Introduction to Fluid Mechanics and Fluids Engineering
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Lecture - 43
Potential Flow (Contd.) and Flow Past Immersed Bodies of Special Shapes

We continue with our examples of super position of different plane potential flows.

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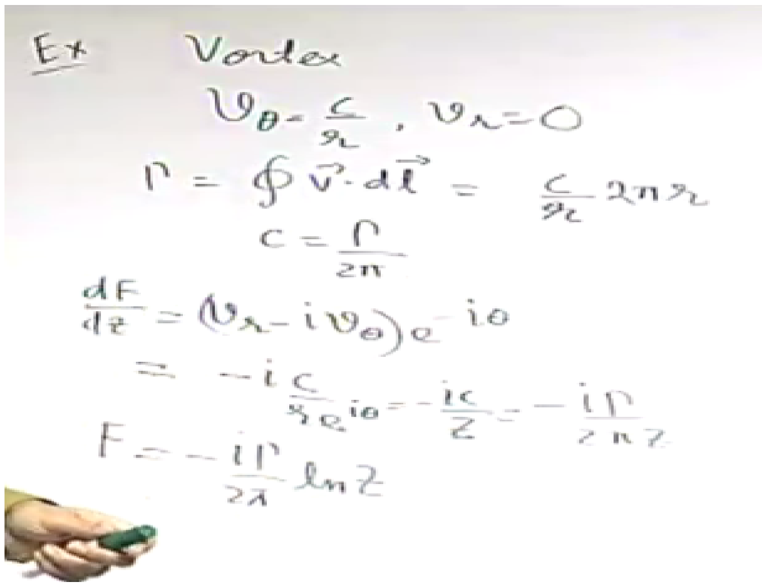

So the next example that we will consider is a Vortex. Okay. So when we consider a Vortex remember that we are talking about an irrotational flow and a Vortex. So it is an irrotational Vortex or a Free Vortex. So what should be its velocity components, we already know V theta = c/r and V r = 0, right. So now this C maybe related to the strength of the vortex, strength of the vertex is a circulation. So what is the circulation, if you recall this one, right.

So this is c/r*2 pi r if you take a circle of radius r and find out circulation around that. So C=gamma/2 pi where gamma is the strength of the vortex of the circulation. Now your df/dz that should be V r- i V theta* e to the power of –i theta, okay. So –i c/r e power i theta, so –ic/z so –I gamma/2 pi z. So what is the form of f, f is –i/gamma/2 pi l and z. You have to keep in mind that this gamma we have considered as positive if c is positive and that is anticlockwise.

So here this – sign is for anticlockwise circulation and + sign for clockwise circulation that we have to keep in mind. So if gamma is just a number just a positive number then this – sign will imply that it is anticlockwise and + sign will mean clockwise. So if – where we are considering gamma itself a positive number. Now, let us say that we want to simulate flow past a circular cylinder with circulation.

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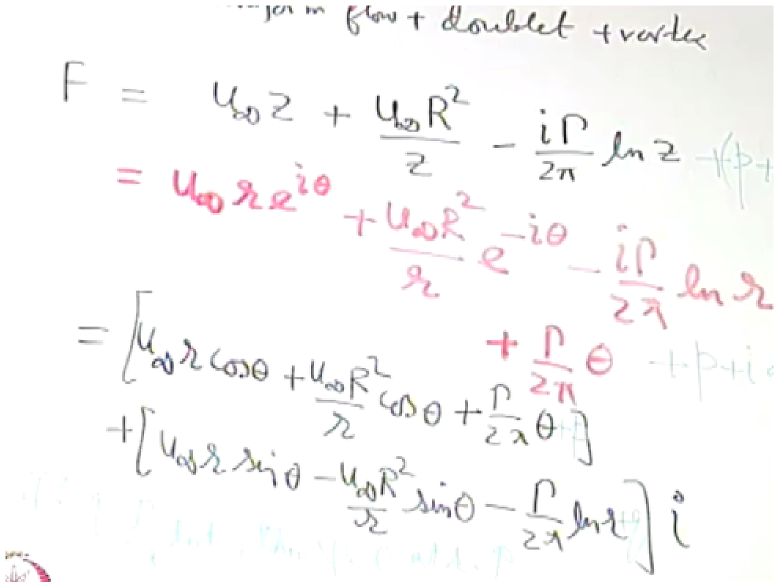

So flow past a circulation in the without circulation was uniform flow + doublet, if you want to introduce a circulation then you have to use this vortex. So it is uniform flow + doublet + vortex, that mean how do you write F. First for uniform flow u infinity z + doublet, for the circular cylinder, flow past a circular cylinder we have seen doublet strength is special u infinity*R square where is the radius of the cylinder.

So u infinity R square/g+ the vortex – i gamma/2 pi. Now let us see that using this can we really simulate the flow past a circular cylinder, because you have to remember that the radial location of the cylinder should represent Psi=0, thus streamline. Let us see whether that is satisfied by this or not. Intuitively that will not be satisfied because up to this part if you consider that satisfied Psi=0 on the radius. Now you have added one extra part, so how do you ensure that. Let us look into that issue.

So we just write this as u infinity e to the power i theta + u infinity/r square/r e to the power-i theta-i gamma/2 pi ln r and + gamma/2pi theta, right. So one i with this i square becomes -1 and that is how this has become +. So if you want to write it in terms of the real and imaginary components u infinity R cos theta, first let us write the real components + u infinity capital R square/r cos theta + gamma/2pi theta. This is the real part.

Imaginary part is more important for because that gives us stream function form, so +u infinity R sin theta – u infinity R square/r sin theta – gamma/2pi ln r. Clearly if you see this form does not satisfy the requirement of-- one i has to be put here. This form does not satisfy the requirement of Psi=0 at small r= capital R, right. Because when small r=capital r this 2 terms will be canceled out. But the third term will remain.

But smaller=capital R should correspond to Psi=0 for flow past a circular cylinder of radius capital R. So what is the way out? The key here is that this definition of F may always be remain, may always remain unaltered if you just add say a constant p + iq, right because it is just a choice of the reference. So just add p + iq so here we will have one p and here we will have one q. The q should be such that it should be gamma/2pi ln of capital R.

Then you may, so if q=gamma/2pi ln of capital R then Psi=0 at small r=capital R. Okay. So in this way if you want to generate the body of a particular shape you have to be very, very careful about the choice of the F. It is the super position but you may also need to add certain constants to make sure that you represent the correct shape of the body. When you have a body of a particular shape what are the important things of the interest?

One important thing of an interest is the stagnation point. So where is the stagnation point. For example, when you have a flow past a circular cylinder, say without circulation. So when you have without circulation this gamma term is not there. So when you have a flow past a circulation within the without circulation then you have when the first term is u infinity- so if you find out df dz, so df dz if you calculate for flow past a circular cylinder you got 1 Vr which is 0 for all Rn theta on the cylinder and one V theta which is proportional to sin theta.

So when theta=0 then or maybe r=capital R and what r=capital R is okay but you see the expression of V theta–2u infinity sin theta. So when theta=0 you have V theta also =0. What about theta=180 degree? Same. So those points on the cylinder are special points where your both Vr and V theta is =0 so those are locations of stagnation point on the cylinder. Stagnation points need not be necessarily be located only on the surface of the body.

They may also be located at outside so the only important condition that you need to find out is where the resultant velocity is 0 that means V r V theta individually 0. Okay. So if you consider the flow past a circulation cylinder you will see that.

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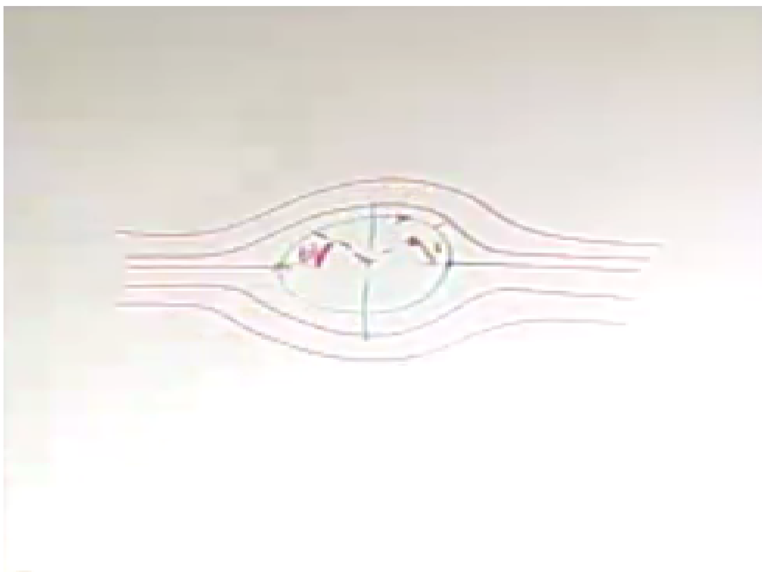

If you just want to consider streamlines without the rotational affect or without a vortex affect. So you have a point here which is a location of a stagnation point then the streamlines follow like this and this point is also like a sort of stagnation point. So here you see that the theta in the way it is measured classically by polar coordinate positive sign convention this should be the angle theta. But even here 180 degree – theta is also fine.

So you may also consider by convention this as the angle theta. The reason is that the answer where it comes in the form of sin theta, so it does not really matter of course in terms of pressure distribution the sin also is not important, it is just the value because you have sin square theta coming in. It is like even more inconsequential what is the sin of that trigonometric expression. So by convention we take this as theta so this is like theta 0, theta 180 degree like that.

And you have such shapes of streamlines such beautiful shapes of streamlines are not there in the real case because in the real case when you have adverse pressure gradient at some point close to this one you have boundary level separation beyond that you do not have such streamlines such symmetric shapes. So we will look into that issue more carefully. Now regarding the effect of circulation let us try to see what is the effect of net effect of circulation.

To understand that what we will do is we will try to calculate the total lift and rack force on a body.

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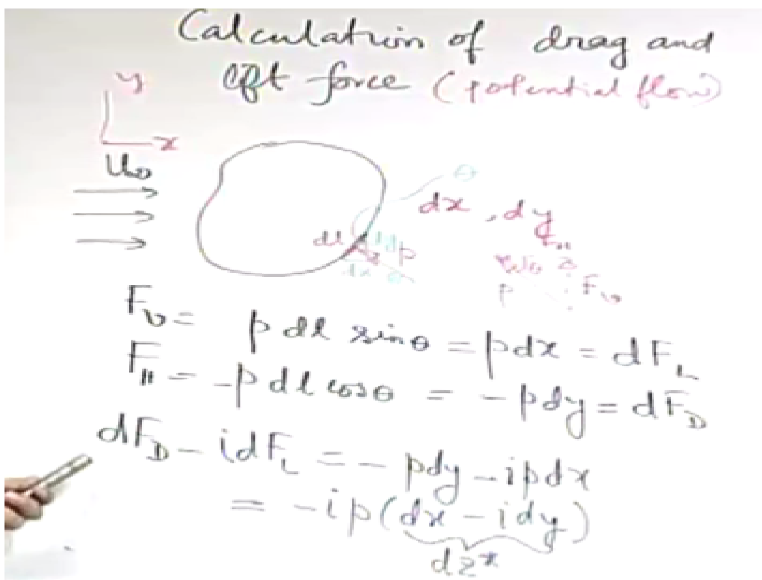

So our next objective is calculation of lift and rack force or drag and lift force. In one of our previous lecture we discussed that if you have a plane flow whatever is the resultant force acting on a body that if it is resolved into 2 components one in the direction of the flow stream another perpendicular to the direction of the flow stream, the force in the direction of the flow stream is known as a drag force, perpendicular is known as the lift force.

So let us say that we have an arbitrary body, let us also consider this as a cylinder but not a circular cylinder. So a very general case of cylinder, we are not committing ourselves whether it is circular or whatever shape. Now let us say that this is the free stream direction and let us try to identify that what are the forces acting on a small element. Let us say this is a small element of length dl. Remember, we are talking about only potential flow not the viscous flow.

So, on this element what will be the origin of the force, only pressure distribution? If it is a viscous flow, then the shear effect will also come. So you have the pressure distribution. So this is strictly only for potential flow, you have to keep in mind. So this is not a drag and lift force for any general case for potential flow. Okay. So now this dl you may say that it is as good as traversing dx along x and dy along y where x and y are these directions.

So you may just break it up into 2 parts, one is like dx along x another is dy along y. Let us say that the angle made by p with the horizontal is theta, therefore we may say that angle made by dy and dl that is also theta, right, dl is tangent to the contour and p is normal to the contour, okay. Now let us find out that what is the resultant lift and drag force due to p. So you have a p force like p here you may resolve it into 2 components.

So one is the vertical component another is the horizontal component and this angle you have to keep in mind is theta, right. So this is the p direction where just separately drawn the sketch for clarity nothing more than that. So we have a vertical component, you have a horizontal component. **"Professor-student conversation starts"** So what is the vertical component? p*dl* what? Cos theta or Sin theta?

Sin theta. What is dl sin theta? dx. Just look into this, triangle of dx, dy and dl. So p*dx. And what is the horizontal component of the force? P*dl*cos theta-, right. That is –pdy. So let us call this as, so this is after all (()) (16:36) bends so let us call this as so this vertical force is nothing but the lift force, because the free stream flow is horizontal so anything perpendicular to that relative to the flow is the lift force.

And this is the drag force, elemental force so we have represented it by differential quantities. Let us now try to find out these complex quantity, again to use the complex calculus dFd – idFL, so that is = -pdy – ipdx. So –ip*dx-idy. Okay. Fine. dx-idy you can just write in a shorthand notation of the complex conjugate of dz, right. Now what is the total force? You have to integrate it over the contour of the body.

So why we are evaluating its see, because if we evaluate if we integrate this in a form of a complex number its component will give us the real and imaginary components will give the lift and the drag force together. **"Professor-student conversation ends"**

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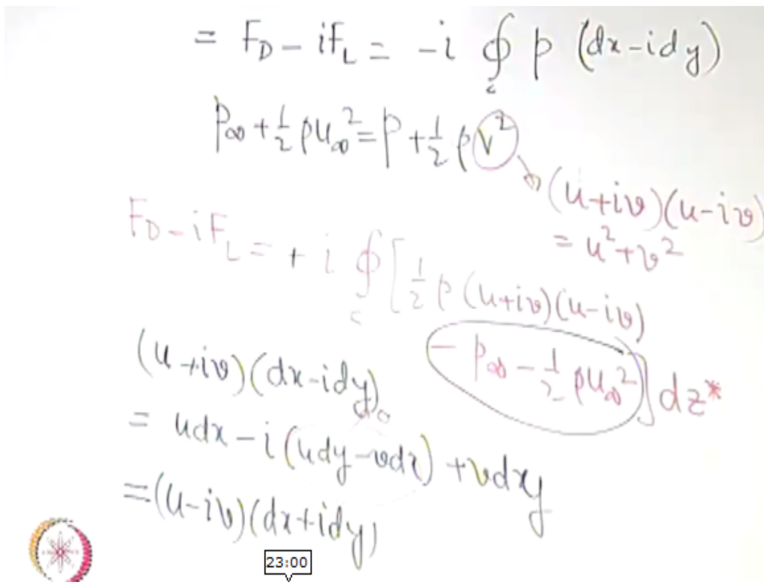

So if you want to find out the contour integral of, so that is nothing but FD-iFL that is = -I contour integral of p dx-idy. Now because it is a potential flow you may write p infinity + half Rho infinity square = p+ half Rho V square at any point, neglecting the elevation effect. This V square is like what, you can write V square in terms of u+iv*u-iv because V square is nothing but u square + v square, right.

So you have the FD-iFL= -I contour integral. In place of p we will write half Rho u+iv*u-iv-p infinity-half Rho u infinity square, this one. Okay. Now what will be the integral contribution of this term? This term is like a constant right. It would come out of the integrals which will be contour integral of something into dx + something into dy, so what is the total dx as you have traverse along the contour, 0.

What is the total dy that you have traverse along the contour 0. So basically what you are doing by evaluating the contour integral is evaluating this integral as your traversing the contour along a path particular direction. So this integral of this term will become 0. Next, let us try to look into say the form u+iv*dx-idy because that is one part of the integrant that you have to evaluate, u+iv*dx-idy. So you can write it as udx-i; udy sorry vdy, right.

Now what can you say about this term? The contour of the body itself is a streamline. So you have dx/du=dy/v. So this term will be 0 on the contour of the body. So this is 0 because contour of the body is a streamline. Similarly, you can show that this will become nothing but u-iv*dx+idy just the conjugate terms because there in place of this –i you will have just +i. And since this term is 0 it does not matter. Okay. So what you are left with?

You are left with the expression in place of u+iv*dx-idy you can write u-iv*dx+idy.

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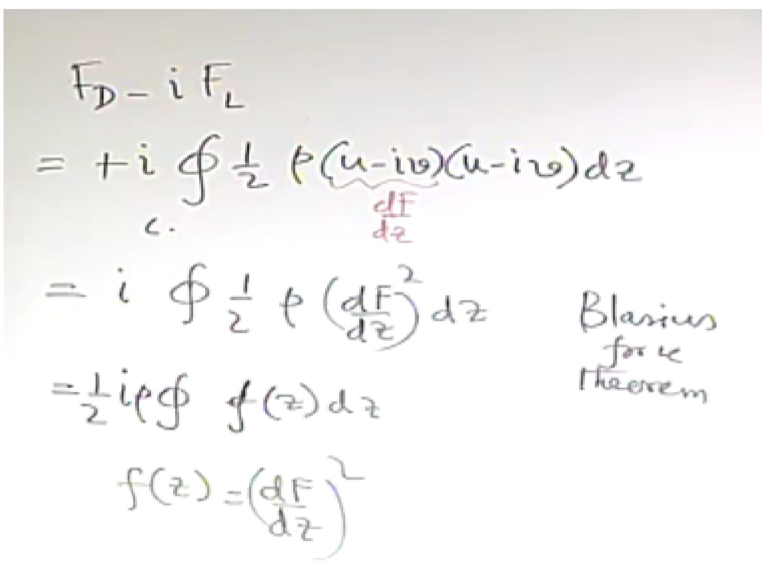

So FD-iFL = -i the contour integral of half Rho u-iv, in place of u+iv*dz star we can write **"Professor-student conversation starts"** (()) (22:54) This i, this will not be there. Okay, it will go to the other side, right, so this will become +, okay. "**Professor-student conversation ends"** So u-iv*dz, right. In place of u+iv*dz start we have written u-iv*dz, this is dz. What is the advantage of this? You see u-iv this is dF dz, right. So we can write this as i, okay.

So what is the advantage of this? If you know what is the complex potential capital F for the super position of flow you may just use this simple integration equate its real and imaginary components to get the lift force and the drag force. This is known as Blasius force theorem.

So it is as good as the evaluation of an integral of the form, so half i, half i Rho * some function of z, dz where the function of z is dF dz square, right this is the form mathematical form. Now let us look into this mathematical form for flow past a circular cylinder with a circulation. Before that we will try to understand one important thing that how to evaluate this integrals, there is a very simple way in complex integration known as Residual theorem by which you may very simply evaluate such complex integration.

How it is possible? let us say that you have a function Fz which has a singularity at z=a okay.

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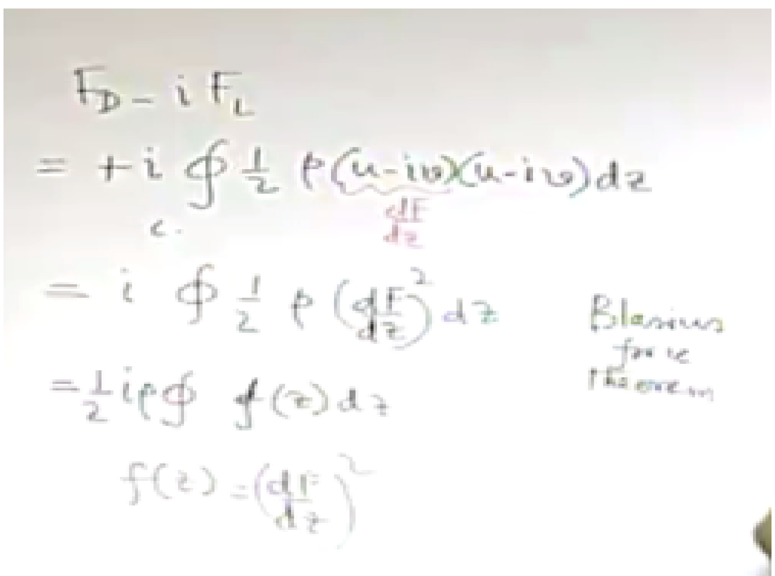

So this you may expand in the form of series like you will have some negative powers and some positive powers, so let us say you have, okay. So it is just like an extended form of a Taylor series where you also have the possibilities of incorporating the negative indices. This is known as Laurent series. So what is known is that integral of Fz dz is nothing but 2pi i* the sum of residues at points of singularity in the domain.

To understand that carefully let us assume that or just as an example let us assume that a=0 as an example, a=0 means, 0 is a point of singularity, like if you have a form of 1/z or 1/z square, right so that has a point of singularity. Now the question is that when you have that as a point of singularity how do you handle that. So let us say that you want to evaluate at an integral of—see when you have a=0 that means you are having to deal with integrals of the form 1/z, 1/z square like that, let us just look into one such form.

Let us say you are interested to evaluate the integral of dz/d square because integral of the function evaluation is as good as like integral of evaluation of the integrals of the corresponding terms, term by term. So if you do that, so let us say z=r e to the power i theta, so we may convert it from z to r theta system. So what is dz? i r e to the power i theta d theta. So this will become, so contour integral of this i r e to the power i theta d theta/ e to the power 2i theta/r square, right.

So what will be this, so now the basis of this is change from theta=0 to theta=2pi, okay. So if you see now, forgot about the 1, forgot about the I r, r square those contributions will get a basic form. E to the power –i theta d theta. So cos theta and sin theta both integrated from 0 to 2pi. So what will that give? That will give 0, right. so this will be 0. Similarly, the higher powers of z will do the same.

But if you just consider integral of dz/z then you will have ir/, now in place of z, r e to the power theta so ir/r integral d theta 0 to 2pi, right. So that is 2pi i. That means the key is this term. All other terms of that type 1/z square or whatever, but those terms do not contribute to the integration. And the residual is nothing but its coefficient so c-1. So it is as good as if you have just one point of singularity the residue is c with subscript -1.

If you have many such points of singularity just it will be sum of coefficient. So it is very straightforward actually, I mean it is not a very complicated theorem but not a very convenient and very powerful theorem. So now for our special case of flow past a circular cylinder with rotation or with circulation we have to just evaluate what is dF/dz and what is the coefficient of 1/z in that; that should be good enough for us to evaluate this integral. So let us do that.

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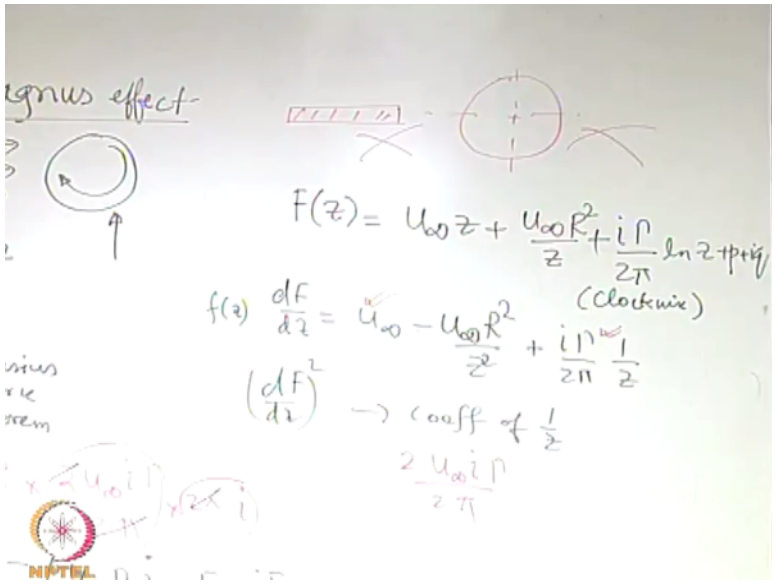

So for that flow you have a V=u infinity z+ V infinity R square/z + or – i gamma/2pi l and z. Let us consider that we are assuming a clockwise circulation. So if we assume a clockwise circulation we just put a + sign here with an understanding that we are assuming a clockwise circulation. So the circulation is clockwise, + you have that p+iq whatever. What is important for us is dF dz, that is u infinity+ sorry – u infinity R square/z square + i gamma*2pi 1/z, right.

What is our matter of evaluation? That in the square of this one what is the coefficient of 1/z, right, that is the integration that we have to do for the Blasius force theorem. So what will be the coefficient, so it is just like a+b+c whole square. So the corresponding coefficient will come only when you have this 2 (()) (32:36) that term. So what will be that? 2 u infinity i gamma/2pi, right. So this will become i rho/2* just 2u infinity i gamma/2pi. Okay * there is a 2pi, 2pi i.

By the residue theorem it is 2pi i * residue, right. So what are the terms that get canceled out, one is this 2pi get canceled out this 2 gets canceled out so you have this as –rho u infinity gamma I, because i square is -1 so one i remains. So this is FD-iFL. So what is the drag force? 0. What is the lift force? Rho u infinity gamma. So the drag force=0 was thought of has a paradox for a long time.

Because if you have a say circular cylinder or cylinder of any shape immersed in a flow when the fluid is flowing on the top of it there is some drag force that is experimentally calculated. But when this was evaluated theoretically they need-- gave rise to a 0. And this was a paradox for apparent paradox for long time and it was called as a D'Alembert's paradox, the whole idea of this paradox—see this is not at all paradox it was a paradox only at that time.

And the reason is that this neglected the affect of boundary layer. So it just considers a drag force with fewer potential flow based calculation where viscous effects were neglected all together but although the Reynold number maybe large but there is always a very thin layer called the boundary layer within which the viscous effect is very, very important. So we cannot disregard that and calculate the drag force. So that was the whole origin of the apparent paradox.

But this lift force was something which was found to be quite accurate even later on and one of the important consequences or conclusions of this is you see the lift force expression, of course there are forces per unit length perpendicular to the plane of the figure that you have to keep in mind. This force is apparently not having anything in the expression which is the function of the shape of the body.

So this type of lift force expression is not just valid for a circular cylinder although we established this case through an example of a circular cylinder but even for bodies of other shapes. And one of the interesting shapes for which the flow-- induced lift force is very, very important is an aerofoil shape. And even for an aerofoil shape this similar expression for the lift force is valid and this is therefore known as a very important theorem.

Or very important expression in aerodynamics known as Kutta–Joukowski theorem. So it just gives basically an expression for the lift force. See this expression for lift force is something which may be quite interesting because what is happening here, you have a circular cylinder, you have a uniform flow like this, you make the cylinder rotate like this, okay. Once you make the cylinder rotate like this there is a force which is exerted on the cylinder towards the top.

Just in place of a cylinder if you replace a shear, qualitatively does not change much, so think of a case you have a ball which you are rotating with this type of axis and it just goes towards that is the top spin of a ball. So this effect is known as Magnus effect in fluid dynamics. So with background of potential flows we have come to a stage when we may appreciate that what is the consequence of the pressure distribution around the body at least.

And till the boundary layer gets separated the consequence of the pressure distribution is something which is consequential for flows of past bodies of walls shapes except flat plate where the pressure gradient it is not important. Now let us look into some bodies other shapes will not calculate similar expression because we have just seen a strategy of doing this. And in fact there are many interesting transformations by which you may convert from one shape to the other.

So for example you have a transformation which may transform-- a geometric transformation which may transform a very thin flat plate or a very thin foil like this into the shape of a circle. So there are similar, so this transformation, I mean there are certain special transformation which to do that and in general these types of transformations are known as Conformal mapping or Conformal transformation. What they do?

Basically, if you have set of lines in the original configuration and a set of line in a transformed configuration it retains the angle between that to preserved, in the original and in the transformed coordinate system. But we will not go into that transformation we never, one of our homework problems we, we will be giving you a problem to solve related to this transformation, but I mean that you may do by considering all the basic theoretical background that we have already developed.

Now what we will try to see next, is that, that keeping in mind that shapes of different, I mean what is the different shapes when we generated by suitable transformation from a simple shape to maybe a complex shape and so on. Generically, we are in a position that if we know how to generate potential flows past simple shape bodies we way generate such flows even for bodies of complicated shapes. But what are the important consequences?

To do that, we will study the flow past 2 important shapes of bodies. One is like an aerofoil section and another is flow past a circular cylinder in somewhat more details than what we have done till now. So when we consider the real flow you have to keep in mind it is not the potential flow, you have the boundary layer of it + the affect due to the pressure distribution that we are seeing.

So the pressure distribution is there which is imposed from the free stream into the boundary layer so that you can calculate from the potential flow, more or less it will not be very, very inaccurate till there a boundary layer separation. But in reality there is boundary layer separation therefore in reality whatever the pressure distribution is predicted by the potential flow does not work. It works approximately well till the point of separation or very close to the point of separation but not far away from that.

So first of all we will consider the case of say flow past aerofoil sections.

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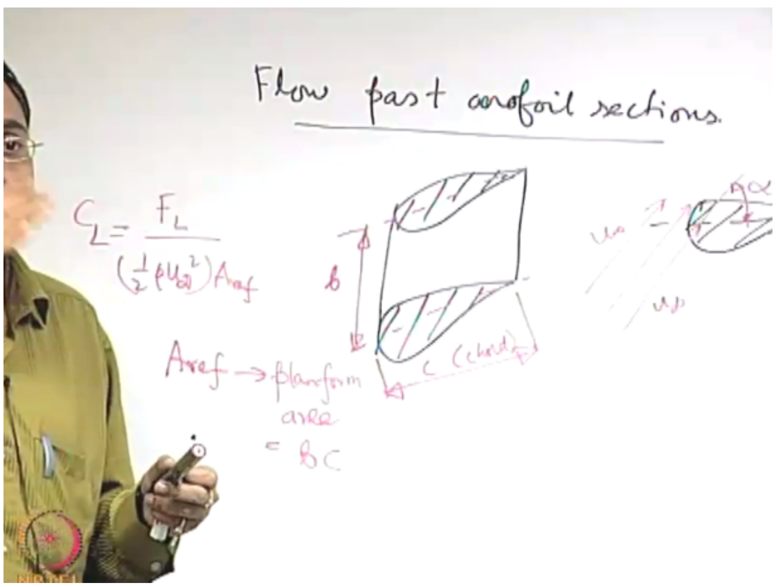

We will just learn some important terminologies. So flow past aerofoil sections. So let us first make a sketch of the section that we are considering. Just consider it like a section of like an aircraft wing, okay. So now there are certain important dimensions which are given to it. One is the dimension of this transverse one and another one this one which is called the chord of the aerofoil, this is called the chord.

If you calculate the lift and the drag force and the corresponding coefficient, say you calculate the lift coefficient, so what is that, that is a lift force/half Rho u infinity square*reference area.

We have seen that for flow past bodies layer of like shapes like cylinder or sphere like that the reference area is what, the reference area is like the projected area. In this case, where the bodies are of much cylinder shape and slim shape so to say the reference area is called as a planform area.

Reference area is different by engineering convention for different thing. Like if you have a ship in water there is the submerge area that is considered as a reference area, so it depends on what is the object that you are looking about, so reference area the landform area that is the area where you view from the top that is b*c. Okay. So that is the reference area. And the second thing is that, what is important is to know that what is the, so if you just draw the sketch of a section.

So this is the chord of the aerofoil. Now what is the angle made by the incoming flow with this one. So let us say the incoming u infinity is like this. So the angle made by the u infinity with this chord, let us say alpha, this is known as angle of attack. This angle of attar is very, very important because if the angle of attack is very small then then original flow is almost aligning with the direction of the chord.

And then the flows boundary layer separation takes place almost at the trailing edge I mean almost the entire flow which is unseparated. But as it increases the angle more and more we will see that beyond a critical value of the angle there will be very quick boundary layer separation and because of the very quick boundary layer separation the lift force will go down. So if you just draw the coefficient of lift versus the angle alpha.

So it will increase then it will attain maximum layers roughly say alpha= roughly 10 to 12 degrees, let us say 12 degree as an example and then it falls down. Because then there is a large separation region because of the great mismatch between the direction of the flow and the direction of the chord the flow separation takes place quite quickly, very hardly. And this kind of situation that is at this angle it is called as that the aircraft or that aero foil is stall.

So this is known as a stall. So one of the important objectives is to generate a lift force. So you may increase the value of alpha to generate a greater lift force if you want to go to a greater elevation. So you can see that if the aircraft has to have a different elevation say higher elevation it should have its aerofoil wind section oriented in a particular fashion with the relative velocity of the wind that is there outside or the relative velocity between the outer ambient and the aircraft. So that angle has to be adjusted properly to get the proper lift.

But once it is in a particular height or a particular level then you do not expect make net thrust or net lift that means the weight is balanced by the lift force and it is just in sort of equilibrium. Similarly, the drag force is balance by the thrust, so if it is in a sort of an equilibrium and moving, so these are some of the important things. Let us look into some animated descriptions of how fluid flow takes place past an aerofoil.

So first let us, so we will see quickly that what are the impacts of different angles of attack. So if you calculate the lift force obviously that will come from the proper evaluation of the expression. So if you evaluate the proper lift force for flow past and aerofoil you will see that it will depend or it will increase with the alpha so long as you do not have a large separated region. That is quite clear, so just think very physically.

If there was no separation and the flow is like from the bottom it to lift it very significantly, if it is horizontal an inclined with it how can it lift it. So as you increase the incarnation you have a chance of lifting it more and more but if you have boundary layer separation altogether then the effect is lost. So let us look into the different cases of aerofoil flow. This is like sort of a potential flow type of situation. So this is a simulated condition of course.

This is the flow visualization example so see carefully some dyes injected around the section and how the dye takes its turn, so these are like examples of street lines; in the steady flow these are light streamlines. Okay, let us look into the next example. So now we will see the effect of different angles of attack. So this is aerofoil section where you have 0, angle of attack. Now let us say 15-degree angle of attack. So with the increase of the angle of attack you see what is happening.

You see a flow separation that is taking place quite quickly so whatever is that circulating region that is a region of low pressure beyond a separation. Look for the angles of higher and higher orientations you see that the affects of the separation become more and more severe. Okay, so these are just, this is a 60 degree angle of attack. So you can see that as we increase the angle of attack the severity of the consequence of separation becomes more and more prominent.

So that is one of the important things that we learned from this. Now if we go on to study the flow past differentiate bodies till now in the boundary layer theory we have considered cases with high Reynolds number. The reason is quite obvious, that if you have high Reynolds numbers then only the boundary layer theory you may apply. But there may be interesting cases when the flow is very, very slow.

Or the relative motion between the fluid and the solid is very, very slow, that is also flow past the body. And therefore let us quickly see what is the consequence of very low Reynolds number flow.

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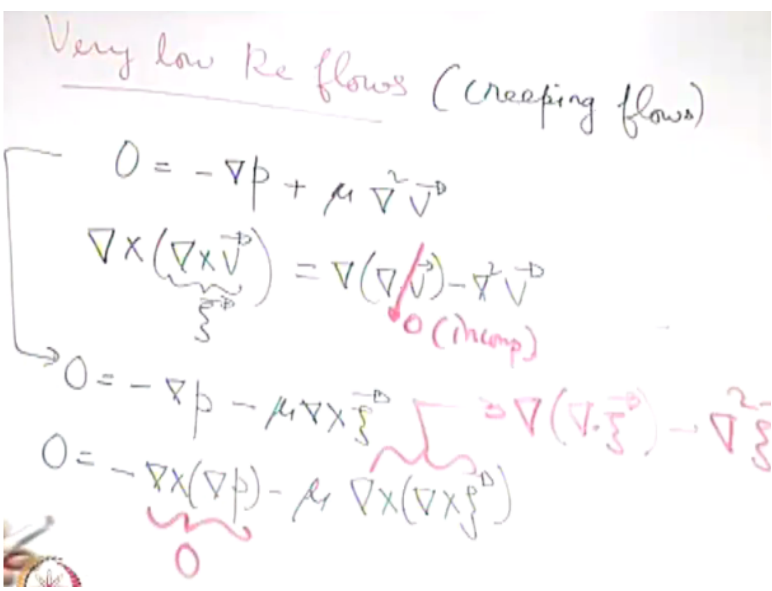

Very low Reynolds number flow, okay. So when we say very low Reynold numbers flows it of course depend on the length scale what we are talking about to describe the Reynolds number but let us say much, much <1 that type of Reynolds number we are talking about. So such flows are called as Creeping flows. As if there is an object which is creeping or moving at a very slow pace relative to the fluid.

So when you have such a situation thing of the Navier-Stokes equation. In the Navier-Stokes equation the left hand side is a representative of the inertia forces; right hand side the viscous term is a representative of the viscous force and then you have a pressure gradient force due to pressure gradient.

So the Reynolds number is very, very small, then obviously the inertia effects you may neglect. Now there will be definitely be some error because there will always be some inertia effect may represent but if the flow is a very, very low Reynolds number that effect is not predominant. So then the Navier-Stokes equation will take of the form 0=-gradient of p+Mu. Now we will, what we will do is we will proceed further with the curl of both sides.

So to do that we will keep in mind that if you want to evaluate for example this curl of this one that is del cross, del cross V so—this identity we have used earlier also for developing some other theory. So now if you have a situation where this, it is an incompressible flow as an example, so this will be 0 if it is incompressible. And the curl velocity is what, it is a virtuosity vector.

So you can write this equation as 0=-gradient of p –Mu* curl of the virtuosity vector, right. We are replacing this del square V by this one. Now let us take a curl of both sides. So 0=, okay. Clearly, what is this? Curl of gradient of a scalar is 0, so vector identity this is 0. And this again we may write in the form of this identity so we will write, okay. Now what is this one? Now what it is, curl of the velocity.

Divergence of a curl of this is 0, right so this is 0 and therefore you come up with the final very simplified form the Laplacian of virtuosity is 0. Okay. Now starting from this form so one can start with a velocity function, evaluate the virtuosity in as a function of that so get a governing differential equation which does not have pressure. So this in a way what it has achieved, it has just eliminated the pressure.

So from this it is possible to get the velocity field. So once you get the velocity field it is possible to evaluate the drag forces and for a very low Reynolds number flow it is you will have the shear effect very, very important so you have the shear force you have also the force due to pressure and then you may use that to calculate the net drag force. And see for very low Reynolds number flow you do not have boundary layer theory value.

Because, if you call something as boundary layer that is extended till infinity that is the viscous affects propagate far and far away from the body. So there is nothing called such a—of course technically you may say that still it is a boundary layer infinite thickness. But fundamentally the boundary layer theory is not valid because it is valid only if delta/l is very, very small. So for such cases you do not have the boundary layer theory valid.

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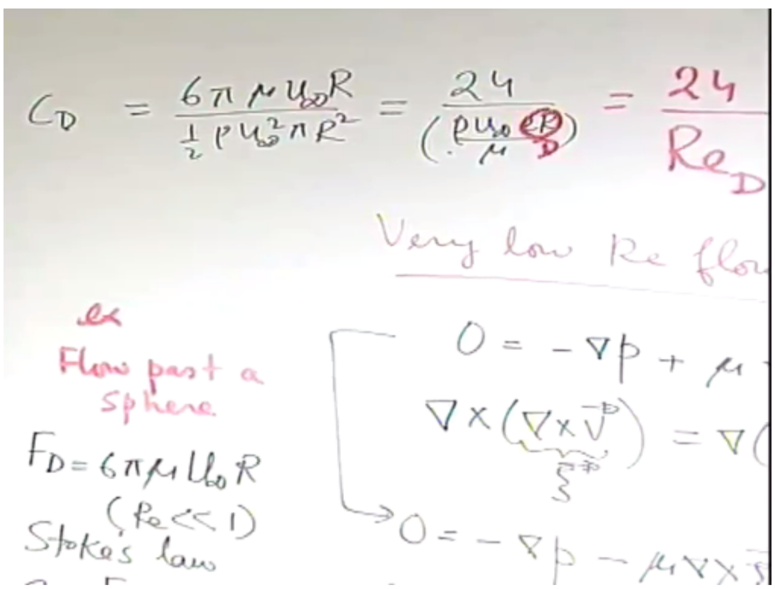

So if you take an example of Flow past a sphere, if you go through this calculation you will find out that the drag force is 6pi Mu*u infinity*R, where R is the radius of the sphere provided that the Reynolds number is much, much <1, this is known as Stokes' law. So in one of the assignments that we have given you we have asked you to prove this by solving the Navier-Stokes equation and it is correspondence I mean through this virtuosity that is given in one of our assignments, if you evaluate.

So it will give you a practice of using this spherical polar coordinate system for solving the Navier-Stokes equation. Now if you want to calculate the drag coefficient the CD, CD is the drag force/half Rho u infinity square* reference area. The reference area is the project area. So what is the projected area of a sphere? Pi r square right. So what will be the expression for CD? 6pi Mu*u infinity*R/half Rho u infinity square*pi R square.

So this will become 24/Rho u infinity*2R/Mu. Why we use 2R because the diameter or the 2R is usually comes there as a reference lane scale, so the Reynolds number. So this is 24/Reynolds number based on the lense scale of the diameter of the sphere. So no Reynolds number flows are the other extremes of the high Reynolds number flows where you have certain very interesting facts.

And in reality the combination of low and high Reynolds number of flows are important, that is we may cover a wide range starting from very low Reynolds number and we go further and further to very high Reynolds number. In our next lecture we will see a lot of video demonstrations on how the flow past body say a circular cylinder shape we will change, as you change the Reynolds number from a low value to a gradually very, very high value.

And whatever we get inference from that we will look into the second set of the demonstrations where we will see the dynamics of sports ball that is how you may control the movement of the sports balls by utilizing the basic aerodynamic that we have learned, so that we will do in a next lecture, thank you.