Introduction to Fluid Mechanics and Fluid Engineering
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Lecture – 36
Introduction to Turbulence (Contd.)

In our previous lecture, we were discussing about the statistical averaging of the fluid flow equations to model the phenomenon of turbulence and we eventually came up to a conclusion that it is giving rise to a problem of closer that means you are getting some extra quantities known as the so-called Reynolds stress but there are no obvious expressions by which you may evaluate the Reynolds stresses.

Therefore, one has to model it by some physical intuition or physical understanding and the model may be as good as your physical understanding and it is not so trivial or not so obvious to come up with a very accurate or a very correct model and that is why in terms of understanding the statistics of turbulence, it is still an unsolved problem. So what we will like to see is not that what has been the most recent advancement on these topics.

Because those are mathematically very involved but we will look into some of the basic physical features or so to say some of the most primitive models. But for most of the physicist, the most primitive models were the best ones because they could give the most important physical insight or the turbulence stresses or the Reynolds stresses but before going into that, let us just recapitulate that how the different length scales are involved in the process of turbulence.

So we were talking about in our initial discussion of turbulence, we were talking about the concept of energy cascading and let us just revisit it, just let us try to say that we take an example.

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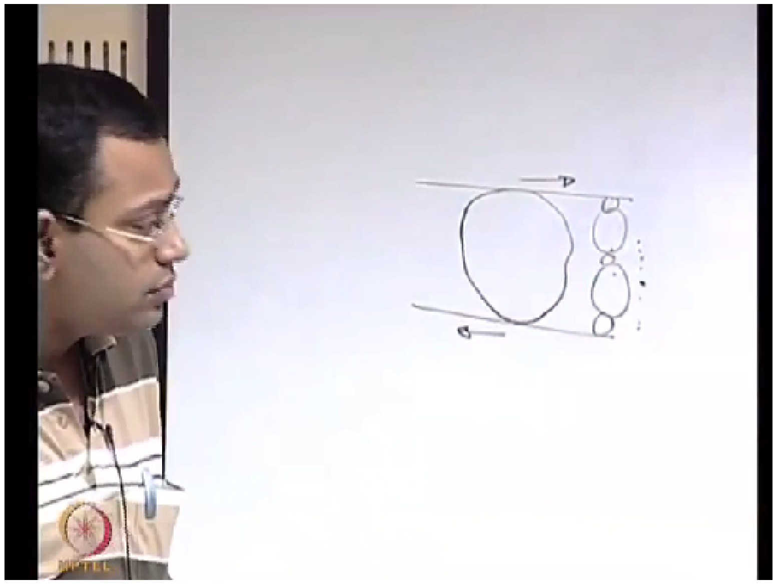

We are not talking in terms of example of a turbulent flow but let us say that you have 2 plates. In between the 2 plates, you have a big piece of stone, okay. So this is not flow. This is just analogy. Now what you try to do? You try to apply a relative shear between these 2 plates. When the shear becomes very strong, this may break up into small pieces. So as if it is extracting some sort of energy from this shear mechanism and getting broken into may be smaller granules and these smaller granules when they are, the entire thing is under shear.

So as if it is a crushing machine and in that way, this is getting broken into smaller and smaller pieces continuously till it comes to really smallest of the granules and energy as if has passed from by shear mechanism from the larger scale to the smallest of the granules. How it has passed? It has passed to the energy cascading mechanism. Equivalent to that also, something is happening in this particular hypothetical example.

So what is happening in a turbulent flow, this large piece of stone is like a large fluid mass. It is extracting energy from the mean flow. So turbulent flows are often characterised with high Reynolds number and that is why it can have a high mean kinetic energy so that the large Eddies can continuously extract the kinetic energy from the mean flow and sustain their rotationality. So always the question is that when the large Eddy passes on its energy to the smaller Eddy then how does the large Eddy itself sustain.

It sustains because it continuously extracts energy from the mean flow and it passes it on to the smaller Eddies that again passes it on to the smaller Eddy, so it is a continuous process. It is not that the process is stopped at once and that is how energy is passed on from larger to smaller to small length scales. Eventually when it goes to the length scale where viscous effects are very very important, then this entire energy is dissipated in terms of viscous dissipation.

And that is how sort of its cycle where energy is taken from the flow and energy is sort of dissipated by viscous action and this cycle goes on. So one important understanding from this cascading mechanism is that in a turbulent flow, interaction between Eddies is very important and interaction between Eddies makes the exchange of momentum fluctuation, energy fluctuation, all these things.

Therefore, we must have a sort of at least overall idea of how you have the exchange of momentum because of fluctuating components between several Eddies or may be 2 Eddies taken at a time and what Prandtl did is? Prandtl tried to draw an analogy between this and the exchange of momentum between molecules and that is how he appealed to the kinetic theory of gases which was substantially developed at that time when Prandtl started looking into the problem of turbulence. So what was the whole idea?

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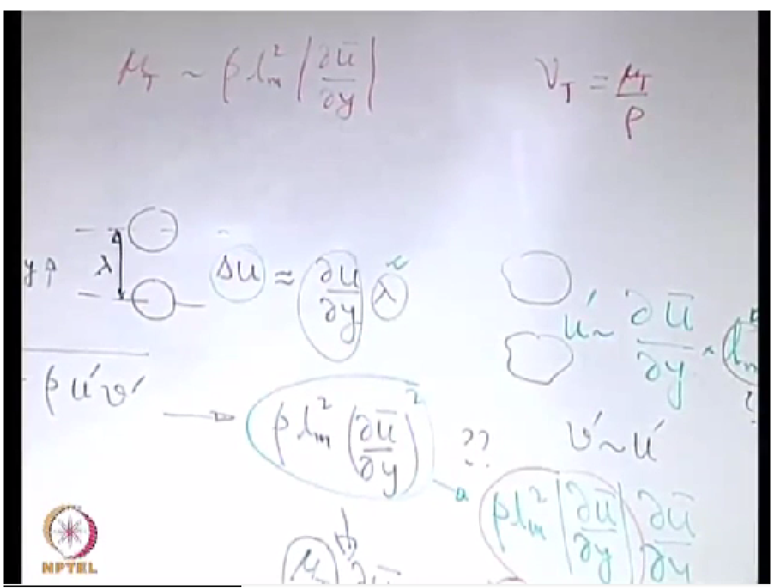

The whole idea is that if you have 2 molecules, you know that there may be a characteristic change in the velocity of a molecule when one molecule traverses a threshold distance which is likes say a mean free path and collides with another molecule because the smallest resolution that you can think of in terms of a molecular characteristic length scale is the mean free path because the change in characteristics of a system of molecules may be possible only with collision and collision can take place only after a mean free path is traversed.

So in terms of a molecular length scale, the mean free path is the characteristic length scale over which one molecule will go and interact and have a change. So if there is a difference delta u between the velocities of these 2 molecules and let us say that this distance is, this coordinate direction is y, then these delta u is approximately as good as the gradient in u*the length, right. This is the molecular picture.

Now in turbulent flow, the molecular picture is not important. It is just the analogy that we are drawing. Now imagine that instead of these molecules, we are having interacting Eddies which are lumps of masses with some sort of rotationality, some sort of vorticity. So in a turbulent flaw, basically what happens? There is a chaotic advection of vorticity with respect to position and time.

So it is like the vorticity is one of the very important issues in turbulence. So these are strongly rotating structures. Now whatever it is, these have fluctuations in there, random fluctuation in their velocities, just like they have u prime v prime, double prime like that. So what happens, if you have say one Eddy interacting with another Eddy. So we have to find out one such gradient and one such length scale.

So the question is what is the gradient that may be straightforward because on a statistically average sense, we are only keeping track of the averaged quantities. So the average quantities when we are keeping track of, maybe we describe the gradient in terms of the average. So instead of the gradient in the velocity as we had for the molecular picture, here we are talking about the gradient in the average velocity.

Because anything beyond that is taken care of by the fluctuation which statistically gives rise to the interaction between Eddies but average of the fluctuation is 0 itself. Now if you want to see that what is the relative fluctuation between these 2. So that is analogous to this delta u. So let us say that is u prime. So when you are describing u prime, approximately say or the scale of u prime, you have this one, you have to multiply this with the length scale just as you did for the molecular picture, the mean free path.

Here you do not have an obvious mean free path type of a concept but if you think of an equivalent length over which may be one Eddy has interacted with another, then that equivalent length Prandtl introduced as lm and he called it as mixing length. We should always keep in mind that a good work of physics is not always that you may represent exactly the reality but create a sort of a picture, physical picture and try to develop maybe a sort of simplistic mathematics to represent an equivalent reality.

And that is what Prandtl tried to do, not that this is what exactly happens in a turbulent flow but he tried to have a sort of a qualitative picture which is represented by the simple quantification. Again this is very very simplistic because Prandtl never had a clue, in fact till now there is no clue that exactly how these varies. There are again approximations but it is not as straightforward or as obvious as a molecular mean free path for a flow of gas molecules.

Now why this type of quantification was important? Because eventually Prandtl wanted to model the Reynolds stress term that is -rho u prime v prime as an example with an average. So we could see that this is an extra equivalent term of the same dimension as that of stress which came into the picture because of the averaging of the Reynolds averaging of the Navier-Stokes equation and since these are not known quantities, it gives rise to a tensor with 6 unknowns.

He had a desperate attempt of writing it in terms of some equivalent quantity which sort of is a pseudo-known. So you have this u prime and what Prandtl said or hypothesised that in terms of order of magnitude, u prime and v prime, the fluctuations should be equivalent and then this thing in terms of an order of, see it is an attempt of writing it in terms of a scale, not really because you are not really knowing that what is this correct length. So it is just a scale but exact value is not known properly.

So then it boils down to the form of rho, just the square of these but this form, I mean, in terms of the constituents of the equation as a form, it is fine but we have not given any due consideration to the algebraic sign of this. So we have to give a consideration to the algebraic sign of this. Remember at the end, we want to write this term eventually as some equivalent viscosity, turbulent viscosity we called it, * the rate of deformation.

We are just taking a 2-dimensional example where you have fluctuation components u prime u prime. Now if you want to do that, then the obvious way should be that and one of the things we concluded is that, this mu t has to be positive. So if it has to be positive, instead of writing it in this way, it is better to write in the following way. Because then it is quite clear that this part of the expression you are assuring to be positive.

See this is just an analogy between terms and therefore, it is important to preserve the physical sense. We discussed earlier in our previous lecture that u prime and v prime are correlated in such a way that in an isotropic turbulence case, you have their average, product of the average 0 but anisotropic, the positive u prime will be associated with the negative v prime and the other way.

So that minus effect will be adjusted with this minus effect so that eventually you should get a positive mu t if you have a positive dudy. So what it means, is that so we discussed that example with a positive dudy. With a negative dudy it will just be the opposite one but whatever is the example with a positive dudy that gives us a clue that if dudy has to be positive and the entire term has to be positive, that means mu t has to be positive.

It is the other way that if dudy is negative, then this term will be negative but still mu t has to be positive. So the positivity of mu t is what has to be preserved and that may be preserved by writing this term in this way. So this becomes the mu t, turbulent viscosity. Sometimes it is also called as Eddy viscosity, mu e and the name is very clear because it is because of the interaction between fluctuating components of Eddies, fluctuating velocity components of Eddies that is why it is often called as Eddy viscosity.

So the summary of Prandtl's initial work is that mu t or the Eddy viscosity, Prandtl said that this is related in this way. Sometimes sort of a kinematic Eddy viscosity is also considered that is you divide mu t with the rho. So that is written as a nu t/rho. What kind of insight Prandtl's hypothesis could give us? let us try to make an assessment. First of all, we have to realise that this is a simplification and one must confront that this is actually a huge amount of oversimplification.

Despite that oversimplification, it gives us some remarkable understandings and one such understanding is that how the velocity varies very close to the wall in a turbulent flow. So we will now try to develop sort of physical picture of the velocity variation close to the wall in a turbulent flow.

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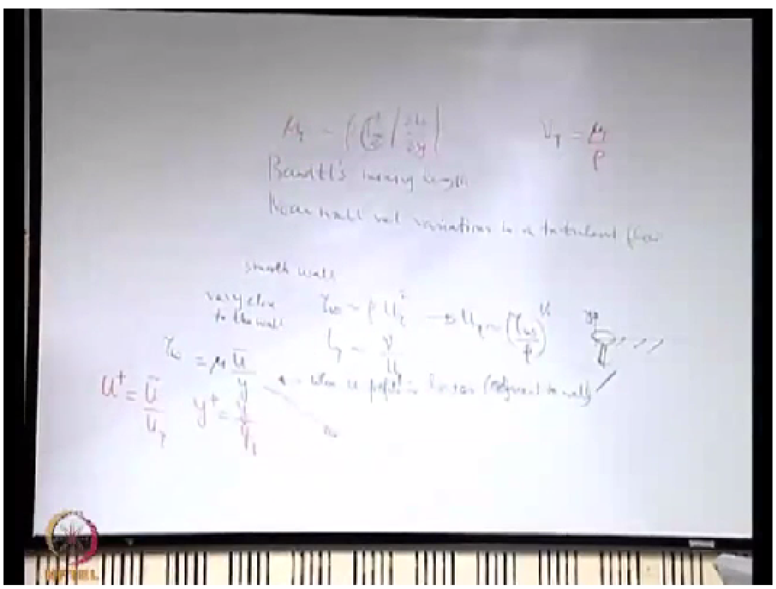

So wall velocity variations. We look it from different angle but first the angle from which you will consider would be a sort of a follow-up of the Prandtl's mixing length model. So this is Prandtl's mixing length model, this lm. So when we talk about the near wall velocity variations, we have to keep certain things in mind. The first thing is that, no matter whether the flow is turbulent wherever but very close to the wall, it is always laminar.

So the turbulent structures are important as you go little bit away from the wall but adhering to the wall because walls are excellent dampeners. So adhering to the wall and if you look into the picture of the flow close to the wall, we will see that what is that thickness what we are talking about as adhering to the wall. When we say adhering to the wall, it is very qualitative but we will come to its quantification slowly but at least we should recognise that very close to the wall, over a very very thin layer, how very very thin it is.

We will see that the flow will be always dictated by a laminar behaviour. So turbulent flow does not mean that it is globally having the same picture. Very close to the wall, it is having a sort of a different picture, that is the first thing. The second thing is that when we are talking about a region very close to the wall, we should also be bothered about the roughness of the wall because the region very close to the wall, the roughness elements of the wall interact very significantly with the flow. So the question is, how smooth the wall is?

Because if the walls are rough, there are protrusions from the wall into the flow and those may disturb the flow. In reality, those are some on the triggering mechanisms of turbulence. So we have to understand that what is the effect of the roughness but first we isolate the effect of the roughness and assume that we are having a sort of smooth wall. So if you are having a sort of a smooth wall.

Let us see that what are the important velocity scales and important length scales very close to the wall. So let us say that you have a smooth wall. Let us try to draw a physical picture that you have a wall, fluid flow is taking place over it, the coordinate normal to the wall into the fluid is y. So very close to the wall where say the effects are virtually laminar. Now at the wall itself you can calculate a wall shear stress, right.

So wall shear stress gives a sort of a picture at the wall and since that is exactly at the wall, no doubt that it has to be driven by laminar behaviour because wall shear stress is calculated exactly by velocity gradients at the wall. Calculating that for a turbulent flow is difficult because that slightly away from the wall is still affected by turbulent fluctuations. So it is not so easy to measure it but if it is accurately measured.

Then this is one of the important parameters that we can get from the wall and just from the scaling arguments, tau wall is given by some rho*u square, is just from the dimensional analogy. So if you know what is tau wall, then whatever u that you get, let us call it u tau, that is a correct velocity scale very close to the wall because that velocity is derived from the wall shear stress, okay.

So we come up with a velocity scale very close to the wall as u tau that is tau wall/rho to the power of 1/2. Next length scale. So what is the important length scale. So close to the wall, if it is a smooth wall, the wall roughness may not be an important length scale but if it is a rough wall, then the wall roughness itself may be an important length scale but here since wall roughness does not come into the picture.

We are having a length scale that is solely dictated by the physical mechanism within the fluid and very close to the wall, whatever is happening is a sort of an effect of energy cascading from the large Eddies to the very small Eddies. So small Eddies, we have also discussed it earlier that large Eddies have lot of anisotropy but small Eddies have virtual isotropy but small Eddies are not perfectly isotropic but they have greater isotropy than that of the large Eddies.

Not only that, small Eddies are having certain important characteristics. Sometimes they appear in patches and disappear. These are known as intermittency in turbulent flows and very involved concepts are related to this. But whatever we get as a gross understanding from the behaviour of the small scales is that, as you go to the smaller Eddies then these things are dissipated if the viscosity that dominates the mechanism and more importantly the kinematic viscosity.

So if you want to find out what is the important length scale that is dictating that, then the length scale over that should be the kinematic viscosity/this velocity scale. Just you look in to the units, this is like meter square per second, this is metre per second, okay. So correct length scale is governed by the kinematic viscosity because in the small scale, the dissipations become more and more important and so this is very close to the wall.

Not only that very close to the wall, we may have a sort of a simplified physical picture. What is a simplified physical picture? The simplified physical picture is that if you are say focusing your attention on a very small region close to the wall, I am trying to draw the velocity profile. Velocity profile means mean velocity profile because we are talking about the statistical quantities. So if the mean velocity profile very close to the wall will just be linear.

And the reason is straightforward because you are really considering a very very short length over which you are considering the velocity variation. That means a linear velocity profile will mean that the wall shear stress is a constant because tau wall is like local mu dudy. So if u varies linearly with y, dudy is like a constant. So if we calculate tau wall, over that very very thin layer, then the tau wall will be just mu*u/y, this is where u profile is linear, very close to the wall or adjacent to the wall, okay.

So when you have this wall shear stress and the related expression, now let us try to write the velocities and the lengths non-dimensionalised in terms of the velocity and the length scale that we are talking about. So we introduce the non-dimensional velocity. So we introduce some non-dimensional velocity say u+ which is u average/u tau. This is a non-dimensional velocity, so this is a scale.

Always you non-dimensionalise with the proper physical scale because the scale gives what? Scale gives the maximum value. So that is you expect this to vary between 0 to 1. When it is maximum, it is 1; minimum it is 0 and y+ as y/y scale, okay. So these are 2 important non-dimensional quantities that we introduce.

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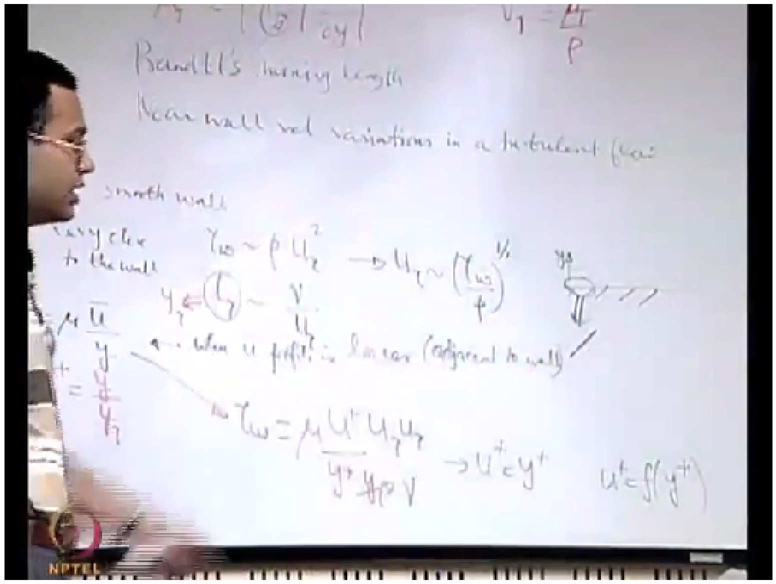

So let us write these wall shear stress in terms of these quantities. So we will write tau wall=mu, in place of u bar, we will write u+/u tau, in place of y, we will write… u+*u tau sorry. In place of y, y+*y tau. Y tau is like this l tau. This is as good as y tau. Since we are using the y coding, we are just calling it y tau. This is a characteristic length scale. Because of the apparent isotropy, it is y or x or whatever, it is just length scale that is important, not the directionality so much.

But close to the wall, the directionality is important because the normal gradient gives rise to the shear stress. So when you have this one, now you may write u tau as a function of y tau. So y tau is what? Mu/u tau. So replace y tau with, replace these with nu/u tau and mu/nu is the rho. So you have right-hand side rho*u tau square, left-hand side tau wall and tau wall scale is rho*u tau square.

So these gives u+=y+, okay. Now this is like a sort of non-dimensional way of writing the velocity profile very close to the wall. If you go a little bit away from the wall, u+ may not be exactly = y+ but over some distance, u+ will be some function of y+, that function is a linear function very very close to the wall. It may deviate from the linear function a little bit away from the wall.

There will be some region away from the wall when this will not work at all and a different form of the functional relationship will come. So we will try to look into that what is that different form of the functional relationship and for that, we will appeal to the Prandtl's mixing length. So when we appeal to the Prandtl's mixing length, we will keep in mind that we are not talking about a region which is really infinitesimally adhering to the wall but slightly away from the wall because the turbulent effects are more and more important as you go more and more away from the wall.

Slightly away from the wall, see if you go, it may be an interesting transition because if you go farther and farther away from the wall, the turbulence effects are important. If you are very close to the wall, adjacent to the wall, the wall shear stress is the dominating factor. So that is the laminar effect that is important. So this we may qualitatively call as sort of inner region and outer region.

So inner region is like a region very close to the wall, outer region is somewhat away from the wall where the turbulence effects are more and more important but these regions are fussy, so there is a transition and it is a sort of overlap. So wherever there is a transition, these effects are, one effect is almost taking over the other. So if you have the total stress, total stress at the wall was solely due to the wall shear stress because of the laminar effects and the turbulence stress was negligible or tending to 0, that is the -rho u prime v prime average was 0.

As you go somewhat away from the wall, you will find a threshold location where the wall shear stress effect is not directly there except that the effect of the wall has got propagated to the inside because of the molecular viscosity but in terms of the turbulence, the turbulence stress is the solely dominating factor because fluctuations become more and more as you go close to the wall.
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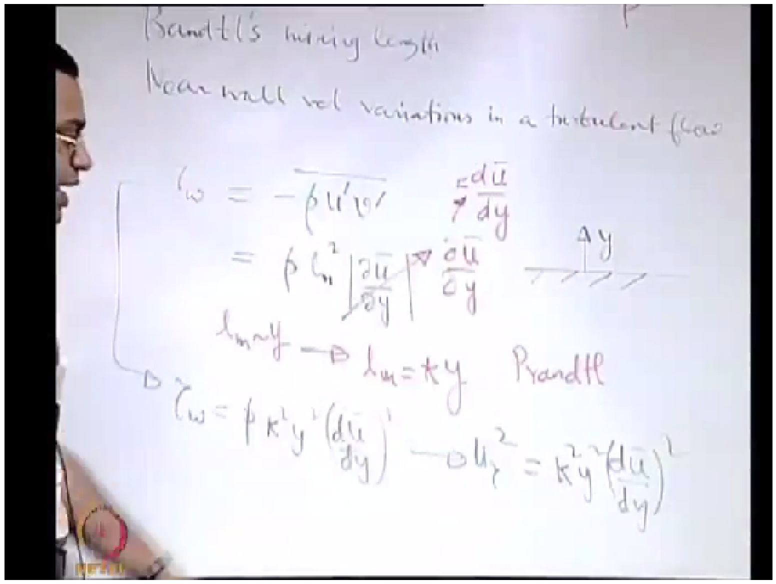

So at that threshold limit, you can say that whatever wall shear stress was there that wall shear stress has been transmitted to a layer where the value of the stress is now been dominated by the turbulent fluctuations. So at that overlap or transition, whatever is true is this type of a relationship, this one. So these we are writing for a sort of a transition from the wall dominated behaviour to the turbulent fluctuation dominated behaviour and it is as if the same momentum flux is being transmitted across those 2 layers of the transition.

That is what is the physical understanding behind this equation. So writing an equation is not important. Therefore, this is not a universal equation, we are writing it at a location for a transition by keeping certain physical constraints in mind and it is important to keep that physical argument in mind when we are writing this equation. Next is we can write this if you are now using the Prandtl's mixing length model, maybe you can write this as rho lm square.

Now see if you are modelling the flow close to the wall and you are going along the y direction, now you know that along the y direction, u increases. So dudy is positive. Therefore, it is just possible to write it as dudy without going for the mod in this type of a case, right. Because here u bar will increase with y from the wall. At the wall, it will be 0 because of no slip condition. So that is the first thing that we do by appealing to the physical picture.

Now you may also take a simplification by considering that there may be fluctuations in all directions but the mean flow is like unidirectional with only x component. So then this may be approximately like dudy with… dudy means du mean dy. So whenever we are writing for a turbulent flow, we are writing it for the mean quantity. So if it is that, the mean over the other directions is u, then only we can write it in this way but otherwise, the partial derivative you have to write.

No matter whatever you write, you may have to stop here because you do not know what is the mixing length and see that is where Prandtl gave another hypothesis. What is he said is that, this mixing length is sort of proportional to the distance from the wall. What was the physical argument? This term is not at all important at the wall. At the wall, the laminar effect is there that which solely dictates the factor.

So at the wall, whatever is the turbulent stress that has to be 0, right. So when y=0, this becomes 0. As you go more and more away from the wall, it has to be more and more important. So physically this term will be increasing as y is increasing and a simple increasing law may be a linear law. That is what was the logic of Prandtl and accordingly, see these type of logical thoughts are important because it is not just a formula that at the end we are going to learn.

We are going to learn basically how these famous mathematicians or engineers or physicists tried to think in attempting a problem which is a very complicated problem in terms of having a simplistic picture and that gives a lot of training to even the present generation of how to approach an unknown problem. So that means you can write this as a sort of proportionality constant into y.

Again this is a model. So this was another hypothesis of Prandtl following up his mixing length concept. Now with days if you try to simplify the equation now further, so tau wall=rho*k square y square and may be… Now if you divide tau wall/rho, you should keep in mind that that will give you u tau square, what is the velocity scale, square of the velocity scale close to the wall. So that means you have u tau square=k square y square dudy square. Now you may extract the square root by referring to the proper sign by keeping this positive y axis in mind.

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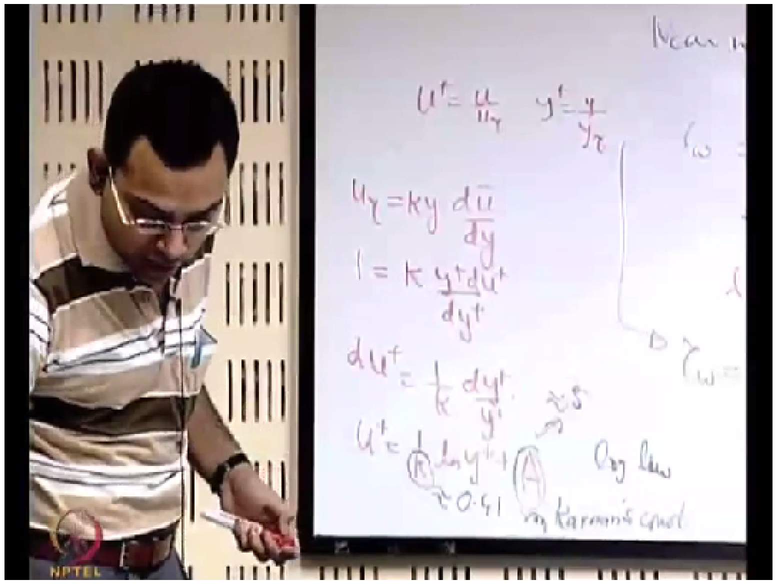

So if you do that, we will get u tau=kydudy, okay. So now what you may do is, you may recall that you have u+=u/u tau and y+ as y/y tau. So let us try to nondimensionalise this equation in terms of u+ and y+. So clearly you can see that this becomes k, you may divide both numerator and denominator by y tau, so it becomes y+ydy+ and this is du+. This u bar you absorb with u bar/u tau, that becomes u+, okay.

So du+=1/kdy+/y+ and if you integrate this, you get u+=1/klny++some constant say capital A. This tells that at a distance somewhat away from the wall, the velocity profile should vary longer rhythmically and there are important constants appearing. The constant A will of course depend on many things but for a wall which is very very smooth, from experiments, these A came out to be very close to 5. See this is not an exact picture; therefore, the constant should be fitted with experiments.

So lots of experiments were conducted and from all the experiments which have been conducted from that time till now, this value of A for a very smooth wall is a sort of like very close to 5. More importantly, although this parameter might vary according to the roughness of the wall but this parameter k or in some books written has kappa, this parameter is sort of universal and it does not vary from one condition to another condition, remarkable thing.

And the value of this is roughly=0.41 which was obtained by a lot of hypotheses and experimentation conducted together by the group of Von Karman and therefore, this is given in the name, in the honour of Von Karman as Von Karman's constant. So it is not like a theoretically derived constant but perhaps nature has created things in that way that no matter whatever is the roughness, no matter how the turbulence structures are distributed but wherever this law is important, this is known as the logarithmic law, log law.

And this law is having this constant kappa which is sort of universal. So it is like a universal constant but not a fundamentally derived universal constant but all the experiments have justified it. Of course, I mean, there have been people who argued that it could be 0.39 or 0.4 or 0.42 or whatever but roughly 0.4 is something which has been obtained from all experiments and that is one of the very remarkable things.

Now we have discussed about the 2 limiting cases but let us just stretch it a little bit to have the picture of the entire near wall velocity distribution, not just the overlapping case or the limiting case.

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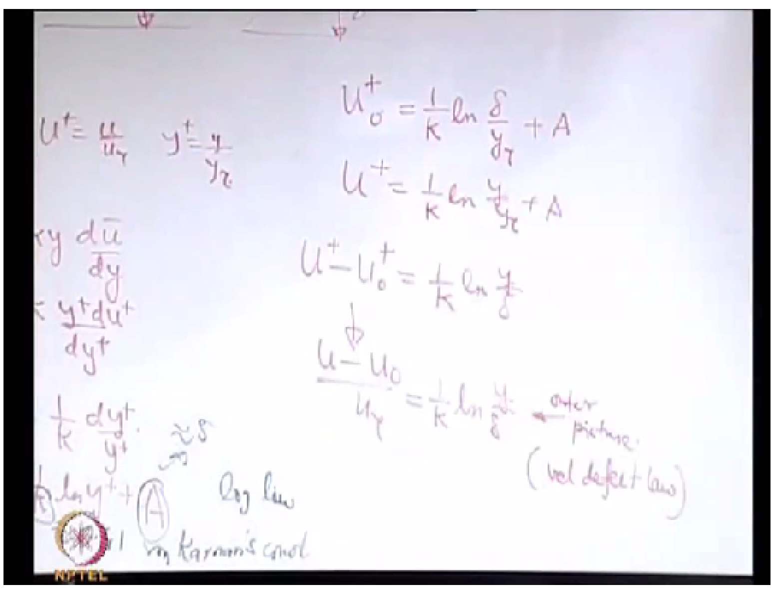

So to understand that let us say that you have a flow where say maybe flow between 2 parallel plates where you have a central line which sought of represents outer behaviour or you may have a boundary layer for flow over a surface or flow on a flat plate where you have a boundary layer thickness which gives us sort of like length scale of the inner behaviour. So either may be, say let us call 1/2 height of the channel as say delta or the boundary layer as delta depending on like what we are looking for as an internal flow or flow over an external surface.

Now when you go to that length, let us write this u+. So u+0, let us say u+0 is the u+ at that length where the outer behaviour comes into the picture and that will become 1/kln, y+ is y/y tau. So y=delta here, so delta/y tau+A and at any other y, you have u+=1/klny/y tau+A. So if you subtract these 2, you will get u+-u outer+=1/klny/delta. So that means you can write this as u-u0/u tau by writing in the dimensional form=1/klny/delta.

So it is a sort of outer picture. Sometimes it is also known as a velocity defect law. Why such a name? Such a name is there because there is a deviation of u from u0 or the outer layer because of some effect of turbulence. Because of the effect of the fluctuating components. Now this is a picture where much away from the wall, maybe outer condition you get such velocity defect. Very close to the wall, you have u+=y+ and u+ as a functional of y+ in general.

In between, you have a logarithmic description in general and maybe outer description is this one. Now what did experiments give?

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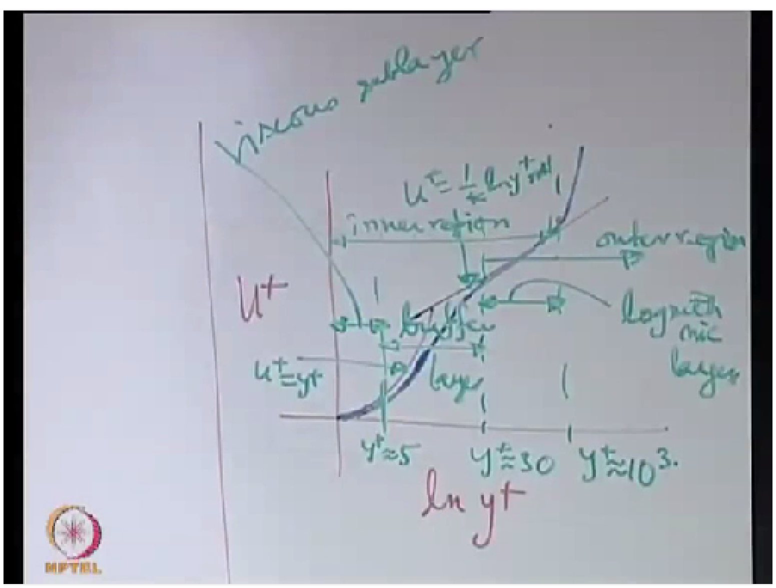

So let us try to make a plot of u+ as a function of lny+. We will try to make up (()) (40:11) these approximate forms with what we get from experiments. So what are these approximate forms?

First of all very close to the wall, you have u+=y+, that is our conjecture. u+=y+ will be like this, not a straight-line because we are plotting with log, okay. Then somewhat away from the wall, you have u+=1/klny++A.

So somewhat away from the wall, u+ versus lny+ will look like a straight line. So let us say that that straight line is this one, okay. So these are the 2 important derived quantities from this physical model or from this simplistic model. Now what is the experimental picture that we usually get? The experimental picture is that very close to the wall, it almost satisfies it, then maybe it undergoes a deviation, sort of a deviation from this way, then it almost satisfies the log law and then it starts deviating away.

The blue line is a sort of our experimental picture and you see that it is qualitatively, of course quantitatively there will be deviations but qualitatively, it is not so much deviating from what has been derived from a very simplistic theoretical conjecture. So there is a limit over which the experimental and theoretical very close to the wall, exactly match this is u+=y+, this line and this line is u+=1/klny++A.

So this limit is roughly like y+=5, okay. So roughly up to y+=5, so what is y+ doing? See y is a region very small, close to the wall but what is y tau? Y tau is nu/u tau. Nu is very small. U tau let us say it is like say nu is 10 to the power -6-meter square per second and u tau is like say 1 meter per second as an example. So this ratio is small, so when it is divided with y, it blows up y. So what is does, it gives a new length scale which zooms or blows up the phenomenon very close to the wall.

Because this behaviour is very close to the wall, you do not have sufficient resolution until and unless you zoom it up or blow it up with a stretching. So this scaling gives you a sort of a stretching. So it allows y to be magnified that is as if you are now sitting with a magnifying glass and seeing very in a vivid detail that what is happening very close to the wall. This new coordinate allows you to do that and you see there is a sort of like a Reynolds number.

It is some u*y/nu. So this y+ is a sort of an equivalent Reynolds number. So the corresponding

y^+ very close to the wall till which the experimental and this behaviour is valid is roughly = 5. Then you come to the other layers. You see that like you will come to locations where these laws are more or less not that effectively valid. Maybe this is like sort of location at which you will get that there is a very nice matching with the logarithmic law.

So this is roughly like $y^+=30$ and these are just obtained from experiments. There is nothing very fundamental about it. Then you have a region where like this log law is valid no more. This is roughly like y^+ of the order of 10 to the power 3. So you see that there are different zones and different zones are given different names. So you have the name of say so called this region as the inner region.

So inner region means, it has a part with the $u^+=y^+$ and it has a part with $u^+=1/k \ln y^+=b$. So there is in the inner region, what are the sorts of behaviours you have? You have behaviour where solely the molecular viscosity is determining what is happening, that is why $u^+=y^+$ and this is called as a viscous sublayer.

In very old books, it is written as a laminar sublayer but laminar sublayer is not a very correct term because that gives a false indication that the entire behaviour here is laminar. It is not so, there may be cases where there are some turbulent fluctuations in this layers. They maybe intermittent. They sometimes appear and disappear but it is not perfectly laminar but whatever it is, it is dominated by viscous effect, molecular viscosity that is why viscous sublayer is a better name for this.

So then you have this layer where you were transitioned from these viscous sublayer behaviour and this is a sort of a patch where you have a transition from one behaviour to another behaviour. This is known as a buffer layer and the layer over which the logarithmic law is valid is known as logarithmic layer and from the logarithmic layer to the entire thing is usually known as the outer region and you can see therefore there is a region which is an overlap between the inner and the outer region.

So it is not that where the inner region finishes, the outer region starts. I would say that the names are not very important because these names have been traditionally given in this way and that is what we are portraying here but what are the important physical phenomena that are occurring over these different length scales are what are the matters of concern for our understanding. Now what we will do now?

We will just have a short description of these in an alternative viewpoint or in an alternative picture. What is that alternative picture? Let us say that we do not know what is the form of velocity profile very close to the wall.

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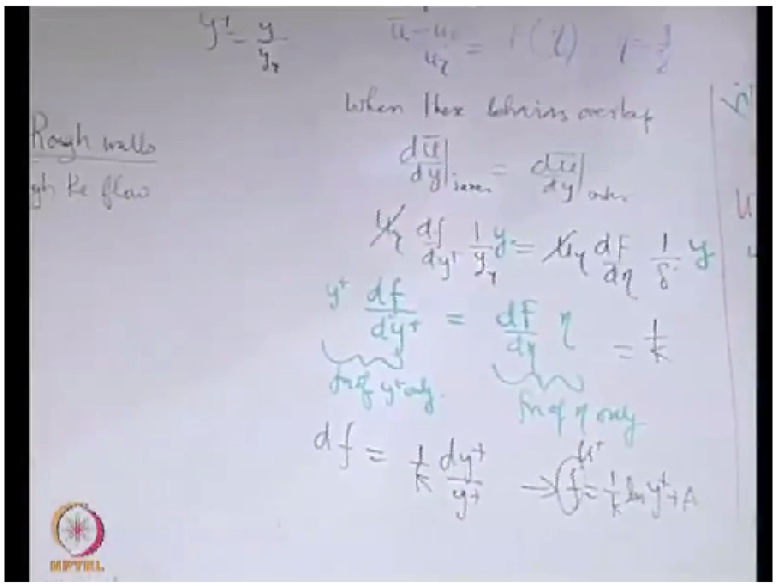

Just from dimensional arguments, we can write that for smooth walls, you have u+ as a function of y+ in the inner region very close to the wall, right. We do not know say the functional relationship and that is quite true because only in the viscous sublayer we know that u+=y+. The remaining region u+ is solely a function of y+ over at least a given threshold length but we do not know the exact function relationship. If you go somewhat away from the wall, you know that, so u+ is what? u/u tau.

Somewhat away from the wall if you go, you have u-u0. u tau, let us say it is a function of what? It is no more a function of the viscous effect. See the thing is in the y+, the important thing is y+ is dominated by a viscous length scale. A length scale based on the kinematic viscosity? As you go in the outer region you see here, the velocity profile is governed by y and the system length scale but not the nu.

So this is in another function of, say let us call this as eta where eta=y/delta. This is what is an understanding that we get from this Prandtl's mixing length analysis. The exact functions we may challenge but the forms we may not challenge because forms are the same. Now at the overlap, this function should behave the same way. That means wherever they are overlapping in terms of their physical behaviour, you can see that one is taking over the other at a region and at that overlapping condition, you must have what?

You must have that these functions giving the same behaviour, not only that, for smooth transition, the derivatives also should give the same behaviour. That means when they overlap, when these behaviours overlap, you have dudy from the inner=dudy from the outer, okay. When we say u, these are all u bar, u average, that we have to keep in mind. So what is dudy in a... So dudy, we can write u tau*dfdy+*dy+dy, right. U is what, u tau*f.

So what is dudy, u tau*dfdy. U tau*dfdy+*dy+dy, by chain rule. So what is dy+dy? y+ is yyy tau. So dfdy+*dy+dy is 1/y tau. What is dudy at the outer? So u tau*dfd eta*d eta dy, d eta dy is 1/delta. So you may cancel this from both sides and what you can do then? You can write, you multiply both numerator and denominator by y again because let us recover the y+ and eta, that will be better.

So let us write it in terms of that. So what you do is, you multiply this by y, you multiply this also by y so that you get back the variables, y+ and eta. So what you get as dfdy+*y+=dfd eta*eta, right. Look into this form, this is a function of y+ only and this is a function of eta only. Eta and y+ are 2 different variables, y+ is the inner variable and eta is the outer variable. One does not directly know the other.

Why one directly does not know the other? The reason is straightforward. The reason is, this is dominant by viscous effects, this does not understand so much the viscous effects. This is dominated by the turbulence effects. So that means each=a constant, say k. So what you can write here, therefore, the df or maybe 1/k to have the analogy with the previous form that we got.

You see remarkably the form is the same and if you integrate it, you will get f=1/klny++A, this form where f is like u+. Similarly, the outer law form also you can get. So just by dimensional arguments, it is possible to get the form and it is the more general way of doing it than the Prandtl's mixing length because this does not assume any form of the mixing length. This just assumes the dimensional dependences of the velocities in the inner and the outer regions.

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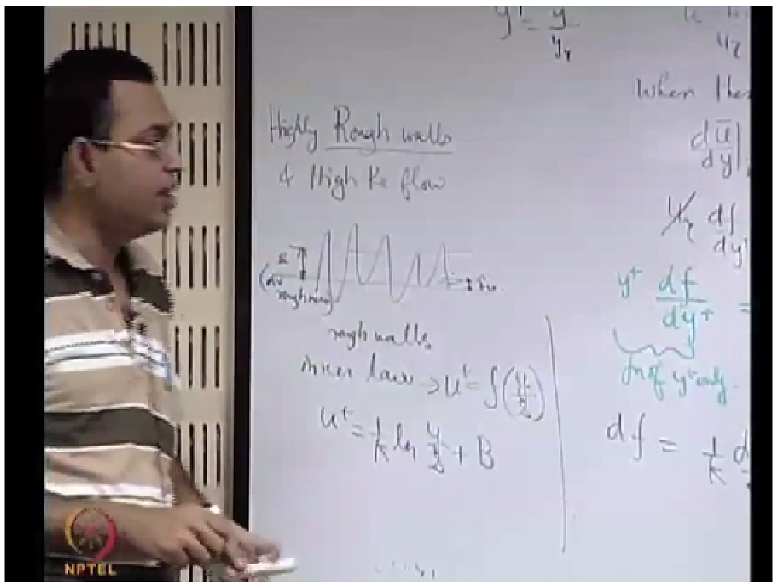

Very briefly let us see what happens for the rough walls? For the rough walls, so rough wall means, let us say highly rough walls and highly Reynolds number flow as an example. So we are not talking about a general case of rough wall, that is very very complicated but an extreme case of rough wall where it is very very rough and a high Reynolds number flow is taking place over that.

So when you have that, see the roughness scales, the wall is say something like this and if you consider the average roughness, let us say that maybe s is the average roughness. This is much much greater than the thickness of the viscous sublayer. So viscous sublayer is very very thin and when the roughness is much larger than the viscous sublayer then it almost nullifies the effect of the viscous sublayer.

So what dominates the behaviour close to the wall for a very rough wall is not the effect of the viscous sublayer but what the effect of the wall roughness. This effect becomes more and more at higher and higher Reynolds number because at higher and higher Reynolds number, the viscous sublayer becomes thinner and thinner. If you go to very very high Reynolds number, the viscous sublayer almost is vanishingly very, very small.

So almost the entire roughness elements are exposed to the flow and therefore the roughness dictates the flow then. So then for rough walls, the inner law is changed. Is changed to what? u+ is now a function of y/s, not a functional y/y tau. That is the only change, okay. The reason is clear that because of these extreme roughness, it is not that laminar or the viscous length scale that is coming into the picture but the wall roughness length scale is dominating, very high Reynolds number makes the viscous sublayer very very thin, maybe this is the thickness of the viscous sublayer, let us say delta v.

So now then if you use the same form, then you will get here that u+=1/kln now y/s+another constant. For very high Reynolds number flow, these v may be close to 8.5 or something like that, roughly like that. That exact value is not important but the form is important. See the dependence of the wall roughness for extremely rough wall and a very high Reynolds number flow, the combination that has become important.

So we can have a broad picture of what happens very close to the wall and why that is important? That is important because of the following reasons. That many of the flows in engineering or wall bounded flows, maybe flow over a plate or a surface or maybe flow in a channel or a pipe. So effect of the wall in dictating the turbulence is what is important and somehow we could develop a qualitative picture or a very simple quantitative picture from the broad understanding.

And you see that Prandtl's mixing length helped us a lot in understanding the actual functional dependencies on the various parameters. Now to end up with the discussion on turbulence, we should also touch upon something which makes turbulence unique. First of all like what are the important characteristics of turbulence that we have understood. Wide range of length scales and timescales and it is very difficult to capture all those in a modelling strategy.

So one has to go for certain statistical description. That is a first thing that we have learnt. The second important thing is even when it is statistically tractable, there are certain non-intuitive picture that it gives. See we have till now developed a sort of a feeling, may not be by explicit understanding, but by intuition that viscosity dampens out disturbances.

The viscosity sort of stabilises the flow but this is not always true. I will just briefly talk about an example where viscosity in a turbulent flow destabilises the flow. That means it instigates the instabilities. Qualitatively, it is likely this. Say if you have a velocity profile, say a parabolic velocity profile.

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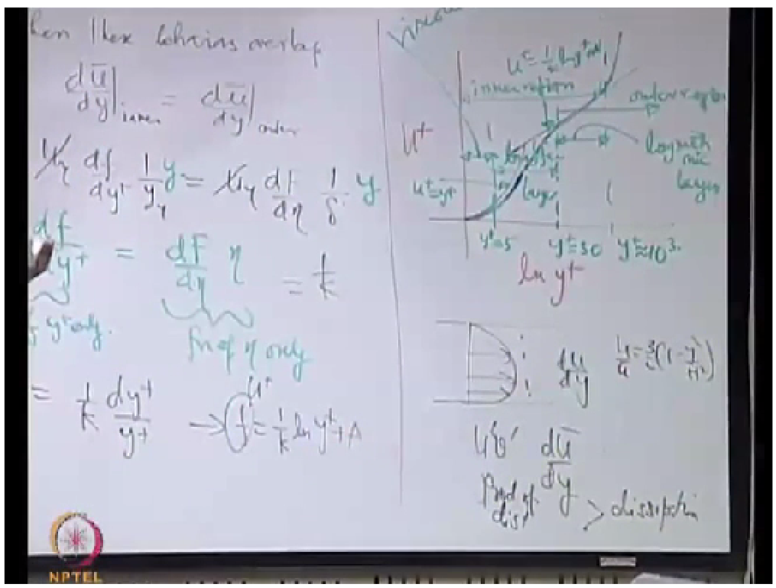

Somehow you have a parabolic velocity profile in a channel flow and this velocity profile say is there in a medium where the viscosity is 0 or say it is an inviscid flow, then if there is any perturbation, the perturbation will dampen out. There is a fundamental theorem which says that why it will dampen out. Because this velocity profile does not have a maxima in the vorticity. That is maxima in dudy, anywhere within the domain other than at the wall.

See if you see this u is like u/u average is 3/2*1-y square/A square. So if you find out maxima of dudy which is the vorticity here, maxima of that means d2udy2 is 0 and that is not achieved anywhere. So that means this does not have a maxima in the vorticity and therefore if is there in a medium, flow medium where vicious effects are not important, any perturbations will die down but if it is there in a viscous flow, then what happens?

Now there is an interesting interaction. There is an interaction between the fluctuating velocity components u prime v prime and the shear in the mean flow dudy and when the shear in the mean flow interact with this one, that interaction is by the mechanism of viscosity and it is just because it is fundamentally like transfer of momentum. Transfer of moment between this fluctuating components and the mean gradient because of shear.

And because of this exchange, there is a production of sort of disturbance energy. The production of disturbance energy, so there is some production of disturbance energy and on the top of that, there is a dissipation of disturbance energy, that is also by viscosity. So at the end the turbulent kinetic energy or the fluctuation kinetic energy will be more if this becomes greater than the dissipation.

Dissipation is also by viscosity. So without viscosity there is no such production and there is no such dissipation. With viscosity, there is the production and there is a dissipation and if the production is greater than dissipation, then the fluctuation component kinetic energy will grow. That means viscosity actually destabilises the situation. It becomes a perturbed situation.

So that is what we have understood that it may be a very nonintuitive situation where vicious effects actually destabilise or may destabilise the flow rather than stabilising the flow and turbulence is one of the mechanism that triggers it, okay. Let us stop our discussion today and from the next day, we will start with the new chapter the boundary layer theorem. Thank you.