Introduction to Fluid Mechanics and Fluids Engineering

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Lecture - 42
Potential Flow Past Immersed Bodies

Till now, we have discussed about the effect of growth of boundary layer over immersed body. And we have concluded that depending on the shape of the body the pressure gradient has a strong role to play. So if the body has a particular curvature or maybe a bluff shape then the pressure distribution on the body may give rise to interesting and important drag forces.

Now the-- regarding the pressure distribution it has to be remembered that there is a distinctive behavior if the boundary separation has occurred. But before the boundary layer separation has occurred, the pressure distribution is something which should follow as a consequence of what happens in the outer stream or the fluid stream because one of the important assumptions or one of the important conclusions that we got from the boundary layer theory is that.

The pressure gradient which acts on the boundary layer is same as the pressure gradient what is imposed on it by the free stream. And if the pressure gradient has a strong role to play in terms of detecting the drag force we have to quantitatively understand that the pressure gradient acts on the body of a given shape. And for that a clue maybe that what is the pressure gradient imposed on the body from the outer stream or outside the boundary layer.

This is valid so long as the boundary layer theory itself is valid, that means so long as there is no boundary layer separation. But till there is boundary layer separation till that limit one may use the boundary layer theory and provided other assumptions are valid that is Reynolds number is large and then we may evaluate the effect of the pressure gradient without referring to the boundary layer but just by referring to the free stream or far stream outside the boundary layer.

And outside the boundary layer the flow does not understand the effect of viscosity. So what is the typical characteristic of the flow outside the boundary layer?

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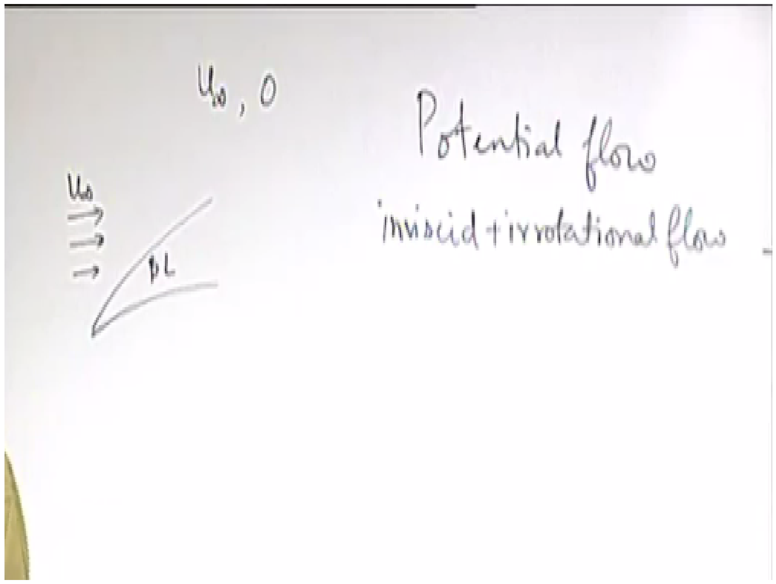

So if you have a boundary layer like this, outside the boundary layer you have some free stream the free stream velocity is u infinity but it goes past the body, this may change because of the curvature of the body. And whatever is the important thing we have to keep in mind that initially say if it was u infinity that means if you consider say 2-dimensional flow for example, you have the x component of velocity as u infinity and y component of velocity as 0.

Now originally is it an irrotational flow? This is an irrotational flow. Now an irrotational flow will remain irrotational if it is inviscid. So outside the boundary layer the viscous affects are not filled and therefore whatever flow was originally irrotational will tend to remain as irrotational. And when it remains irrotational always we may always describe it through the gradient of a scalar potential.

The velocity field, you may always express as the gradient of a scalar potential if the velocity field is irrotational, and such a flow is known as potential flow. So Potential flow is therefore as good as inviscid and irrotational flow. The key here is that if it is irrotational initially as the free stream condition but there are viscous affects inside, the viscous affects will make the irrotational flow to rotational one.

Therefore, you must have an inviscid condition on the top of this one to make sure that it is irrotational forever for all conditions, so that for all conditions you may find out that for all conditions you have the velocity as a gradient of a scalar potential. It is important to look into the potential flow. See, when we discuss about potential flow the first bottleneck or the first mental block that comes to our mind is that therefore, we are talking about a case when viscous effects are not present and when viscous effects are not present it is an ideal fluid flow.

Will such a flow exist? The question is not, whether viscosity is 0 or not. The question is whether effects of viscosity are important or not. So we have seen that outside the boundary layer the effect of viscosity is not directly important. It does not mean that the fluid has no viscosity it simply means that the viscosity is not coming into the picture in terms of dictating the fluid dynamic characteristics.

And therefore it is as good as inviscid flow outside the boundary layer. So when it is as good as inviscid flow outside the boundary layer. It is also irrotational as the free stream condition it will remain as irrotational for the entire region over the body till, till you come to a condition where these conclusions you may never draw, thus, what are those conclusions? Boundary layer separation, because the boundary layer separation has occurred.

There is no question of a boundary layer, no question of something outside the boundary layer, so all those things become irrelevant. So these considerations we may apply till the boundary layer is growing, till there is no boundary layer separation. But there is always a limit up to which the boundary layer is growing at least up to that limit if we want to evaluate what is the affect of a pressure gradient.

It is important that we calculate the pressure field outside the boundary layer because the same pressure field is imposed on the fluid in the boundary layer. And the pressure field outside the boundary layer may under these conditions be calculated by the potential flow theory and that is why the potential flow is important even in the context when you have the affect of the viscosity or boundary layer.

The next question is that how should we approach for evaluating the velocity and the pressure fields in such a flow field. See the potential flow cases that is where you neglect the viscous effects--A really mathematician's paradise and that is how the entire subject of fluid mechanics initially developed through the mathematicians. And one of the important techniques by which this entire thing was entire mathematical development occurred for potential flows.

One of the important mathematical techniques was by using the theory of Complex Variables. So we will be using the theory of complex variables. Very briefly we will discuss that why theory of complex variables maybe important in such a context and how it may be utilized in this context. But we will mainly look into the developments using that theory that how you generate different types of flow, different types of potential flows by using the calculus of complex variables. So we start with defining a function.

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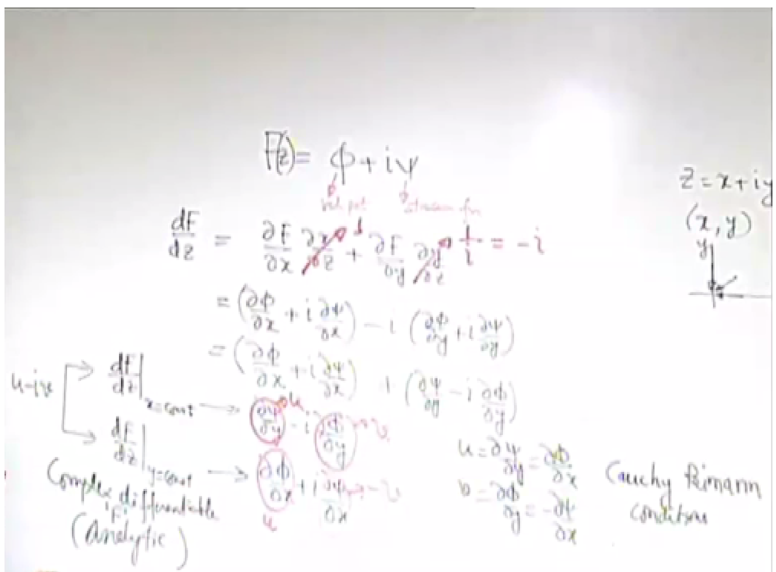

Say capital F which is = Phi + i Psi. What is Phi and what is Psi? Phi is the velocity potential and psi is the stream function. So what kind of flow we are talking about? We are talking about case where both Phi and Psi are defined. So 2-dimensional, incompressible irrotational flow. Okay. So that is the example that we are taking. Now when you have this as F, see why we have chosen such an F.

See when you choose a complex number say Z=x+iy. Fundamentally it is ordered pair of real numbers x and y, this ordered pairs are chosen by some orthogonal basis x and y, okay. Here Phi and Psi are also 2 orthogonal bases because we have seen lines of constant potential and the streamlines, there is lines on constant or orthogonal to each other everywhere in the flow field except the stagnation point, right.

Therefore, this is sort of a mapping from say, a complex plane with x and y as coordinates to Phi and Psi coordinates, nothing more than that, but the orthogonal it is preserved. So because they are orthogonal we can choose such a complex function even preserving the general characteristic of what we have for such a complex number. Now, the next thing is what is the important advantage of this?

See if we write so this is therefore a function of Z. It is a sort of a transformation on Z. So if we have such function what is the advantage that we extract out of it? The advantage is that in straightaway in one sort we are writing the velocity potential and stream function together through a single complex function. So if we write the complex function then we may find it this real and imaginary parts one will give the stream function and other will give the velocity function. That is one of the straight mathematical advantages.

Next, let us look into the differentiability of these functions. We should remember that Z is x+iy. So let us try to find out what is dF/dZ. The first question that we will like to answer is does dF/dZ exist or not. So to test that we will soon see criteria that what should be the corresponding test, but just let us try to look into it in a simple way, okay. Now the simple reason is Z is a function of x and y, right.

So what is this one? This is 1, right because Z is x+iy and what is this? 1/i that means you multiply both numerator and denominator by i so this becomes –I. So now next step is let us partially differentiate this function with respect to x, so you have, okay. So now, what is the big test of the resistance of these dF/dZ for a complex function. One of the important test is that the evaluation of the derivative should not be dependent on the direction which it is calculated.

See now the direction is important because you are approaching say you may approach a limit as delta Z tends to 0 by using the definition of the limit. The delta Z tends to 0 you may approach from various directions because now it is not a single direction it is a plane of points. So from—

for example you may choose to approach the limit as Z tends to 0 by going in this way. You may choose by going in this way you may choose by going in this way.

So you may choose various directions in which you may approach towards the limit as Z tends to 0. Now the existence of this derivative requires that the value of this dF/dZ should not be dependent on the direction in which you approach in the limit to calculate the derivative. So that means that if you approach it along say x=constant line that is along y-axis. So what you get, if you approach it along y=constant.

These are special directions and if we satisfy these directions any combination of these directions will satisfy because these are 2 mutually orthogonal coordinate directions. Interestingly you see that if you recall our basic fluid kinematic discussions this is what, this is u, right. This is also u. What is this? V. And what is this? –V. So that means because we have u=, and v= this one you are having that no matter in whatever direction you calculate this it comes out to be u-iv, right.

And that, that means this dF/dZ exists. Because we may calculate it to be the same irrespective of the direction in which you are calculating. The whole key is because these conditions are satisfied. So not in general but only for such cases when these relationships do Cauchy Riemann conditions. So when this Cauchy Riemann conditions are satisfied we can see that this dF/dZ it exists that means it is called as complex differentiable.

That is it is differentiable in a complex sense so that means it is complex differentiable and such a function which is complex differentiable is also known as an analytic function in complex theory that means F is analytic. That is these are the terminologies which is important to know these terminologies. So you see that if F is analytic and yes from our basic fluid mechanics considerations we show that yes it is analytic if we frame F in this way.

Then you see the next advantage is if you differentiate F with respect to Z, this is once you know that this is differentiable, you write as a function of Z its differentiation is just ordinary differentiation. So you write F as a function of Z, find out dF/dZ exactly in the same way as you could find out dy/dx when y is a function of x. Then in one stage you get another complex number, if you equate a real and imaginary parts you get u and v the velocity components.

So the strength of using this approach is if you frame a complex function then it is derivative—where it is Phi+i Psi then it is derivative directly will give you u-iv the velocity components u and v. Therefore, it is a very convenient way of extracting the velocity components. Now this u-iv is in the context of description through the Cartesian coordinate systems. But we may describe the same thing in terms of Polar coordinate system.

So let us try to see that how we describe it equivalently in terms of a Polar coordinate system.

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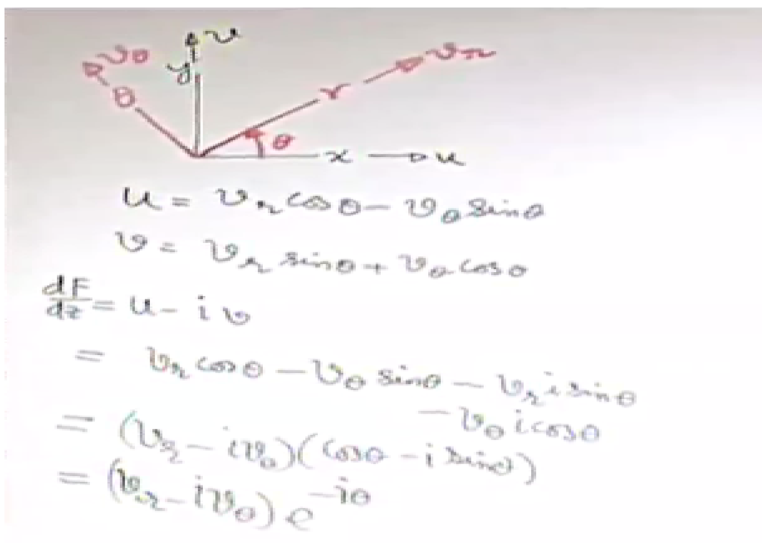

Let us say that, you have this as x, y coordinates and let us say that you have this as r theta coordinates. Along the rn theta direction you have the velocity components as say vr and v theta. Along the x, y components you have the velocity as u and v. Our objective is to write vr and v theta in terms of u and v. So what is vr? Or if we write u in terms of vr. So u=vr cos theta – V theta sin theta. What is v? vr sin theta + v theta cos theta, right.

So our complex function dF/dZ was u-iv. So let us just write u-iv. So vr cos theta – v theta sin theta – vr i sin theta – v theta i cos theta. So this you can write vr – i theta*cos theta – i sin theta, right. That means by using the De Moivre's theorem you can write this as vr –i v theta* e to the power of –i theta. So we should, we remember that alternative way of writing this in terms of the Polar coordinates u-iv as good as vr-i v theta* e to the power-i theta. Okay.

Now the next thing is that with this mathematical background how do we may utilize this to generate different flows? To understand that we will go through some examples.

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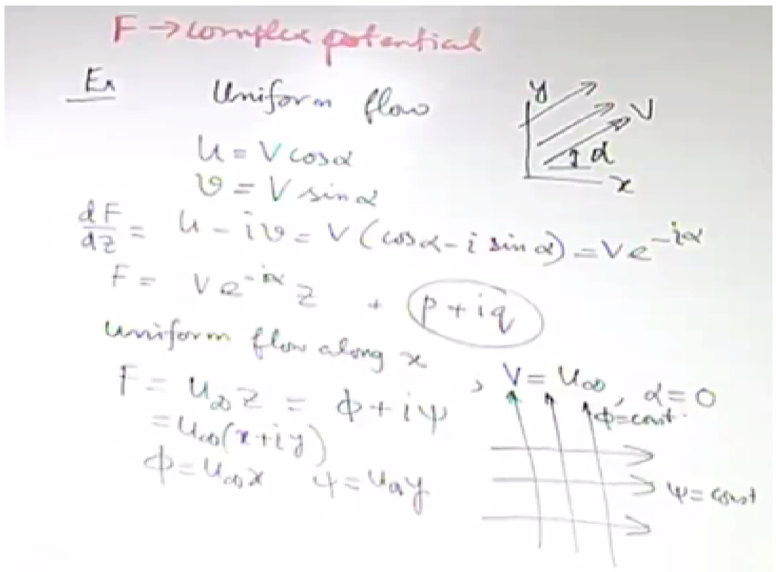

So the first example say our objective is to generate a uniform flow. Uniform flow, so you have to keep in mind that what is your objective, our objective is to generate mathematically some physical flows by finding out the proper expressions of the capital F function. So all these flows are inviscid and irrotational type of flows. Let us say one is uniform flow. So uniform flow maybe in different directions.

Let us say that this is x and this is y and you have a uniform flow which is having a velocity V making an angle theta with the x-axis or say alpha with the x-axis. So you have such uniform flow. A free stream flow inclined at an angle alpha with the x, that is the flow which we want to generate. So what is u here? u is V cos alpha. What is, that is small v, v sign alpha. So u-iv is what V cos alpha-i sin alpha, so V e to the power –i alpha.

And this we want as dF/dZ. So F is V e to the power –i alpha Z maybe plus something plus some p+iq you may put. It is not always this is necessary to put that, but we will see that if you want to generate a body of a particular shape then this constant will be useful. But just for a free stream flow you need not use this because then you need not generate a body of a particular shape so this is not necessary.

So you may choose a reference such that, so you have to keep in mind that this F is like F is called as complex potentials. So the name of F is a complex potential. So it contains both the stream function and the velocity potential and you see those are not fundamental quantities those are just mathematical quantities. Fundamental quantities of the velocities for which gradients of those parameters are important, absolute values are not.

That is why if you add some extra constant it does not matter. When you calculate the gradient of the derivative this does not matter at all. So this is just a choice of reference depending on your choice of reference you may put this as 0 also, it makes no difference. It gives still the same velocity field. Now if you, so using this example let us say that you want to generate a uniform flow along x.

If you want to generate a uniform flow along x with a velocity say capital V= u infinity and Alpha=0. So what is F in that case? u infinity*0, right. Now this is Phi + i Psi, right, Z is x+iy. So what is Phi? Phi u infinity x, Psi is u infinity y. So how do the streamlines look like, constant Psi lines, lines with y=constant parallel to x-axis. So these are the streamlines, these are Psi=constant. Lines of Phi=constant x=constant that is lines parallel to y-axis.

So these are Phi=constant. And you can clearly see they form a net of orthogonal lines. Okay. These are very simple example but this demonstrates the use of the method from this we will go to more and more complex examples.

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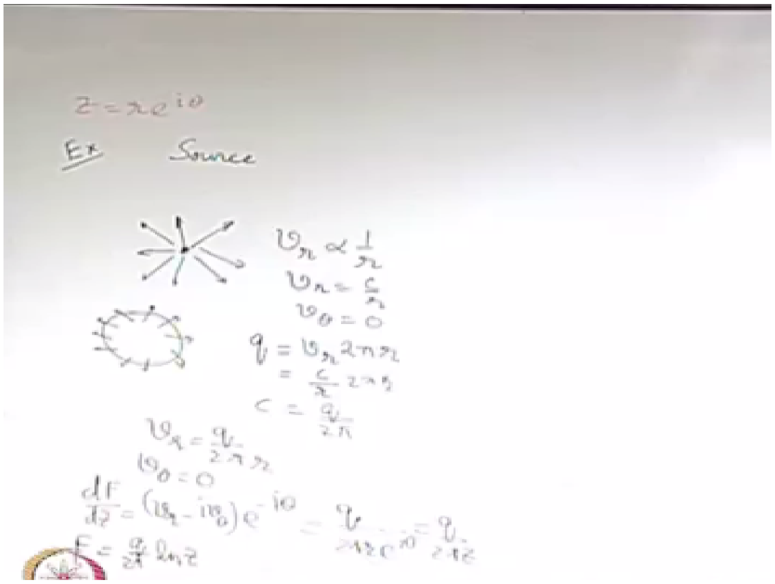

The next example we will talk about is something called as a Source. What is the Source? Source is something like this. Let us say that you have a point, this point is radically emitting some flow in the radial direction, so this is a point through which you have flow emerging radial from all directions just imagine like that. So that the velocity component vr is inversely proportional to the radial location.

That means at r=0 this point if it is r=0 then vr theoretically tends to infinity tends to infinity. And therefore r=0 is like a singularity point, so that we have to understand. Now let us vr=c/r and v theta=0. So this is a pure radial flow. Okay. Now how is the force defined or designated? It is defined or designated by the rate of flow from the source. So how do you find out the rate of flow from the source?

If you consider at imaginary circle located at a radial r from the center. Then what is that rate of flow? See this is a plainer thing so rate of flow when you consider you have a circle like this you have flow perpendicular to the face of the circle. It is a plainer thing. So when you consider that area the area just become sort of the perimeter over which the fluid is flowing for a plainer 2-dimensional case.:

So say you consider unit length or a length of L it becomes 2pi r*l. Here it is a plainer case so just 2pi r. So the flow let us say small q is vr*2pi r so that is c/r*2pi r that means c is q/2pi, this q is called as strength of the source. Why it is strength of the source? It physically indicates the rate of flow from the source. So if q is larger and larger you have the higher and higher rate of flow emitted by the source. So this is called as the strength of the flow.

So what is the velocity field here? Let us just write the velocity field. You have vr=q/2pi r, right v theta=0. Remember dF/dZ is vr-iv theta*e to the power –i theta, right. So this is q/2pi r e to the power i theta, right. Now remember that r*e to the power i theta is just another way of representing the complex number Z, right in the Polar form, right. So Z is r e to the power i theta. So this is q/2pi Z.

So you look into this thing that although dF/dZ exists, it is an analytic function but that has a singularity at Z=0, right. So at Z=0 it is see as to be complex differentiable or analytic because of this division by 0, so these such points are known as points of singularity. Now what is the form of F? That you can find out straightaway as it by integrating this. See once you write as a function of Z on a single variable it is as good as the simple differentiation.

So F becomes q/2pi ln Z. So this is the form of the potential – complex potential function F and you can write it in terms of the Phi and Psi.

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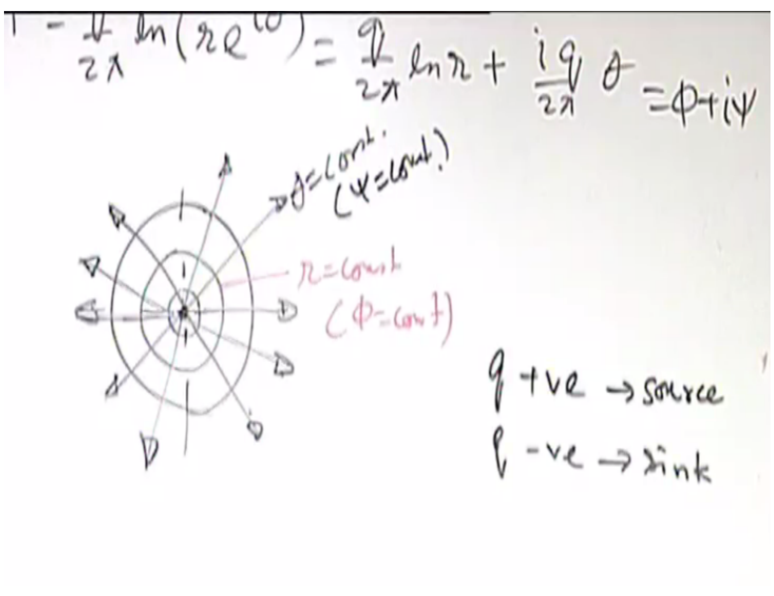

So you can write for example F as q/2pi ln r e to the power i theta, right. So that is q/2pi ln r+ iq/pi theta, right. So it is just like Phi+i Psi by definition of F. So what are the Phi=constant lines? Phi=constant lines are r=constant lines, that is the radial lines. And Psi=constant lines are the theta=constant lines. So what are theta=constant lines? So if you have, if you consider a net like this, just different lines like this.

So what do these straight lines indicate? These are what, these are theta=constant, right. So theta=constant are basically the radially diverging lines and Phi=constant sorry r=constant are these lines. So r=constant lines are what? r=constant lines are the equipotential lines. So these are Phi=constant lines. And theta=constant lines are Psi=constant lines. So you can see that physically it represents a sort of that is a source at the origin and their maybe flow like this.

So you can see that the shape of streamlines gives you an intuitive idea of the nature of the flow if the flow. If the flow is in the opposite direction that means instead of radially diverging it is converging to the point the origin then we call it a sink, just in place of a source, so in case of a positive we call it a source till now we have implicitly presumed that positive but if q is negative we call it a sink.

Now we have seen that what are the shapes of the streamlines and the equipotential lines for the source and the sink. Let us now consider a super position of the source and the sink. Say we consider now an example like this.

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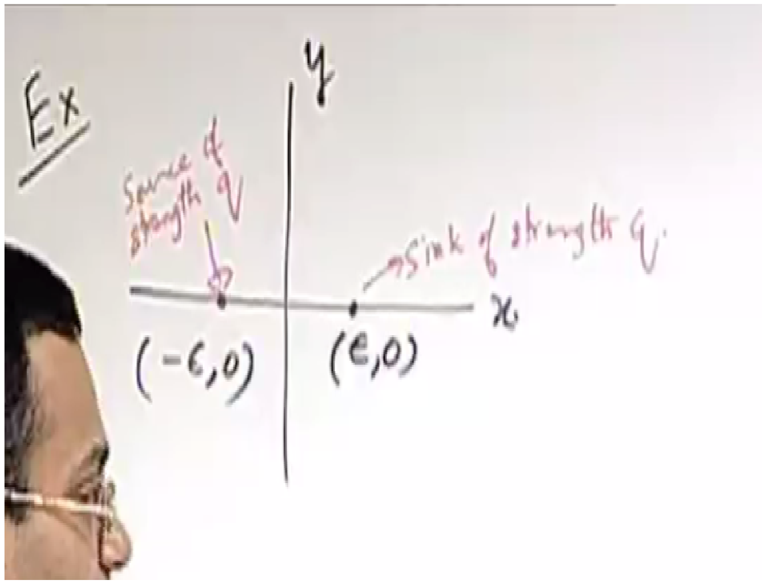

That let us say that this is a real axis the x-axis then you have a point epsilon, 0 and you have a point –epsilon, 0. At the point epsilon, 0 you have a sink of strength of q and at the point –epsilon, 0 you have a source of strength q. Okay. Our objective is to find out the velocity field and the stream function velocity potential all these things. Okay. So we can see that whatever we have learned till now it is a combination of that effect.

But the question is if it is a linear combination or not. To understand that keep in mind.

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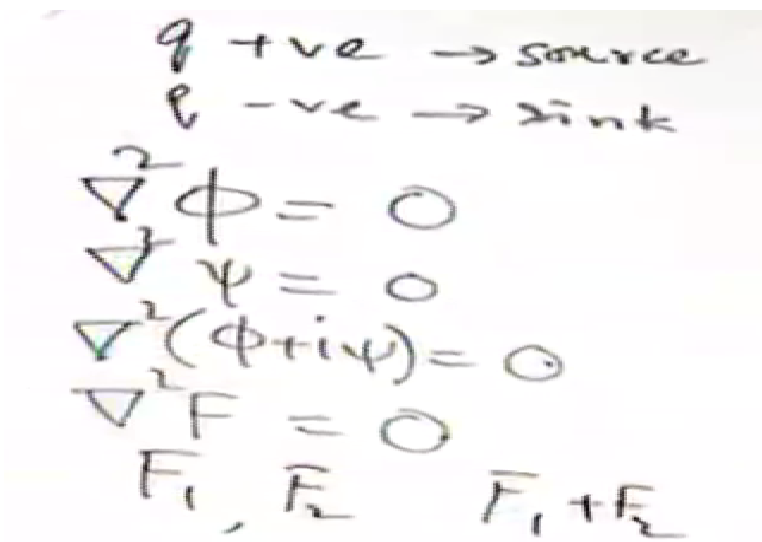

That both the stream function and the velocity potential satisfy the Laplacian equation. So you have Laplacian of Phi=0, Laplacian of Psi=0 that means Laplacian of Phi + i Psi also=0, right.

That means Laplacian of F=0. This is a linear differential equation. That is why if F=F1 is a solution and F=F2 is a solution then F1+F2 is also solution, right.

That means you can consider this problem as a linear super position of the effect of the source at a point and a sink at a point. So you can just add those together. That is the advantage of the linearity of the problem. So let us try to do that. So we will now write try to write F. So now you tell what should be the F, what should be the continuation of F, say F1 for the source of the strength q.

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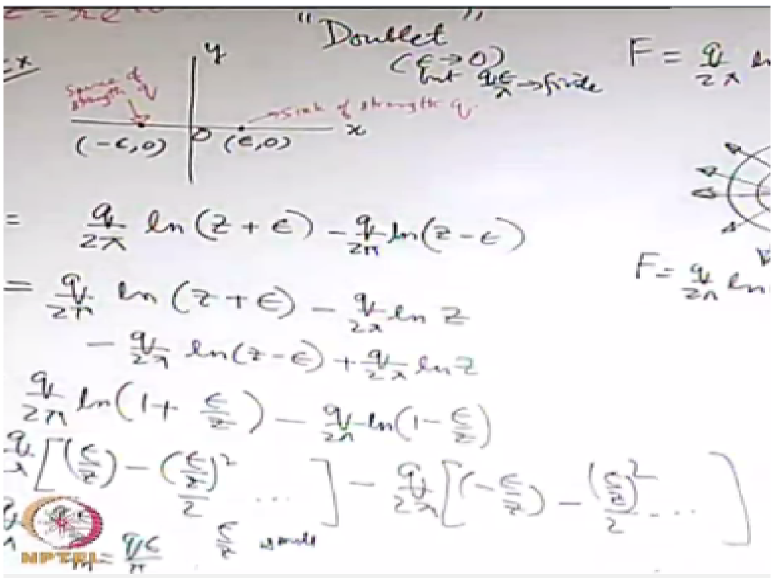

So q/2pi ln of what? See it is a shift of origin sink, earlier we considered the source of sink at the origin. Now it is located at a different point. "**Professor-student conversation starts"** (()) (36:16) No, no we will write it in terms of the Z. "**Professor-student conversation ends"** So remember that F is q/2pi ln Z. so we will write Z, we will make a transformation or a translation of Z to something Z+ something or Z- something, what is that + or -. Yes, Z+ epsilon, Z right.

So as it you have used the Mu coordinate system capital Z with its origin located at the source, right then for the sink –q/2pi ln of Z-epsilon. Now it is located at epsilon. Let us consider that epsilon is small but how small or how large we will see, that is what we need to consider. Before that we will just do a bit of a manipulation on this. The whole objective of this manipulation is we will write the terms in the form of ln of 1+ something.

So that we can use the logarithmic expansion. Okay. So q/2pi, right. So let us just try to expand this log 1+ this one we assume that epsilon/Z is small. So log1+x, what is the expansion? x-x square/2 and so on I mean we would need not write all the terms –q/2pi. Then this is –epsilon/Z then –square terms become keep remain the same and so on, right. In the limit as epsilon/Z is small then like you may write it approximately as q/2pi*2 epsilon/Z, right in the limit a epsilon/Z small. So you have q epsilon/pi Z.

So this we just write in a shorter notation as sum m/Z where m is q epsilon/pi. Such a case when epsilon is tending to 0 this is known as a Doublet, so that is the name of this one, when you have epsilon/Z r epsilon tends to 0. But important thing is all the epsilon tends to 0 we are assumed that q epsilon/pi is finite but q epsilon/pi is finite, that is very important, because you have to understand that if epsilon tends to 0 you always have a chance of q epsilon/pi also tending to 0.

But maybe q is so large so it is a product of 2 quantities that is very important, q is very large, epsilon is very small but q*epsilon is finite, that is what we are thinking about. And this is known as a doublet that is a name. So let us try to make a sketch of or try to identify that what should be the corresponding natures of the streamlines and the equipotential lines for a doublet. So what is F?

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$$F = \frac{m}{z} = \frac{m}{r} e^{-i\theta} = \frac{m}{r}(\cos\theta - i\sin\theta)$$

$$= \phi + i\psi$$

$$\psi = \frac{-m\sin\theta}{r} = \frac{-my}{x^2+y^2}$$

$$x^2+y^2+\frac{my}{\psi} = 0$$

$$x^2+y^2+\frac{2m}{2\psi}y+\left(\frac{m}{2\psi}\right)^2 = \left(\frac{m}{2\psi}\right)^2$$

$$x^2+\left(y+\frac{m}{2\psi}\right)^2 = \left(\frac{m}{2\psi}\right)^2$$

F is m/Z that is m/ or mr e to the power m/r sorry—m/r e to the power – i theta, right that is m/r cos theta – i sin theta. This is Phi + i Psi. So let us try to identify what are Psi=constant lines. So what is the expression for Psi then? –m sin theta/r right. We may easily convert it into Cartesian coordinates by multiplying both numerator and denominator by r. r sin theta will become y and r square will become x square + y square, right.

So if you want to identify what are the streamlines, streamlines are Psi=constant lines. So you can easily do that by noting that x square+ y square + my/Psi that is =0, right. So you can write this as x square + y square + 2m/2 Psi y +, right. So x square + y+m/2 Psi whole square so it represents a familiar circle. x-a whole square + y-b whole square = r square, that form. So represent the family of circles, the values of course depend on mn Psi. So let us try to make a sketch without going into the values.

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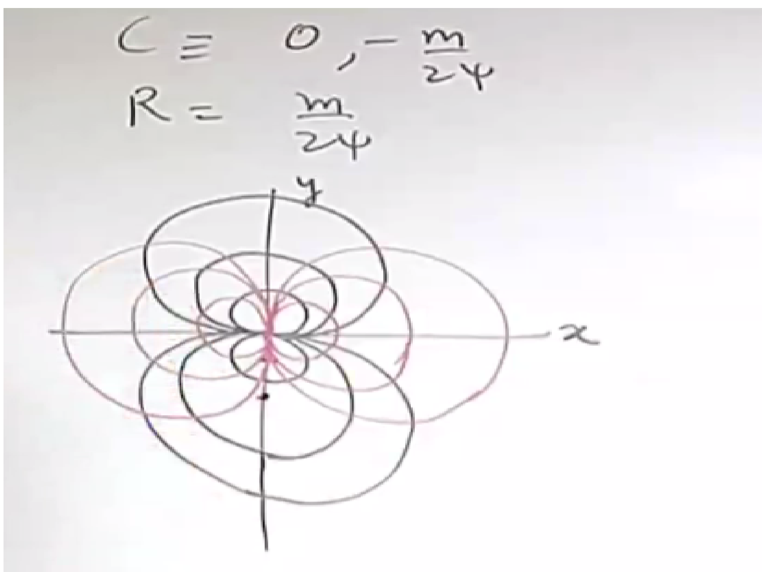

So what is the center of this circle? 0-m/2 Psi and the radius m/2 Psi. So let us try to make a sketch assume name as positive and Psi as Positive. So this is x-axis this is y-axis 0, -m/2 Psi and the radius m/2 Psi, right. So you have—whatever is this distance that is same as the radial distance. Even if you have a smaller radius it is like this. Similarly, you may also depending on the value of m and all these you may also have it apart of it at the top, right.

So you may also have exactly the same way. So as you have a source which is located very close to the origin and you have a sink located very close to the origin on the other side and there is a flow from the source to the sink, that is what it is sort of qualitatively representing. Now Phi=constant lines they will just be similar and orthogonal you can easily calculate by looking into that Phi=m/r cost theta again multiply both numerator and denominator by r.

So it will be mx/x square/y square. And that will represent sort of familiar these types of lines. Okay. And these sets of lines are orthogonal to each other. And you can see that just if you look into plane mathematics it is a very interesting way of generating orthogonal lines which is just a way of thinking a bit differently from just the fluid mechanic prospective. But for us the fluid mechanics prospective is the important concern that we are talking about. Now we will consider another example.

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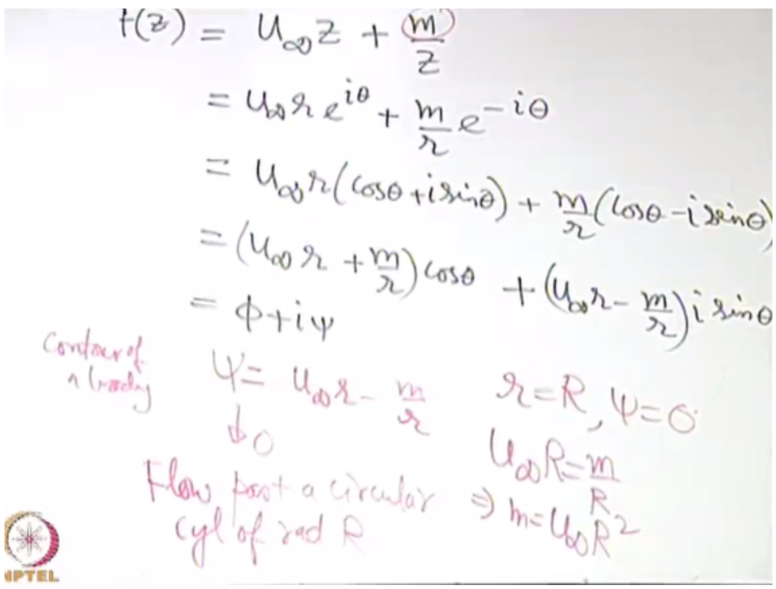

Uniform flow along x + doublet. Okay. So it is a super position of uniform flow along x-axis + a doublet says of strength m. So what will be the resultant complex potential? First, due to uniform flow along x that is u infinity*Z + m/Z, right. So this is as good as u infinity* r e to the power i theta + m/r e to the power –i theta. So u infinity*r cos theta + i sin theta + m/r cos theta –i sin theta. Okay. So u infinity r+m/r cos theta + u infinity r-m/r i sin theta.

So this is we should remember that Phi + i Psi. Okay. Now let us say that we want to generate a body of a particular shape over which the flow is flowing by using this. We are going just one step forward. When you want a generate a body of a particular shape we always have to keep in mind that the contour of the body is a streamline because there cannot be flow across it, right.

So if you want to generate a contour of a body it should represent a Psi=constant by convention that constant is taken as 0 that is the reference. So if you want to generate the shape of a body the shape of a body should come out from the condition of Psi = a reference which is 0. So the Psi which is here, you can see that if you now generate a contour of a body, this should be 0. You can see that this Psi solely depends on the radius r, right.

So you can fix up a radius such that on that radius the Psi is 0. And what is that geometry which is of a fixed radius, that is a circle, right. So let us say that you have a r smaller=capital R Psi=0. You can see that from this super position that this in this example it is solely a function of a radius not a function of a theta, okay. So that means in that case you have u infinity*R=m/R that means m=u infinity *R square.

What is the implication? and implication is very interesting. If you choose the strength of your doublet as u infinity*sum radius square, then you represent a particular flow. What is the flow? The flow in the plane is like past a circular body that means in a 3-dimensional sense it is flow past a circular cylinder. So see starting from very simple mathematics we have generated a flow past a circular cylinder by combining a uniform flow with a doublet.

So this represent flow past a circular cylinder o f radius r. okay. So what is the key, the key is you have chosen your strength of the doublet in a particular way, that is =u infinity*R square. What is the velocity field? So let us calculate the velocity field.

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$$= u_\infty - \frac{m}{r^2} e$$

$$= (v_r - i v_\theta) e^{-i\theta}$$

$$= \left(u_\infty e^{i\theta} - \frac{m}{r^2} e^{-i\theta}\right) e^{-i\theta}$$

$$= \left[u_\infty(\cos\theta + i\sin\theta) - \frac{m}{r^2}(\cos\theta - i\sin\theta)\right] e^{-i\theta}$$

$$= \underbrace{\left(u_\infty - \frac{u_\infty R^2}{r^2}\right)\cos\theta}_{v_r} - i\underbrace{\left(-u_\infty - \frac{u_\infty R^2}{r^2}\right)\sin\theta}_{v_\theta}\Big] e^{-i\theta}$$

The velocity field will be calculated by differentiating F with respect go Z. So dF/dZ is u infinity – m*Z square, right. So this is u infinity – m/r square*e to the power-2i theta. Just writing Z as r e to the power i theta. m is u infinity*capital R square that we will just keep in mind. Now we know that this we may write as vr-iv theta* e to the power of –i theta. So what we do, we write this as u infinity* e to the power i theta then –m/r square e to the power –i theta.

The whole thing multiplied by e to the power –i theta. So here it is multiplying by e to the power i theta* e to the power –i theta because to get just the same form of this one. So it is u infinity*cos theta + i sin theta-m/r square cos theta-i sin theta * e to the power-i theta. so u infinity-m is u infinity capital R square/small r square cos theta-i u infinity sin theta-u infinity sin theta-u infinity-m/r square sin theta where m is u infinity*capital R square. Okay.

So it is just like, this has become vr and this has become v theta, right. So when you have when you consider flow past a circular cylinder.

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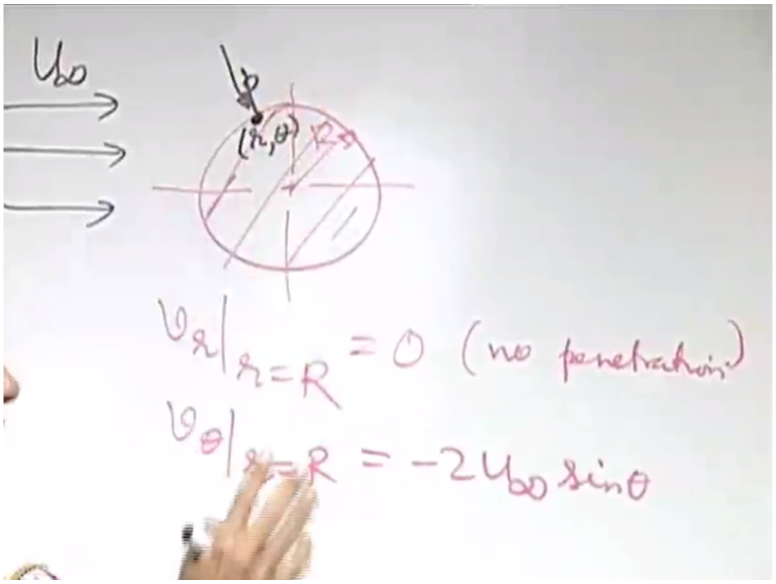

So we have a circular cylinder say of a particular radius capital R. We are interested to know that is the velocity at the surface of the cylinder. Remember, this is not a viscous flow this is the potential flow analysis therefore No-slip boundary condition will not be satisfied. Okay. So what is vr at small r=capital R. This has nothing to do with the No-slip. This should be 0 at small r=capital R. No penetration boundary condition.

So there is a circular cylinder on which fluid is flowing, so fluid should not penetrate the circular cylinder. So you can clearly see that at small r=capital R vr is 0. So this is no penetration, so no penetration is satisfied. What about v theta at small r=capital R? That is -2u infinity sin theta, right, so you can see that there is a tangential component of velocity of fluid on the circular cylinder surface, this is so called as slip type of velocity.

And that is there because the potential flow does not have a consideration of No-slip boundary condition. Now how do you find out the pressure distribution? See we have found out the velocity field, so what is the situation you have a uniform flow coming with a velocity u infinity, now the fluid is flowing past a circular cylinder and we are interested to find out what is the pressure distribution and the velocity field.

So find out the pressure at some points. Let us say we are interested to find out the pressure at this point p which is given by the location say some R, theta. Since it is like, the theta is even 180 degree–theta if it is replaced it is the same, so it does not matter whether you choose theta in the proper Polar coordinates sense or you choose theta from the left hand side sense, all that same.

So see, because it is an irrotational flow and all other conditions are for the Bernoulli's equation that is incompressibility all those things are there, so you may use the Bernoulli's equation between any 2 points. So you may use it from a point at infinity to a point which is marked on the cylinder.

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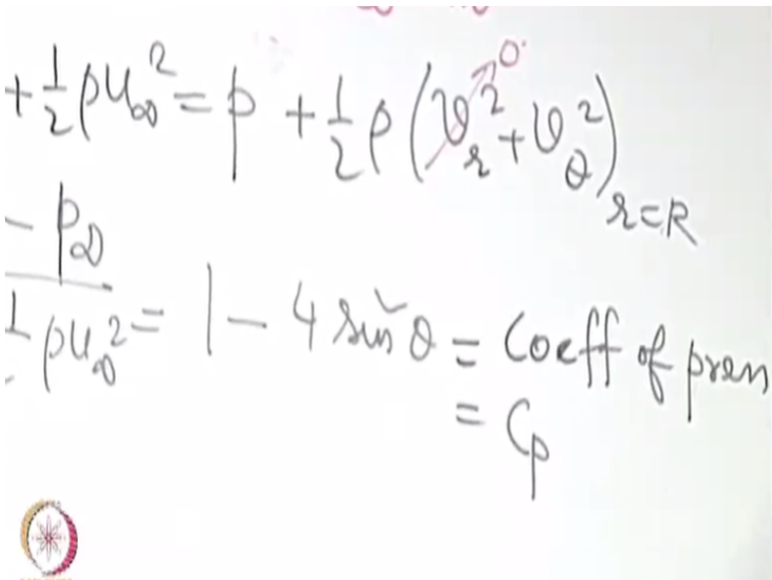

So you have p infinity + half Rho u infinity square=p square, sorry p + half Rho v square at this point, v square is vr square + v theta square at small r=capital R, right. vr=0 and v theta is -2 infinity sin theta. So you can get a pressure distribution p-p infinity in a non-dimensional form/half Rho u infinity square=1-4 sin square theta because 2 sin theta squared will be 4 sin square theta, right. So this is known as coefficient of pressure so the non-dimensional pressure distribution Cp.

So that means, see we have got a pressure distribution and we may use it even for a boundary layer theory till there is boundary layer separation, because whatever is the pressure imposed by the free stream from outside the boundary layer which is like a potential flow same pressure is acting within the boundary layer.

But of course this is the pressure at a point on the cylinder and this pressure the-- what you calculate even in the presence of a boundary layer will not be too much different from this one till you come closer and closer to the apex point and you encounter adverse pressure gradients. So till the pressure gradient is very, very favorable the actual pressure distribution and this pressure distribution will almost remain the same.

We stop here in this lecture and we will continue again in the next lecture. Thank you.