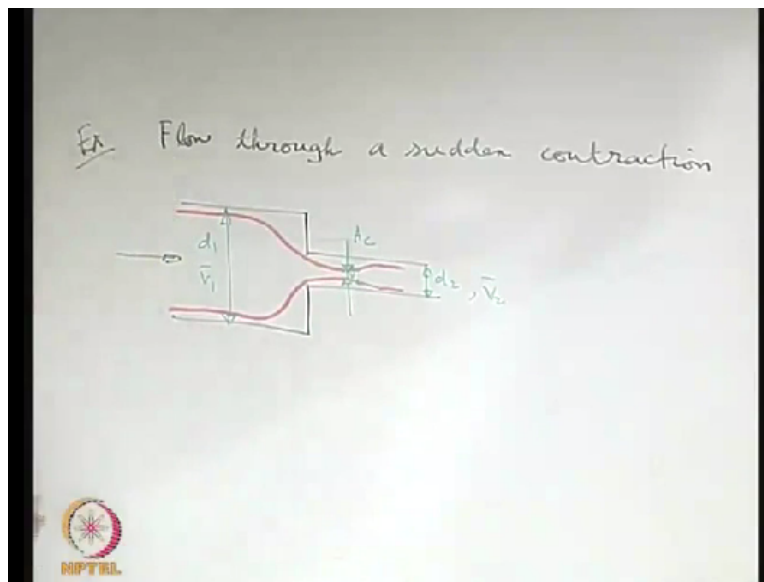


Introduction to Fluid Mechanics and Fluid Engineering
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Lecture – 47
Pipe Flow (Contd.)

We were discussing about the minor losses in a piping system and we took an example of flow through a sudden expansion.

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We will take another example, flow through a sudden contraction. So for flow through a sudden contraction, the idea is that the fluid is flowing from a pipe of a larger size to a pipe of a smaller size and when the fluid is flowing in that way, let us see that what happens to the streamlines. We have encountered such cases earlier and from our previous experience, we know that the streamlines first of all they will tend to get converged because the area is suddenly reducing.

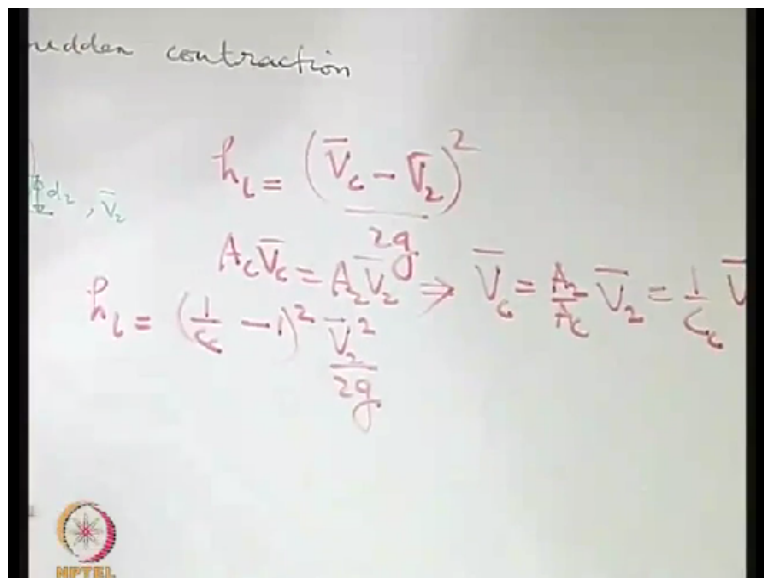
Then that tendency of convergence will continue till it comes to a minimum distance of separation between the extreme streamlines. That location is known vena contracta. So the same thing happens here and beyond that the streamlines diverge. So let us say that this is the location of the vena contracta. So we call this area as say A_c and let us say the average velocity of flow across this one is V_c .

And let us say that d_1 and d_2 are the diameters of the pipes 1 and 2 and V_1 and V_2 are the corresponding velocities, average velocities. So you have v_1 average and v_2 average. Now interestingly we may observe one thing that when you have this type of flow, then in the first part, the streamlines are sort of converging. So that means when it is converging, the area is reducing and the velocity is increasing.

So it is a sort of accelerating flow and as if there is a favourable pressure gradient that is accelerating the flow. Beyond the vena contracta, it is expanding and the situation gets completely reversed and the situation beyond the vena contracta is as if it is flow through an expansion. So it is not a geometrical expansion induced by the configuration of the system but because of the expansion in the configuration of the streamlines.

So here whatever losses may be there, may be attributed to that expansion. So somewhat nonintuitively, for loss for flow through a sudden contraction is basically because of an expansion. So the loss is mainly attributed to whatever is happening here which is nothing but an expansion. So whatever expression that we could derive for loss because of sudden expansion, that the same expression we may apply here.

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Handwritten derivation of head loss for sudden contraction:

sudden contraction

$$h_L = \frac{(\bar{V}_c - \bar{V}_2)^2}{2g}$$

Continuity equation: $A_c \bar{V}_c = A_2 \bar{V}_2 \Rightarrow \bar{V}_c = \frac{A_2}{A_c} \bar{V}_2 = \frac{1}{C_c} \bar{V}_2$

$$h_L = \left(\frac{1}{C_c} - 1 \right)^2 \frac{\bar{V}_2^2}{2g}$$

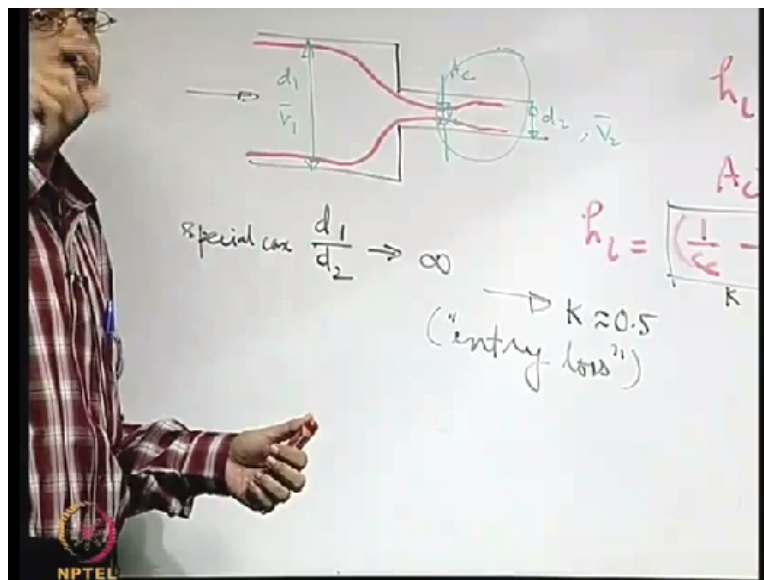
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So what was my head loss expression that we used for this one or we derived for this one? That was like sort of here V_1 will be $V_c - V_2$ whole square/2g, right. Now you can always write, V_c

and V_2 in terms of A_c and A_2 . So you can write $A_c \cdot V_c = A_2 \cdot V_2$ which means you have $V_c = A_2 / A_c \cdot V_2$. If you recall the area of the vena contracta/the area of the corresponding conduit known as a contraction coefficient, C_c .

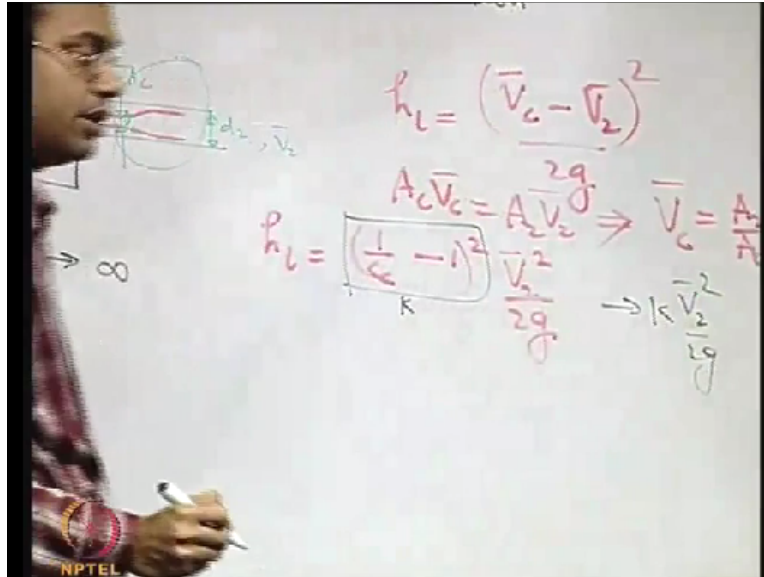
So this is $1/C_c \cdot V_2$. Therefore, the head loss will become $1/C_c - 1$ whole square $\cdot V_2^2$ square / $2g$. Therefore, this head loss will explicitly depend on what is the value of the contraction coefficient and accordingly one may write it in the form of as a fraction of the kinetic energy at 2. Now there are extreme cases like as you have cases for sudden expansion where the ratio of the diameters is greatly varying.

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Here also you can have a case where you have, say d_1/d_2 tending to infinity. This is a special case.

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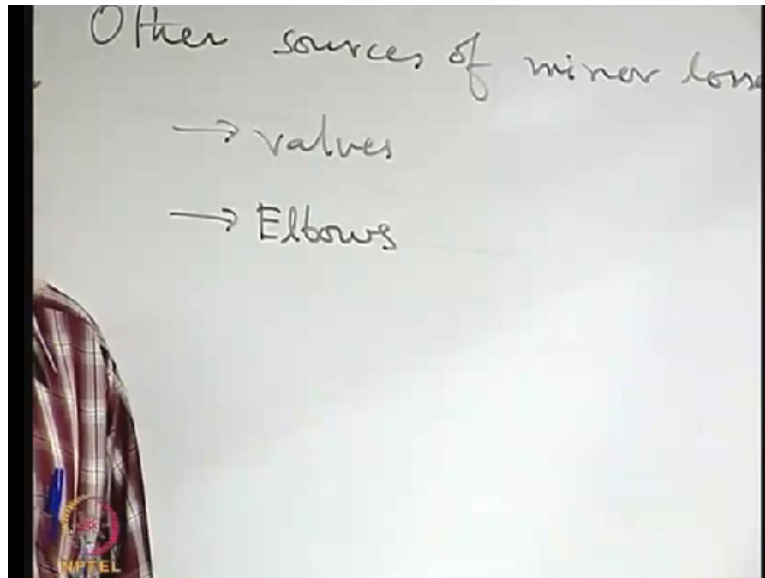


So in general the form is like, this is like k , so it is like $kV_2^2/2g$. The value of K will depend on the contraction coefficient in such a case when d_1/d_2 tends to infinity. It is found experimentally that k is very close to 0.5. So this case like d_1/d_2 tends to infinity, what does it show? It represents an equivalent situation that there is a reservoir from which fluid is entering a pipe where the reservoir size is much much larger than the diameter of the pipe.

So in that case, this loss is known as entry loss because we have discussed about an exit loss. So entry and exit are always relative to the pipe. So here the fluid is entering the pipe from a larger reservoir. So this is known as entry loss. So the concepts of entry and exit loss are somewhat similar. One is like fluid is entering the pipe from a reservoir, so that is entry loss and exit is fluid is exiting from the pipe to another reservoir.

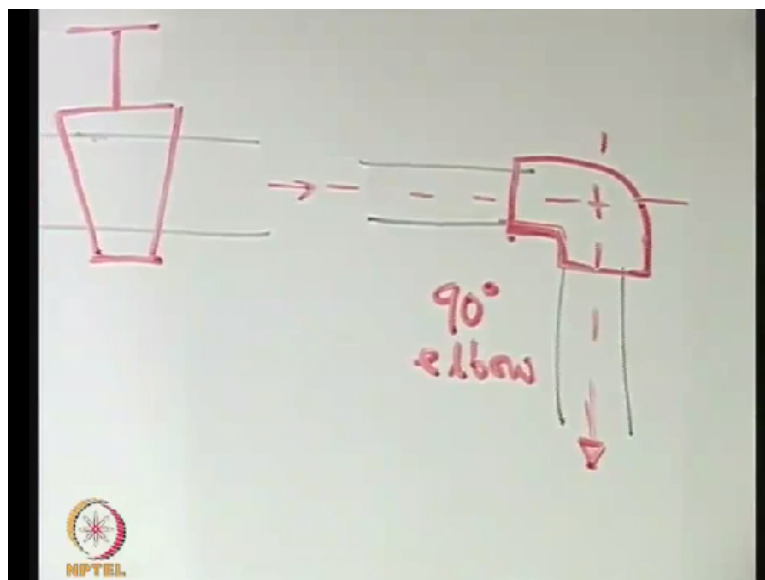
And in either of these cases, it is a sudden expansion and contraction that we are keeping in mind and as we have just discussed that loss due to sudden contraction is basically due to a sudden expansion. Now it is not always that you have a sudden expansion and contraction as the only possibilities because of which there are minor losses. So minor losses may be present because of many other things.

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So other sources of minor losses are like presence of valves. So if there are valves in a pipeline. So what does a valve do?

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So if you have a pipeline say like this and you put a valve in the pipeline. We are not drawing a valve in a proper detailed manner but just let us say this is a schematic way of representing. So let us say that this is a physical obstruction. So when this is lifted, the entire fluid may flow. When this is put down, depending on the extent to which it is put down, it will restrict the motion of the flow or motion of the fluid.

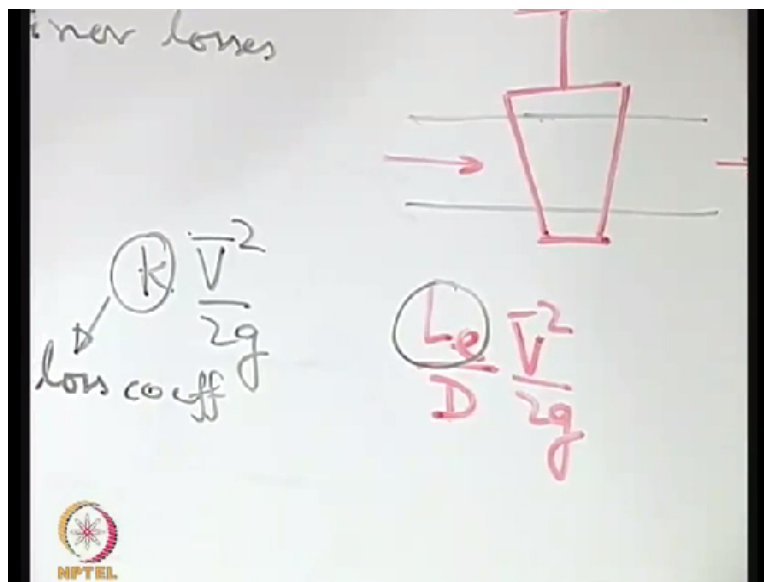
Therefore, valves somehow they may restrict the motion and because of that there may be a

pressure drop across it. So it may act like an orifice where it is an reduced size of the area available for flow and because of that as we have seen in our earlier examples in flow measuring devices, in such cases you may have losses. So valves will also have losses. Then you may have elbows.

So what are the elbows? These are fittings which try to accommodate a change in direction of the pipeline. So you have a, say a pipeline like this and you want the flow direction to change like this. So what you do? You fit a piece which may be somewhat like this. So this type of piece is known as a 90 degree elbow. The name 90 degree is quite clear that the change in angle here that is experienced by the flow is 90 degrees.

This is a 90 degree elbow. So in this way you may have elbows of various degrees. So such things like valves, elbows, these things are known as fittings of a pipe. So when you have a pipeline, you just do not have an isolated pipe but you have certain things which fit with the piping system and those are known as fittings.

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So for all these, it is not so easy to calculate or rather write analytically exactly the expression for the loss but one may have whole amount of experimental or computationally available data and from that, the loss is somehow characterised as $k \cdot V^2 / 2g$. The idea is straightforward that you are trying to write the loss as in proportion to the kinetic energy rate and the motivation is

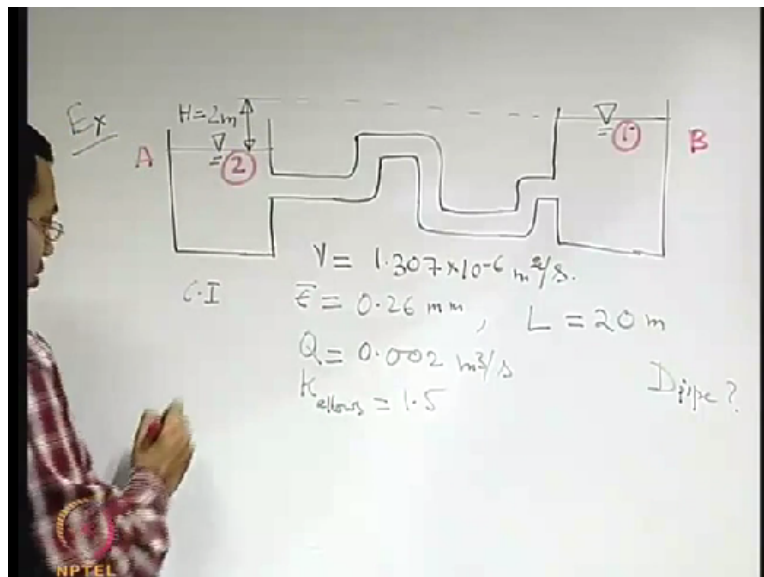
that, in the previous cases, in all cases we could successfully write the loss in this form, $k \cdot V^2 / 2g$.

But here the k is not something which is straightforward or analytically determined but it comes from experiments and many other considerations. So this k is known as a loss coefficient. So typically whenever one is dealing with an engineering analysis of a piping system and there are fittings, there are piping handbooks which refer to the loss coefficients based on what fittings that you are using and one may refer to that data from the piping handbooks.

And those databases, those have been created by lots of experimentation or these days also by computer simulation and important is to get a value of this one. In many other cases, it is also written in an equivalent form like it is written in some L equivalent $/ D \cdot V^2 / 2g$, this form because we have seen that this is also another way of writing the loss, $fL / D \cdot v^2 / 2g$.

So if it is written in that way, then this sometimes is known as equivalent length of as if it was replaced by a pipe of some length and that would have created some loss but more commonly, it is of the loss coefficient that is quoted and that is used. So let us consider one problem where we will illustrate how to make use of the concepts of major and the minor losses.

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So we have a piping system like this. It is connecting 2 reservoirs of large extent. The values of

the kinematic viscosity, the value of the kinematic viscosity is given. The pipe is made of cast iron with following characteristics. The average surface roughness 0.26 millimeter. The total length of the piping system is 20 meter and then the flow rate that we expect from the system is 0.002 meter cube per second.

And the loss coefficient for the elbows is 1.5 and thing is that what should be a good design of the diameter of the pipe, okay. So pipe of uniform size but it has sudden bends and turns, okay. So first of all, let us say that the name of the reservoir in the left is A and the name of the reservoir in the right is B, okay. Let us say this is reservoir A, this is reservoir B. What would be the direction of the flow from reservoir A to B or B to A? B to A, right.

Because you have a natural head available in form of a potential energy head and if you want to have a flow from A to B, that also could be possible if you had a pump at some place which will energise it to overcome that deficit in the height. So when there is flow from B to A, let us say that you write the energy equation with losses which is like the equivalent modified Bernoulli's type of equation for flow from say 1 to 2.

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$\nu = 1.307 \times 10^{-6} \text{ m}^2/\text{s}$
 $\epsilon = 0.26 \text{ mm}$, $L = 20 \text{ m}$
 $Q = 0.002 \text{ m}^3/\text{s}$
 $K_{\text{elbows}} = 1.5$ D pipe?
 $\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + \sum h_{\text{loss}}$
 $H = \sum h_{\text{loss}} = \frac{fL}{D} \frac{V^2}{2g} + 0.5 \frac{V^2}{2g} + 6 K_{\text{elbows}} \frac{V^2}{2g}$
 $f_h \text{ of } (f, D) = 0$

So essentially what you write, $p_1/\rho g + V_1^2/2g + Z_1 = p_2/\rho g + V_2^2/2g + Z_2 + \text{the summation of head losses}$. We are technically to be correct; you have to use kinetic energy correction factor at these places but if you see here that will not be important. Let us write just

for the sake of writing it properly. If you assume, so first of all we are neglecting the unsteadiness in this case.

So we are assuming that these are very large diameters as compared to the diameter of the pipe. So it is as if there is a slow change and whatever change in the height of these reservoirs, that is not very significant. That is almost like negligible. So if that is there, that means the corresponding velocities of this free surfaces are much negligible as compared to the flow velocities in the pipeline.

Then you basically neglect these terms, these are small. Both p_1 and p_2 are same which is p atmosphere. So you have $Z_1 - Z_2$ that is h is summation of the head losses. So what are the head losses? Now you tell. First of all, let us consider the major loss. So head loss first of all, if you write the major loss, what is the major loss? It is of the form $fL/DV^2/2g$ where V is the velocity of flow, average velocity of flow through the pipe.

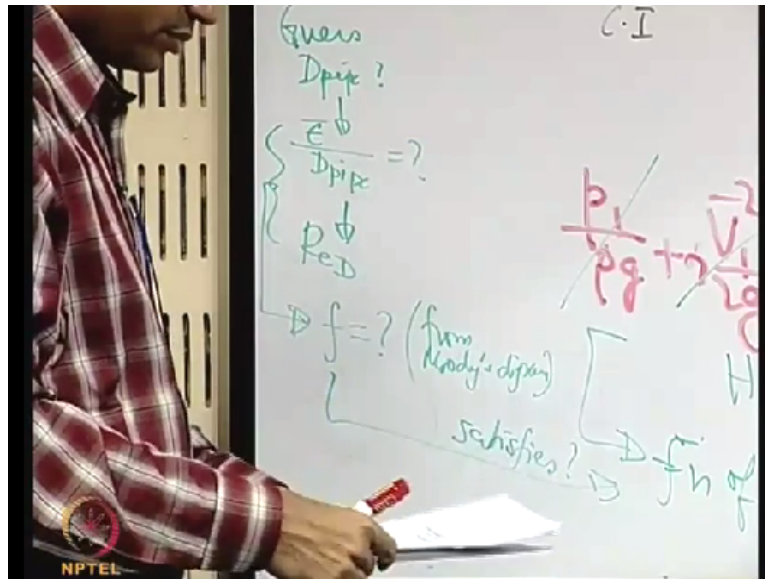
Total length of the pipe is given. So this is major loss. Then minor loss. What are the sources of minor losses? Do not do it haphazardly. Follow the path of the flow. So first when the fluid enters here. This is an entry loss. So what is that? $0.5V^2/2g$. Then it encounters some number of elbows. How many are there? 1 2 3 4 5 6, right. So you have 6 90 degrees elbows for which you have each as k_{elbow} as the loss coefficient $\times V^2/2g$.

Then there is a exit loss, okay. So what is the exit loss? $V^2/2g$, right. So this is major loss. This is entry. This is elbow and this is exit. So from this what you can find out is of course Q is given. So you can replace V will $4Q/\pi D^2$ but D is the diameter of the pipe. So this equation will boil down to what form? Some equation which is a function of f and D , that is $= 0$, right.

So function of f and $D=0$, it will be just a polynomial function and what will be the power of D in that expression? $L/2D$ to the power 5 because here V^2 will bring $1/D^4$ to the power 4 another D , so it will be a polynomial D to the power 5 and some function of f , together that will be 0. So how do you then go ahead? What extra information you have? You have information on

the epsilon. So what you may do?

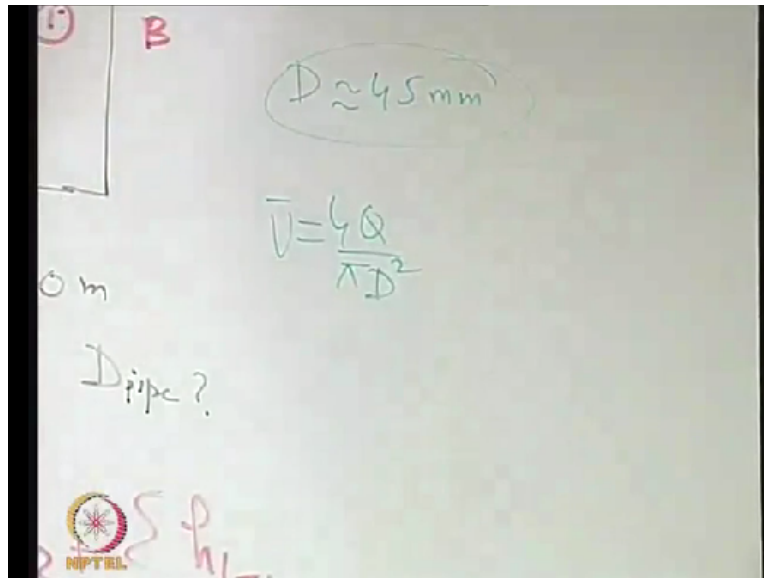
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You may guess value of the diameter of the pipe. If you guess a value of the diameter of the pipe, that will give you what is epsilon/the diameter of the pipe and then you can calculate what is the Reynolds number based on the diameter of the pipe and these 2 together should give you a value of f from the Moody's diagram.

You have to check whether this f , it satisfies this or not because this equation is a function of f and D . So if you substitute D , you will get a value if f . So if it does not satisfy, you have to go through this iteration process again and again till it converges. So let me give you the answer to this problem so that at least you can check whether answers are coming properly.

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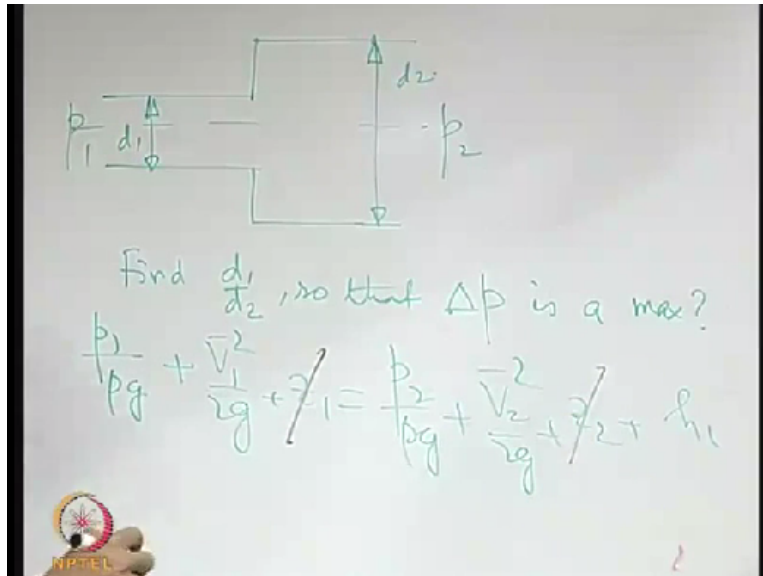


So for this one, the answer is the diameter=45 millimeter roughly. **“Professor - student conversation starts”** (i) (22:19) What value you will guess is up to you, that is why it is guess. Yes, sir, but... I mean, it is obviously the question is when you guess a diameter of a pipe, if you want to have your wildest expression of imagination, 1000 kilometer, you may start, may be if you want or may be 1 nanometer if you want.

So all of you have certain common senses and you will always like to exercise a common sense. If you say that to exercise the common sense, that seems most of us do not have proper common commonsense. So let us see how we do it. **“Professor - student conversation ends”** So in this equation, you have a function of f and D , right. You have commonsense values of the friction factor. If you look into the Moody's diagram, you will see 0.002 0.0002 like that.

This is just a linear function of that one, substitute that f and see what order of magnitude of D satisfies that. So it will give you a reasonable order of magnitude of D , okay. So that is a commonsense way of going for a guess. So whenever you go for a guess solution, it does not have to be a wild guess. I mean it has to be a bit of a civilised guess to get some kind of quick answer. So let us work out another problem.

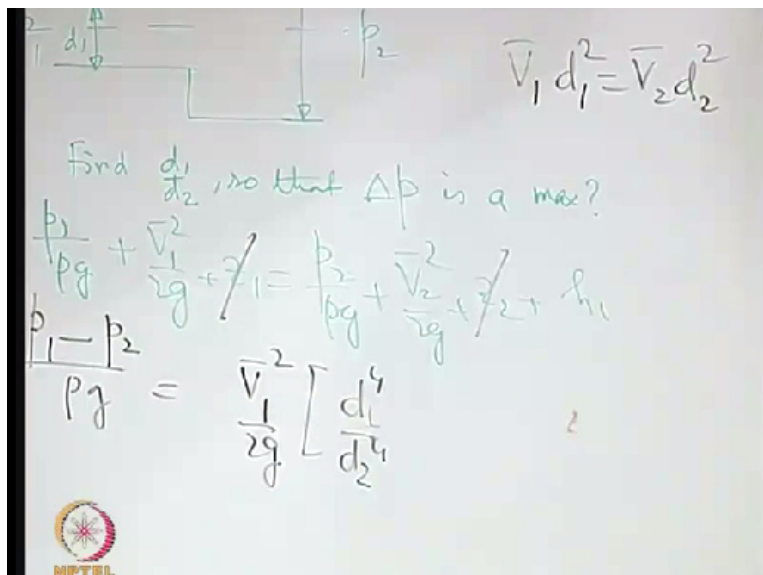
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Let us say that you have a horizontal pipeline with diameters of d_1 and d_2 and the pressures at the 2 ends are p_1 and p_2 . So what you have to find out is find the ratio of d_1/d_2 so that Δp is a maximum. Δp is the difference between p_1 and p_2 . So this is very straightforward. We will just outline the procedure. So if you write the modified energy equation between the sections 1 and 2 that is modified equation considering the losses, you have $p_1/\rho g + V_1^2/2g + Z_1 = p_2/\rho g + V_2^2/2g + Z_2 + \text{head loss}$.

So it is a horizontal pipeline that is given. So Z_1 and Z_2 are the same, that you cancel from the 2 sides.

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Then you can write say $p_1 - p_2 / \rho g =$, now you can express V_2 in terms of V_1 by noting $A_1 V_1 = A_2 V_2$, that means, you have $V_1 \cdot d_1^2 = V_2 \cdot d_2^2$. So you can write $V_2^2 / 2g$ as $V_1^2 / 2g \cdot d_1^4 / d_2^4$... sorry the other way. d_1 to the power 4 / d_2 to the power 4.

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$$v_1 d_1^2 = v_2 d_2^2$$

if Δp is a max?

$$\frac{p_2}{\rho g} + \frac{V_2^2}{2g} + \frac{z_2}{2} - \left(\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + \frac{z_1}{2} \right) = h_L \rightarrow \frac{(V_1 - V_2)^2}{2g}$$

$$\left[\frac{d_1^4}{d_2^4} - 1 \right] + \frac{1}{2g} \left[1 - \frac{d_1^4}{d_2^4} \right]^2$$

and then you have -1 for $1 \cdot V_1^2 / 2g$, then what is this head loss? This is just $V_1 - V_2$ whole square / $2g$, this is sudden expansion loss. The lengths are not substantial to have major losses as important. So here this is an example where we will see that minor loss is the dominating one. So the length is so short that the loss due to the length is not. Of course it is there but that may be neglected as compared to this loss.

So this also you can write in terms of $V_1^2 / 2g \cdot 1 - V_2 / V_1$ that is d_1^2 / d_2^2 square with square, right. Then I need not work it out further. It is a very simple exercises. You just consider say $d_1 / d_2 = x$, so it is a function of x only for maximum of this, the derivative with respect to x should be 0. So that will give you the value. Now next what we have seen in these examples that what are the major and the minor losses and how they are taken into account and again that important consideration that minor losses need not always be minor.

Minor losses sometimes are much much more significant than the so-called major losses. Next we will look into cases. So here we have till now considered cases of isolated single pipes but in a system, in a piping system there may be a number of pipes and these pipes may be connected in

series or parallel, just in the same way as electrical resistors are connected. So then what would be that equivalent piping network is just like the equivalent electrical circuit network and we will see briefly the corresponding ideas for pipes in series and pipes in parallel.

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Handwritten notes on a whiteboard titled "pipes in series". The notes show a schematic of two pipes in series with parameters d_1, V_1, l_1, f_1 and d_2, V_2, l_2, f_2 . The flow rate Q is indicated. The head loss h_f is shown as a function of Q, d, l, f . The equations derived are:

$$h_{f1} = \frac{f_1 l_1}{d_1} \frac{V_1^2}{2g} = \frac{16 f_1 l_1 Q^2}{2g \pi^2 d_1^5}$$

$$h_{f2} = \frac{16 f_2 l_2 Q^2}{2g \pi^2 d_2^5}$$

$$h_f = h_{f1} + h_{f2} = \frac{16 Q^2}{2g \pi^2} \left(\frac{f_1 l_1}{d_1^5} + \frac{f_2 l_2}{d_2^5} \right)$$

$$\frac{f_e l_e}{d_e^5} = \frac{f_1 l_1}{d_1^5} + \frac{f_2 l_2}{d_2^5}$$

$$\frac{f_e l_e}{d_e^5} = \frac{f_1 l_1}{d_1^5} + \frac{f_2 l_2}{d_2^5} \Rightarrow \frac{f_e l_e}{d_e^5} = \sum \frac{f_i l_i}{d_i^5}$$

So first pipes in series. So pipes in series, it means that you have let us say that you have 2 pipes like this. The name series is obvious. They are connected one after the other. So you have let us say that the diameter of the first pipe d_1 , the average velocity V_1 , length l_1 , friction factor f_1 and for the pipe 2, corresponding things are there. So when we consider these pipes in series and parallel, in this analysis, the analysis that we are presenting as a theoretical development, we are not considering the minor losses.

We are considering only the major losses. So the head loss for the pipe 1, what is that? $f_1 l_1 / d_1 * V_1^2 / 2g$. What is V ? V is $4Q / \pi d^2$. So in terms of the flow rate, so $f_1 l_1 / d_1 * V_1^2 / 2g$ will be $16Q^2 / (2g \pi^2 d_1^5)$. So $16f_1 l_1 Q^2 / (2g \pi^2 d_1^5)$, then, $2g \pi^2 d_1^5$ to the power 5, okay, where Q is the flow rate which is going through each of these pipes. So when they are in series, what is the common thing for them is the flow rate.

The same flow rate is going through the 2 pipes. So if you have h_{f2} , you have similar thing, $16f_2 l_2 Q^2 / (2g \pi^2 d_2^5)$. Now what is the concept of an equivalent pipe, that is you replace this 2 pipes in series by a single pipe of some diameter, let us say d is the

equivalent diameter, l_e is the equivalent length and f_e is the equivalent friction factor such that you have the same flow rate and the same head loss, okay.

So it is just like an electrical circuit where you are considering the same voltage and same current flowing through that. So you find out an equivalent resistance sort of thing. So here it is like the head loss is like the pressure drop which is like a potential drop sort of thing and the flow rate is like a current so to say. It is not exactly analogous mathematically but is just another qualitative way of looking into it.

So when you have this h_f expressed as the head loss in this equivalent situation then h_f must be = the sum of h_{f1} and h_{f2} . So if you write h_f for the equivalent pipe, it is a single pipe of length l_e . So from this you can write $16f_3l_e$, same Q is there, $\frac{1}{2}g \pi \text{ square } d_e$ to the power 5 = $16f_1l_1Q^5$ square $\frac{1}{2}g \pi \text{ square } d_1$ to the power 5 + $16f_2l_2Q^5$ square $\frac{1}{2}g \pi \text{ square } d_2$ to the power 5. So from this, what we can get?

We can get a very important expression, that $f_e l_e / d$ to the power 5 = $f_1 l_1 / d_1$ to the power 5 + $f_2 l_2 / d_2$ to the power 5. So in general if you have n number of such pipes in series, you have $f_3 l_3 / d$ to the power 5 = summation of $f_i l_i / d_i$ to the power 5, $i=1$ to n . So as if it is like an equivalent resistance as the sum of the resistances, that is a simple way of looking into it. Now let us look into pipes in parallel.

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$Q = Q_1 + Q_2 \Rightarrow \sqrt{\frac{d_e^5}{f_e l_e}} = \sqrt{\frac{d_1^5}{f_1 l_1}} + \sqrt{\frac{d_2^5}{f_2 l_2}}$

Pipes in parallel

$Q = Q_1 + Q_2$
 $h_{f1} = h_{f2} = h_{fe}$

$h_{fe} = \frac{16 f_e l_e Q^2}{2g \pi^2 d_e^5} = \frac{16 f_1 l_1 Q_1^2}{2g \pi^2 d_1^5} = \frac{16 f_2 l_2 Q_2^2}{2g \pi^2 d_2^5} = k (say)$

$Q_1^2 = \frac{k d_1^5}{f_1 l_1}$
 $Q_2^2 = \frac{k d_2^5}{f_2 l_2}$

So when you have pipes in parallel, let us try to make a sketch of may be a situation like this. So you have 2 pipes which are sort of connected in parallel. That means say they are branching off from, just let me sketch it in a of a different way. Let us say that that you have pipes through which some fluid flow, Q is coming there. Now you have 2 pipes with say diameters d_1 , length l_1 .

So length l_1 means not just straight portion+also the curved portion, all those taken together. $d_1 l_1$ and the friction factor f_1 . Second pipe, $d_2 l_2$, friction factor f_2 . So these pipes both are connected across these 2 points which are shown as cross. So what you can say that let us say that Q_1 is the flow rate through this one. Q_2 is the flow rate through this one. So you can say that $Q = Q_1 + Q_2$.

If you consider the node which is given by the cross, just like Kirchhoff's current law. So the Q is distributed as Q_1 and Q_2 . Then what about the head loss. Head losses are the same because eventually we were talking about the difference in energy between these 2 points, no matter whether you traverse by the upper pipe or the lower pipe, eventually you end up at the same point and the loss of energy therefore should be same as what you calculate from here or what you calculate from here.

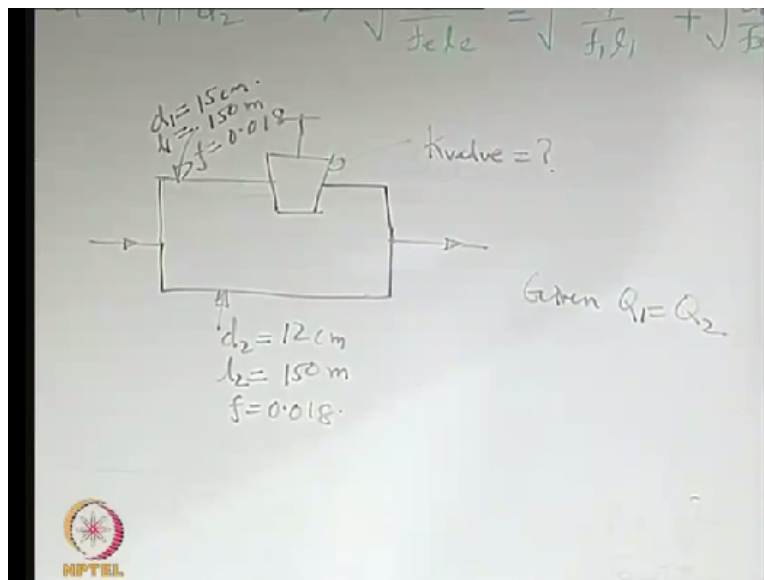
So you have $h_{f1} = h_{f2}$. So these are basic equations and from that you can find out the equivalent

length of the pipes and for the equivalent pipe, you have say $h_f = h_{f1} = h_{f2}$ and $Q = Q_1 + Q_2$. So what is the h_f of the equivalent pipe? $16 f_e Q^2 / 2g \pi^2 d^5$, right. This is the h_f of the equivalent pipe. This = h_{f1} that is $16 f_1 L_1 Q_1^2 / 2g \pi^2 d_1^5$ and this is also = h_{f2} , okay.

So this is h_{f1} . This is h_{f2} . Let us say that $h = \text{some constant } k$ and this $16/2 g \pi^2$, this is a term which is like a constant for all. Let us call it as c . So you can write, this is Q_2 , sorry. So you can write $Q_1 = \text{or } Q_1^2 = k * d_1^5 / c f_1 L_1$, right. Similarly, $Q_2 = k * d_2^5 / c f_2 L_2$ and Q is $k d$ to the power $5/c f_e L_e$. **“Professor - student conversation starts”** (()) (38:14) Q_2^2 square, yes, right. **“Professor - student conversation ends”**

Since you have $Q = Q_1 + Q_2$, you have, from these expressions, root over d to the power $5/f_e L_e = \text{root over } d_1 \text{ to the power } 5/f_1 L_1 + \text{root over } d_2 \text{ to the power } 5/f_2 L_2$, okay. The other terms get cancelled out. So these are expressions for the equivalent, the relationship between the equivalent and the original ones in terms of the respective diameters and the friction factors. So with this background, let us try to work out a few problems where we have the pipes connected in may be series or parallel.

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So you have 2 pipes, 2 pipelines and these 2 pipes, the upper one is $d_1 = 15$ centimeter and length is 150 meter, the friction factor is a constant which is 0.018. The other pipe is, the diameter d_2 is

There is a formula which is readymade available with you and you might be tempted to use that. What should prevent you from being tempted with that is that here you have a minor loss, that is not considered in this formal, okay. So to use that formula, it will give you erroneous solution but obviously the concept of pipes in parallel, you may use. So what are the things, you have $h_{f1}=h_{f2}$, not just h_f , the total head loss. Not just the friction loss.

$K_{valve} = ?$

Given $Q_1 = Q_2$

$h_{loss1} = h_{loss2}$

$\frac{16 f L_1 Q_1^2}{2g \pi^2 d_1^5} + K_{valve} \left(\frac{V_1}{2g} \right)^2 = \frac{16 f L_2 Q_2^2}{2g \pi^2 d_2^5}$

$Q_1 = Q_2$ (given)

Ans. $K_{valve} = 18.62$

So you can cancel that from the 2 sides and get the value of the k valve straightaway, a very simple exercise. The answer is k valve is 18.62. Next we work out another problem.

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$Q = Q_1 + Q_2 \Rightarrow \sqrt{\frac{d_1^5}{f_1 l_1}} = \sqrt{\frac{d_1^5}{f_1 l_1}} + \sqrt{\frac{d_2^5}{f_2 l_2}}$
 Ex. 2 pipes length l , diam d_1 & d_2 // same f
 given $\frac{d_1}{d_2} = 2$ Find $\frac{h_1}{h_2}$
 series: $\frac{f_1 l_1}{d_1^5} = \frac{f_1 l_1}{d_1^5} + \frac{f_2 l_2}{d_2^5}$
 $h_f = \frac{16 f_1 l_1 Q^2}{2g \pi^2 d_1^5} = \frac{16 Q^2}{2g \pi^2} \left[\frac{f_1 l_1}{d_1^5} + \frac{f_2 l_2}{d_2^5} \right] = h_2$
 parallel: $h_f = \frac{16 f_1 l_1 Q^2}{2g \pi^2 d_1^5} = \frac{16 Q^2}{2g \pi^2} \left[\frac{1}{\left(\frac{d_1^5}{f_1 l_1} + \frac{d_2^5}{f_2 l_2} \right)^2} \right] = h_1$

You have 2 pipes of length l and diameters d_1 and d_2 and they are arranged in parallel. When they are arranged in parallel, the loss of head for a particular flow rate Q , Q is the flow rate, the loss of head is h_1 and the same pipes when they are arranged in series, the loss of head is h_2 . It is given as $d_1/d_2=2$. Find h_1/h_2 . Neglect the minor losses and assume a constant friction coefficient to be the same for all the pipes.

So there are 2 important assumptions, that minor losses are neglected and number 2, friction coefficient or the friction factor is a constant and that constant value is same for all the pipes, okay. Under which conditions, friction factor you have a constant virtually? For very high Reynolds number, highly turbulent flow, it will become only a function of ϵ/d . But here the diameters are changing.

So we are assuming that ϵ is also different for the 2 pipes such that ϵ/d remains the same so that the friction factor remains the same. So when the 2 pipes are connected in series, so you can have brought this out through the equivalent resistance concept. So when they are in series, what is the condition for the equivalent, $f_1 l_1 / d_1$ to the power 5 = $f_1 l_1 / d_1$ to the power 5 + $f_2 l_2 / d_2$ to the power 5, this is for the series and now the equivalent thing, the equivalent thing has combinations of 3 parameters.

And see it is not important what are the individual values of these parameters. It is important that

you collectively choose them to satisfy this constant, that should be good enough. That means you may choose your equivalent friction factor or equivalent length in such a way that you will get some equivalent diameter or you may choose equivalent friction factor and equivalent diameter as to be something so as to get some equivalent length.

So you may take any of these out of 3, 2 very freely and the third one you get from this expression. Let us say that we assume that the 2 pipes are of the same length, right. So let us consider that l_e or in fact, if you see that it is $f_e l_e / d$ to the power 5, that is going to be solely important for the head loss. So even if you do not assume any particular value, that will not matter.

So if you consider the head loss, what is that? $16 f_e Q^2 / 2g \pi^2 d^5$, okay. So you can clearly see that you get an expression where you have $f_e l_e / d$ to the power of 5. So let us say that you write in place of that, $16 Q^2 / 2g \pi^2$, then you write f_{l1} / d_1 to the power 5 + f_{l2} / d_2 to the power 5. This is given as h_2 . This is series. If they are in parallel? Again h_f formula is the same but expression for, so this you have $16 Q^2 / 2g \pi^2$, then you have $1/d$ to the power 5/ f_e , right and that you can substitute in place of this one, right.

That is d to the power 5/ f_e and this is given as h_2 , sorry this is given as h_1 . Just you divide by these 2 and you will get a ratio, when you divide you will get a ratio of d_1/d_2 and l_1 and l_2 are the same. So that ratio will give a number. So this when you divide, you will just get a number. f_1 and f_2 are the same. So those effects will cancel and it will be expressed solely in the as a function of d_1/d_2 , if you write h_1/h_2 .

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$$d_1 \& d_2 \quad || \quad h_1$$

$$\frac{h_1}{h_2}$$

$$\text{Series} \quad h_2$$

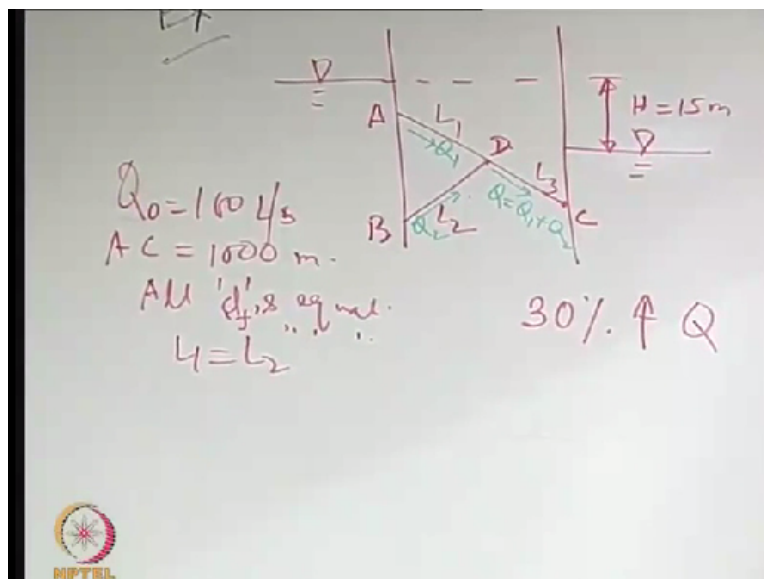
$$\frac{h_1}{h_2} = 0.02188$$

$$L_e$$

$$\left[\frac{f_1 L_1}{d_1^5} + \frac{f_2 L_2}{d_2^5} \right] = \frac{h_2}{L_e}$$

So the h_1/h_2 , the answer is 0.02188 that is the answer. Let us work out may be another problem.

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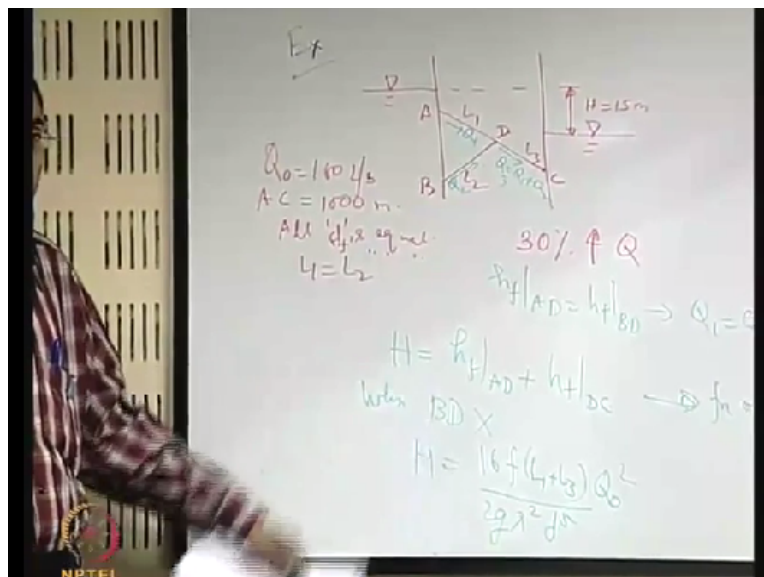
The problem statement is like that initially, so you can see that here there are pipes AD BD and DC. So this is just shown by schematic, so not the with being shown. So here initially only the part AC was there. There was no branch BD and then the flow rate was 100 liter per second that is given. So Q_0 is 100 litre per second and the length of AC is 1000 meter that is 1 kilometer. To increase the flow rate, another pipe BD is added, okay.

Estimate the length of the new pipe. That is the problem. All diameters are equal. So all diameters are equal and assume the same length for all the pipes, not for all the pipes, that is

$L_1=L_2$ that is same length for the 2 parallel pipes and same friction factor for all pipes. So friction factors are also equal, okay and it is given that there is a 30% enhancement in the flow rate because of this.

So you have to find out basically L_1 and L_2 that is the question. So let us say that there is a flow rate Q_1 through L_1 and Q_2 through L_2 and the total Q is sum of Q_1 and Q_2 . So then you can write, so the head loss is if you neglect this elevation difference. The head loss should be what? The head loss for AD and head loss for BD, they should be the same. They are like pipes in parallel. So if their head losses are same, head loss is function of Q , f and L . So you have f and L are same. Therefore, Q should be same.

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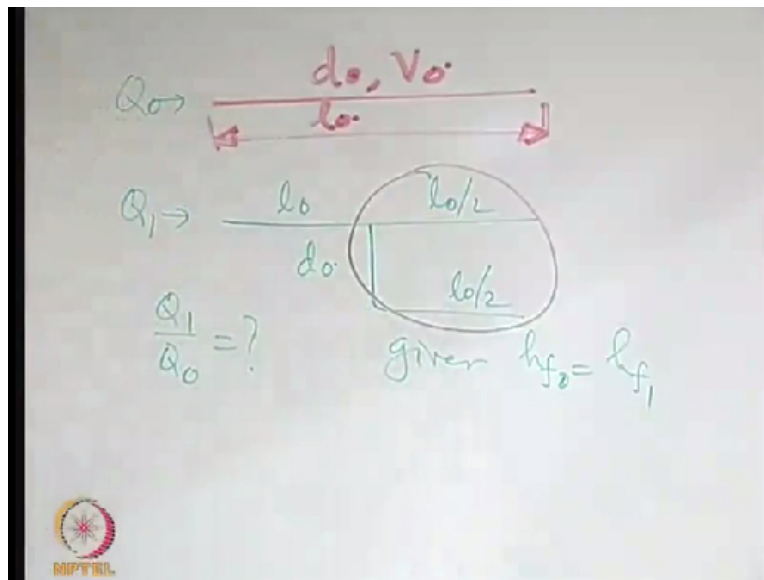
So $h_{fAD}=h_{fBD}$, that will give you $Q_1=Q_2$ and therefore you have Q_3 which is either $= 2Q_1$ or $2Q_2$, all the same. Then what is the total head loss? That is capital H . So we will not write the modified equation in all details. You have just seen that this capital H should be compensating the total head loss. So the head loss in AD+the head loss in DC, right. So this will be a function of Q_3 because head loss in AD is a function of Q_1 , Q_1 may be expressed as a function of Q_3 .

And head loss in DC is a function of Q_3 and the head loss when this branch system was not there, still the head loss would be the same, right. So when BD is not there, then the head loss is the head loss for the length AC with the original flow rate as Q_0 . So $16fL$, L is L_1+L_2 with Q_0

square/2g pi square*d to the power 5 and it is given as that there is a 30% enhancement in Q, that means Q_3/Q_0 is 1.3.

So from that you can find out the missing length. You have to keep in mind that total L_1+L_3 is 1000 meter. So just assume this as some x and this is $1000-x$ and this is also then x. You can solve for that, remaining things are given. Let us, may we look into another problem very briefly. So let us say that you have 2 pipes or a pipeline.

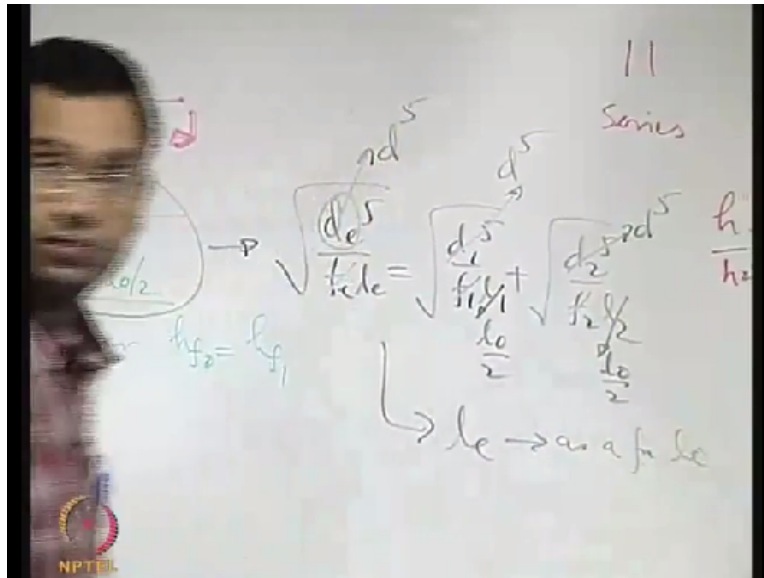
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It has a diameter say d_0 and the velocity V_0 . It is having some length say l_0 . To increase the flow rate, a new arrangement is made. What is the new arrangement? The new arrangement is a branch is taken away from the midpoint of this one. So this is l_0 . This is $l_0/2$ and this $l_0/2$. So the diameter is the same. The diameter is d_0 for the second arrangement as well.

You have to find the change in flow rate. Say here flow rate is Q_0 , here the flow rate is Q_1 . So you have to find out what is Q_1/Q_0 given $h_{f0}=h_{f1}$, okay. So this is a straightforward pipe series/parallel problem. So only thing is, what you do? You replace this by the equivalent pipe. So if you replace this by an equivalent pipe, these are 2 pipes in parallel.

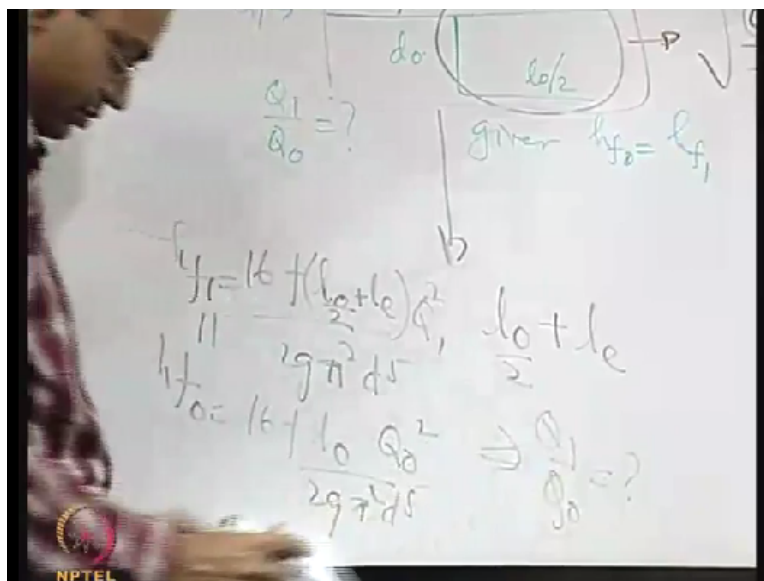
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So root over d to the power $5/f l_e = \text{root over } d_1 \text{ to the power } 5/f l_1 + \text{root over } d_2 \text{ to the power } 5/f l_2$. Here all f 's are the same. Let us consider that the equivalent friction coefficient are also the same, l_1 is what? $l_0/2$, l_2 is $l_0/2$. d_1 and d_2 are the same which is $= d$, same diameter pipe. So this is d to the power 5 , this is d to the power 5 . So let us say that the equivalent diameter is also d . So you can find out an equivalent length in terms of as a function of l_0 , right.

So then this entire pipe as if is replaced by a pipe of length $l_0 + l$ equivalent.

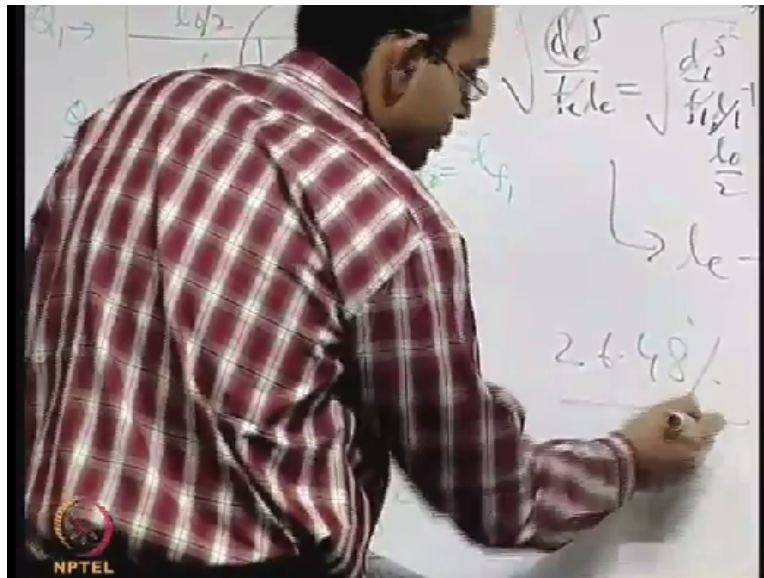
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And you have $h_{f1} = 16 f l_0 + l$ equivalent $\cdot Q^2 / 2 g \pi^2 d^5$ and h_{f0} is $16 f$, sorry this is $l_0/2$. $l_0/2 + l$ equivalent. Sorry this is $l_0/2$, just correct it. This is $l_0/2$, half half. So

$16f_1Q_0^2 \text{ square}/2g \pi^2 d^5$. From here, since these 2 are equal, you can find out what is Q_1/Q_0 .

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The answer is that the increment is 26.48%, okay. So this is just very simple equivalent pipe system analysis. So let us stop here today or for this lecture and we will continue with the next lecture with a new topic. Thank you.