Introduction to Fluid Mechanics and Fluid Engineering Prof. Suman Chakraborty Department of Mechanical Engineering Indian Institute of Technology – Kharagpur

Lecture - 52 Introduction to Fluid Machines (Contd.)

We continue with our discussions on hydraulic machines. We were discussing about centrifugal pumps and let us work out couple of problems before proceeding further. So, let me tell you the problem statement. Centrifugal pump lifts water against a static head of 32.067 meter.

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the= 32.067 , Suction Eff =127 cm

The static head is 32.067 meter out of which 3.054 meter is the static lift. Sorry 3.054 meter is the suction lift. This is not the static lift this is the suction lift. The suction and the delivery pipes it is given that the suction and the delivery pipes are both 12.7 centimeter in diameter. The loss of height in the suction pipe is 1.07 meter of water and that in the delivery pipe is 5.955 meter of water. These are given data.

The impeller is 30.54 centimeter diameter and 2.54-centimeter-wide at the outlet. It refolds at 1,450 rpm and the blade angle at the exit is 35 degree. Hydraulic efficiency of the pump is 80% and overall efficiency of the pump is 68%. We have to find out number 1, what is the discharge from the pump and what is the power required to drive the pump? These are the data given.

The first data is the total lift that is the static lift in the suction and the delivery side. So, these are some practical data, I mean some of the data will with which certain decimal points are because of conversion from our old system like system with the new system. For example, you can see 2.54 centimeter that is the, this is the width of the blade at the outlet B2. So, this is just like a blade with 1-inch width.

Now, let us see that what information we have from these data. Keep in mind that let us quickly draw the schematics. So, if you have this as the pump, so you have the reservoir, the suctions side and then the delivery side. So, what we can say is what is the total height developed by the pump. That is nothing but Pd/rho g + Vd square/2G – Ps/rho g + Vs square/2g.

So you can write Ps and Pd in terms of hs, hd, hsf and hdf. We have written all those by applying the energy equation between 1 and s and then between d and 2 and the corresponding result was, what? So if you write Pd that is you will get if you apply the energy equation between d and 2 you have P2/rho g + P2 square/2g. So you have this one as hd, V2 and Vd are the same and P2 is p atmosphere.

So you have this as hd + hdf. This is 0 at atmosphere. So, if you are writing the gauge pressures. Then what about Ps? So, if you write Ps/rho g + Vs square/2g + hs = P1/rho g + V1 square/2g + hsf. So, this is again 0 atmosphere, so then this is very small. So, Ps/rho g + Vs square/2g is – sorry this is – this is a loss. So this will be – of what, this one then + Vd square/2g, right.

So 1 Vd square/2g coming from here and this is seemed total Ps/rho g + Vs square/2g. One important thing you can see that we could have the liberty of putting the gauge pressures here because here the pressure difference is important, so the reference is immaterial. But if the reference was important then obviously we could not do it.

For example, if this was dependent on a single pressure but not the difference between 2 pressures then we should not have taken this as any arbitrary reference. Now, so that means this is basically hs + hsf or rather hs + hd + hsf + hdf + vd square/2g. These data are already given to us. So, you can substitute the values. So it will be some numbers hs + hd that is

given I have already written that symbolically so that you can substitute the data from the given values.

So this will be some of these constant numbers + you know the diameter of the pipes ds or dd. So this is therefore Q square/Vsq/a. So, this is of the form a + bQ square. So this is the head that is eventually developed by the pump. Now, if you consider the hydraulic efficiency, so what does the hydraulic efficiency give? It gives the ratio of the head developed by the pump to the head input to the impeller.

So what is the head input to the impeller? That is the theoretical head.

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That is U2 V theta 2 - U1 V theta 1/g. Here we assume that there is 0 swirl at the inlet. 0 inlet swirl. This is the technical way of meaning that you have V theta 1 = 0. That is alpha 1 = 90 degree. So, if you then simplify it, it will boil down to U2 V theta 2/g. Let us try to construct the velocity triangle at the outlet and see that how we can relate this with the flow rate. So, this is the blade you have U2 W2 and V2 this angle is beta 2.

So, what is V theta 2? This component of V is V theta 2. So, V theta 2 is nothing but U2 – Vr2 cot of beta 2. Because this is nothing but Vr2 cot of beta 2. This is Vr2. So, we have H/U2 V theta 2/g that is the hydraulic efficiency that is given as 80%. So from this we can say that H/U2/g * V theta 2 is U2 – Vr2 cot beta 2 that is 80% or in place of H you have all the form a + bQ square and U2/g * U2, Vr2 is Q/pi d2 * b2.

That is the radial velocity that is the volume flow rate / the area over which it is flowing. This got beta 2 this is 0.8. Now, what you know here you also know U2 what is U2? U2 is, it is just like V = omega R. So that is pi, so omega is n is the rpm, so omega is 2 pi N/60 and R is D/2, right. So this is the U2. So, if you substitute U2 here you will see that it is just a quadratic equation in Q all other parameters unknown.

So from here you can solve for the value of Q2. So, if you solve for the value of Q. So, from here you will get Q = 0.039-meter cube per second. This is the answer to the first part. The second part is what is the power input? Power input is related to the power output by the overall efficiency. So, the overall efficiency is the power output. Power output is based on the output head H.

So rho gQ * H is the power output and power input is the thing that you have to find out and the ratio of this Q is the hydraulic efficiency which is 68%. So from here you can find out what is the power input, 22.27 kilowatt, okay. Let us work out our next problem. So, you may take down the problem. The basic design of a centrifugal pump has a dimensionless specific speed of 0.075 revolution.

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This is the dimensionless specific speed 0.075 revolution. The blades are forward facing on the impeller and the outlet angle is 120 degree to the tangent. With an impeller passage width at the outlet = one tenth of the diameter. The pump is used to raise water through a vertical distance of 35 meter at a flow rate of 0.04-meter cube per second. The suction and the

delivery pipes are each 150 millimeter in diameter and a combined length of 40 meter with a friction factor of 0.02.

Other losses at the pipe entry's, exits and bends all together are 3 times the velocity height in the pipe. If the blades occupy 6% of the circumferential area and the hydraulic efficiency is 76% neglecting the slip, what should be the diameter of the pump impeller? So, this is again quite similar to the previous problem but with a few subtle differences. So, first of all you are given a hydraulic efficiency, so you need to make use of the hydraulic efficiency.

So, how do you make use of the hydraulic efficiency? So, as you recall the hydraulic efficiency is H/U1 V theta 1/g - U2 V theta 2/g - U1 V theta 1/g. So, you neglect V theta 1 considered that it is 0 swirl at the inlet. So, now you can express U and V theta in terms of the given parameters, so if you just recall what we did for the previous problem V theta 2 is U2 - Q/the area over which it follows * cot beta 2.

Which one? ds = dd, yes, I think it is in centimeter, there is too much, yeah. No, this is 150 millimeter, sorry not meter. So, now here we have to figure out first what is H? So, how to figure out what is H? H is nothing but the total lift + the total losses, right. So, total lift is hs + hd + total losses are the major losses and the minor losses. Major losses are fl/d, now here the diameter of the suction and the delivery pipes are the same.

So, you can use a common d and the friction factor is also common. Velocity is also common, so this is as good as velocity in the suction pipe or the delivery pipe. Because flow rate is the same and the area of cross section also the same and then the minor loss 3 * V square by 2g. How do you find out V? You know Q and you know the diameter of the pipe. So, V is where V is Q/pi d square/4.

So, that means with the given data you know what is the numerator? What is the H? You also know the hydraulic efficiency. U, regarding U2, U2 is pi D2 N/60. D2 you do not know. N also you do not know but you have an information. What is the information? You had given a dimensionless specific speed. So, what is the dimensionless specific speed? N root Q/g H to the power 3/4, right. This is given as 0.075.

So, from here you can find out what is N? So, that means N also you know. Here in the other term you know Q then Q / the area of flow. One important information is given in terms of the relative occupancy of the blades. What is given? Blades occupy 6% of the circumferential area. That means whatever is the circumferential area available for the flow that is only 94% of the theoretical one.

What is the circumferential area available for flow? That is pi * D2 * V2 that is the theoretical one, that multiplied by 0.94 is the actual one. Because some part of the area is occupied by the blades through which of course, I mean that is the solid through which fluid is not flowing. Then if you referred to this equation you have one unknown here in terms of D and in the second term also you have only one unknown in the form of D that is D2.

So, you get an algebraic equation in terms of D2 from which you can find out its value. If you solve for that you will get D2 = 0.214 meter. Let us workout our next problem. So, you may note down the statement of the problem.





When a laboratory test was carried out on a pump it was found that for a total height of 36 meter and a discharge of 0.05-meter cube per second cavitation began when the sum of the static pressure and the velocity head at the inlet was reduced to 3.5 meter. This is given data for the onset of cavitation. The atmospheric pressure was 750 millimeter of mercury and the vapor pressure of water was 1.8 kilo Pascal.

If the pump is to operate at a location where the atmospheric pressure was reduced to 620 millimeters of mercury that is a new atmospheric pressure and the temperature is reduced so that the new vapor pressure is 830 Pascal then, number one, is it necessary to reduce the height of the pump and if so by how much? And number 2 what is the value of the critical cavitation parameter?

So, if you recall that if you considered the cavitation for a pump we have to consider the suction side because at the suction the pressure may fall below the vapor pressure giving rise to the onset of cavitation. What is given is the pressure + the velocity head at the suction for the onset of cavitation is this one. Now you may apply the energy equation between 1 and s. Then what you get out of these, so you have P1/rho g + V1 square/2g, okay.

So, you have P1s, P atmosphere. You have V1 negligible and when you have the onset of cavitation this you have same as the vapor pressure. So, at the onset of cavitation you have P atmosphere by rho g - P vapor/rho g - hs - hsf = Vs square/2g. This is not always but only at the onset of cavitation when the suction pressure becomes the vapor pressure. This is onset of cavitation.

Why this parameter is important because we have seen that the cavitation parameter is this quantity divided by H. So, then our cavitation parameter for the onset of cavitation becomes Vs square/2gH. How do get Vs square? You have to keep in mind that it is given that at the onset of the cavitation the sum total of this is 3.5 meter. When its onset of cavitation this is just the vapor pressure because it is given that it is onset of cavitation.

And the vapor pressure is given. So, from this information you can find out what is Vs square/2g because the vapor pressure is already given. So, when you know what is Vs square/2g and what is H it is just a matter of substitution in this expression to get what is the critical cavitation parameter. The answer to this is 0.092. Then in the other part the only change is that you have a different atmospheric pressure and you have a different vapor pressure.

But the critical cavitation parameter which is a sort of normalized or non-dimensional indicator of the onset of cavitation that should not be altered. Therefore, you have the critical cavitation parameter same as that of the original one. So that is you have P atmosphere nu/rho

g - P vapor nu/rho g – hs nu – hsf nu/h, h remains the same. Same total head it is operating then it is same as P atmosphere/rho g - P vapor/rho g – hs – hsf/H.

See, the piping system is kept unaltered and the flow rate is kept unaltered which is very important. So, if the piping system is kept unaltered and the flow rate is kept unaltered the head loss will also be unaltered. That means in the new location, new elevation of the pump you have hsf and hsf in all location they are the same. The logic is you have the same piping and you have the same flow rate.

Then H is being the same, so from here you can calculate that what is the difference between hs prime and hs. Because the new atmospheric pressure and the new vapor pressure those are given. So, when you have to convert keep in mind that atmospheric is given in terms of millimeter of mercury. So, from that convert unit of Pascal by using the density of mercury you can easily do it. So, this answer to this is -1.67 meter.

Let us look into another problem on the pump because before we move on to the next item. (Refer Slide Time: 34:00)



There is a pump in an aquarium for which the head and discharge characteristics of the pump these are given by the manufacturer. The discharge in the unit of meter cube per second and head in the unit of millimeter of water. So, some data are given. This is the information on the head discharge characteristics. These are the pump characteristics and therefore these are supplied by the manufacturer. The pump is located in a way which is shown in the figure just schematically and the difference in elevation between the suction and the discharge reservoir is 0.8 meter. The properties of water are given that is rho = 998 kg per meter cube, mu = 0.001 kg per meter second. The tube which connects the pump and the reservoirs is 5 millimeter diameter is smooth and the total length is 29.8 meter and neglect minor losses.

The question is, what is the maximum flow rate that you can achieve in this system, okay. So, you have to consider first of all look into the HQ characteristics. See these closely resemble to the HQ characteristics which we have theoretically studied. So, when you have Q = 0 there is a particular H and then H drops, so for a particular Q the H becomes 0. So, you can develop no more head.

So, if you consider these HQ characteristics you can see that there is a maximum Q but this maximum Q may not be achievable by the system because this is just the pump characteristic. When the pump is there in the system it has to satisfy the head losses everything in the system and then it has to satisfy, it has to supply that particular Q. So, we have to understand what is the system characteristic?

To understand the system characteristic, the first thing that we have to understand whether the flow is laminar or turbulent in the system. So, one of the key issues is the Reynolds number. So, you know the Q of course there are many Qs. So, to be conservative which Q should we use to determine whether the flow is laminar or turbulent?

If for the largest Q the Reynolds number is less than the critical Reynolds number, then the flow will remain laminar irrespective of the level of disturbance. So, if you consider this particular Q and consider the Reynolds number on the basis of that, so Reynolds number based on the diameter is rho V average is Q/pi d square/4 * d/mu. So, if you calculate this one this will come out to be 1,270.

And this is therefore much below the threshold around 2,000 is the threshold for the pipe flow. On the top of that it is a smooth tube that is given but even the smoothness will not matter here because it is less than the critical Reynolds number, so it will remain laminar irrespective of the level of disturbance. If it was a turbulent one, then you could have use the formula for the head loss for the turbulent flow through hydraulically smooth pipes. So, depending on in whatever regime it is you are corresponding expression for head loss will the laminar. So, the head loss is what? 128 mu QL/rho g pi d to the power 4. The Hagen–Poiseuille equation for the head loss. So the total heads that is developed by the pump it has to satisfy what? See what the system demands?

The system demands that it has to first satisfy the static lift that head should be good enough to give the static lift should be good enough to overcome all the losses. The losses are major losses or minor losses. Minor losses are neglected therefore this is the only loss that you are having. So the head developed by the pump as demanded by the system should be the h static + hf. So, that is of the form of a + bQ, okay.

So, if there was no head loss only a would have been there, only the static lift would have been there. But now you have a Q dependent thing. So, you have this as the system characteristic a straight line. System hQ characteristic and this is the pump characteristic and the point of intersection is the operating point. So, that is the maximum flow rate that you could get in a system. Not the maximum flow rate that the pump could deliver.

So, this is the Q max. So for the pump characteristic given this you have to fit a polynomial and then solve that with the system characteristic and find out what is the point of intersection. So, the corresponding this one the answer is 1*10 to the power – roughly 1 * 10 to the power – 6-meter cube per second, okay. Now, we have looked into the characteristic of the pumps, next we will look into the characteristics of the turbines.

So, what is the difference between the pumps and the turbines as hydraulic machines? The difference is very, very obvious, what the pump will do? The pump will take an input energy an input power and transmit the power to the fluid and therefore the fluid will go to may be different height. On the other hand, what the turbine will do? The turbine will take power or extract power from the water.

The hydraulic turbine will take energy from the water and will do useful using that. So, it absorbs energy from the water and then produces at the expense of that energy. So, in terms of energy transfer it is just the reverse one. Whereas, the pump basically inputs energy to the water the turbine takes energy from the water and converts that into it

So, there are different types of turbines but broadly we have 2 important classifications. One is known as the impulse turbine another is known as the reaction turbine. So what happens in an impulse turbine?

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So we have to just look into these names, impulse turbine and reaction turbine. So, in an impulse turbine first of all see when you want to run a turbine you must have some head available. So, how is the head available? Let us say that you have a reservoir which is located at a large height. So, let us say that we have such a reservoir like this where you have water stored.

So, it may be in a hilly region, in a high elevation region where you have the storage of water. Now you have a pipe, a large pipe which conveys this water to a lower elevation which transmits this water to a lower elevation. The whole objective is to utilize the head of the fluid which is there in this reservoir. So, when it is coming down when it is say being discharge there is a drop in the potential energy here.

So, you expect that to convert into kinetic energy first. To do that you put a nozzle here. So, this nozzle what it tries to do is it takes the input potential energy of course the entire input energy is not available because there is some head loss in this pipe. But the total height difference between this reservoir level and this nozzle level – the head loss in the pipe is the total head that is available at the nozzle.

And the nozzle tries to convert that totally into kinetic energy. So, the nozzle the area of cross section reduces and therefore the velocity increases. So, with that very high kinetic energy what the nozzle will do as an example let us say that the nozzle will have an impinging jet on something known as a bucket. So, what it is trying to do is it is trying to import some energy on a bucket and there are many such buckets.

And if you look into the practical configuration we will look into that practical configuration in a moment. These types of buckets are mounted on a wheel with a particular pitch like that. So, when the jet is falling on the, on one of the buckets then its direction is changed. So, what the bucket does? It changes the direction of the jet. When you change the direction of the jet there is a change in linear momentum that rate of change of linear momentum has a moment with respect to the axis of the wheel and therefore there is net rate of change of angular momentum.

Because of that net rate of change of angular momentum there is a torque that is acting with respect to the axis of the wheel. So, if there is a shaft which is connecting the wheel that shafts start rotating. Now with that shaft is connected the shaft of the generator that generates electrical power. So, there is a shaft, so this is the turbine basically. So, you have a shaft there and there is another shaft which couples with this one.

So that shaft is the shaft of the generator. So, when this shaft rotates synchronously the shaft of the generator also rotates and that is how electrical energy is generated. So, this is the way in which the hydraulic form of energy is converted into electrical form of energy in a hydel power plant having working on this principle of impulse. So, this type of turbine is known as a Pelton turbine and the wheel which is there is known as a Pelton wheel.

So, sometimes these turbines in a technical perspective are also given a broader sense of turbo generator, because you have a turbine that is not only good enough. So, the turbine generates some power output but that is a mechanical power output that is the rotation of the shaft but you have to convert that into electrical power. So, you also need to have a generator. So, you have turbine and a generator couple through a common shaft coupling.

And that is why the integrated system is known as a turbo generator. Now there are certain technical names or issues associated with these. First of all, this reservoir from which the

fluid is supplied is known as the head race and the corresponding level, this level is known as head race level. Now this wheel it is not enclosed but it is not sealed completely but somehow it may be enclosed with a casing with the vent.

The whole idea is that you do not allow the water to splash in all sides. Because with the impact of the water with the bucket now there will be there may be splashing of water. So, this is, these are called as buckets, okay. Now, there is also a reservoir to which the all the water may be eventually dumped and in that way that is like that is called as the Tail race level.

But the net head although it appears that there is this elevation difference between the head race and tail race will matter but the most important thing that matters is not that difference but the difference of this height. Let us say this is H0. So, if there are no head losses in the pipe then the net head available for the turbine is H0. But if there are head losses in the turbine then it is H0, if there are head losses in the pipeline then it is H0, if there are head losses in the pipeline.

This large pipe which conveys the water from the head race level to the turbine level this is known as Penstock. And these buckets are there in a circle and they are center line the circle drawn with their center line that is known as the pitch circle and the corresponding diameter is known as the pitch circle diameter and all the calculations are made referring to the pitch circle diameter.

We will go into the construction of the bucket subsequently but first let us understand that why this called as an impulse turbine. We have just given an example but why the name impulse? So, from the working principle it is quite clear that it is the impulse of a jet with a high kinetic energy that is it is only utilized to do the work.

So, here when the fluid is having an impact with the bucket then there is a change of linear momentum, change of angular momentum and hence the torque but that is only affected by the impulse of the jet because there is no change in pressure as the fluid is flowing through the bucket. So, there are 2 ways in which energy could be readjusted. One is by the kinetic energy.

Here of course in the same level you cannot have a change in the potential energy but you could have a change in the pressure also. That is also another way of imparting ahead. But here you are not having any change in pressure. So, the entire thing is just like open to atmospheric pressure. It is just a casing but not a sealed one.

So, the important thing is it is important to recognize that when you have an impulse turbine you do not have a change in pressure as the fluid is moving through the turbine passage. That is what is very, very important. It is only the impulse of the jet that is being utilized. One the other hand when you have some turbine called as a reaction turbine. So, let us give an example of the reaction turbine.

Then an example of a reaction turbine may be this one which is in a sense it may look like, the constructional features may look like the centrifugal pump but there are many, many differences. But when you draw schematically you will have similar blades and this is the rotor and this is basically representing case when fluid is entering the passage in the blade from the radially outward direction to the inward direction.

So, this is just opposite to the movement of fluid in a centrifugal pump, centrifugal pump from the lower diameter to the higher diameter. This radial flow turbine from the outer diameter to the inner diameter and it is guided by the blade passage exactly in a very similar way. And there is certain technical name difference like this rotor in a turbine is called as a runner. In a pump it was called as impeller.

So, in a turbine that is known as a runner and this type of radial flow turbine is known as a Francis turbine, just like that impulse turbine was the name of that was Pelton turbine. So, here we are giving an example of a reaction turbine a radial flow reaction turbine example is this one the Francis turbine. Now, again the question comes why is it a reaction turbine. So, when the fluid passes through this blade passage it has, basically what it does?

The fluid is coming with some energy. It is giving its energy to the rotor and living with a lower energy. That is what it is doing and that is how the rotor is energize it is rotating. That is the basic thing which is happening. If you explain to a school boy or school girl what is happening. It is what is basically happening.

Some energy, the fluid is coming with some energy it is transferring some energy to the rotor in the process so that the rotor starts rotating and the fluid is living with a lower energy. That is how the energy transfer is taking place. Now, when this energy transfer is taking place the mechanism is that there is not only a change in kinetic energy but more importantly there is a continuous change in pressure as the fluid is flowing through the blade passage.

And that makes it distinguishable from the impulse turbine, where there is no change in pressure as the fluid is flowing across the rotor. So, when there is a change in pressure because why there is a change in pressure because totally this is concealed. So, when you have, it is not exposed to a common atmospheric pressure. So, with a change in velocity and the blade passage geometry it has to adjust by having a change in pressure.

So, there is a continuous change in pressure through as the fluid is flowing through the blade passage of such a turbine and that is why this is known as a reaction turbine. The name reaction is like it is based on sort of an action reaction principle. Again it is same. Why it is because first of all there cannot be a turbine which is 100% reaction turbine. You must have at least an impulse of the jet to begin with.

So, something here is coming with some kinetic energy and then the pressure change is there taking place inside as well as the kinetic energy change is also taking place inside. So, there is some impulse effect but the reaction effect, the reaction effect is not literally like action reaction but the change in pressures through the blade passage of the turbine. So, if there are changes in pressure we called that as the reaction turbine.

There is a turbine which, I mean there are many types of reaction turbine this is a radial flow reaction turbine there maybe axial flow reaction turbine. Just like axial flow pumps we could have axial flow reaction turbine that is one of the examples is Kaplan turbine. So, that is this is radial flow and Kaplan turbine is axial flow. So, let us look into some visuals to understand that how these different turbines may operate.

So, we will first look into a very simple visual, a simple demonstration. You can see there is a wheel that is rotating. This is not really a real turbine but it shows you the principle. So, look into the right hand side you will see that some water is falling on the wheel it is creating a

change in the direction of, change in motion of the wheel. So, this is one of the examples but we will look into the bit more details of the constructional features through the next visual.

So that the next visual will give us the detail of how the different blades of the turbines. These are the buckets you can see. These are the buckets of a Pelton wheel. So, you can clearly get the idea of the constructional features of the buckets which are there and then we will see different types of turbines. So, this just shows the 3 dimensional view of the different ones. This is what type? This is the radial flow, the Francis turbine.

So, you can see that the blade passage and the shaft, etc. all these things are mounted. So, it is important to get a visual feel first that how these turbine blades look like because that will help us a lot in understanding the working principles or some of the basic like mathematical expressions. This is what, this is the axial flow. This is the propeller type or the Kaplan one. So, if you look into this one it is just like a propeller.

And it is having a rotation with respect to its axis and the fluid is flowing along the axial direction. And we will look into some other visuals. So, let us say that we look into the Francis turbine in terms off more engineering detail perspective. We have not discussed about like what are the manners in which the fluid enters the turbine.

So this shows that the fluid enters the turbine we will see why it does not enter through a uniform area passage but a continuously changing area passage and that is what is important. And if you look into the Kaplan turbine you will see that this is the direction in which the fluid flows through the Kaplan turbine. So, it is sort of enters radial and then leaves axial and this is having a lot of importance with regard to that design of the blades of a Kaplan turbine which are typically twisted in Nature.

So, let us stop the discussion for the time being and we will continue in the next lecture. Thank you.