \beta_1)$$

$$\Rightarrow \beta_1 = 180^\circ - 84.77^\circ$$

So you can write this one $v \theta 1$ as $u_1 + V_r \cot 180 \text{ degree} - \beta_1$. So from here you can find out what is β_1 . So β_1 is actually in answer $180 \text{ degree} - \beta_1$ is given. So β_1 is $180 \text{ degree} - 84.77 \text{ degree}$. What is a loss?

(Refer Slide Time: 56:58)

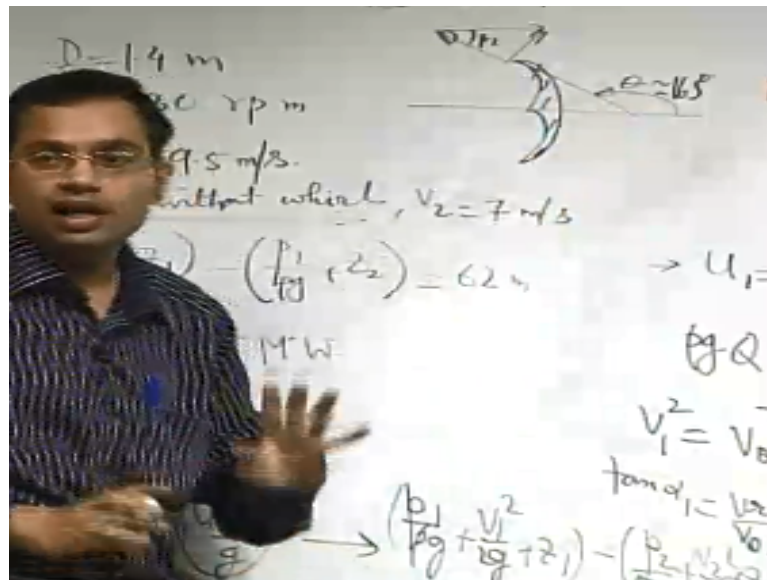
Handwritten notes showing the derivation of head loss. The notes include the equation $H_1 - H_2 = \frac{u_1^2 - u_2^2}{2g}$ and the velocity triangle relationship $u_1 = V_1 \cos \alpha_1$. The final result for head loss is 13.49 m .

The third part is the loss that is $H_1 - H_2 - u_1 v \theta 1 / g$. If there was no loss $H_1 - H_2$ would be $U_1 v \theta 1 / g$, but because of a loss you do not have $H_1 - H_2$ as $U_1 v \theta 1 / g$. So U_1 you know $H_1 - H_2$ because you can write $H_1 - H_2$ as $p_1 / \rho + g + v_1^2 / 2g + v_1 - p_2 / \rho + g + v_2^2 / 2g - u_1 v \theta 1 / g$. And at least the difference in $p / \rho + g$ and z that is already given and v_1 and v_2 you can calculate.

Because you have $V_{r1} = V_{r2} = v_2$. So that if you substitute here you will get or in fact yes. So if you calculate this the head loss that you come up with is 13.49 meter that is a head loss. So

if you look into this basically some head is input to the system and there is some head loss also because of some sources of inefficiency within the blade passage maybe friction as one of the examples. Now when you consider this blade angle just keep one thing in mind just think of the Pelton turbine.

(Refer Slide Time: 58:40)



So if you consider the Pelton turbine just for an analogy. See this angle this beta 2 what was there for the Pelton turbine or let us say this theta you can see if you refer to the expression for the efficiency the blade efficiency it was a function of $1 + k \cos \beta_2$. So what would be the maximum efficiency when beta 2 is 0. If you may vary beta 2 as a design parameter that means this theta is 180 degree.

But in practice this theta is not kept as 180 degree but something optimally as close to 165 degree that is a design practice for a Pelton turbine why. The reason is like a practical reason if you keep beta 2 as 0 the fluid that is coming from 1 bucket will hit the back of the other bucket directly. So to avoid that so you cannot have like hitting at back of another bucket. So that is why this is not totally change in direction, but something in between close to 165 degree as an optimal design parameter.

So the blade angles no matter it is a Pelton turbine or a Francis turbine or whatever these are very important parameters. So with this we conclude our discussion on fluid machines. Thank you.