Introduction to Fluid Mechanics and Fluid Engineering
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Lecture - 41
Flow past Immersed Bodies (Contd.)

We will now consider external flows with pressure gradients.

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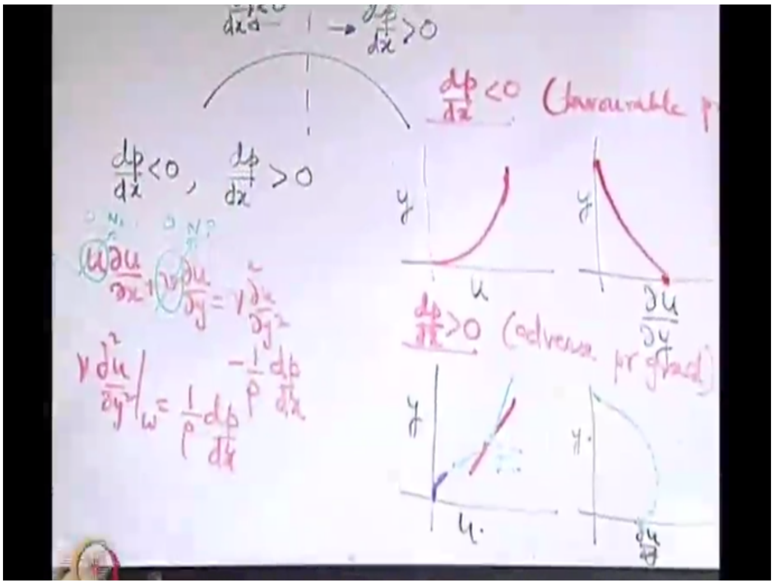

So, the whole understanding is that if you have a surface with such a curvature we have seen in the previous lecture at least qualitatively that because of the curvature of the surface, pressure gradient may be developed along it. Our objective is to have an assessment of the effect of pressure gradient. We have to keep in mind that, pressure gradient for an external flow may be induced by such a curvature effect but for an internal flow, when it is say flow in a pipe or a channel, you automatically have a pressure gradient.

There it is not a curvature induced pressure gradient but natural consequence of the confinement that is there. And here, it is the curvature that is creating the pressure gradient. Now, our important objective first will be to see that what is the effect of or what is the demarcating effect of dp/dx<0 and dp/dx>0. And we will look into these cases separately. So, when we say x, remember we are talking about a boundary layer coordinate.

So, this x need not be a rectilinear coordinate x, and a rectilinear coordinate y. So, let us say that when we are talking about dp/dx<0, maybe we are talking about any flow in this part and whenever we are talking about dp/dx>0, we are talking about the part in the right hand side. SO, here you have dp/dx<0 and here you have dp/dx>0. We will try to draw the following profiles say u verses x or just the other way we will plot u verses y, del u del y verses y and del 2 u del y 2 verses y.

We will try to make a qualitative sketch of these. Let us first do it for dp/dx<0. So, for dp/dx<0 we have already seen dp/dx, even dp/dx=0 we have seen for flow over a flat plate. So, the situation is quite similar even for dp/dx<0. And what you will get is, see this is how you will vary with y till you come to a condition where like when y tends to delta, u will tend to u infinity something like that. Now, if you want to plot du/dy versus y.

So, first of all at the wall, what is du/dy, positive or negative? Positive right, u is increasing with y. So, you have a positive du/dy at the wall. What is du/dy at the fast stream? As y tends to infinity, it tends to 0. So, may be like this. In between what should be the variation? The variation is monotonic. There is no separate physical behavior. What is d2u/dy2 at y=0? Negative, because du/dy is decreasing with y.

So, you have a negative d2u/dy2 and eventually it will come down to 0 as y tends to delta. So, that is the qualitative picture. And this kind of qualitative picture is general hallmark of a flow with favorable pressure gradient. So, this is favorable pressure gradient. Next, let us consider a case of dp/dx>0, which we have seen is known as adverse pressure gradient. Let us try to make such plots. Velocity profile du/dy versus y.

Before making the plot, let us try to look into the corresponding boundary layer equations. So, the boundary layer equation what you have with pressure gradient? U del u/del x+v del u/del y=nu del 2 u/del y 2-1/rho dp/dx right. Let us see that how we applied these for dp/dx<0. See, at the wall, let us say we are interested to apply it at the wall. So, when it is at the wall, u is 0 by no slip, v is 0 by no penetration.

So, basically at the wall, what you have is 1/rho dp/dx right. So, what we conclude out of these is that, when dp/dx is negative, this is negative and that is what we got here. So, this has to be consistent with the governing equation. This we did intuitively from our previous experience in dealing with such cases. But, it agrees with the governing equation. When it comes to dp/dx>0, you must have d2u/dy2 at the wall>0 right.

So, at the wall, let us start with this diagram. So, d2u/dy2 is >0 at the wall right. The next question is that, what is d2u/dy2 at the fast stream far, far away? It has to be 0 right. At the very fast stream away from the wall, there should be no sort of gradient. U is like u infinity constant. So, far away, you must have like asymptotically d2u/dy2 is 0. The question is then; would we have d2u/dy2 something like this?

May be something like this or even could we have width cross the y axis somewhere. There could be many possibilities, that is this is one value at the wall, this is the value at the far stream, far, far away. Think of a case, could you have in between a point where u is a maximum. You could in between you should have a point where u is a maximum. That means, u just has attained the maximum and then may be further its change is very less. Typically, just outside the boundary layer.

So, when u is a maximum, you have du/dy=0. And what is d2u/dy2? Negative. So, you have its value positive somewhere, its value negative somewhere because at some point away from the wall, you must have u has maximum. And this is a continuous function so it must cross 0 somewhere right. So, the correct variations will not be this one's right. Try to understand this very carefully by logical reasoning that you have d2u/dy2>0 at the wall, there must be some point away from the wall where u is a maximum du/dy is 0 and d2u/dy2 is negative.

So, that means there is a variation from and d2u/dy2 to the second derivative is the continuous function. So, it has a variation from positive to negative somewhere it must have crossed the 0 right. And that is why its variation should be like this. So, that means, we have identified a point where you have d2u/dy2 may be 0 and not the fast stream but somewhere in between. See, the fast stream d2u/dy2=0 is obvious. This is not something which is obvious. This is not an obvious conclusion.

So, now if you want to extrapolate this plot that is you want to get du/dy variation. What will be the du/dy variation? At the wall, you have a du/dy something okay. We will see that there may be a limiting case when you may also have du/dy at the wall in the limiting sense 0 and that what happens we will see that what is that. But, in a general sense, du/dy at the wall is say positive. Then, you know that at this point, you will have du/dy as the maximum. That is why d2u/dy2 is 0.

So, you may have something like this okay. So, du/dy is maximum therefore d2u/dy2 is 0. What happens to the u verses y? Just look into u verses y. So, now if you draw the velocity profile, here at this point, you have d2u/dy2 is 0 that means it is an inflection point. That is there is no rate of change of du/dy with respect to y. So, you may have a curve may be something of this sort, where the curvature changes.

But what you have is the slope from by looking from one end is same as at this point, if you consider the slope like this and a slope like this. So, it is may be just show it in a different way something like an S shape. So, where this will come to a limit of course where it will not change any further. But, this is a point where no matter it takes slope from the bottom part of the curve or the top part of the curve, it is the same.

So, that point we call as a point of inflection pi. So, the point of inflection in the velocity profile is one of the hallmarks of the dp/dx>0, which is not there for dp/dx<0. This is the first thing that we are understanding okay. Now, next thing is that now, what happens at the wall? See, when you have a velocity profile of these particular shape, it is highly possible that, it comes to a limiting condition when du/dy is 0 at the wall.

Just you look into this particular form. So, if you consider an extreme case of this will represent what? An extreme case of these may have a situation when like the velocity profile takes off almost in this way, almost like a vertical takeoff. That means, with change in y, u does not change.

So, du/dy with a shape of velocity profile, it is possible, with such a shape it is not possible. So, you may have a limiting case, when du/dy with the adverse pressure gradient that du/dy at the wall may even become 0. See, in a favorable case, it started with >0, when it is coming less and less favorable, at the wall you may have du/dy=0 and then if it continues in that way, it may be <0.

And <0 is such a case when actually u will be not in the positive direction but in the negative direction. So, reverse flow will occur. To understand that, let us try to draw a schematic, say for flow over may be circular shape object or object with a curvature.

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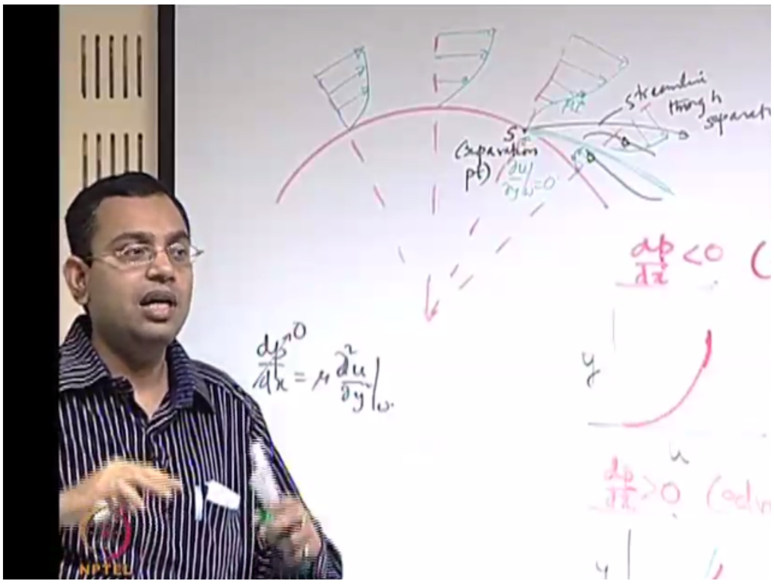

So, let us say that we have an object like this. So, we are interested to draw velocity profiles at different radial locations. So, initially, when the dp/dx is <0, you see that it is a intuitive way in which the velocity profiles are there. May be it maintains it like this in the process the boundary layer grows. Now, when you come to the dp/dx>0, you will come with locations of inflection points in the velocity.

So, now, if you draw the velocity profile at some point, you will come to a state, let us say that, at this point we come to a limiting condition when the velocity profile is something like this. We have just now seen that it is possible with dp/dx>0. That is what we learned from this experience. So, this is a special case. Of course the velocity profile has an inflection point, that is the first thing. The second important thing is, here you are having at the wall=0, that is a limiting case.

When it is 0 at the wall, and then the pressure gradient is going on getting more and more adverse, then what is happening? The flow may reverse in its direction. So, you might have sort of a reverse flow like this. In this way, you may get points, so, if you have a reverse flow, we will see physically that why such a reverse flow is occurring. But, if you come to a location let us say, this point represents what? This point represents a location where the velocity is 0.

Similarly, you may draw such velocity profiles at all other points and you will find that, there will be such points. So, if you join such points, get a locus of may be all the points where you have such cases with velocity as 0 and above that, velocity is >0, that means as if the wall which was a location of 0 velocity has got shifted to this new location. Because beyond which you are having the velocity as >0 and it is maintaining its direction. And here what is happening is a local back flow.

That is a loss. So, to say because it does not contribute to the main energy of the flow. So, this point where this all initiates that is known as a point of separation. And when this has occurred, we say that the boundary layer has got separated. So, you no more have a monotonous growth of the boundary layer beyond this point and the name of this point is the point of separation.

So, we call this as S, this is separation point. You may have different stream lines passing through the separation point may be like this or whatever I mean that is possible. This is a stream line passing through the separation point, just as an example. And it may possible that you have locally re circulating flows so you may also have other stream lines like this. Because, in some case, you have a negative flow, then you may have a positive flow like this.

So, the question at the emphasis is not really that how should the stream lines look there. The big emphasis is that, from the till the point S, the boundary layer did grow as we expected and as we discussed may be like for flow over a flat plate. Flow over a flat plate is a special case, what is the special case? Dp/dx=0. So, when dp/dx=0 for flow over a flat plate, where lies the point of infliction.

Look into the governing equation, dp/dx=mu at the wall. So, if dp/dx is 0, that means, this is =0, that means point of inflection for flow over a flat plate is at the wall at y=0 okay. So, that case is a limiting case, it is safe from such a phenomenon because if you have a point of inflection within the flow domain, these type of limiting situation does not occur, where you might have the velocity gradient 0 at the wall.

So, when you have the velocity gradient 0 at the wall, from the point S what we have come to a conclusion that the boundary layer separation has taken place from point S onwards. That means, because of these type of reverse flow, the boundary layer is not monotonically growing as it was doing earlier and as if it has got detached from the wall. As if the new boundary that now represents the growth of whatever, apparent growth of boundary layer is this one, it is the new one.

So, the very important conclusion we get out of this is that, this line sort of represents a wall that has got detached from the main wall like that. So, it is therefore considered as the flow separation or boundary layer separation when you consider the case of a boundary layer theory. So, very important restriction is that, beyond this point S, you cannot apply the boundary layer theory.

So, when we were discussing about the boundary layer theory and its assumptions, one of the important assumptions that we made is Reynolds number is large, that remains. But the other important assumption is that there is no boundary layer separation. Because if boundary layer separation occurs from the separation point onwards, the boundary layer theory is no more valid. And that we have to keep in mind.

So, these are the 2 most important fundamental assumptions behind the boundary layer theory that we have developed. Now, what happens beyond the separation point? Actually there is a low pressure region that is formed beyond the separation point. And because of this formation of the low pressure region, this low pressure region behind the separation point, it is called as a wake.

And what is important here is to note that because of the formation of that wake, or the low pressure region or the separation, the symmetry in the pressure distribution around the body is disturbed. So, if there was no such separation, the pressure distribution with respect to the vertical axis would have been same for the left and the right. But, because it has occurred, now it gives rise to a low pressure region and the pressure distribution does not remain uniform.

And therefore, there occurs a net resultant force because of the pressure distribution that acts on the body. And the pressure distribution that acts on the body, it is very critical because depending on the act, you may have resultant force that acts on the body. We have seen that the resultant force that acts on the body is a combination of the effect of the shear as well as the pressure gradient.

The pressure gradient is absent for flow over a flat plate, but for flow with such surfaces with such curvature that effect cannot be neglected. That is one of the important conclusions that we should keep in mind. Now, before moving on to the consequence of this separation, let us may be look into one example to see that how the velocity profile gets effected because of it.

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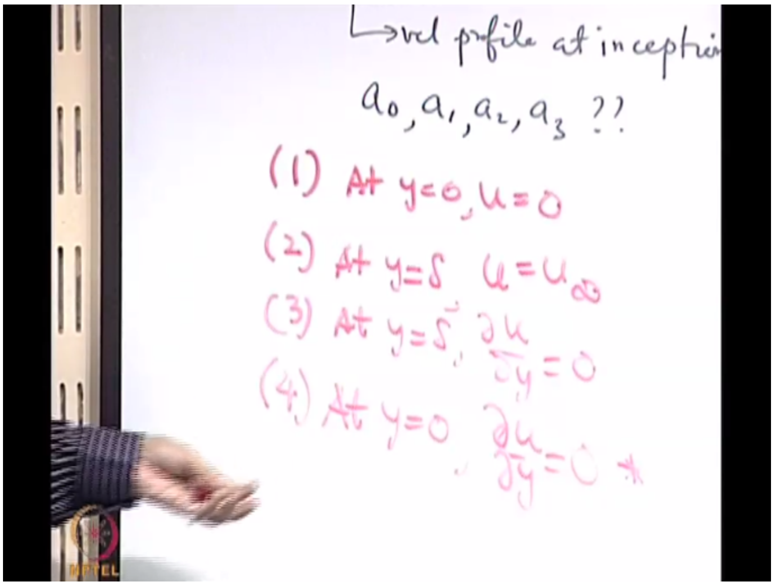

Let us say that, we want to have an estimation of the velocity profile at the point of separation and let us say that it is given in this form u/u infinity=a0+a1y/delta+a2. This is an example of assume that this is velocity profile at the inception of separation. Our objective is to find out a0, a1, a2, a3 consistent with the physical conditions. So, it is just like estimating a velocity profile or fitting a velocity profile in a polynomial form that we did for flow over a flat plate.

But the boundary conditions which should be consistent with this one, the boundary conditions are somewhat altered. So, what are the boundary conditions that we need to consider for this. Again there are 4 constants. So, we should consider 4 boundary conditions to specify the velocity profile completely. So, what are the boundary conditions that we should consider? Number one, at y=0, u=0 fine.

Then at y tends to infinity or at y=delta here which is like tends to infinity at y=delta u= u infinity, at y=delta you have no further change in u and 4th one, at y=0 this is =0, this is very, very important because this is the hallmark of separation. At y=o, you cannot apply d2u/dy2=0, that was for flow over a flat plate, because dp/dx was 0. Here dp/dx is not 0 because you are given that the velocity profile at the inception of separation. When there is a chance of separation, it must be dp/dx>0.

Now, the big question is that why such a separation will occur. See, when the fluid is there within the boundary layer, it has some energy, but because of the reduction in the velocity that is being imposed by the shear effect within the boundary layer, the fluid in the boundary layer has a tendency to get slowed down. Now, when it has a tendency to get slowed down, still it has to move forward, then what is that which will try to make it move forward. One is, the shear interaction with the outer layer.

That is what it tends to make it move forward. So, it takes some energy from that and tries to retain its energy and make its forward movement. And if have $dp/dx<0$, that on the top of it helps it. If $dp/dx=0$, then also it is okay there is no extra effect from dp/dx as a help. But, there is no extra effect also in terms of resistance. But, when $dp/dx>0$ then, the pressure gradient resists the forward movement.

So, there is a poor fluid element, which is located within the boundary layer has a tendency to get slowed down and the pressure gradient externally on the top of it tends to slow it down further and further. So, it has a tendency of continuous reduction in momentum and that may give rise to a shift in the direction of the velocity. So, that is how, you can get a back flow or a reverse flow. So, the momentum in the boundary layer is not sufficient and external pressure gradient which is acting adverse to it if $dp/dx>0$.

May be just sufficient to make it flow in the reverse direction. But the question is, if $dp/dx>0$, is it necessary that it has to flow in the reverse direction. The answer is no, it may not. Because, still it might have sufficient momentum to overcome that adverse pressure gradient and somehow maintain the flow. But if the pressure gradient is so adverse that, it does not have enough momentum to overcome that, then only it will be a reverse flow.

So, existence of an adverse pressure gradient is a necessary condition for boundary layer separation. But it is not a sufficient condition. So, you cannot say that there is an adverse pressure gradient and because of that there will be boundary layer separation. But, you can definitely say that if there is no adverse pressure gradient, there will no boundary layer separation.

So, adverse pressure gradient is a necessary condition for boundary layer separation but not sufficient. The sufficiency will come from what is the strength of that adverse pressure gradient. Is it good enough to overcome the forward momentum and create a reverse flow. That is the important concept that we need to learn. So, what we have understood is that, if you have a solid body in a fluid.

The solid body in a fluid may be subjected to different forces and this forces may come from, one is the skin friction drag which we have identified for flow over a flat plate, even without pressure gradient that is because of the shear + there may be a force due to the pressure gradient. So, that force due to pressure gradient may give rise to a net effect which may also be a drag force. But, to understand that, what is the net effect?

Is it just a drag force or something more than that we have to formally define that, what are the forces which may act on a body which is immersed in a fluid. So, let us do that. So, we have now at least got the origin of the forces that is the pressure gradient and the shear. But how we combine these to get the net force.

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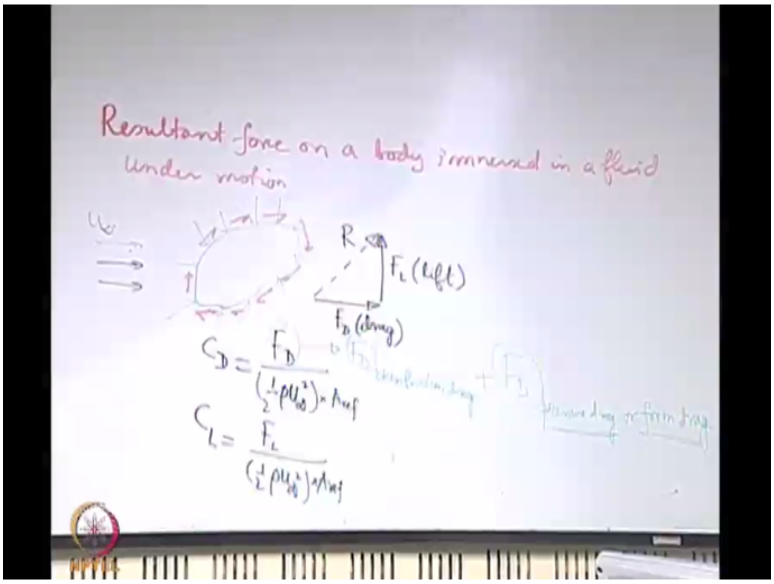

So, the resultant force on a body immersed in a fluid under motion. So, do not confuse it with fluid statics. It is not fluid at rest. So, we are not talking about forces like buoyancy forces like that. So, this is solely because of the relative motion between the fluid and the solid. Whatever is the force that force we want to identify. So, generally the convention is that, let us say you have an arbitrary shape body.

Let us consider a 2 dimensional flow. So, what are the forces which are acting on the body? One is, there is a pressure distribution around the body. The pressure distribution is not uniform. Why it is not uniform? Because for example, there may be flow separation because of the flow separation, after the flow separation has occurred, it will be a low pressure region. And that means, there is asymmetric pressure distribution that is created because of flow separation.

So, the net effect is that, the pressure distribution will be non uniform, there will be a force due to pressure distribution + there will be a force due to the viscous shear effect. So, you will have some shear effect also. Let us say that, there will be some force due to shear. So, you can see that even mathematically, whatever we considered as tangential forces and normal components of forces.

Those are automatically coming physically through the effect of pressure as the normal force and the effect of shear through the tangential force. The resultant force whatever if you integrate that net effect over the body, you may resolve it into 2 components. Of course you may resolve it into 2 components in many ways. But, what is the important convention? Let us say that this is the free stream direction.

Let us say, this is the u infinity direction. So, the force on the body what is usually done is, it is resolved into 2 components. One is in the direction of the free stream and another is perpendicular to the direction of the free stream. And let us say that this is the resultant force. So, this force which is along the direction of the free stream is known as the drag force and which is perpendicular to the direction of the free stream is known as the lift force.

This is where we are talking about a 2 dimensional flow, but if it is a 3 dimensional flow, there also may be a side wise force in the third direction. If there is also a component perpendicular to the plane where we are drawing the figure. But, otherwise, these 2 components must be there. Because the flow at least must be a planar flow. And these are very important.

Because this drag force is something which will come in the form of a frictional resistance and the lift force is something which may give an upward motion to the body. And this is very important for wings of aircrafts. So, if you have an aero foil section, like which represents the wings of aircrafts, then because of the lift force generated, it may go from a lower elevation to a higher elevation.

So, both the lift force and the drag force are very important and it is very important to characterize those. So, when we characterize the lift and drag force, we have seen that the drag force we may characterize by a coefficient of drag CD. So, the coefficient of drag is the drag force by half rho u infinity square*some reference area. We will discuss that what should be the reference area.

Similarly, you have a coefficient of lift, this is the lift force divided by some half rho u infinity square. So, the coefficient of the lift and the coefficient of the drag are just non dimensional ways of representing the lift and the drag force. So, when you have this drag force, this drag force has 2 contributors, because see the resultant force along the direction of the flow is a resultant force due to shear having a component along the direction of flow + resultant force due to pressure having a component along the direction of flow.

So, when we have the drag force, its physical origin is divided into 2 parts. What are this parts? One is known as skin friction drag, which originates because of the shear and another is known as pressure drag or form drag okay. Then from the name pressure drag or form drag, it is clear first of all why from the name pressure drag it is clear that it is because of the pressure distribution on the body.

So, just like the skin friction drag is because of shear force distribution on the body. The pressure drag is due to the pressure distribution on the body. Why it is also called as form drag? Because it is a strong function of the geometric form of the body. Because the geometry of the body is what induces the pressure gradient. So, that is why it is called as a form drag.

In engineering, one of the important objectives may be to minimize the drag force between a fluid and a solid. And if one wants to do that, then one has to see that which one is the stronger contributor? The skin friction drag or the form drag. And it all depends on whether the flow is laminar or turbulent. So, we will now try to look into the issue that first of all if the flow is, if we consider the skin friction drag is it greater for a laminar flow or turbulent flow.

If we consider a form drag or pressure drag, is it > for turbulent flow or laminar flow. So, let us look into that. So, if we consider say, a skin friction drag depends on what? It depends on the wall shear stress distribution. Wall shear stress depends on what? One is the viscosity of the fluid that does not change if the flow becomes turbulent from laminar, what changes is the velocity gradient at the wall.

So, now you tell that if you consider the skin friction drag for a flow, which is laminar and for a flow which is turbulent. Then which should offer more skin friction drag? Obviously that will have more skin friction drag for which du/dy is more. If you consider mu du/dy, the wall shear stress. So, which should have more du/dy? Think of a situation. Consider that you have flow in a channel, if you have a laminar flow, what is the velocity typical fully developed laminar flow velocity profile like parabolic distribution right.

This is what we have derived when we were solving this by using Navier stokes equation. But, if it is a turbulent flow, of course when you say turbulent flow, we are drawing only the average component of the velocity. How it will look? See, for the turbulent flow, the important thing is that there is a great level of mixing in the turbulent flow because of the interaction between Ads of different scales.

So, because of very efficient mixing in the turbulent flow, the velocity so mixing, what it tries to do? It tries to homogenize it. So, it will try to have uniform velocity profile almost throughout. But, at the wall, it should satisfy the no slip boundary condition. So, it should abruptly come down to 0 velocity at the wall. So, the turbulent flow average velocity profile will be something like this okay. So, if you now consider that where du/dy is more at the wall for the turbulent flow.

Because here, there is a high change in u within a very short y distance close to the wall. So, this is the turbulent and this is the laminar. So, we can see that since du/dy at the wall for turbulent is >du/dy for the wall for the laminar flow considering all other conditions un altered and when we say turbulent du/dy, you have to keep in mind this is the mean velocity that we are talking about.

So, that means you have the wall shear stress for the turbulent is > that for the laminar. That means you have the skin friction drag is > for the turbulent flow right. Now, let us come to the form drag or pressure drag. When you come to the form drag. So, for the form drag we are bothered about the geometry of the body. So, the form drag depends on the pressure distribution around the body. So, form drag occurs because of what? Because of a boundary layer separation.

Boundary layer separation will occur when? When the fluid in the boundary layer has insufficient momentum. So that adverse pressure gradient strongly over comes that and creates a reverse flow okay. So, now in which case within the boundary layer you have more momentum, for a turbulent boundary layer or a laminar boundary layer? You expect a greater momentum in the boundary layer in the turbulent boundary layer.

And since you have more momentum within the boundary layer for the turbulent boundary layer, you have a greater chance of resisting the adverse pressure gradient. Therefore, flow separation may be delayed by virtue of having a turbulent boundary layer. So, if the flow separation is delayed, what will happen? The separation point will shift to further downstream. And the pressure distribution which is there around the body that asymmetry will be less and less.

Therefore, the drag force because of the difference in pressure between the points which are located upstream and downstream of the separation, since the separation point shift towards the end towards the downstream, then what happens? Then obviously you have a much smaller wake, wake is the low pressure region beyond the separation point. So, beyond the separation point, the region if it is very small, then that is in a way better because it will give rise to less form drag or less pressure drag.

So, if you have a turbulent boundary layer, it has > momentum, it will delay the separation and in the process, it will have less form drag. So, the turbulent boundary layer has higher momentum so it delays the separation and that means it has less form drag or pressure drag. See, the engineering design has therefore s sort of conflicting things. So, if you have a laminar boundary layer, you have less skin friction drag, but a greater pressure drag or form drag.

Whereas, if you have a turbulent boundary layer, it is the other way. Your question is not to minimize this individually but the total drag force you have to minimize. So, when you need to minimize the total drag force, you have to see which one is dominating. And when you see that which one is dominating, it depends on what is the shape of the body. If the shape of the body is such that it induces a separation much earlier, then the form drag is what dominates.

If the shape of the body is such that it does not induce the separation so early, then the skin friction drag is what dominates. Let us look into some movies, where we will try to figure out that what are the important characteristics of flow behind different bodies.

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So, if you look into this flow. This is flow past a cylinder okay. So, if you consider this that circular thing has a cylinder where the length is perpendicular to the plane of the figure. If you play it once more, you see that after the separations, the separation has occurred somewhere like maybe at somewhat in the middle section. Beyond that, you have the region in the back, where you have this rotating structures and all, those are low pressure regions.

But, if you consider a different shaped body, you may also see a very similar thing.

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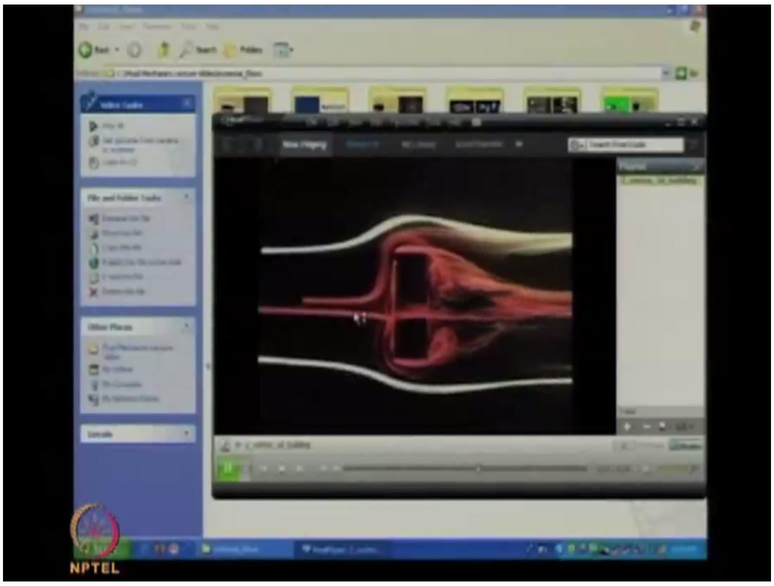

This is flow past a building. So, it is basically a building simulated and if you see that there is a flow separation and there is low pressure region in the back. So, whatever is the region which is towards the right after the boundary layer separation has occurred that is a low pressure region. And it strongly depends on the shape. So, let us look into the case if you change the shape, what happens?

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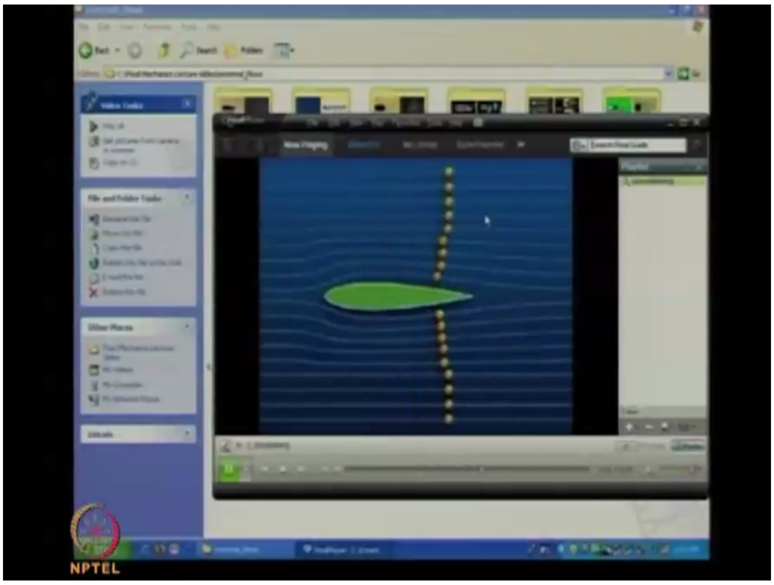

Let us have a shape like this. So, if you see now, this is such a shape where the boundary layer separation is not there. It is almost avoided. It of course depends on that what is the direction of the relative velocity between the solid and the fluid. And that we will see but just you can see that because of the section, the section is such that it is not blunt. That you can see, the previous bodies were sort of blunt and those are technically called as bluffed bodies.

These type of a shape is called as a streamlined body. So, a stream lined body will have a much less tendency of boundary layer separation. And that let us try to see an example where we have a combination of streamline and bluffed body.

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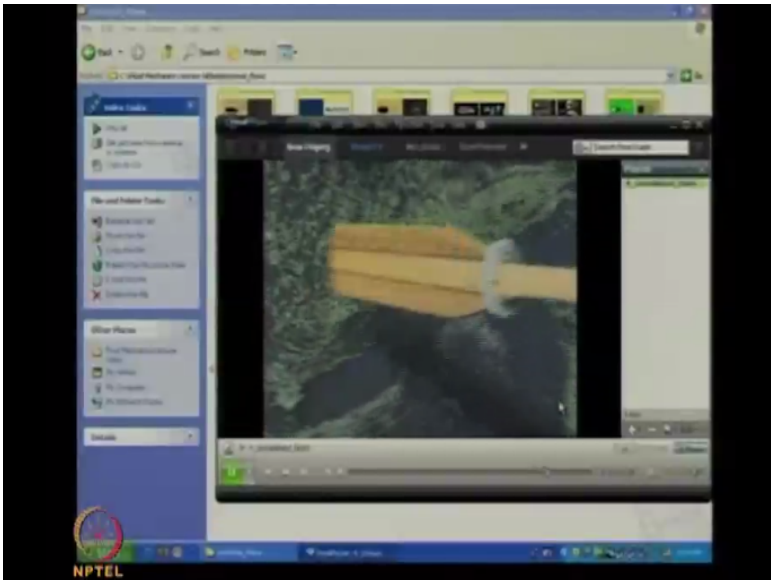

So, the sort of board that is moving, it is like a streamlined shape and it is sailing fine in the water, but, if you see now closely the flow around the stirrer, you see that this is quite bluff and t he flow around that is something which is having the effect of flow past a bluff body. So, I mean, you may have combinations of bluff and streamlined body together in the same effect.

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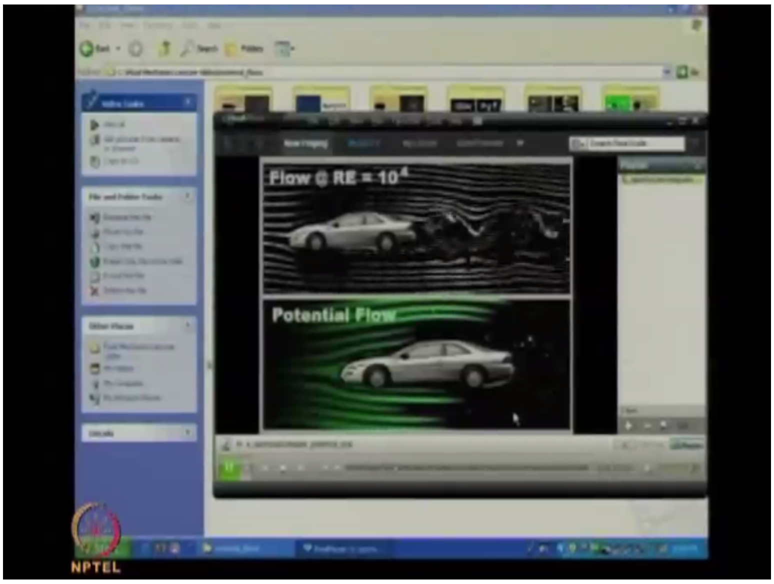

And if you want to see that what is the consequence in engineering design. Of course we will try to understand this later that what is the consequence in engineering design. But, just to see that if you do not consider any boundary layer separation effect. Boundary layer separation effect comes if you consider boundary layer effect. Now, if you do not consider a boundary layer effect, it is like an in visit flow, which is a potential flow. You see that case at the bottom.

It does not show the boundary layer separation. At the top you see, that you can get a boundary layer separation and you have a low pressure region. But if you design the car, the backside like a stream lined shape, then you may delay that separation. And that will reduce the drag. So, you see that many of the racing cars are designed in that way. So, to avoid or to minimize the drag force.

We will come into this examples one after the other, when we will look into more details of flow past different shape bodies. But, our important conclusion is that, if you have bodies of different shapes depending on that, different drag forces may dominate. So, what are the important measures that you may take to delay the boundary layer separation. Because boundary layer separation is an important issue that gives rise to a form drag or a pressure drag. So, what could be the measures?

There are several measures which are possible. One of the measures is known as boundary layer suction. So, let us just note this down that what are the possible measures to delay boundary layer separation.

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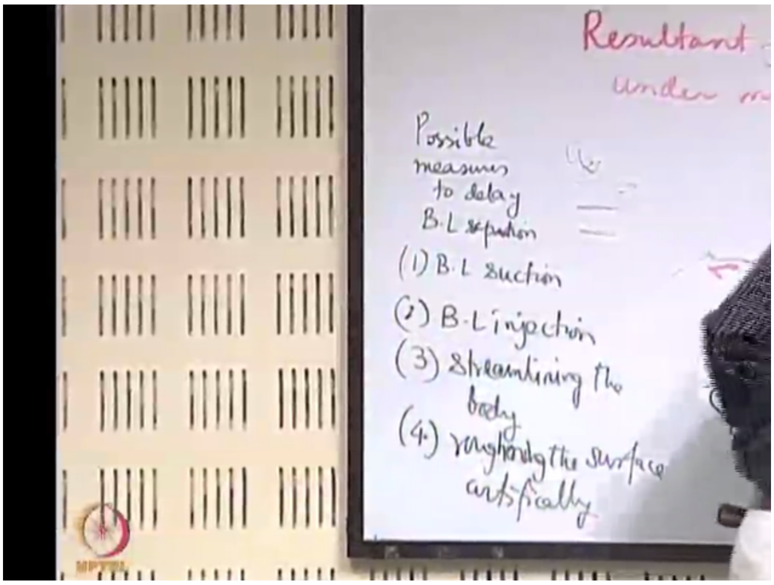

Number one is boundary layer suction. Boundary layer suction is what? Let us say that you have a solid boundary and there is boundary layer that is formed. Of course not for such a straight surface, the form drag will be important. Let us say that you have a curved boundary. Boundary layer is formed close to it. And somehow what you do is, you suck away fluid from the boundary layer into the solid. In the process what you do?

You are basically increasing the momentum of the fluid within the boundary layer. Basically you are sucking the fluid from the boundary layer. In the process, what you are doing? You are enhancing the momentum of the fluid within the boundary layer. We have seen that, one of the reasons of boundary layer separation is that the fluid in the boundary layer does not have sufficient momentum to overcome the adverse pressure gradient.

So, in this way we have looked into one example earlier in the previous class. Say, where you have some fluid which you put through the holes from the top and you suck that from the bottom. So, this is one of the ways in which you can create greater momentum in the boundary layer. You could also do it in the other way. You might inject fluid into the boundary layer.

So, increasing the momentum is not so much dependent on whether it is upwards or downwards. So, you might have boundary layer injection. Then of course changing the shape of the body, that is stream lining the body. So, if you have a bluffed body, you may redesign it to make it of a stream lined shape okay. Then, may be by artificially roughening the surface, if you roughen the surface, what happens?

You may create sufficient perturbations to trigger the onset of turbulence. So, roughening the surface artificially. These are some of the very practical things and later on in one of our lectures we will see that sometimes the cricket ball is roughened in one side, what effect does it have on creating the swing in the cricket ball in a particular direction. And these are some of the basic scientific tools that go behind.

We will just work out a problem quickly on the calculation of the resultant drag force on a body.

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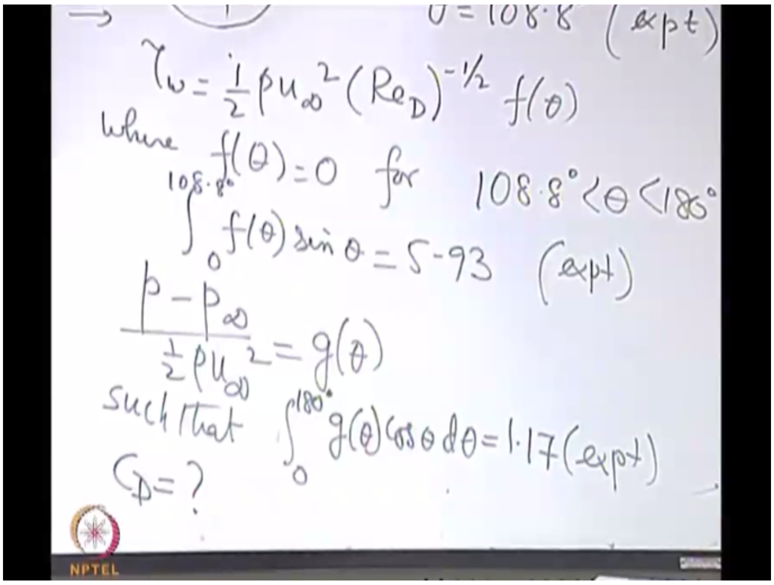

Let us say that, you have a circular cylinder with p infinity, u infinity as the free stream conditions, measure an angle theta from this direction. And it is given that boundary layer separation occurs. We will see that when boundary layer separation occurs in such a case in detail later on. But, say for this problem, it is given that boundary layer separation occurs at theta=108.8 degree, this is from experiments.

The wall shear stress is given by tau all=half rho u infinity square*Reynolds number to the power -half*a function of theta. Where the function of theta is=0 for a range of theta between 108.8 degree to 180 degree and integral 0 to 108.8 degree f theta sin theta=5.93, this is also from experiments. Also, it is given the pressure distribution in a normalized way is given by p-p infinity/half rho u infinity square=g theta, such that integral 0 to 180 degree g theta cos theta d theta=1.17.

This is also from experiment. So, experimental pressure distribution and shear distribution is given as a function of theta. From that you find out what is the net coefficient of drag. So, you can clearly see that after the boundary layer has got separated say 108.8 degree, beyond that, you do not have a wall shear stress effect. Because as if the effect of the wall has got detached from the boundary.

So, now how do you find out the resultant force? So, if you just consider a small element located on the surface. Let us say that, you consider a small element d theta located at an angle theta. So, the length of this one is what? Length of this one is Rd theta. If you have a total force on this, you have one as the shear tau all and another as the pressure. So, what is the force due to the wall shear stress df due to the shear.

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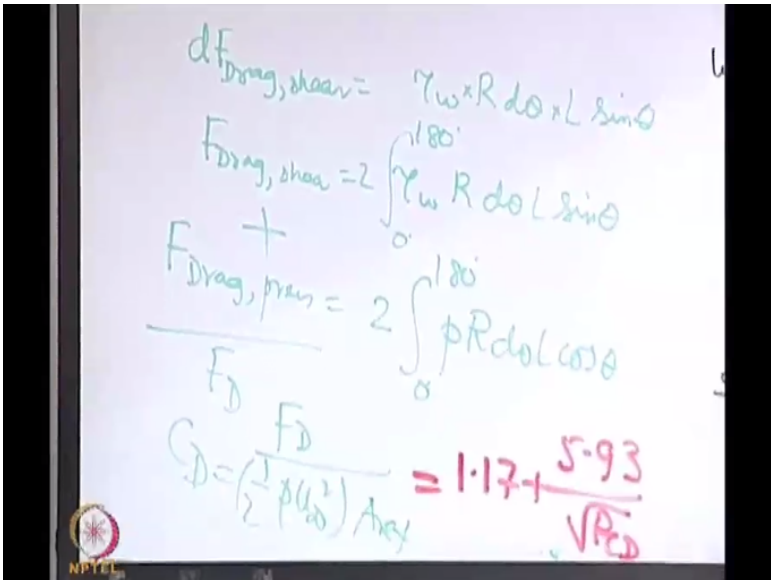

So, that is tau all*Rd theta*the length of the cylinder. That is number one but, what you want is the drag force, coefficient of drag so, you want the force component of this in the direction of the flow. So, what will be that? So, basically this*sin theta. So, you will have dF drag due to shear=tau all*Rd theta*L*sin theta okay. So, you have the total drag force due to shear=tau all*Rd theta L sin theta integrate 2*0 to 180 degree right.

Tau all, you know as a function of theta and so you substitute that. So, tau all=this*f theta. F theta sin theta d theta integral is already given to you. So, you can substitute that here. It becomes a very simple calculation. Similarly, what will be F drag due to pressure? In place of tau all, the effect will be because of p and sin theta will become cos theta. So, 2 integral p Rd theta L cos theta from 0 to 180 degree. So, the total drag is the sum of these 2.

That is the total drag. And what is the CD? CD is the total drag force divided by half rho u infinity square*reference area. For this kind of a case, so the circular cylinder is like this, and flow is taking place cross it. So, the reference area is taken as the projected area perpendicular to the flow. So, what is the projected area perpendicular to the flow? L*2R. So, this a reference that is called as a frontal area or projected area, that is=2R*L.

So, if you do that, let me give you the final answer, which you may check. The final answer is 1.17+5.93 divided by square root of Reynolds number. So, what we can see here, is that the coefficient at very high Reynolds number may be virtually independent of Reynolds number. And what are the important thing that we see? This 1.17 is from where? This is from the form drag; this is from the skin friction drag.

So, very high Reynolds number flow may have the form drag is the sole determining factor for very high Reynolds number flow. So, you can clearly see the effect of the form drag and the skin friction drag. So, let us stop here today. We will continue again in the next class. Thank you.