Introduction to Fluid Mechanics and Fluid Engineering
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Lecture - 38
Boundary Layer Theory (Contd.)

In the previous lecture, we saw the order of magnitude analysis for boundary layer over a flat plate. Now that gave us an estimation of the thickness of the boundary layer, the wall shear stress parameters like that, but what if we exactly want to solve those equations without going for any order of magnitude analysis. So to understand that how that might be possible.

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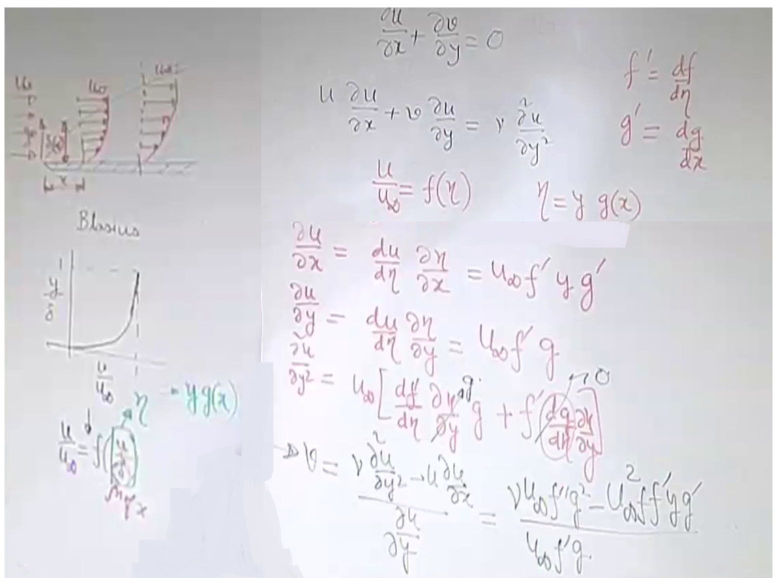

Let us rewrite the momentum equation, that is we are still considering the flow over a flat plate that is the momentum equation that we were dealing with, our objective is to solve this equation, we keep in mind that it is not that you have just one equation and 2 unknowns, because you also have the continuity equation to support the momentum equation. Now you can clearly see that even in such a simple form, we see this is non-linear partial differential equations.

So the question is that solving this just by using this particular form may not be very, very simple. But there are certain techniques in which under certain circumstances, the partial differential equations may be transformed into ordinary differential equation, and one such important transformation is known as similarity transformation or stretching transformation. We will try to see that whether that means, what we are trying to do?

We are trying to investigate whether it is possible to convert it as a function of a single variable not 2 variables x and y, but a single variable where the single variable will contain both the information of x and y. And if we are successful in doing that, what is the adjective that we will achieve? Because of having function of a single variable the partial differential equation will be converted into ordinary differential equation.

So to highlight whether that is possible, let us just look into first a qualitative way in which historically this phenomena was understood, when this phenomenon was understood historically a lot of effort was given by a famous scientist and engineer known as Blasius, basically mathematician. So Blasius, what he tried to do? He saw one of the important behaviour that is if you look into the velocity profiles.

You see that these velocity profiles are not same as you go along x the velocity profile change, but the velocity profile looks as if it is a stretched version of what it was at an x before that, so this motivated him to make a plot of say u/u infinity versus y/delta, because it sorts of normalizes the stretch, because this is always confined between 0 to 1, this also always confined between 0 to 1. And then it was found that if you make such a plot then u/u infinity as a function of y/delta is such that this functional variation, this is one, this is one, this variation is same at all sections.

That means behaviour at all sections may all be combined and normalized in this particular functional form, and this gives a very important physical insight that u/u infinity is the single valued function of y/delta. And you see that we have just seen from the order of magnitude analysis that delta is some function of x right, in fact it scales with square root of x that is what we have seen. So that means see the dependence of both y and x these are there.

And you may introduce a new variable which is like sort of y/delta, let us consider it a new variable eta, which has a single variable it is dictating the behaviour. So this physics is see how physics is related to mathematics that is what is very, very interesting, because we will rigorously derive and come up with the same conclusion which from a very different physical insight could be obtained that this gives the motivation that this velocity behaviour is the function of a single equivalent variable.

Where that equivalent variable carries the information of a both y and x, y explicitly, x implicitly through delta, and that means that this variable eta maybe of the form of y into some function of x right, because delta is some function of x, and y is there. So let us call eta or let us say introduced eta as y*g x okay. Now let us say that we write u or say u/u infinity as a function of eta, because that is what we get from the normalized picture.

Based on this one and we will remember that what is this eta? eta=y*g x, we will try to understand physically that what this transformation is trying to do, we will do that once we get an estimate of what is g x. We will see that mathematically that g x will scale with 1/delta, see physically there is no physically we are seeing that this eta is a function of separable function of y and x.

So this similarity transformation is a special case of method of separation of variables that you have learnt in mathematics course. So the variables are separated, so you have effect of y and effect of x separated, and because of a particular physics that is occurring this separation is possible. And we will see this separation will become mathematically consistent separation of variables whatever physics is governing this it will also become mathematically consistent.

That means this g x will indeed come out from the mathematics to be of the order of 1/delta and that we will show just from pure mathematics without going into the physics, and that will give us a sort of equivalent between these 2. So the objective now is to use this similarity variable and make a transformation of this partial differential equation to ordinary differential equation, to do that let us say that we want what first del u/del x, then so different terms that we are looking for.

So what is this? We will just using the chain rule right, so du/d eta is what? u infinity*f dash, so when we write f dash what we are meaning is df/d eta that is the shorthand notation we will use okay, and then the partial derivative with respect to x y into dg/dx right. So we will write it g dash, so we will use again shorthand notation g dash is dg/dx, so both dash but the variables are different for f it is eta, for g it is x okay.

What is partial derivative with respect to y, so u infinity okay first let us write the chain rule description and then you will write, so du/d eta is u infinity f dash, then that will be g. We also require a second derivative with respect to y, so let us do that, second derivative with respect to y is like, first of all you have this f dash term, so let us consider it second derivative so you have df dash/d eta that*g+ f dash is there and what? dg/d eta okay.

Let us see g is explicitly a function of x right, if you want let us write and see whether it is consistent or inconsistent I do not mind let us just write, if you feel that this type of chain rule is going to work let us just keep it as it is. So now see I mean before going into further let us investigate whether this sort of chain rule is going to work or not, see one important thing is you have to look for the description of the function in terms of explicit representation and implicit representation.

See you have g as a function of x both explicitly and implicitly, so you have to think eta and x and 2 different variables, 2 different sort of independent type of variables. And then if you look into it in this way, see g does not understand what is x, so this is what is you are writing in terms of explicit, so g explicitly is a function of x only, so it does not understand eta maybe there is an implicit interlinkage between eta, g, y whatever but it does not understand explicitly what is that.

So this is clearly=0 okay, so this one d eta/dy is what g, so this becomes g square. **"Professor - Student conversation starts"** (()) (11:41) yes and that is what see you are not writing del g/del eta; you are not writing partial derivative that is what you have to understand okay you are just writing the ordinary derivative. So if you see this is where knowing too many things is bad, so you have started with a very basic calculus what?

You know g as a function of x, if you are not asked to find out a derivative of g with respect to anything else other than x that will be 0, so that is what is we are doing. So you have g=2 x, you are asked you to find out what is dg/dy, so what you will say? So just like that, so this is not a variable which is contained within g and ordinary derivative not partial that we have to be careful. **"Professor - Student conversation ends."**

So now let us try to write this expression see what would should be our strategy, see v we do not know, so we will write v from this expression, and then eliminate that from the continuity equation by using the continuity equation. So what will be v? v=this one okay that is the first term nu u infinity f double dash g square. See there is some lot of algebra in it, so if I make any mistake please correct -in place of u it is u infinity*f, and partial derivative of u with respect to x.

So you have u infinity another u infinity so infinity square f f dash y g dash/u infinity f dash g right.

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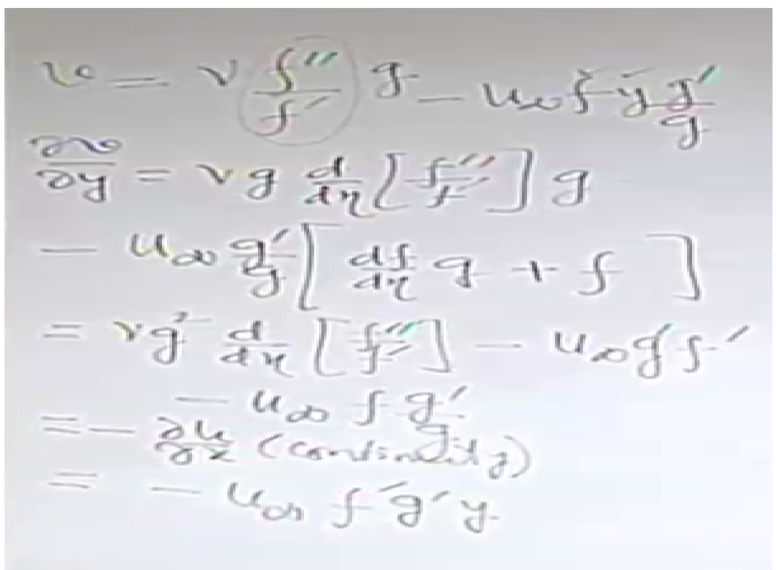

So let us write v then term by term, so nu f double dash/f dash g right that is the first term, second term-u infinity f y g dash/g. If you want to eliminate v, then basically we have to find out what is del v/del y, and then equate that with –del u/del x in the continuity equation, then v will be eliminated. So we have to differentiate it once with respect to y, so when you want to differentiate it with respect to y, g is a function of x that is like a constant for that partial derivative.

So nu g, then basically you are dealing with this is a function of eta, so d/d eta of this one f double dash/f*del eta del y that is g, so another g has come, this g*this g will make it g square, then next term -u infinity g dash/g is like a constant. So for it so for it there are 2 variables, one is f, another is y. So for f it is like df/d eta **"Professor - Student conversation starts"** (()) (16:10) I mean if there is some explanation has to be given that that is by what algebra.

So if I have made a mistake in algebra you let me know otherwise no explanation, so u/u infinity is f, so when you have substituted u that is u infinity*f that is how f has come okay. **"Professor - Student conversation ends."** So -u infinity cheaters by g then f*y you differentiate, so this df/d eta*del eta/del y is g +f* the partial derivative of y with respect to y, so that is 1. So nu g square d/d eta of f double dash/f-u infinity g dash f dash, df/d eta is dash, g and g get cancelled out,-u infinity f g dash/g.

"Professor - Student conversation starts" (()) (17:33) which one d/d eta yes this is f dash right okay. **"Professor - Student conversation ends."** Now this del v/del y=-del u/del x from the continuity equation, so that =-del u/del x from the continuity, and del u/del x expression we already have, so that=-u infinity f dash g dash y, this term has y right yes g*y right yes okay, because it was a product of f*y, so y has to be there right okay.

So now if you look into these equations, so we have this as in one side and this in the right hand side right. So if you just compare these 2 you will see that first of all, of course there is another term in the left hand side this one also okay, so if you compare these 2, these 2 terms get cancelled out right. So we are left with the form of the equation let us just write it.

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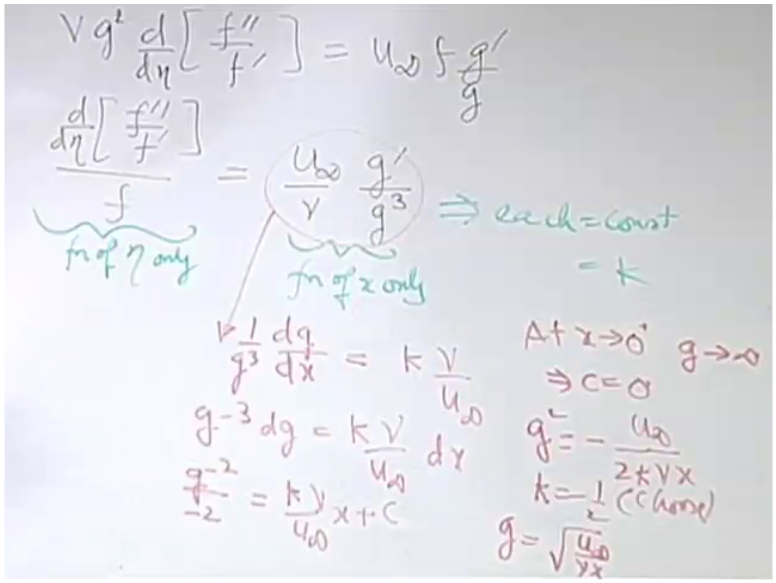

So we write this as nu g square d/d eta of f double dash/f dash=u infinity, there was no u infinity okay that is fine, u infinity f g dash/g okay. So we may just isolate the effects of the variables eta f in this way right, the purpose of the way in which this was done is to show that now you are able to rearrange it in a way that the left hand side is a function of eta only, right hand side is a function of x only, these 2 variables do not explicitly know each other.

So this is a function of eta only, this is a function of x only, this implies that it has to be a constant. Let us say that the constant is K okay, so long as this proportional relationship is satisfied it does not matter what constant we take that is what we have to understand, because we are satisfied with this equality, equal to what is not going to matter as a lot. So we will use this K in a way that just helps us in our algebraic simplification.

To get a clue of that what should be a good K for that, let us consider this equation and maybe try to find out g as a function of x, so now if you integrate this this will give u g as a function of x, so what is this one so you have dg/dx 1/g cube dg/dx=K*nu/u infinity, that means g to the power-3 dg=K nu/u infinity dx. So if you integrate it g to the power-2/-2=K nu/u infinity to x+ some constant of integration right.

How do you know what is the constant of integration? You must know g as at some point where you know x, so at some point you must know the relationship between x and g. So think of the flow over a flat plate, this is the flat plate the boundary layer is there, can you tell what is g at x=0, yes try to remember the physical meaning of g, g is like scales with 1/the boundary layer thickness, so g tends to infinity as x tends to 0.

One of the important things that you have to remember is the boundary layer theory is singular at x=0, that means you do not really have at x=0 delta=0, you only have at x tends to 0+delta tends to 0+ that is all, but exactly at x=0 that is not always you see in books whenever we say loosely say at x=0 delta=0, obviously it is okay in an approximate sense. But if you rigorously want to state what is the boundary condition, then the important thing is at x tends to 0, you have that delta =0.

Because at x=0 it is a singular behaviour it does not have any definition of the boundary layer thickness. So that means if you utilize that at x tends to 0+, g tends to infinity, then C will be=0. So from this what we conclude g square=-K/2 nu sorry -u infinity/2 K nu x. One of the important physical restrictions is that K has to be negative, because g is a positive quantity, it is meaning the inverse of the boundary layer thickness.

So any negative K choices is fine, but what we may chose may be a good number is K=-1/2 that somehow nullifies many of the bad numbers, so let us say K=-1/2, this is just choose not a must not a ritual this is just like a convenient way of doing it algebraically. So when you do that then g becomes square root of u infinity/nu x, you see if you recall from the order of magnitude analysis we got delta scales with square root of x, so this is these scales as 1/delta.

So from mathematics you are getting back the same physics that you got from order of magnitude analyses, now with the same K important is with the same K, because so long as you keep the same K does not matter what K you take by satisfying this condition, you may solve for the f, because f is what is important for f gives you a velocity profile u/u infinity is f.

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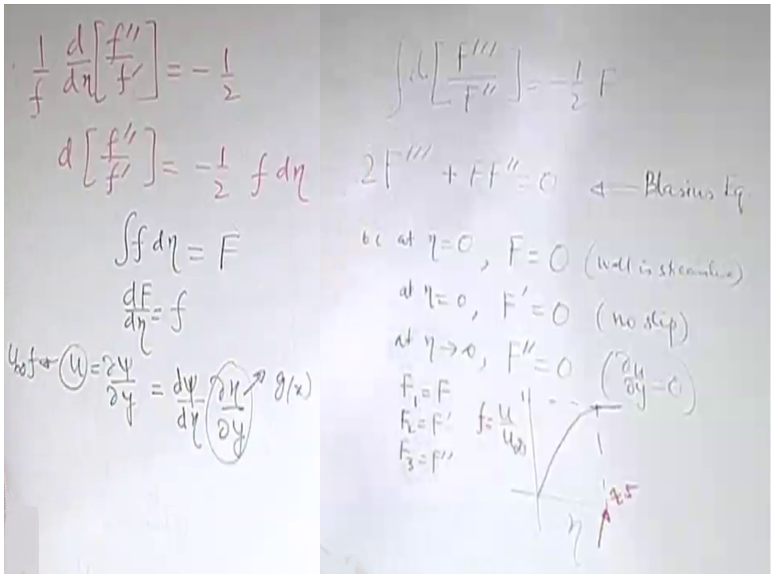

So let us do that, so you have d/d eta of f double dash/f*1/f=-1/2 sorry f dash, then d of f double dash/f dash is equal to-1/2 f d eta. So in principle it is an integral form, only thing is you do not know explicitly as the function of eta, but the form is separable and integral. Now to make it a bit more convenient, let us say that we say that integral of f d eta =capital F, just because this is sort of an integral form, we want to convert into to a new variable which will give us the convenient form where as if this integral is already there.

Question is what could be physically this f? So we can write df/d eta=small f, see this f includes also constant of integration, see because when you integrate this there will be a constant of integration, but the constant when differentiated will give back the same thing. So we will not explicitly use any constant of integration, when we integrate this constant of integration is inbuilt with this capital F that we have to understand or keep in mind.

So if you see that f is velocity, this is another spatial derivative of velocity, so it is equivalently like a stream function. So let us try to see, let us recall the definition of stream function, this is a 2 dimensional incompressible flow, so stream function definition is valid. So u=del psi/del y right, so you can write this as say a if u is a function of eta only, then stream function will also be eta in that way or u/u infinity is a function of eta only, then this d psi/d eta*this one.

So this is like g x, but most important thing is that when you write u d eta, so u is what? u is u infinity*f. So when you write f d eta by separating variables here, then whatever you get as d of that that is the equivalent of a stream function with a of course of multiplying factor, that means capital F has a significance of a stream function, in a transformed manner but it has the significance of a stream function.

So basically as if we have eliminated v by using the stream function, so keeping that in mind let us complete the description of the equation. So you have d of small f double dash means capital F triple dash/capital F double dash=, that means 2F triple dash +F F double dash=0 right, this equation is known as Blasius equation. This is an ordinary differential equation, but it is not a simple linear ordinary differential equation with which most of you are very, very familiar.

So this does not have any analytical solution, but this may be solved numerically by many ways, and we will not go into those the numerical techniques of solving this equation, this not a course on numerical analysis. But I mean there are straightforward ways, for example it is possible to decompose this equation into 3 coupled ordinary differential equation of first order, these are third order, you may decompose this into 3 coupled first order equations.

And then for each of those you may use a technique known as 4th order Runge-kutta method or any Runge-kutta method is fine, so long as you can cast this in the form of an initial value problem. We will see that what are the boundary conditions first let us see, and we will find out whether it is possible to cast it in an initial value problem, so what are the boundary conditions, this is a third order equation, it will require 3 boundary conditions.

So boundary conditions first at eta=0, eta=0 is what? Eta =0 is the surface of the plate, what is the value of F capital F? Now you remember that capital F is having the significance of a stream function, along a streamline the value of stream function is a constant, and the surface of a solid boundary is always like a streamline, why? Because there is no flow across it. Streamlines are imaginary lines in the flow field where there is no flow across it.

So the solid boundary here the flat plate is like a streamline, question is when it is like a streamline, what is the value of the stream function? That is up to you, because stream function is the relative quantity, see you have u=del psi/del y, so with psi and psi +C by both of this u is satisfied, your basic variable is u, stream function is just a mathematical way of looking into it. So any constant value of stream function that you can choose at the solid boundary.

Standard convention is we choose the constant as 0, obviously that is the most simple for calculations. So we chose this=0, fundamentally this is any constant, but the choice is arbitrary because it does not depend on the difference, also at the wall you have no slip foundry condition. So no slip boundary condition means u=0 that is small f=0, that means d capital F/d eta=0, so at eta=0 you have f dash=0 capital F dash, this is no slip, this is wall streamline.

And what else, when you go at a distance theoretically infinity from the solid boundary, what do you face you come up with a situation when there is no further gradient of you with respect to y, that means small f is a constant that means d capital F/d eta is a constant, that means d to d capital F/d eta is 0, that means as eta tends to infinity F double dash=0, that is basically okay. So it is a well posed mathematical problem.

And if you are write, if you decompose this into 3 first order equations, let us say you will have 3 variables say F1 =F, F2 is F dash and F3 is F double dash. See these are like initial value problems for F1 and F2, because at eta=0 you know the values of the variable, so it is like as if eta is like a time coordinate that is you have at time=0, you march with time and get the solution instead of time it is the eta as the variable.

For the third one, you see you do not know it as eta=0, but you know as eta tends to infinity, so that is not a sort of initial value problem where you know the value at 0, you know at value at the other end. So what you may do you may guess with the value at eta=0, so you guess with what is the value of F double dash at eta=0, and solve the 3 coupled initial value problems by some method say Runge-kutta method.

Once you solve that you will see that at the other end, when you go to infinity of course numerically you cannot treat infinity, how will you get infinity you will just see that beyond some eta there is no further change that numerical value is like infinity for you. So infinity is what? Infinity is what that does not understand the effect of the plate, for us infinity is anything >delta that does not understand the effect of the plate.

So once you get the value of F double dash at that end, you will see that you will not get this=0, because you started with a guess, but you will see that it will be somewhat deviated from 0 more accurate your guess was more close it will be to 0. So what you do is you have a guess you get something as non 0 say it is shifted to one side of 0, you make a different guess, it will be shifted to other side of 0.

Then interpolate with a new guess which is the sort of in between these 2 until and unless your guess is such that it matches with the boundary condition at infinity, this is known as shooting method, that means you have a target and you are having a shot of a bullet which you want to shoot the target, the target is this boundary condition. So you are shooting with different initial conditions, till you come back with an iterated solution, where you are really shooting the target you are getting the condition satisfied at infinity.

So this is known as shooting method, so once you use a shooting method the advantage is by this method you make convert a boundary value problem into an equivalent initial value problem. So you will get 3 coupled initial value problems from these 3, and then you may use the well-known solvers of initial value problems, there are many standard solvers of initial value problems, and you may also write your own simple code and solve this.

So if this is solved numerically how it looks? So if you find out capital F, you made differentiate it once with respect to eta to get small f that is your velocity profile, so we will get say you are plotting u/u infinity as a function of say, other way let us plot say u/u infinity as a function of say eta. So if we plot these the general behaviour is like that it becomes=1 beyond this of course it does not change further.

And we may therefore conclude that whatever is this value of eta this corresponds to the location of the boundary layer right, because the velocity has become almost the free stream velocity, so this is f so this is f versus eta plot. And this value is roughly 5, if you do it do numerically.

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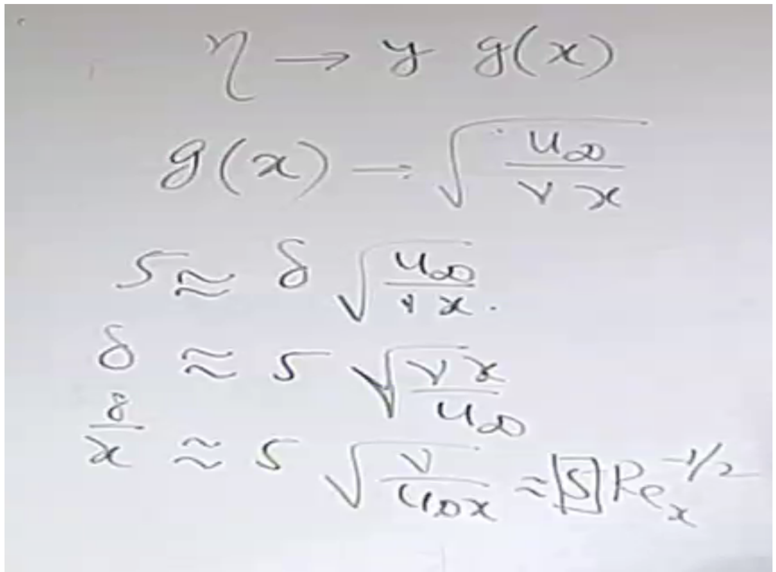

So what we may conclude out of this, what was this g x or what was this eta? Eta is y*g x, so what was g x? 1/ no no in terms of x, square root of nu infinity nu x/u infinity, u infinity/nu x because you have to keep in mind it is scaling with 1/delta. So whatever it is 1/square root of x should be there, so there are such see this is one thing like for all the exams you tend to mug up, there is nothing wrong with it, but you get a lot of pressure in mugging.

The problem is there is nothing wrong, if you intelligently mug up, so when you intelligently now you see I mean you are just chanting it liker mantra that g x is this, g x is this, keep in mind that it scales with 1/delta, and then you need not mug up, and then just a simple physical 1 or 2 line of work will give you what would be the scaling variation of g x. So that means when eta=5 say y=delta roughly, so 5 is approximately delta*root over u infinity/nu x.

So if you want to find out that what is delta/x, you have to, what you have to do? So delta is approximately 5*square root of nu x/u infinity, so delta/x is 5 square root of nu/u infinity x, that means 5*Reynolds number to the power-1/2. See by order of magnitude we estimated up to

Reynolds number to the power-1/2, only this 5 multiplier is something what we get from the solution. So with a lot of toil, lot of chance of doing bad algebra and all this.

We came up with something which is important, but more qualitative interesting and important things we could get from the order of magnitude in a really very simple way. Now of course if you know, what is this delta as a function of x, you can calculate the other things. Next what we will do is see this is a method of solution which was never liked by primitive engineers, because engineers never wanted to look into non-linear differential equations.

And they found that well in practice it will make our life much more tough, of course like we should not say that engineers are dull sets of people, all of us are engineers but at the end what happened is that people look for some approximate solutions, the whole idea of the approximate solution was like this, that I do not care how delta varies with x, for me as an engineer what is the most important conclusion is how wall shear stress varies with x, because that comes directly to my design.

As a designer, I do not want to understand delta, of course this a very bad approach. But the thing is that it may be a working approach for an engineer, who does not want to go into mathematics, and sometimes in practice that is okay, so when that is the objective, the objective is somehow we could come up with an approximate analysis where we avoid solving such equations, but come up with a reasonably good estimation of wall shear stress.

But the basic of that method is still mathematically very rigorous, the application is not approximate and that rigorous method we look into in a very careful manner that is known as momentum integral method.

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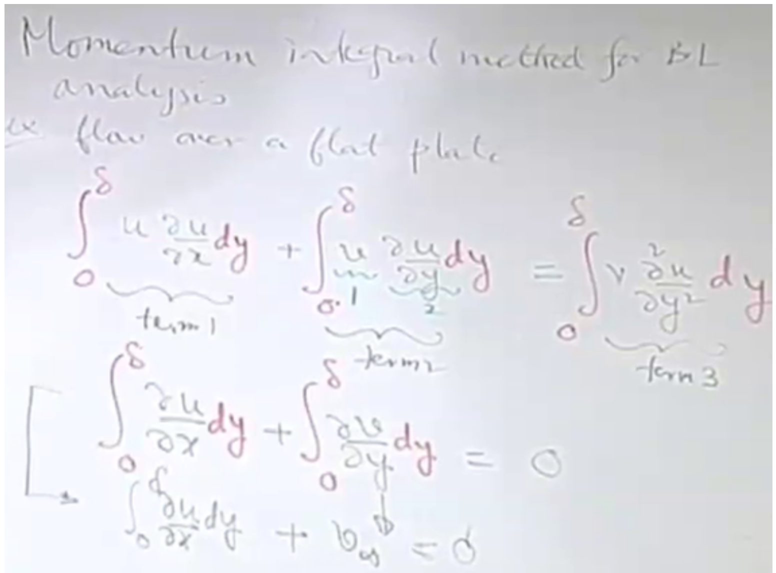

For boundary layer analysis we will take the example of flow over a flat plate, let us understand what is the essence of the method, essence of the method is again like we start with the boundary layer equation. Let us say we start with the momentum equation, see, what is the method? The keyword is the name of the method, momentum integral momentum equation, integral means it has to be integrated.

So what we are basically trying to do in this method is we are integrating the momentum equation over something, what is that something? That something is across the boundary layer, so we are integrating this with respect to y all the terms with the range of y from y=0 to y=delta okay. Now when you do that let us see that what simplification we get out of this, first of all let us consider this term, so let us call this as term 1, term 2 and term 3.

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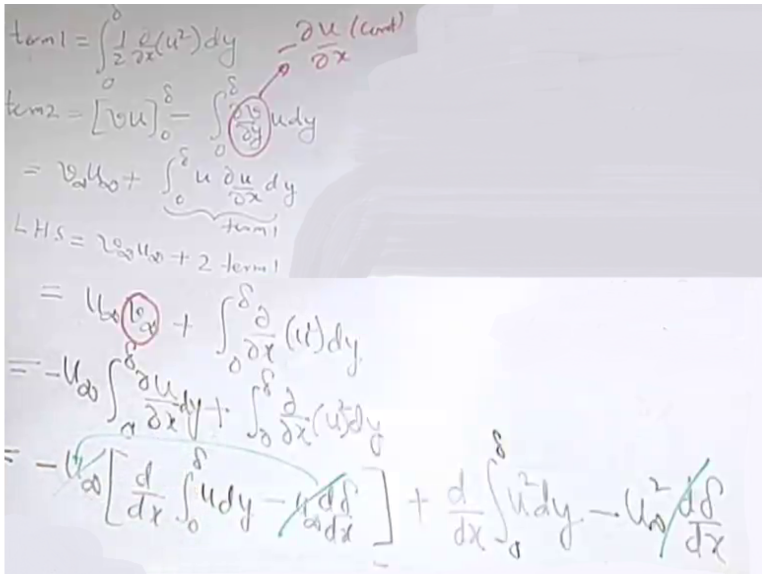

Let us write separately terms 1, 2 and 3. So term 1 this one right. Term 2, for term 2 let us integrate by parts, because you have already a du/dy type of thing, when it is integrated it will give you u, so we integrated it by parts by considering these are the first function and these are the second function okay. So the term 2 will become first function*integral of second-integral of derivative of first*integral of the second right.

So in terms of the first see it is v at delta*u at delta, so what let us say v at delta is v infinity that is what symbol we have used *u at delta is u infinity-v at 0*u at 0, both are 0 v at 0 because of 0 because of no penetration, u at 0 because of no slip, -now you may use the continuity equation and write this as -of del u/del x by continuity. So this becomes this +integral of 0 to delta, so this is term 1 right. So the left hand side becomes v infinity u infinity+2*term 1.

So u infinity*v infinity +this one, what is that we do not know in this equation, we do not know what is v infinity, to know that let us use the continuity equation, because continuity equation will relate v with u. So next what we do is we write the continuity equation, and integrate it across the boundary layer just as we did for the momentum equation. So let us write the integrals of the continuity equation, this term we do not disturb we just write it as.

This term what we do? This will be v at delta-v at 0, so this is v infinity, so we have got an expression for v infinity that will substitute here. So this will be u infinity*v infinity that is u infinity so-infinity integral 0 to delta okay, so at least we have eliminated v sorry, at least we have eliminated v. But another important objective which we want to satisfy, and before that we ask ourselves a question that can we take in partial derivative with respect to x outside the integral.

We have to see then the general rule by which you may flexibly take the differentiation from outside the integral to inside the integral or inside the integral to outside, and for that the rule of mathematics is the Leibnitz rule. So let us consider that Leibnitz rule.

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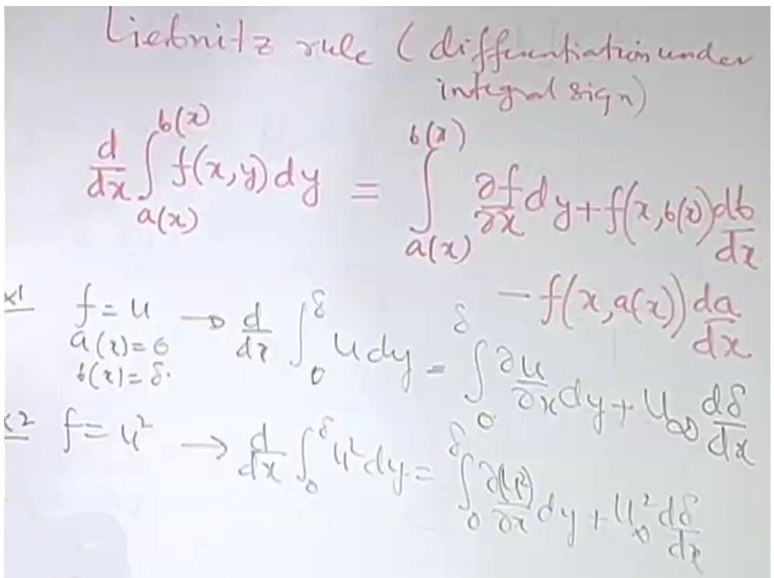

Leibnitz rule is for differentiation under integral sign, what is this if you have a function say f, say d/dx of integral f x, y dy where the limits of integration are one function of x to another function of x. This is the Leibnitz rule okay, so you can clearly see that if the limits of integration are not functions of x, then you may easily keep the derivative inside and outside without any problem, but here what are the limits of integration?

See there is a limit of integration delta which is itself a function of x. Therefore, let us try to apply this say in one case our function of concern is u squared, another case is u. So let us take an example, example 1 say f=u, so you have d/dx of f=u a x=0 and b x=delta okay. So you have d/dx of integral 0 to delta u dy=0 to delta +f x delta okay, so f is u, u at x and delta what is that? So the function is substituted at x=x and y=delta.

So what does it become? u infinity*d delta/dx - it is 0 because a is 0, so that is fine. If you take a second example as f =u square, then similarly, you will get this as d/dx of 0 to delta u square dy +u infinity square d delta/dx. So let us substitute that here, so if you substitute that here this is -u infinity*this one is d/dx of 0 to delta u dy-u infinity d delta/dx that is for the first term. For the second term -u infinity square d delta/dx right.

Then you can clearly see that this u infinity square d delta/dx I mean this one combined, and this time they get cancelled. So the left hand side becomes d/dx of u square-u*u infinity dy that is the left hand side. What is the right hand side? Right hand side is just one line let us just write the right hand side.

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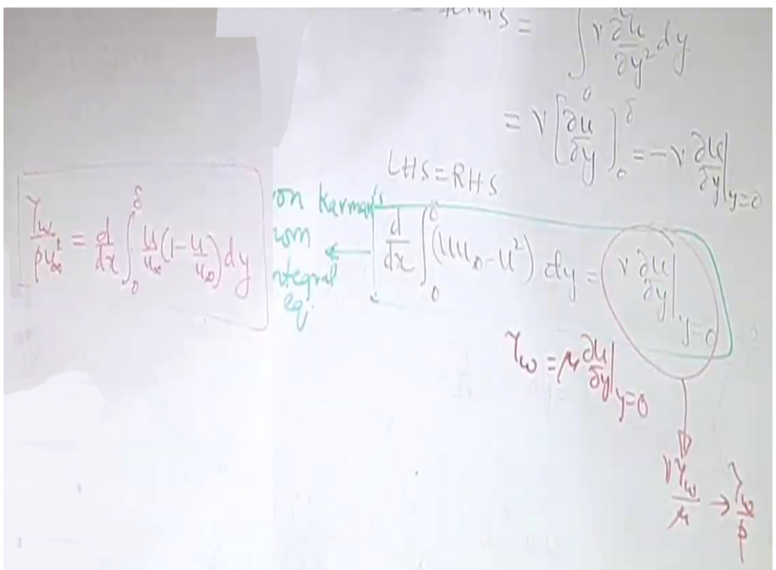

So the right hand side is that is the term 3 that is integral of nu okay, nu is a constant that we have assumed these properties are constant, so this is from 0 to delta. What is du/dy at delta? 0 because u does not vary with y, so this becomes -nu del u/del y at y=0. So the left hand side=the right hand side, that means we can write d/dx of say integral 0 to delta u*u infinity-u square dy=nu del u/del y at y=0.

This equation was first derived by Von Karman and that is why the name of this is known as Von Karman's momentum integral equation. So this is basically an integral form of the momentum equation when integrated within the boundary layer or across the boundary layer. What is the advantage that it is giving? Let us just look into it briefly, so you know that, what is the objective of us to calculate the wall shear stress?

What is wall shear stress? That is mu del u/del y at y=0, that means the right hand side we may write as nu*tau wall/mu, so tau wall/rho. If you non-dimensionalize tau wall with rho u infinity square, because that is what we commonly do, then just let us write one more step for that equation. So tau wall/rho u infinity square =d/dx of integrals 0 to delta u/u infinity*1-u/u infinity*dy.

So this is a very useful engineering expression or a mathematical expression with no approximation. See where will approximation come? This is perfect approximation will come, when you substitute any approximate velocity profile. So if you do not know a velocity profile you may still make a guess of u/u infinity, do this integration and that will sort of give you a relationship between tau wall and delta.

So if you know how delta varies with x, you will get variation of tau wall with x. So that depends on the substitution of an approximate velocity profile. So how to do that that means how to make the use of the momentum integral equation to get an expression estimation for tau wall and say even delta approximately, that we will take up in the next class, thank you.