Introduction to Fluid Mechanics and Fluid Engineering
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Lecture - 37
Boundary Layer Theory

We will start discussing on the boundary layer theory today. And before going into the theory of the boundary layers, let us briefly recapitulate, what is the boundary layer? We introduced this concept qualitatively in one of our introductory lectures, and just to take it up from there we take this example.

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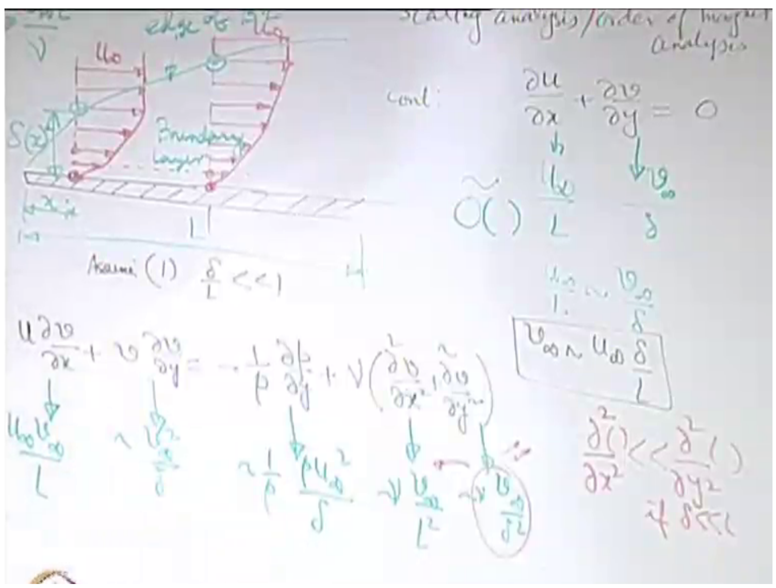

That you have a flat plate, the plate is confronting a free stream of fluid with a velocity of say u infinity, now when this fluid is coming in contact with the plate, what is happening that is what we are interested to see. Clearly, when we consider say one section, and we go further and further away from the plate, the physical phenomena tend to get changed. How it tends to get changed?

Adjacent to the wall say if you consider a fluid element which is adhering to the wall just as marked in this figure, this fluid element is stationary relative to be plate by the no slip boundary condition. As you go little bit away from the plate on the same vertical section, you see that you may have fluid elements which are subjected to the slowing down effect of the wall, but not as severely as the wall aerating fluid layer.

Because this fluid element is somewhat away from the wall, so on the top of it, it has a faster moving layer, and at the bottom of it, it has a slower moving layer. So the net effect is that it is slowed down, but not to that extent as the fluid element adhering to the wall, so it will have some sort of a velocity. Now if you go further away, then you will find that there are fluid elements which are further away from the wall.

So they are slowed down by the wall but not explicitly but implicitly, how implicitly they have a layer at the bottom which is moving at a slower pace and which is closer to the wall, then the fluid elements which are above that and which are moving faster. So as a consequence as a competing consequence what happens is this fluid layer tends to move at a velocity which is somewhat larger than the velocity of the layers which are there at the bottom.

In this way, the velocity increases till one may reach a point where we reach a point is a question that we will see that one maybe points somewhere where the velocity is almost equal to u infinity. And let us say that we are happy with the location where it is 99% of u infinity, it is as good as u infinity, and beyond that the velocity virtually does not change, beyond that the velocity remains almost the same.

So if you join the tips of the velocity vectors at this location, this is what you get as the velocity profile at this location. These are certain terminologies that we introduced earlier, and we could also qualitatively understand that despite not being in contact the plate, you will see that somehow there are fluid elements which are feeling the effect of the plate. The fluid element directly in contact with the plate is supposed to feel the effect of the plate.

Others are not directly in contact with the plate, but to some extent within a distance from the wall they are feeling the effect of the wall. And how they are feeling the effect of the wall? It is through the fluid property viscosity, which is propagating the message of the wall from the wall towards the outer fluids. Now if we draw the same thing that is the velocity profile description at a section further away from the inlet, then what we see here the velocity is 0.

Then in the vertical sections as you move above, what you see? The qualitative picture is same as what is there in the first section, but quantitatively what you expect that at this layer the velocity should be even < what it was in the corresponding previous section. The reason is very, very obvious that now more and more fluid is in contact with the plate, so the effect of slowing down by the plate is more and more severe.

So you have velocities which are less at a given vertical section location, in this way here also the u infinity condition will be reached, but the distance over which it will reach will be somewhat more than the distance over which it was reached in the previous section. Let us say somewhere here the u infinity condition is reached, beyond which the velocity remains the same. So you can clearly see that if we want to demarcate the characteristics of the velocity profile we are having 2 important characteristics.

One characteristic is where the velocity profile has it sort of a gradient, and another characteristic is that the gradient cease to exist. So if we demarcate these 2, one way of demarcating is to find out, to what distance this velocity profile exists, let us say that to this distance the velocity profile exists for this section, and for the next section maybe this distance the velocity profile exists that means gradient in the velocity profile exist.

So in this way the plate maybe as large as you want along the axial direction, but like these 2 representative's sections are good enough to have a sort of a physical picture. So if you want to figure out that what happens subsequently of course this physically this type of behaviour, it continues along the plate, so as if like this effect of the wall gets more and more propagated. Now it is possible therefore, to have a region within which the velocity profile has a strong gradient, so that the effect of the wall is felt very explicitly.

And a region which as if does not understand the effect of the wall, and to demarcate that one may just draw a locus of this mark points, because this indicate the distance from the wall up to which the viscous effects are explicitly felt. So if you draw the locus of such thing this gives the imaginary line, this line is not something is which is existing in the flow field, it is just for our conceptualization that we are having such a demarcating boundary.

So this demarcating boundary demarcates the flow domain into 2 parts, 1 part adjacent to the wall where wall effects are important, or so to say the slowing down effect of the wall or the viscous effects are important, and outside this viscous effects are not important. It does not mean that the fluid outside this has no viscosity, simply the effect of viscosity is not manifested in the form of shear stresses, because of lack of existence of velocity gradients.

So within this layer, the viscous effects are sustaining shear stresses, and transmitting the disturbance of momentum from one layer to the other, so that is what is called as momentum transfer. So the layer where this physical behaviour is occurring is known as boundary layer, and outside the boundary layer is in this case is the behaviour of the free stream, and this line therefore, is a demarcation between the boundary layer and the region outside the boundary layer, and this is known as edge of the boundary layer.

So boundary layer is just a concept which helps us in demarcating the flow domain into parts where the path that we are focusing on that the boundary layer is an interesting part, because that is where the viscous effect is explicitly felt. And the entire purpose of the boundary layer theory is to have a detailed understanding of what happens in this layer, so the whole objective is there is a scientific objective, scientific objective is we must understand.

So if say if we call that this is x direction, and the transverse direction is y direction then at a given x, there is a thickness of the boundary layer which we call as delta. So we see qualitatively that as x increases delta also increases, but the question is how thick or how thin the delta is, so that is one of the very interesting things. Because that will dictate us that what is the extent within which this velocity gradient exists.

From an engineering point of view what is important? From an engineering point of view what is important is what is the total shear force or the shear stress that is acting on the wall, and to know that you must have a detailed picture of how the velocity gradient are shaping up within the boundary layer. So from an engineering point of view the shear stress is important why? Because it will give rise to your drag force.

And based on the drag force one may intelligently design engineering systems, where you have interactions between fluids and solids, and not only that it is also possible to have a clear idea of what is the extent over which the wall shear stress effects are predominant. So for all these reasons it is important that we emphasize on the characteristics of flow within the boundary layer, and the studying of the boundary layer theory has one of those things as important objectives.

So we start with the boundary layer theory by considering simple case, and we will adhere to that simple case in a major part of this chapter that is we will consider the density=constant, and we will consider steady flow, not that there is no relevance of boundary layer theory for variable density or unsteady flows, yes they are definitely. But for an introductory course this is what is going to give us a lot of insight on the very, very fundamentals of the theory.

So as usual what we will consider we will consider that our basic governing equations which we have developed, these governing equations will be the basis with which we will start the analysis. So what are the governing equations, first the continuity equations is there, so we will not write it in the index notations is just a 2-dimensional description, so you will consider a 2-D flow also with velocity component as u and v.

So u is u1, v is u2, so we will write the continuity equation under these assumptions you have. So let us write the continuity, this is the continuity equation, we will write the momentum equation subsequently, but let us first start looking into the continuity equation. Now when you look into the continuity equation, what we are trying to guess from the continuity equation, first of all we will try to have a picture on the order of magnitudes of different quantities.

That means what is the order of magnitude of u? And what is the order of magnitude of v? This is very important because before getting into the exact quantification order of magnitude will give us an idea of the range of values that this may take. So for that one important style of analysis is known as order of magnitude analysis or scaling analysis, so we will look into that scaling analysis or order of magnitude analysis.

This is a very powerful tool in scientific analysis because we will soon see that without solving the equations, we will have a fair idea of the order of magnitudes of various quantities which are governing the physical behaviour. So how we do that? To do that we basically refer to the governing equations, and we refer to the characteristic values of various parameters appearing in the governing equations. So what are the characteristic values?

The characteristic values are called as so called scales, so when we have say this du/dx type of term, we are looking for so this term if you write in terms of an order of magnitude, order of magnitude is given by this type of a symbol, this one or maybe order of either way. So order of this one is what order of this one is order of u/order of x so to say, so order of u means what is the maximum u so to say, and order of x is, what is the maximum x?

So first of all x is like it is apparently mathematically it is unbounded towards a particular direction, and these type of problem is physically like a marching problem that means you start marching along the plate as you march further and further you see that the boundary layer is growing and growing. So if the plate was at infinity maybe it would have grown, may not be mathematically exactly in the same fashion.

Because maybe initially it is laminar but after some distance the boundary layer may become turbulent, because the Reynolds number here is defined on the basis of the characteristic length scale which is the axial length scale. As you go further and further, as your axial length scale becomes more and more there may be a place where the location where the Reynolds number is so high that slight disturbance will trigger a turbulence.

So qualitatively the boundary layer will grow but quantitative manner in which it will grow will change, but whatever it is here we are focusing on the laminar boundary layer, for turbulent boundary layer we will separately have some consideration although not in details, but at least we will have some consideration. But irrespective of the way in which the boundary layer grows, the growth of the boundary layer will be dictated in a strong way by what is the total length of the plate.

Because the characteristic Reynolds number here that you are looking for a boundary layer description is this one, rho the characteristic velocity u infinity*L/mu or in terms of the kinematic viscosity u infinity*L/nu, where nu is the kinematics viscosity. Therefore, the characteristic length which dictates the problem is the length of the plate here as an example, for internal flows like flow through pipes and channels.

What we have seen is that it is like say if you have a parallel plate channel is the distance between the plates or the depth that is the important characteristic length scale, whereas here it is the axial distance. So for a flow through a channel or a pipe the axial length is not an important characteristic length, but here the axial length is an important characteristic length. So the scale of x is the maximum x and that is L, what is the scale of u? u infinity.

So the scale of the first term is u infinity/L okay, what is the scale of the second term see it again requires the knowledge of the scale of v and scale of y. What is the scale of v? We do not know directly, but let us say that the scale of v, we may estimate that scale of v is the maximum v and which exists maybe at so-called at the edge of the or outside edge of the boundary layer. Let us give it a name some names of say v infinity.

There is nothing as such physically v infinity, but just for a nomenclature say v infinity. And what is the scale of y delta right. Now look at the governing equation there are 2 terms, these 2 terms are somehow adding an effect that the net result is 0 that means each of these terms should be of same order of magnitude, and one term would be of opposite sense than the other. So that they get nullified, what we mean by this?

So if this is like 10 meter per second/1 meter, this cannot be 1 meter per second/1 meter, because then these 2 cannot cancel each other, to cancel each other they must have the same order of magnitude, and if they cannot and cancel each other, then the result cannot be 0. So when we talk about order of magnitude, see this is what we are looking for is the velocity 1 meter per second order or 10 meter per second order or 100 meter per second order.

If we say 1 meter per second order we do not literally mean that it has to be 1 meter per second, it could be 1 meter per second, 2 meter per second, 3 meter per second like that. But order of magnitude wise it is like a sort of a single digit meter per second something like that, if we express it in a single digit. So if it is order of 10 meter per second it might be say with anything between 10 to 99 meter per second.

Of course there is a fuzziness, what is the fuzziness? If it is 99 meter per second, I would say that it is much closer to 100 meter per second than 10 meter per second. So these kind of fuzziness exists, but the important thing is it is your perception of how you get rid of the fuzziness and get a correct description of the values. The fuzziness that we found is because of the use of the decimal system, so it has nothing to do with the order of magnitude analysis.

It has something to do with our way of expressing the order of magnitude, so if we do not have any specific bias towards the system of representing the order of magnitude, at least what we can say from this relationship that you must have u infinity/ L of the order of v infinity/delta, so that these 2 terms somehow nullify their effect, and from here we get v infinity of the order of u infinity*delta/L okay.

So when we have this we should try to have an estimate of the relative estimate of v scale with respect to u scale, and the we are saying that what is the parameter that is governing it is delta/L. So we will we may have all sorts of possibilities, so in a boundary layer you may have delta >>L delta <<L, even delta of the order of L all those are plausible boundary layers, but the theory that we are going to develop we will develop for a special case.

And we will see that what is the relevance of the special case, the special case is delta/L is very, very small that is one of the important assumptions of the boundary layer theory. But again see as if we are forcefully doing delta/L is <<1, when there is a physical problems say you are a person working in an industry you are always entitled to say that I do not understand what is delta, so delta/L is <<1.

What is this I mean how from the system parameters can I tell that whether delta/L is <<1 or not. So there must be something from the system description system parameter description that should tell us whether delta/L is larger or smaller or whatever, but we will now go through an exercise which will tell us how the system parameters will be linked with the smallness or largeness of the delta/L.

So the important assumptions, the first assumption that we are making is that delta/L is <<1 that is small okay. Now with this understanding, let us go to the momentum equation, before going into the momentum equation along the x direction which is the dominating effect here, we will first look into the momentum equation along the y direction. So y momentum equation, since density is a constant we will divide both the sides of the momentum equation by the density.

So that the viscosity will be converted to kinematics viscosity, so we will have again we are not considering an effect of body force, but if there is somebody force that may be added. Now our job is to estimate the order of magnitudes of various terms, let us consider term by term this term first term, what is the order of magnitude of this? So order of magnitude of u is u infinity v is v infinity/L, this one v infinity square/delta, this one 1/rho see this is some pressure scale by some length scale.

So the question is what is the appropriate pressure scale that we do not know, so by lot of experiments it was understood that in such cases where delta/L is <<1, the pressure scale is governed by the kinetic energy scale. So that is almost like an inviscid flow, so that means the pressure scale if we call as delta p, then delta p is of the order of rho*u infinity square, whether this scale is correct or not we will justify at the end.

That whatever scale we are assuming maybe experiments give that, but the question is theoretically it has to be consistent with the remaining part, we will do that but for the time being we assume that the pressure scale is governed by this kinetic energy scale. So rho u infinity square, see when we wrote the scale we did not care whether we write 1/2 rho infinity square or rho infinity square, because objective of the scale is not to give the exact value, it is the order of magnitude.

So with the 1/2 order of magnitude does not change, so rho u infinity square/delta. These terms say the first term nu, what is this you tell the first term v infinity/del square right, and this term is nu v infinity/delta square, all are order of magnitude. Because all these terms are like d/dx or dv/dx, d/dy or dv/dy like that. Now without doing any analysis we can straight forward say that if we assume delta/L is <<1 that is delta <<L, then this term is much much more dominating then this term.

Because here division by delta square at division by L square that means in the first cut analysis without thinking about anything else, whenever delta/L is <<1, the first term here is much much more insignificant as compared to the second term and therefore, may be neglected. So that is see it does not matter, whether it is v or u or whatever, it is the denominator that is what is creating the difference.

And therefore, we may have a general conclusion that no matter of whatever variable if delta/L is small that is delta << L. And see these terms are like the first term is axial diffusion of momentum disturbance physically, and the second term is transverse diffusion of momentum disturbance. So we will say that the axial diffusion term is negligible as compared to the transverse diffusion term that is the way in which we speak this in words.

Now so that is one of the important things we will keep in mind not just for this equation, but for even the x momentum equation we will apply the same logic. So at least we could get rid of one of the terms in terms of the analysis, that well this term will not be important so long as the other term is there. Now let us try to make an assessment of the order of magnitudes of the different terms in the left hand side, and the important terms in the right hand side.

First of all, you may estimate v infinity with u infinity*delta/L, here also like that u infinity*delta/L. So what you come up with is the order of magnitude of these terms eventually becomes u infinity square delta/L square, and even this term is also like that right okay. So we may conclude see order of magnitude of 2 terms if they are the same, adding then the order of magnitude becomes either of them not that you add so if it becomes 2 it makes no difference, 2 does not change the order of magnitude.

So the left hand side order of magnitude is u infinity square delta/L square, and what is the physical meaning of the left hand side, these are like inertia terms, so these are like because these are like the if you recall, these are nothing but the advective components of acceleration or convective components of acceleration. So this like represent the inertia of the flow.

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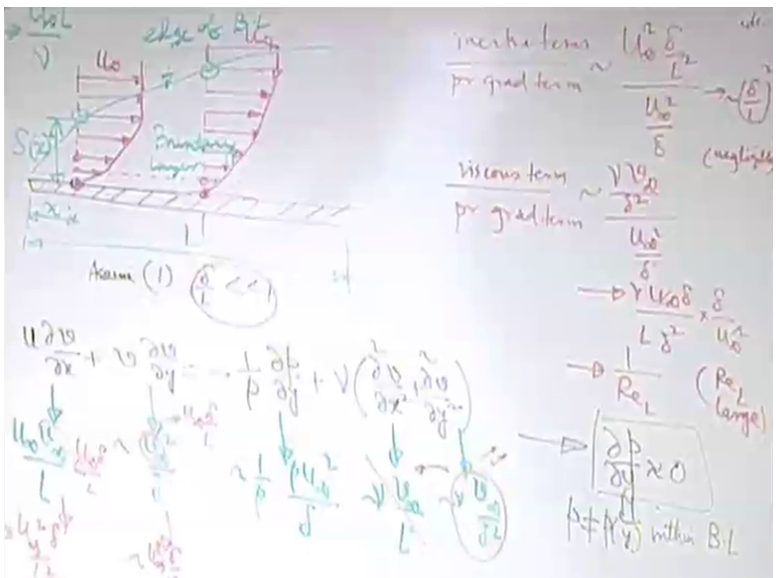

Keeping that in mind let us write the ratio of the inertia terms and the pressure gradient term order of magnitude, this is of the order of inertia term is of the order of u infinity square delta/L square, and pressure gradient term of the order of u infinity square/delta. So this becomes of the order of delta/L whole square right. Therefore, if delta is <<1 see so many simplifications are possible just by this consideration, so this becomes one of the very key considerations.

That means this is a negligible term. So the first conclusion is there in the y momentum equation, the inertia terms are negligible in comparison to the pressure gradient term. Then let us consider the ratio of the dominant viscous term by the pressure gradient, so that is nu*v infinity/delta square/u infinity square/delta, so what is this? v in place of v infinity we will write u infinity*delta/L nu u infinity delta/L delta square*delta/u infinity square.

So that becomes delta/ sorry delta is gone 1 by Reynolds number right, so 1/ReL. So we will also consider that the Reynolds number is large, and we will see very interestingly that is not an additional consideration it should follow from the consistency of this assumption, but for the time being we will assume that because we may only show from the x momentum equations the relationship between delta/L and the Reynolds number.

So one of the important consequences of considering the Reynolds number small, Reynolds number large will be delta/L small, so delta/L small this automatically implies that the Reynolds number is large, so for the time being let us consider it as an independent assumption, we will not assume that this is the truth. Because we will show that this is the truth or if dealt/L is <<1, so for the time being it is as if another independent assumption.

We will see subsequently that is not an independent assumption, it is relationship with the consideration of delta/L. See I am not just now I have told that we will show that there is a relationship between large Reynolds number and delta/L is small, so we have to wait till that right. Now when Renault number is large, so that means that the viscous term by the pressure gradient term is small that means whatever will be dominating in this equation if at all something dominates is only the pressure gradient term.

So from this the important conclusion is that within the boundary layer, so all this order of magnitude estimates we are writing within the boundary layer, so now as if we were zoom or the focused attention is the boundary layer, so then that will give rise to this=0 right, because all other terms are non-dominating. And therefore, this gives rise to a very important conclusion, what is the important conclusion? Pressure is not a function of y within the boundary layer.

What are the considerations that we used for this? The considerations that we used for this are delta/L <<1 and Reynolds number large, these are the 2 considerations that we used. Now one of the important things that we should mention at this stage is that this x and y coordinates are generic, for flow over a flat plate you have natural choice of x and y that means plate is a flat one, so along it you can orient a linear x direction and perpendicular to that y direction.

But if you have flow over a curved boundary, so if you have say flow over a boundary like this, so then x is written in terms of the curve linear x, and maybe local co-ordinate which is orthogonal to that, so the curve linear one will become like a tangential co-ordinate x at a given point, that means it is as if like a tangential x and our normal y at each and every point. So the x and y direction continuously may shift as you are moving along the curve.

So the x and y are generically called as stream wise directions and cross stream wise directions something like that, so these are changing continuously. So in the boundary layer we use x and y co-ordinate for all the cases not just for flow over a flat plate, flow over a flat plate we just give as an example, so it is not the only case where the boundary layer theory will be relevant as we understand.

Because many of the surface over which flows occurred in engineering are not like flat surfaces, so you may have curved surfaces like, so you may have wings of airfoils and so on. Obviously, if you have any curved surface it is possible to have sort of different way of describing the coordinates, but this x and y co-ordinates will preserve in the sense that we have just discussed, and this we call as boundary layer coordinates.

So boundary layer coordinates are these generic x and generic y, so it is not that any orthogonal x, y you choose or any constant x, y you choose. It has a special meaning that is locally at a given point, if you want to assess what is happening with the boundary layer, then along the surface that means tangential to the surface at that point is x, and perpendicular to that is y, and that may vary from one point to another if the surface is curved.

So from the y momentum equation whatever we get, you have to remember that this y is the generic y co-ordinate that we are talking about. Next we come to the x momentum equation which should hold in some sense the key towards assessment of the behaviour within the boundary layer.

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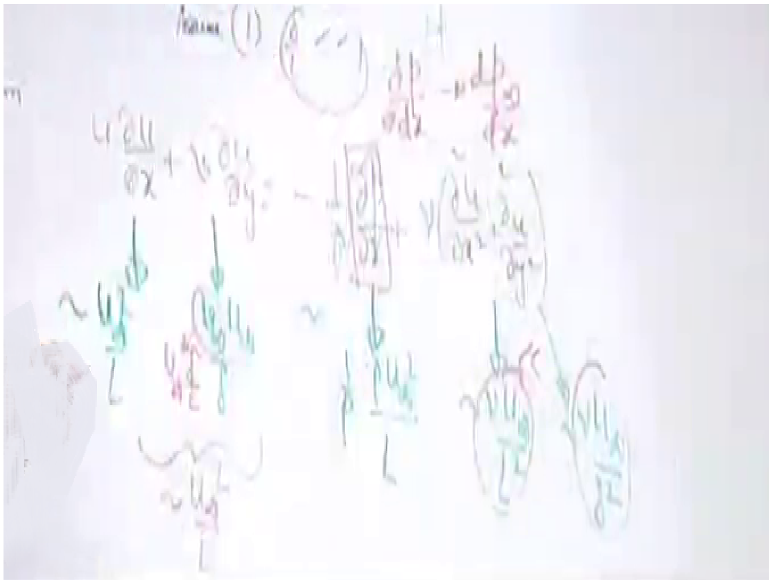

So x momentum equation is that u del u/del x+ v del u/del y-1/rho this one, so let us write the order of magnitude of different terms we have now been familiar in how to write those. So what is the order of magnitude of this u infinity*u infinity/L, so u infinity square/L, this v infinity*u infinity/delta, this one -1/rho the delta p scale that is rho u infinity square/L, this term okay - is not important for order of magnitude, then this term nu u infinity/L square, this term nu u infinity/delta square.

And again just by previous consideration, we know that this term will be << important than this term. Now let us pay a lot of attention to the pressure gradient term, see what conclusion we got from the y momentum equation? The conclusion that we got is that the pressure is not varying with y, and y is the direction along which there is a change in behaviour because of the existence of the boundary layer.

So that means we can say that the pressure variation within the boundary layer is not important, what it means is that whatever is the pressure gradient that is acting on the flow is because of the pressure gradient that is imposed in the outer stream or the free stream by whatever mechanism. And what is the importance of the outer stream or what is the simplification that we have for the outer stream. The outer stream is like an inviscid flow.

So the outer stream is like an inviscid flow that means what? The outer stream is like an inviscid flow that means you may use the equations for inviscid flow for the outer stream. So for the outer stream by using the equations of an inviscid flow, if you find out what is the pressure gradient, the same pressure gradient is imposed on the fluid within the boundary layer. And that means the correct scale of pressure gradient should be rho u infinity square.

Because that comes from the inviscid flow analogy, if you neglect the potential energy part the delta p will be of the order of rho u infinity square. For example, if you use the Euler equation or maybe they Bernoulli's equation when rho is a constant that is what you get. So the important understanding is that this is consistent with the assumption of the pressure scale delta p of rho*u infinity square.

So if these conclusions were not consistent that means that assumption was not correct, so that is the first thing. The second thing is p in this problem could be a function of x and y, now p is not a function of y. Therefore, p is a function of x only, therefore, a very important thing is here we can write this as dp/dx, going one step forward it is dp/dx where it is dp/dx outside the boundary layer.

Because we have just seen physically that the implication of this one implies that whatever is the pressure gradient which is existing outside the boundary layer, this boundary layer fluid is subjected to the same pressure gradient. So that means in terms of symbols we can write this as good as dp infinity/dx where infinity stands for the outer stream that is the notation okay. So that is about the pressure gradient term.

Now we will look into the other terms, so what are other terms? So the other terms you see when you have v infinity here you can substitute the v infinity in terms of u infinity, so this is of the order of u infinity*delta/L, so you can see that combining these terms each of the terms is of the order of u infinity squared/L. So the order of the left hand side which is the inertia term is like u infinity squared/L okay. Now what we will do with these orders, we will very soon see with an example.

But before looking into that example, let us summarize the boundary layer equations. So boundary layer equations are simplified versions of the Navier Stokes equation which we feel that are valid within the boundary layer.

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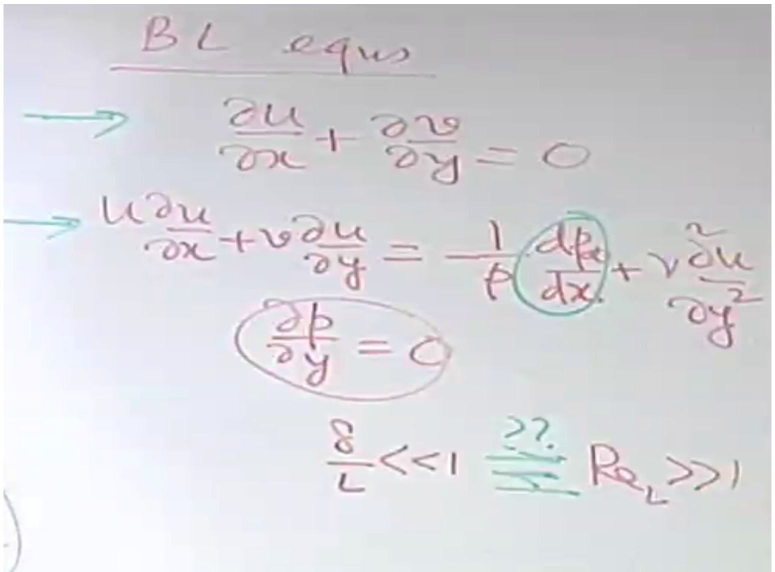

So boundary layer equations that is the summary of the equation, first equation is the continuity equation that is very important that has to be there, again when we are writing the boundary layer equation under the assumption that we have already described, x momentum equation okay. If you feel always write the y momentum equation like this, but it is not always this is ready to explicitly write this, because its effect is already inbuilt in the simplification of the x momentum equation.

So conventionally when people ask that what are the boundary layer equations usually people refer to these 2, not that the y momentum equation is not present it is there but its implication has somehow been inbuilt in the x momentum equation. What are the assumption that we consider for these? other than rho=constant and the steady the special boundary layer assumptions delta/L is <<1 and Reynolds number is large >>1.

And so that now the question that we will like to answer is that are they thought of equivalent does one follow from the other or what they are equivalent to each other or not, that is the question that we would like to answer. When we like to answer the question, before answering the question we have to keep in mind that this sort of relationship should come from the description of the system, and the description of what is happening within the boundary layer.

See the Reynolds number is a system scale description, this does not understand, what is boundary layer and so on, whereas this is related to the boundary layer thickness. So if we sort of describe an equivalent between these 2 we will achieve our first objective that from the system scale variation, we will have an estimate of how thick or how thin the boundary layer maybe. So to do that we will take that the special example of flow over a flat plate.

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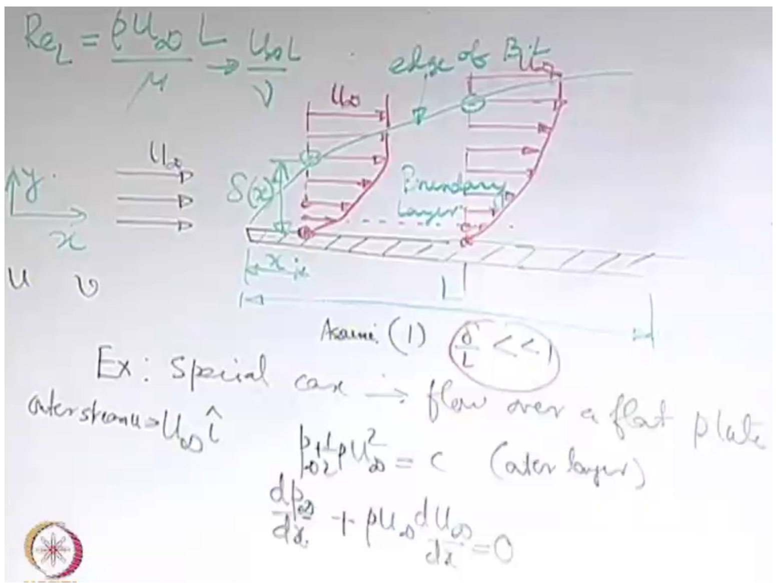

So let us take an example of flow over flat plate, special case flow over a flat plate. First of all, what is our objective is to find out what is this dp infinity/dx okay, let us consider the outer stream. See the outer stream the free stream flow is u infinity, so the velocity field is like u infinity i in the outer stream, the outer stream u is u infinity i, you can clearly understand that this is an irrotational flow, because no gradient in the velocity.

So when this is an irrotational flow, we have discussed earlier that an irrotational flow may remain irrotational if there are no viscous effects. So outside the boundary layer, so this is, what is the importance in conceptualizing the boundary layer, outside the boundary layer viscous effects are not important. And therefore, the behaviour is like an inviscid one, so a flow which was irrotational will remain irrotational eternally, because of this effect.

Of course we forget about other possible effects which might make the flow rotational like the Coriolis force and so on, those are not important in this context. So if you have that as a situation that means outside the boundary layer you have inviscid irrotational flow, so it will be when you have an irrotational flow outside the boundary layer that means if you have also density=constant you may use Bernoulli's equation between any 2 points where the points are located at any location but outside the boundary layer.

Within the boundary layer you cannot use the Bernoulli's equation that you have to clearly understand, but outside the boundary layer you may by considering these cases. So when you use the Bernoulli's equation that means you have p+1/2 rho u infinity square is constant in the outer layer, forget about the potential energy difference that is not important here. If the potential energy difference is important you include that in p and call it as a Piezometric pressure.

Now if you differentiate that with respect to x, so this is p infinity this is p infinity outside the boundary layer, so dp infinity/dx while we are doing it we require that in the x momentum equation. So dp infinity/dx +rho u infinity du infinity/dx=0 right, now we have u infinity as a constant here, it is not changing with x. So u infinity may change with x, because of what? Because of one of the things is the curvature of the boundary.

Because of the curvature of the boundary you may have a gradient of pressure, and you may have a gradient of the free stream velocity, but for a flat plate there is no effect of the curvature. But there is an effect of the curvature if you have flow over a sphere or a cylinder we look into those examples later. So there the curvature will introduce a pressure gradient, and that pressure gradient will imply that there will be also u infinity gradient so to say.

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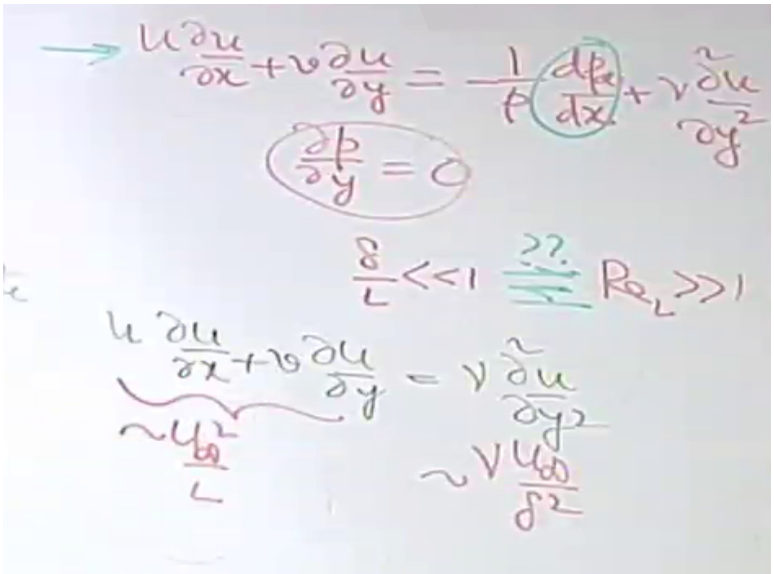

But here you have u infinity gradient is 0, u infinity is constant for flow over a flat plate, so du infinity/dx=0 which implies the dp infinity/dx=0. So for flow over a flat plate the dp/dx is 0, so when you have dp/dx=0, then you are left with your momentum equation with this one okay. So if you write the order of magnitude of the different terms here, the left hand side order of magnitude is u infinity square/L, and the right hand side is nu u infinity/delta square right.

If this equation has to be important then order of magnitude of left hand side and right hand side has to be the same, otherwise these terms cannot nullify each other.

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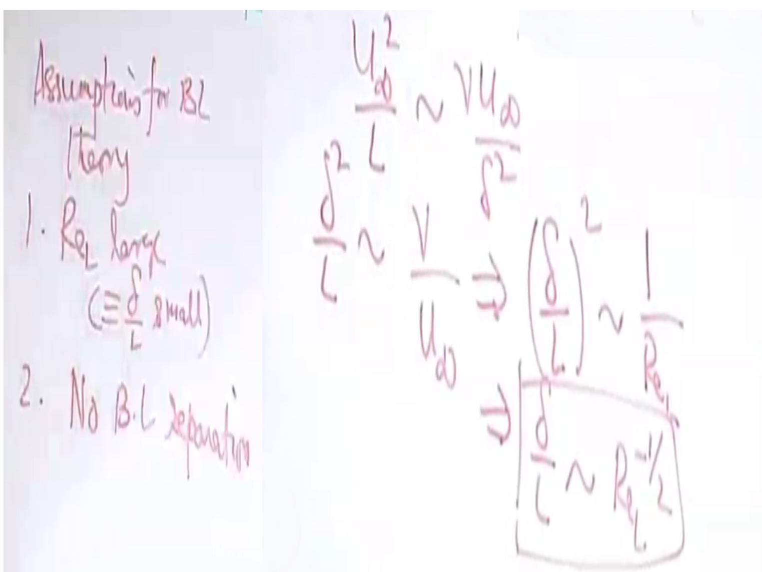

And therefore, the important conclusion that we have from that consideration is that u infinity square/L is of the order of nu u infinity/delta square, so you may write delta square/L of the order of nu/u infinity which implies that delta/L whole square if you divide both sides by L is of the order of 1/Reynolds number right that means delta/L is of the order of Reynolds number to the power-1/2 okay.

So see this is a remarkable thing, because this is a very, very important observation or conclusion that we arrived at no cost, without solving any equation, without going into a computation of equations or numerically or whatever without going into any sort of implication just by looking into the order of magnitude. And we later on see that why this is so important by toiling very hard at the end we will come up with an expression.

After when we solve the questions we will come up with an equation delta/L equal to some constants C into Reynolds number to the power-1/2. And therefore, disregarding that effect of the constant C, the dependence of the Reynolds number will be still the same what we get from such simple analysis, and that is what is one of the important powers of this orders of magnitude analysis. So the important conclusion that we get from this order of magnitude analyze this is very interesting.

What is that if delta is <<L that assumption is consistent with Reynolds number is large, because if Reynolds number is large then only you have delta/L<<1. So these are perfectly equivalent that answer now we have given, so that means what are the assumptions under which the boundary layer theory is valid or either delta/L <<1 or equivalently Reynolds number is large. But we will always say that Reynolds number large is a more fundamental way of looking into it.

Because an analyst who does not know the boundary layer will, or who does not want to go into the details of the boundary layer thickness will always be interested about the system level parameters, and Reynolds number is a system level parameter, if you know the length of the plate, if you know u infinity, if you know what is the fluid and its properties, you can estimate what is the Reynolds number.

And based on the Reynolds number, you can come up to the conclusion that whether this theory that you have developed is valid or not, so you do not have to really deal with delta/L because you know implicitly that if the Reynolds number is large delta/L has to be small. So these are equivalent considerations. The other important consideration which does not come for flow over a flat plate, but may come for flow over a curved surface which we will see subsequently is that is this boundary layer going to grow monotonically.

The question is that this type of growth of the boundary layer monotonically will not always be the situation it will depend on the pressure gradient in the flow, for flow over a flat plate the dp/dx is 0, so it will just grow like this monotonically. On the other hand, if you are considering curve boundary there are 2 possible cases where you are having dp/dx>0 and dp/dx<0. We have discussed earlier that dp/dx>0 is sort of considered as an adverse pressure gradient in the direction of the flow.

It tries to decelerate the flow, and it might so happen that the deceleration effect is so strong that the flow actually might take place in a reverse direction close to the wall, because wall has an effect of slowing down, and on the top of that as it like there is a dead person under pressure gradient is shooting a gun on that dead person. So wall is like slowing it down very severely, and on the top of that there is an adverse pressure gradient.

So the poor fluid element which is very close to the wall cannot sustain all these resistances and may have a reverse directional motion, and then this monotonic growth of the boundary layer is disturbed that situation we called as boundary layer separation. We will look into that in a more physical way subsequently, but important thing that we will understand is that if such a boundary layer separation occurs then the boundary layer theory is not valid.

So what are the important assumptions for validity of the boundary layer theory, one is like the Reynolds number is large which is of course equivalent to delta/L small, and number 2 there is no boundary layer separation okay. So with this understanding what we will do is we will just go on one step forward, and see that whether we may calculate the wall shear stress from this description. So let us see what is the wall shear stress, wall shear stress is what is a very important thing, because it is one of the important engineering requirements from the analysis.

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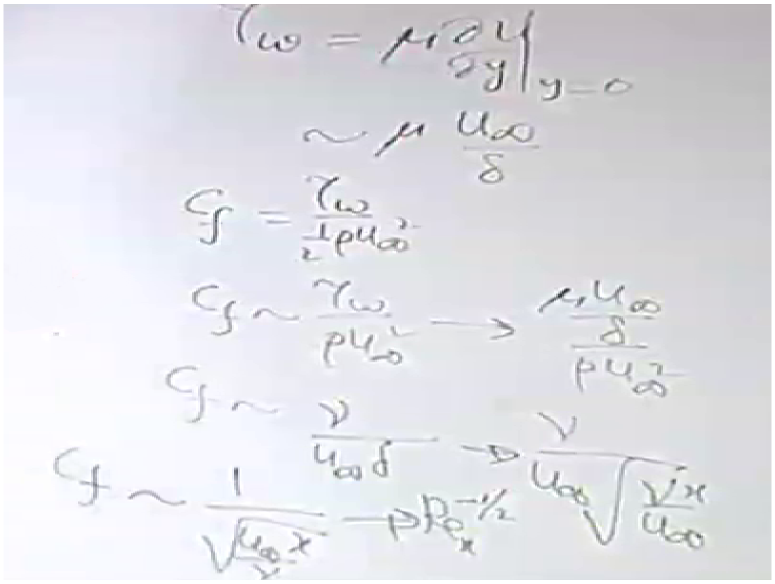

So what is the wall shear stress? The wall shear stress is the mu*del u/del y at the wall y=0, so what is the scale of the wall shear stress? What is the order of magnitude of this? mu*u infinity/delta right. So important non-dimensionalization of wall shear stress is what? We have seen it earlier in an example when we are dealing with Navier Stokes equation that is the friction coefficient Cf=tau wall/1/2 rho u infinity square.

So order of Cf is tau wall/rho u infinity is square, forget about the 1/2 because of the order. So that means this is mu u infinity/delta/rho u infinity square, so Cf the order is mu/rho, mu/rho is nu the kinematics viscosity, see the kinematics viscosity is what is the governing the picture that you can see, nu/u infinity delta. Now you know how delta varies with x, so delta square/some local x will be what? Will be nu/u infinity from this one just replace L with x, local x.

So we are now trying to find out that at some local x that is now our length L, so some x somewhere what is the corresponding wall shear stress. So delta square/x is this one, so we can write replace delta with nu/u infinity*square root of nu x/u infinity, so Cf scales with what 1/square root of u infinity x/nu right, so Cf scales with Reynolds number to the power-1/2.

This is also a very important thing, and we will see later on that if you exactly evaluate the expression it will be Cf=some constant into Reynolds number to the power-1/2. So dependence of all the important quantities on Reynolds number may be obtained in this way. So let us stop for this lecture, and we will continue in the next lecture, thank you.