Introduction to Fluid Mechanics and Fluid Engineering
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Lecture - 40
Boundary Layer Theory (Contd.) and Flow Past Immersed Bodies

In our previous lecture, we were discussing about some of the important concepts related to the boundary layer theory and towards the end we were discussing about the displacement thickness and the momentum thickness. Just to get some idea of how these concepts may be conveniently used. Let us look into one example.

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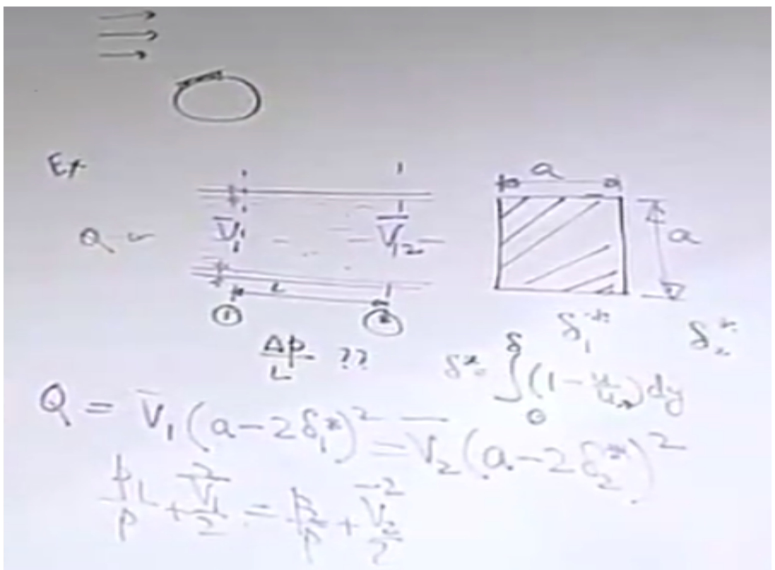

Let us say that you have duct. This duct may of square shape or whatever shape. Let us assume that the section of this duct is a square, just for convenience and we are focusing our attention on 2 sections, 1 and 2, which are located relative to each other by a distance L along the axial direction and our interest is to find out that what is the average pressure gradient acting over this length. Sometimes it is not a very impractical situation. This is a very practical situation actually.

If you somehow want to assess the flow past different bodies in something called a wind tunnel. So if you have heard of something called as wind tunnel, wind tunnel is an artificial place or a location where you have controlled flow of air and you may study the effect of flow of air past bodies of different shapes. So the key is that if you have a body of a particular shape and the body is moving in air, the fact that dictates that what should be the resultant force is the relative velocity between the solid and the fluid.

So you may keep the solid stationary and make the fluid flow past over it. It is as good as having the solid moving through the fluid with the same relative. So if you have a controlled arrangement tunnel, in the tunnel, you put a solid somewhere and have a controlled flow of air over the solid and measure various parameters. Some of the parameters of interest may be say pressure drop.

Let us say that at the sections 1 and 2, we know, by some measurement, we have got an idea of the velocity profile within the boundary layer. That is, you know that what is U/U infinity as a function of Y/delta. Remember here, there is nothing called as U infinity because here it is a confined flow. So it may be U/U outside the boundary layer. U outside the boundary layer is the U corresponding to the central line maybe.

So U/U outside the boundary layer as a function of may be Y/delta. So let us say that that profile is known. Then, also it is known say that what is the total rate at which the fluid flow is entering the system. So that is also known because that is what is controllable. So to get a simple estimate of this delta P/L what one may do is one may get an estimate of the displacement thicknesses at 1 and 2, delta 1* and delta 2*. What are these.

Of course we have the corresponding expressions like as we have derived in the previous class. So if you either have a measured velocity profile or if you have some approximate velocity profile, that is also fine, but you should have a fair enough estimation of the boundary layer thickness and that is possible because if you have a velocity flow across the section, you will find that beyond a certain distance from the wall, the velocity gradients are not there.

That distance you may estimate as the boundary layer thickness. So this is how you get delta *. Once you get delta *, for average predictions, you may forget about the boundary layer characteristics altogether. Because then, your situation is equivalent to that you have, as if displaced the solid boundary by an amount say delta * so that the flow now is taking place within the narrow domain, but almost in an invisible fashion.

So then, you may write, if you consider that v1 is the velocity of the fluid in the inviscid core through section 1 and may be v2 is the velocity of the fluid, which is going flowing through the inviscid core at section 2, then you may write from the continuity equation Q=v1*, what is the equivalent a1, let us say that the square has a section a x a. This is the section of the wind tunnel. So v1*a-2 delta1* square that is how you relate v1 and v2.

So this is one of the relationships you get. Not only that, since these are inviscid core and let us say that you have negligible difference in height, you can always use the Bernoulli's equation if it is an incompressible flow. So P1/rho+v1 square/2=P2/rho, these P1, P2 are not very accurate or local P1, P2. These are like sort of average P1, P2 over a section, but if you get these ones, then you will see that you may write it in terms of v1 square and v2 square.

You get another expression involving v1 and v2 and depending on whatever is measured and whatever is not measured by using this very 2 simple relationships, you can find out the pressure difference or whatever. It depends on what you measure and what you estimate from the calculations. The basic principle is you may use these equations depending on what is known and what is not known.

Again the important understanding is that this is not very accurate, but for many engineering estimations, that kind of local ideation and accuracy of local ideation is not what is required and we required is just an average pressure difference and in such cases, these type of simple estimation is fine. Now the other important part is that what we assume implicitly while looking into this problem may be that we assume that the boundary layer is thin.

Now it might also happen that the boundary layer is thick. Obviously, if the boundary layer is very thick, the boundary layer theory will not be valid. That is one of the things, but despite the validity or invalidity of the boundary layer theory, you will have thick boundary layer, only the corresponding theory that we have developed might not work, but it does not mean that boundary layer may not be there if it is thick.

But what is our expectation in these cases whenever we have almost presumed that the boundary layer is very thin, if the Reynold's number is very, very large. So in these cases, we are looking for experiments of large Reynold's number flows. Towards the end of such discussions, we will come into such cases where the Reynold's numbers are very small and you will try to see that what are the corresponding demarcating features.

We will not go into the mathematical details of such cases. Sometimes the mathematical details of low Reynold's number flows, although apparently they might be easier, sometimes they may be more important. We will not go into that details, but what we will try to see is that what is the characteristic demarcation between the very low Reynold's number flows and high Reynold's number flows.

One of the important hallmarks of high Reynold's number flows that we have identified that the boundary layer is very thick and that is what we have assumed for this case, so that boundary if it is thick and it is thicker than the width of the, of course it cannot be thicker than half of this one, but if it is as good as almost half of this one, then this estimation will be more and more erroneous. Because this entire estimation was based on boundary layer equations, which were based on the assumption that delta is much, much small as compared to the axial length scale.

So all those assumptions we may not be able to justify, if we have low Reynold's number flow. Now the second big question, this question always appears to my mind when I first was introduced into the boundary layer theory, that if the boundary layer is so thin, why not neglect it at all. Because after all if you have a flow domain, say you have some body and fluid flow past it. Now if the boundary is very, very thin and the remaining parts does not understand the effect of the wall to get a gross effect of the flow, why not neglect the boundary layer altogether.

Because at least mathematically when something is very small in comparison to many other things, we have neglected it many times. So why we cannot do it for the analysis of the boundary layer. To understand that, let us look into a very simple problem. We are trying to answer a question that why not neglect the boundary layer altogether if it is so thick, that is the question that we are trying to answer.

That question we will try to answer not through direct use of the boundary layer theory but through a separate example.

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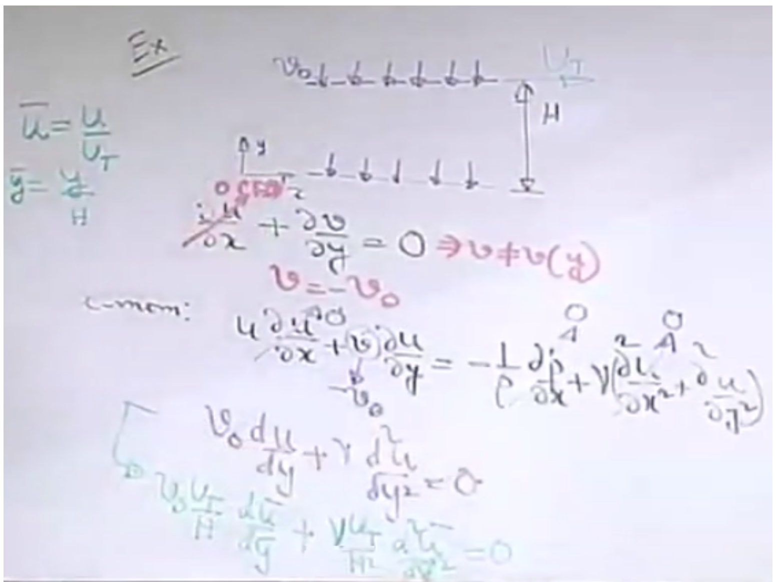

Let us consider this as an example. Say you have 2 parallel plates and let us say that these plates have some holes, through which fluid may flow vertically. That means, let us say as an example that some fluid with a velocity enters the pores in the top plate with a uniform velocity v0 and leaves through some pores in the bottle. That is one of the things, the other thing is let us say that it is basically quick flow, so that the top plate is pulled towards the right with a velocity relating to the bottom plate.

Let us say that the velocity Ut. So let us also assume that it is a fully developed flow. Basically flow between 2 parallel plates just like the quit flow that we have discussed while we were discussing the exact solution of the Navier-Stokes equation. Assume that there is zero-pressure gradient that means the pressures at the inlet and outlet are the same. Only extra thing we want what we considered in that problem is now you are having a transverse motion because of some suction effect, which is sucking some fluid from the top and releasing it to the bottom.

There is forced vertical component of velocity. We will later on see that these type of suction is not something which is very irrelevant. It has some consequence with the boundary layer theory. We will come into that later on, but right now we are just treating it as a simple problem, which may be treated mathematically in an elegant way. So let us write the governing equations for this. The governing equations to set that up, let us set the coordinate axis, say x and y, like this.

Let us say that the gap between the 2 plates is H. So the continuity equation, first let us write, so we are assuming 2 dimensional incompressible flow. These are the continuity equation, also the flow is fully developed. What is the outcome of the fully developed flow? So this will be 0 for fully developed flow. From here, we conclude that v is not a function of y. From this, can we predict that what should be v for this problem, v=-v0.

Because at y=h, v=-v0, this is v is not a function of y, therefore it should be same for all values of y. So v=-v0. That is what we get from the continuity equation. Next, let us go into the momentum equation, x momentum equation. So let us simply it. First of all, we are assuming that no pressure gradient is acting on this pressure at the inlet and exit are the same. It is fully developed flow, so you have the derivatives u and the higher order derivatives all with respect x vanish, so u is the function of y only.

V becomes =-v0, so the governing equation that you get is v0 du/dy+nu d2u/dy2=0. Now sometimes this analysis becomes a bit more insightful, if we non-dimensionalize the parameter, so let us just use some non-dimensional parameters. Let us say u non-dimensional=u/u of the top plate and let us say y non-dimensional=y/h. So this governing equation will become v0u top plate/h du dy+nu u top plate by a square d2u/dy2, all non-dimensional=0.

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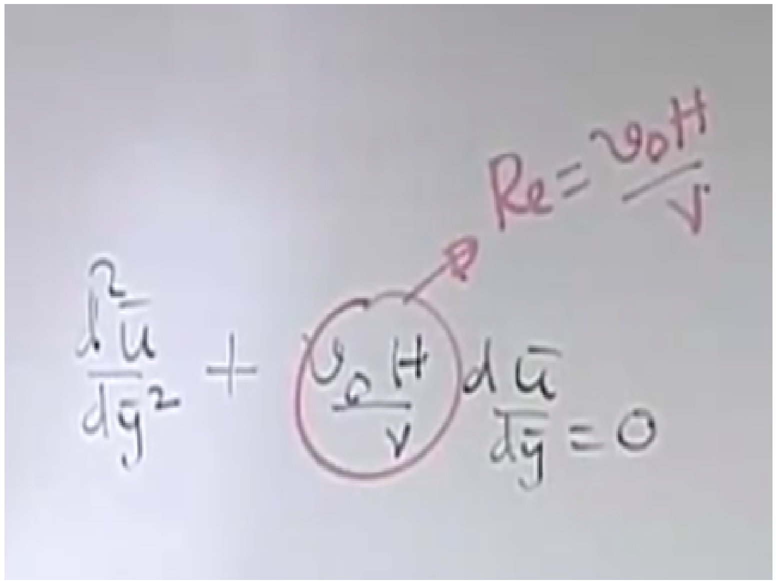

So we may write it in a simplified form as d2u/dy2+v0h/nu du dy=0. V0h/nu is a sort of Reynold's number. If you recognize this, first of all it is a non-dimensional number that you must recognize because all other terms are non-dimensional. So it has to be non-dimensional and it is something as a Reynold's number, we can say, just as this involves here writing this. Our objective will be to solve this equation. Solution of this equation is very easy actually.

Because it is just a second order ordinary differential equation and you may just use the standard technique like substitute u=e to the power my and then get auxiliary equation and so on, get the solution of this equation. We will not go into the solution in that way, but we will try to have a different insight by going through a different method of solution. So what we will do, first of all for writing convenience, let us just omit the bar at the top.

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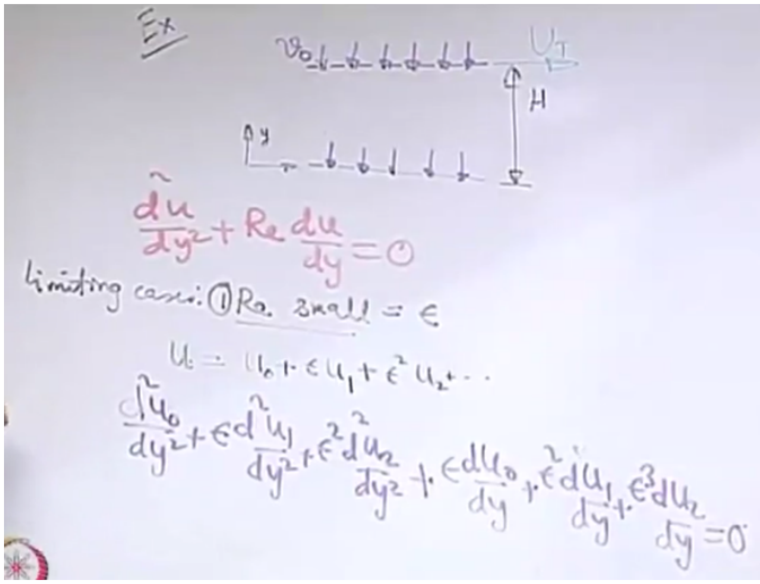

So we will just write d2u/dy2+Reynold's number*du/dy=0. Remember these are actually u bar y bar, just for convenience in writing, we are dropping the bar. Now what we will do is, we will consider to limiting cases. What are the limiting cases? 1 is Reynold's number is small. This is the first case we will consider. Let us say that is= epsilon. When the Reynold's number is small or is epsilon, see this problem may be viewed upon in this way that you have say undisturbed state when there is no vertical component of velocity.

So it is like a regular quite flow. Now you are adding a small disturbance velocity to it and that disturbance velocity may be thought of as a perturbation to the usual quite flow and that perturbation is going to affect the solution of the velocity. In that case, there is a method known as method of perturbation where what you do is you may expand u as u0+epsilon u1+epsilon square u2 like this.

What you are doing, you are expanding u in the form of a series in powers of epsilon, where u1, u2, these are not u0 are not constant. These are functions of y. So what you are having, you are having term by term, higher order terms, maybe less and less important, because epsilon is small. So it is as good as the power series expansion in terms of small number. This is known as perturbation expansion. So what you are doing, u0 is the so called base state, which does not understand the effect of v0.

Because of the effect of v0, the additional perturbations in u0 come into the picture and these perturbations are given by this. Now the way in which you may solve these is very simple. It is just algebra, so what you do is you substitute these expressions of u in the governing equation. So if you substitute that, what you have d2u0/dy2+epsilon d2u1/dy2+epsilon square d2u2/dy2+epsilon du0/dy + epsilon square du1/dy + epsilon cube du/dy=0.

We have done nothing special, just substituted these series expansion of u in the governing equation. Now, what we will do, we may equate the like powers of u, that means what you have.

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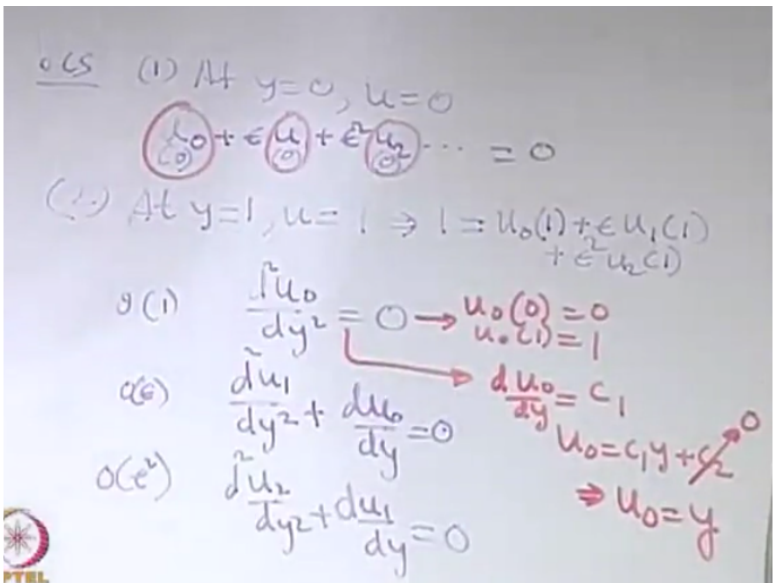

That means you have d2u0/dy2=0 that is 1 equation, that is of the order of 1. Then what is the order of epsilon d2u1/dy2+du0/dy=0, this is of the order of epsilon, then of the order of epsilon square d2u2/dy2+du1/dy=0 and so on. Depending on how many terms you take, you may go on writing terms of various orders, which will give their individual governing equations. Then this u0, u1, u2 should also have their own boundary conditions.

So how do you assess that what should be their conditions. What are the boundary conditions? Boundary conditions are at y=0, u=0. That means u0+epsilon u1+epsilon square u2=0. So y=0, you must have individually u00, u10, u20, like that. What is the other boundary condition? Second boundary condition, this is a non-dimensional y, remember. So at y=1, what is u? u=1, non-dimensional. So that means you have 1=u01+epsilon u11+epsilon square u21, like that.

These are all at 0, you have to understand this. So then what are the boundary conditions on u0. So u00=0 at u01=1 by equating the like coefficients from the 2 sides of this series expansion for the boundary condition. So then what is the solution of this? You have du0/dy=some constant C1, so u0=C1y+C2, at y=0, u0=0, therefore you have C2=0 and at y=1, u=1, that means C1=1. So you have u0=y. Next you come to the second one, the order of epsilon.

What it will give you, d2u1/dy2+du0/dy is 1=0. If you integrate it, du1/dy=-y+C3 and u1=-y square/2+C3y+C4. What are the boundary conditions on u1, u1 at 0 is 0, that means you have C4=0 and u1 at 1=0 from this expansion. So at y=1 this becomes, what is C3, at y=1, this is 0, so C3 becomes half. In this way, you may have expansions of higher and higher terms, so at the end what is the solution that you are going to get.

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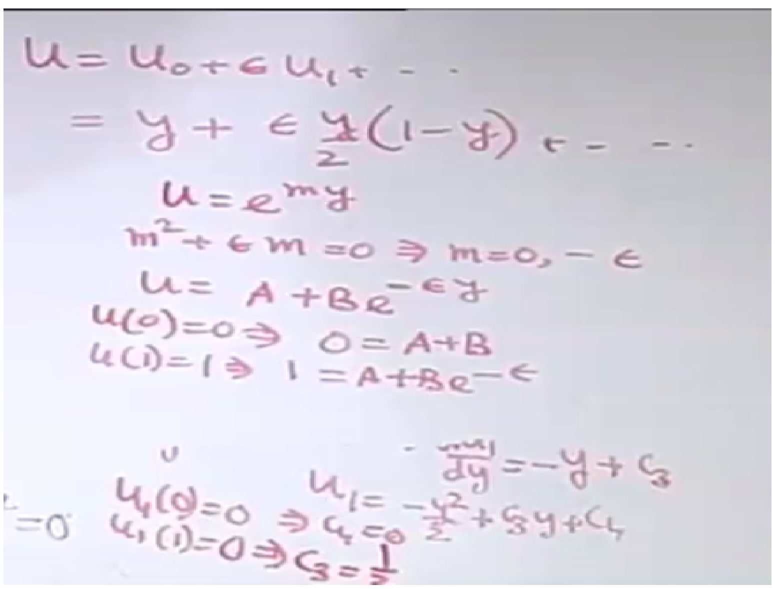

The solution that you are going to get is u=u0+epsilon u1+like this, so u0 is y + epsilon u1 is 1/2*y/2*1-y. In this way, you have series of terms. If you compare this with the exact solution, let us find out the exact solution of this, so if you substitute u=e to the power ny as a trial solution. So you have m square + epsilon m=0, that is auxiliary equation. So m=0, -epsilon. So you will have u=some A+B e to the power –epsilon y.

Boundary conditions at y=0, u=0, so u0=0 will give you 0=A+B and u1=1 will give you 1=A+B e to the power –epsilon. From here, you can find out A and B. It will be nice and interesting to see that it is exponential. Of course, you can clearly see it is an exponential variation. Now you may write it in the form of an exponential series, e to the power x like 1+x/factorial 1, x square/factorial 2 like that.

You will see that these terms will be sum of the terms of the initial part of the exponential series. More and more number of terms you take, you can cover the more and more accurate part of the proper exponential series, but this might be the dominating terms. In this way, you can see that if you have a small perturbation, you may get a solution in this series expansion and that in a limiting sense of small epsilon may match quite accurately with the limiting value of the exact solution, that is fine.

This is for low Reynold's number. Let us consider the other case, he Reynold's number as large. So the second limiting case that we are considering is the large Reynold's number.

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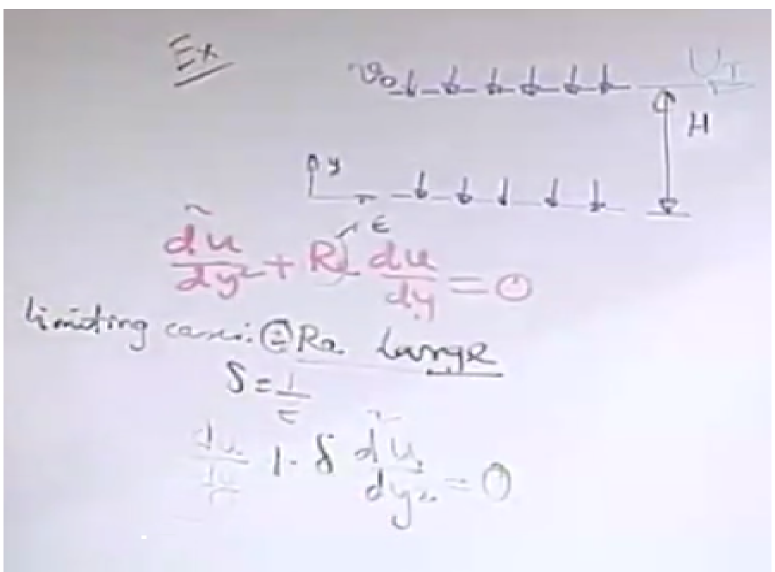

Reynolds number as large, what we have seen is that one of the tricks in which you may use the perturbation method is by expanding it in the form of a series where you have powers of small quantities, like powers of epsilon, where epsilon is a small number. Now here if you treat it as epsilon, epsilon is not small, because Reynolds number is large, but you may introduce a delta=1/epsilon. That will then be small. So then, just replace the epsilon with 1/delta.

So you have du/dy + delta*d2u/dy2=0. We have just switched the variable from epsilon to delta. Why? The expectation is that now delta is small, so we should be able to write a power series expansion as we did for the previous case. Let us try to do that. If we try to do that, try for expansion of u.

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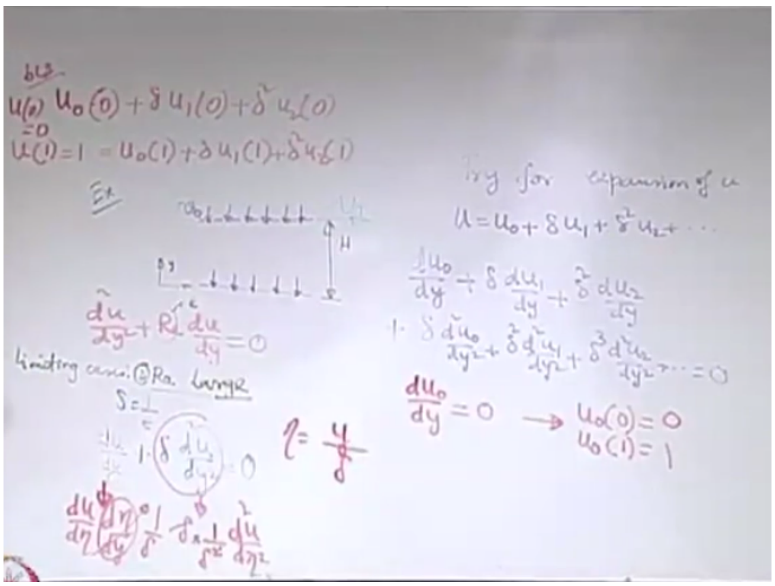

You write u=u0+delta u1+delta square u2 in this way. Now in the place of epsilon, your small variable is delta. Substitute it here. Now you try for expansion of u in this way. You substitute u as a function of y here. So you have du0/dy + delta du1/dy + delta square du2/dy + delta d2u0/dy2+delta square d2u1/dy2+delta cube d2u2/dy2=0. So if you isolate terms of different orders, the leading order term will be du0/dy=0. That is the leading order term.

What are the corresponding boundary conditions? Let us again expand the boundary conditions, so you have u0, which is =0, here we are writing the boundary conditions u0+u0 at 0+delta u1 at 0+delta square u2 at 0, like this in this way. Similarly, u at 1, which is =1=u01+delta u11+delta square u1 like that. So again by equating the like coefficients what are the corresponding boundary conditions for this, what is u00? What u01? 1.

Can you satisfy these 2 boundary conditions by solving this? This gives u0=constant. You cannot simultaneously satisfy these 2 constants. These are very simple examples. How interestingly mathematics gives you beautiful physics. Where are we lacking here? As I was telling you as a part of boundary layer theory, in the boundary layer theory, you have the coefficient of the viscous term may be small in comparison to the coefficient of the inertia term.

This is like an inertia term; this is like a viscous term. So this coefficient is small, because of the smallness of the coefficient, you may be tempted to discard this term altogether and that is what has happened. In the process, what has happened mathematically, mathematically, your second order differential equation has got converted into a first order differential equation. So it cannot satisfy the 2 boundary conditions with the second order differential equation demands.

So in whatever way you make an approximation, you cannot reduce the order of your governing equation, because then you cannot match with the boundary conditions, which must be satisfied or which must be given by the proper higher order governing differential equation. So here forcefully, the governing differential equation has got reduced to first order, although you have to satisfy the corresponding boundary conditions given by the requirements of the second order equation.

That is one of the basic mathematical reasons why you cannot neglect this term altogether although the boundary layer is thin. These are something to do with the equivalence of the thinness of the boundary layer. Because this is 1/Reynolds number. We know that in boundary layer theory, Reynolds number is large. So this has some sort of analogy with the boundary layer theory, full analogy mathematically, partial analogy physically.

But mathematically it is fully analogous because you are having a small coefficient of the viscous term, so to say and that is what we have to remember that in no way, no matter how small it is you can neglect it out, because then it becomes mathematically a heel post problem. Physically, that means what? That means physically also you cannot justify such an assumption. So when you physically cannot justify that assumption that means that you must give a due important to the boundary layer.

What is the difficulty in giving a due importance to the boundary layer? The difficulty is that the boundary layer is so thin. So maybe you are getting an illusion, you are thinking that will, I should not capture the boundary layer, because it is so thin, but if you want to capture the boundary layer properly, one way is like you magnify the boundary layer. So as if the boundary layer is very thin, but you are sitting with a magnifying glass, which zooms up the boundary layer to a large extent.

So that you can see whatever is happening within it. You can resolve whatever is happening within it. So then what you have to do. Within the boundary layer, say the transverse coordinate is y, you have to apply a magnification factor to the coordinate, so that it becomes large enough, low in proportion and although it is thin, within that boundary layer, if you resolve it properly, it has to be given due importance. That is what we are mathematically recognizing.

Now we are trying to see that physically how we can give it its due importance, only by resolving it properly. To resolve it properly, you have to understand that since it is thin, you have to apply a magnification factor to it. If y has a range within the boundary layer, which is 0 to a very small number, which is so small as compared to your flow domain that it is not coming within your resolution. So what you may do?

You may apply a magnification factor and a natural magnification factor is 1/delta. Here if you introduce a new variable eta=y/delta, then what happens, this is the small number. You are dividing it by something small to blow it up or stretch it up. So that means the boundary layer whatever is resolution, you are trying to stretch up its resolution by applying these transformations. So this is known as stretching transformation in mathematics.

The whole idea is that since you are having a very small region, which you desperately want to resolve physically because if you cannot resolve it, you cannot solve the problem, but physically if you want to resolve, you must use a different coordinate for that. That coordinate should be a blown up coordinate, not the original coordinate and that blowing up factor is 1/delta.

Keeping in mind that delta is small. Now if you see, recast the variable. So instead of y, you use the eta. So then what is du/dy. So du/dy is du/d eta *d eta/dy. So this is 1/delta, then this term delta d2u/dy2 is 1/delta square d2u/d eta 2. Now you see from both the terms, you may cancel 1/delta. See 1/delta is not 0, it may be small, but it is not 0. So there is a difference between small and 0, that is what we are trying to highlight here.

It may be limitingly small, but that does not mean that you may have the liberty of taking it as 0. What is the governing equation?

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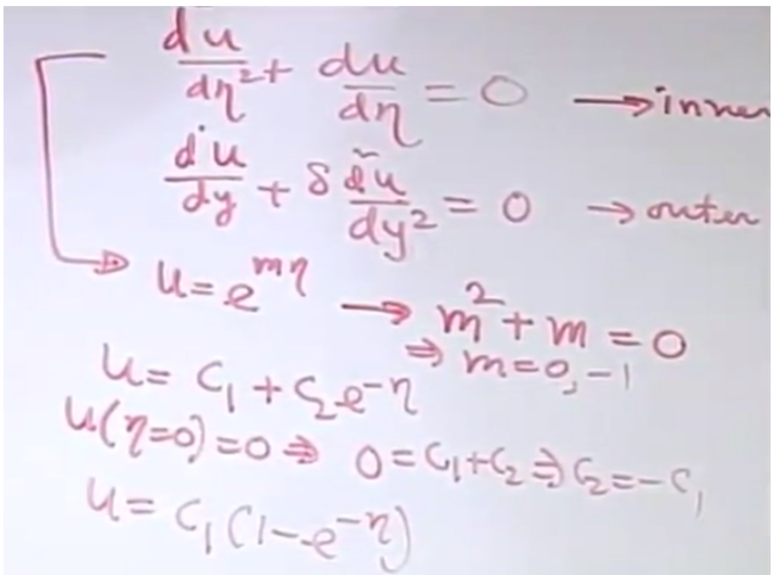

The governing equation becomes d2u/d eta 2+du/d eta=0. This we say is the appropriate governing equation for the inner region. Now we are demarcating the flow region into 2 parts, 1 is the inner region, inner region is what, that is the boundary layer. That means the region very close to the wall where you really want to resolve this high gradient. So that is the inner region. In the outer region, that is outside the boundary layer, this equation is fine.

It should have no problem because outside the boundary layer, the resolution of the boundary layer is not important. So for the outer region, you can still use the original differential equation. You are still using the original differential equation, but with a rescaled value. This is the outer region. So you have 2 variables, 1 is y another is eta. These variables directly do not know each other. Eta is the inner variable, which has the sort of has the responsibility of resolving only the boundary layer.

This is what, this is like the outer variable, which sort of has the responsibility of resolving what is there outside the boundary layer. So what it means is that these 2 variables may be treated independently. When you want to treat the inner variable, you have to keep in mind that anything outside the boundary layer is like eta tends to infinity. Because inner variable is only confined within the boundary layer, outside the boundary layer is something very large for that guy who is sitting in the boundary layer.

For that it becomes a coordinate like infinity. If you want to solve this, it is possible to solve these now with the perturbation. Now if you want to apply the perturbation method with the inner and outer region separately by resolving and rescaling the inner region, this is known as the singular perturbation method. It is different from the regular perturbation method that we have seen.

Singular perturbation method is important because you want to resolve what is happening in a very thin region close to the wall and therefore you are having a rescaled variable. We will not go into the details of the singular perturbation method. Just for giving you the information on the name of the method, I am giving it, but we will just try to solve these 2 equations independently and try to match them and see that what solution we get out of that.

If you solve this equation, if you use u=e to the power m eta as a trial solution. Our whole objective now is not to just do the full perturbation analysis, but to see that how you match the inner and outer solution, because they directly are 2 different variables, but somehow the outer solution must know what is the inner solution. That is how to get the complete solution. So u=e to the power m eta if it is the trial solution, then you have m square + m=0.

That means m=0, -1. So u=some C1+C2 e to the power –eta. What is the boundary condition that you can apply on u? At the wall, definitely this the thing that you were looking for within the boundary layer. At the wall is u at eta=0 that must be =0. That means you have 0=C1+C2 that means C2=-C1. So u=C1*1-e to the power – eta. Question is how do you get C1. For that, you have to match this with the outer solution. So you have to know what is the outer solution.

For the outer solution, you may use the perturbation method. Let us apply the perturbation method for the outer solution at least.

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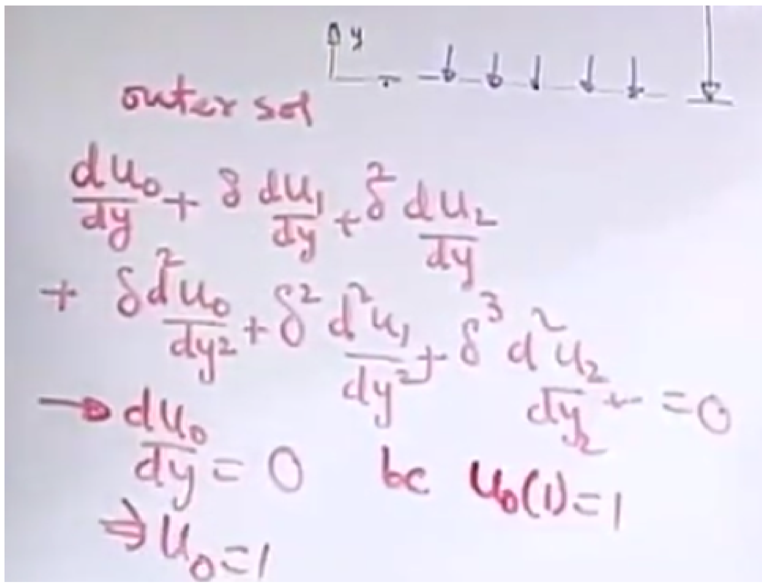

This is the outer solution. This we had just written earlier, but where we failed. We tried to apply it for both the boundary layer as well as whatever is outside the boundary layer. Now we know that it fails within the boundary layer, we are not trying to use it within the boundary layer. But outside the boundary layer anyway, the viscous effect is not there, so you may totally disregard the viscosity outside the boundary layer. That physics we are trying to use here.

If you recall that from here, what we got is du0/dy=0 obviously from the highest order term and what were the boundary conditions for this, if you recall for u0, u0 at 1, u0 at 0 we cannot use because this is not validate 0. This is only in the outer region. So only we can use that u01=1. That means the solution of u0 is 1 because u0 is a constant. Now you have to make a matching. What is the matching?

The matching is y tends to 0 for the outer region is equivalent to eta tends to infinity for the inner region. As if where the outer region starts are beyond the tractability of the inner region coordinate. So this is the limiting sense in which, this is called the matching condition between the outer and the inner region. So when you have y tends to 0, you do not care what is y tends to 0, because this is same for all y, but eta tends to infinity is C1, so u becomes C1.

That means from here you will get 1=C1. That means your complete solution in the boundary layer becomes u=1-e to the power –eta. What is the trick of this method? The trick is you introduce a separate coordinate for the boundary layer, which you call as inner region where basically you stretch up the local coordinate, you use an outer region coordinate for which you may sacrifice even by lower the order of the equation.

Because higher order equation you may preserve within the boundary layer. Because higher order term is important, that is the second order term in the governing differential equation is important only within the boundary layer, where viscous effects are only important and then match of the 2 solutions by considering the limiting values of the inner and outer coordinates. So at least from these exercises this is reasonably simple and tractable mathematically.

But with this, we have understood a very important concept. What is the concept, that you cannot neglect the effect of the boundary layer altogether, although that may be thin and how to mathematically track that, you see that y/delta is something in our similarity variables we have introduced y/delta and see start from the experimental observations of Blasius. Blasius observed that u/u infinity is a single valid function of y/delta. So y/delta did not come up to Blasius as a mathematical entity.

It came up to him as a physical entity from the experiments. Then, similarity transformations were automatically giving rise to a tractable solution if you rescale the variable as eta=y*some function of x, gx where that scales with 1/delta and therefore we see that y/delta is a sort of a magic coordinate system within the boundary layer, which gets conformed by this analysis also. What it essentially does physically, it zooms up the boundary layer coordinate by applying it a stretching by an amount 1/delta.

The fact that we have understood that the boundary layer is important and it cannot be discarded altogether, so we have to understand that what are the importance of the boundary layer. We have to now see that if there is a body, which is immersed in a fluid and if there some force, which is there acting on the body, what are the consequences of these forces under a dynamic condition. That means if a fluid is flowing past a body.

Then there is a boundary may be at high Reynolds number. The boundary layer is very thin, but if there is a fluid element or may be a fluid particle located within the boundary layer what are the forces that the fluid particle feels from within the boundary layer. So let us identify the forces experienced by a fluid particle located within the boundary layer.

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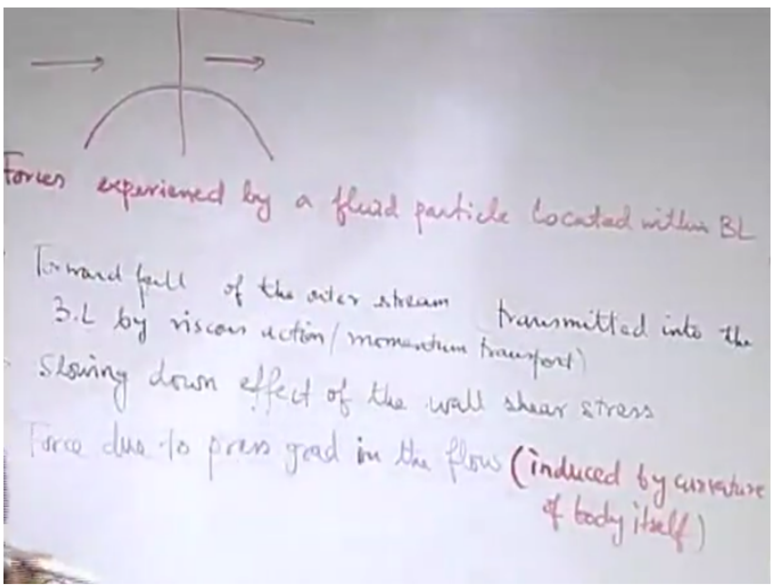

When there is a fluid particle located within the boundary layer, it is being subjected to at least a couple of effects, 1 effect coming from the top of it, another effect coming from the bottom of it. The general understanding is whatever is there at the top of it, is trying to pull it forward because it is there with a higher velocity, that may be the free stream velocity for flow on a flat plate. So you have a forward pool of the outer stream.

How this forward pool of the outer stream is transmitted into the boundary layer by viscous action for laminar flow directly and by the momentum transport between the eddies for turbulent flows. So transmitted into the boundary layer by viscous action or momentum transport. Then, obviously there is a wall at the bottom which tries to slow it down. So slowing down effect by the wall shear stress, that is also one of the forcing parameters.

This effect is always present because the fluid is viscous and at least within the boundary layer, there are velocity gradients, that means you have both the viscosity as well as the rate of deformation, so the shear stress will be there. Apart from that, we have not considered till now, but we will consider subsequently is any effect of pressure gradient on the flow. Force due to pressure gradient on the flow.

When we consider flow on a flat plate, the first 2 situations were stream working. You had the outer stream trying to pull the thing forward, because it is moving with a higher velocity, the wall is trying to slow it down. So these 2 for a laminar flow these are like artifacts of the shear in the flow, but not only have a shear in the flow, you may also have a pressure gradient in the flow. Pressure gradient in the flow is important for flow on a flat plate.

We have seen that because for that if you have u infinity as constant, you have dp infinity/dx=0 and what is the important thing that we could understand of the general boundary layer theory. That whatever is the pressure gradient imposed from the outer stream, the same pressure gradient acts on the fluid in the boundary layer. Therefore, if there is no pressure gradient in the outer stream, obviously the boundary layer fluid is not subjected to any pressure gradient.

But if instead of a flat plate, you have curved boundary, then because of the curvature effect, you will have a pressure gradient. Why because of a curvature effect, you will have pressure gradient, we will come into the theoretical aspect of it, but as engineers we must understand the basic physics out of it, let us say, you have flow over a body of a circular shape. Think in this way, consider a fluid which is hard away from the boundary. It is like being in a free stream like this.

Now when you come close to the wall, the wall will have some effect, so it cannot move like this. If you consider extreme streamlines may be one located at a very far distance, it is not feeling its effect, so it is moving like this and the shape of the body itself is a streamline. Because there is no flow across it, so you have as if a confinement like this, but this is one end of the confinement, this is an outer end of the confinement.

From here to here, you see that the area of cross section is gradually decreasing. So it is like a converging cone of venturimeter, so to say and here you see that the area of cross section, that the flow gets is continuously increasing. So it is like a diffuser and therefore the pressure gradients acting on these sections are different. In one case it is favorable and in another case it is adverse. In which side it is favorable, left or right?

Left side it is favorable on the other hand; on this side it is adverse. So you can clearly see that the effect of the curvature of the body itself is sort of introducing a pressure gradient and that pressure gradient must exert some resultant force on the body. So that we must understand that what is force due to pressure gradient that is there in the flow. So the important thing is that the pressure gradient in the flow may be induced by curvature of the body itself.

Therefore, our next objective will be to study that what is the effect of the pressure gradient in terms of the force acting on a body which is immersed in a fluid and fluid is passing across it. It is very, very important because almost all engineering objects are not like flat plates. So they have certain curvatures and there will be effects of pressure gradient starting from engineering objects, sports balls like tennis balls and cricket balls and so on.

These are having certain curvature. So when they move in the flow, what types of forces act on that and how these things are deviated from their original trajectory because of that will be a matter of great interest and that we will study in our subsequent classes. Thank you.