Introduction to Fluid Mechanics and Fluid Engineering
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Lecture – 35
Introduction to Turbulence (Contd.)

In our previously lecture, we were discussing about some of the important statistical characterizations of turbulent flow and we will just continue with that.

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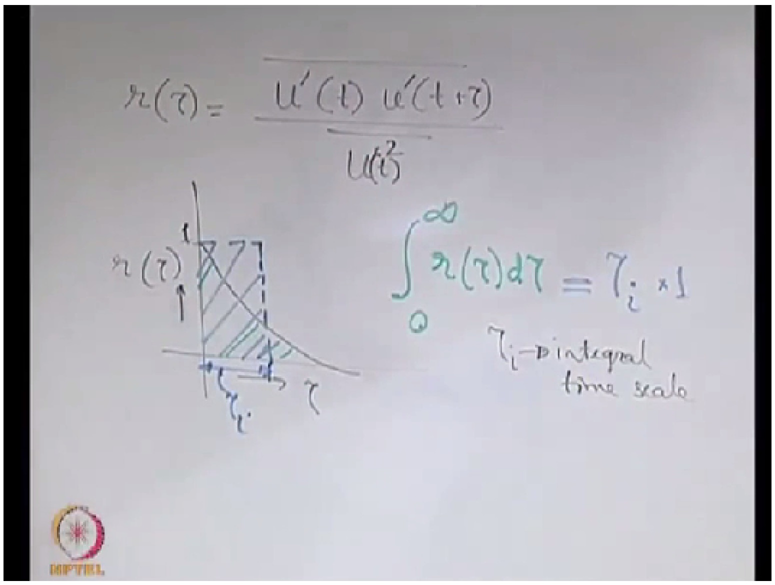

We were discussing about the correlation coefficient towards the end of the previous lecture and that we defined as follows. We have to keep in mind here that this is not like the most general definition of a correlation coefficient but this is a special type of correlation coefficient which we call as autocorrelation coefficient. So what is autocorrelation coefficient? If you have a random variable say at some instant of time.

Here we are talking about some instant of time at a given location and the same random variable considered at the same location at a different instance of time, say t+tau, then how the outcome of the random experiment based on the random variables, they are correlated. If the random variables are same, it is known as autocorrelation. So it is based on the correlation between the outcome of the same random variable just at different conditions.

One is at t, another is at t+tau and here the random variable is u prime which is the fluctuation component of the velocity along x. So this sort of indicates that how strongly the outcome of the random experiment in terms of the random variable, u prime here, at a given point at 2 different instance of time are correlated. So if you have, for example, a plot of this r tau versus tau. When tau-0, you see that you are talking about the correlation of the random variable u prime at a time t with itself at time t.

And therefore they are exactly the same and it is clear from the expression also that the correlation coefficient is 1. So this autocorrelation coefficient is 1 at tau=0. As you increase the tau, you are having the correlations of the same random variable at 2 different instance of time which are differing from one another and in that way, the correlation coefficient will tend to get reduced.

Beyond the threshold time limit, you see that the correlation coefficient is really very very small and to get a feel of the timescale over which the variable is strongly correlated with itself at just a different instant of time, we may consider the following. So if you find out the area under this curve, that is basically integral of the correlation coefficient from tau=0 to infinity. So that is area under the curve.

Now if we have a very strong correlation, say correlation coefficient=1 but lasting over a period of time which is different, say lasting over a period of time given by this tau I such that the area under the rectangle which is tau i*1 is same as the area under the previous curve. Then what it shows is that tau i is obtained as a timescale, a representative timescale over which the variables may be thought of to be very very strongly correlated and that may be obtained from the integral of the correlation coefficient and this tau i is known as integral timescale.

So integral timescale physically is a representative timescale over which a random variable, here the random variable is the velocity fluctuation, the random variable is strongly correlated to itself as an outcome of the statistical averaging over the random experiments. Similar scales may be obtained by considering the other types of correlation functions such as cross-correlation functions where may be you are trying to find a correlation between u prime and v prime.

So then u prime has to be replaced with b prime but we are not going into all those details. Our main emphasis here is to develop a building block for the basic statistical analyses or statistical description of turbulent flows. The big question remains that why should we at all go for the statistical description of the turbulent flows? And we actually try to answer this question in a part of our previous lecture.

That when you have a turbulent flow, the governing differential equations, for example the Navier-Stokes equation, they are very much valid. Only problem in implementing the governing equations as the Navier-Stokes equation to solve for the velocity, pressure and so on, is that number 1, there are multiple length scales and timescales. So you have the largest length scale over which any important physical phenomenon taking place as the system length scale.

And as you go down, you will find that the smallest length scale is the Kolmogorov length scale which is much much smaller than the system length scale and there is a whole lot of physical activity engulfed between this largest and the smallest length scales and all these length scales and similarly all the different timescales, they need to be captured quite accurately so that resolution of the scale is important.

Even if that is resolved, the second question is, how reliable are the results? Because the results will strongly depend on what? The results will strongly depend on the following. How reliable are the input data? So when you say that how reliable are the input data, it obviously depends on many things. The first is how reliable are the initial conditions. How reliable are the boundary conditions?

So how reliable are the initial or the boundary conditions, this depend on the randomness of the physical behaviour and in turbulent flows, the randomness of the physical behaviour is very very strong. So when the randomness in the physical behaviour is very very strong, the only way in which you may have a sort of deterministic behaviour is what? The only way in which you can have a sort of deterministic behaviour is by statistical averaging of the governing equations because the statistically averaged behaviour will be sort of deterministic.

But otherwise each simulation may be like a sort of random experiment where slight deviation in the initial condition or the boundary condition may trigger a large change in the final outcome. So statistical averaging is the only important practical resort and that is why, we try to develop the statistical basis or the statistical description of the turbulent flows. With that background, what we will try to see is that, how we may statistically average the governing equations, that is the Navier-Stokes equation is an example.

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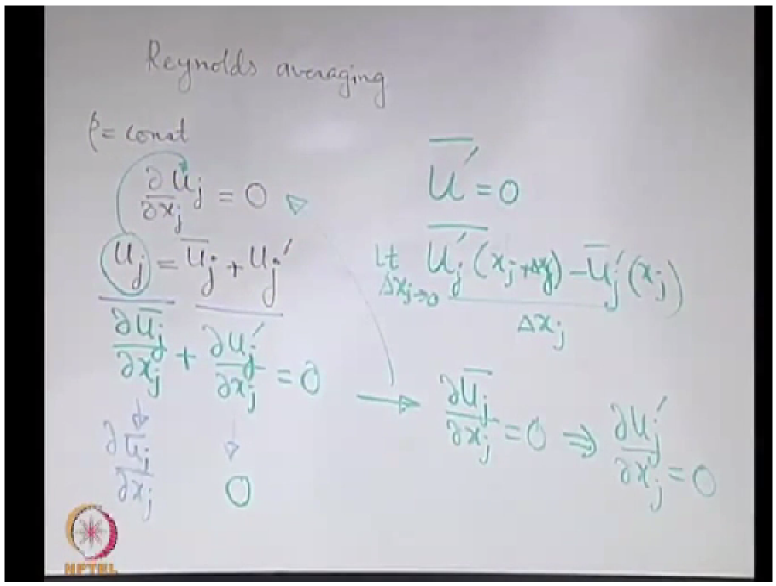

So we will now see the method in which you average the Navier-Stokes equation and since Reynolds contributed a lot towards that, we also call these as Reynolds averaging process. The Reynolds averaging process starts with the basic form or the well-known of the governing equations, for example we start with the continuity question. So let us assume that we are dealing with the case when the density is a constant.

That means you have the continuity question as this one. So in the first step, what we do is, we decompose the variable. Here the velocity is the variable into 2 parts. One is the mean and another is the fluctuation over the mean. When we say mean, we are not getting too specific that what type of mean but as we discussed earlier that it may be for example time average, space average, or ensemble average but here we are just, sorry, yes, but we are basically dealing with some sort of averaging here.

And that averaging is what is going to give rise to some equations in terms of the average quantities. The way in which we derive the equation is very straightforward. So we substitute this uj in the continuity equation. Once we substitute that, what we will get? We will get this = 0 and the next step would be to make an averaging of this equation. That means you average each of these terms.

So if each of these terms is like time averaging, then what you are basically doing? So when you are time averaging a variable, then you are basically multiplying that with dt integrating it from say time=t-t+T/the time period in the limit as t tends to infinity. We have discussed that this infinity is notional, that means it is a timescale much larger than the individual small timescales of turbulence fluctuations.

So that operation when we are doing, we have to see that what is the consequence? The consequence for the first term is straightforward, because it is already averaged. So average, value of the averages that is itself. Now if you look into the other term, we have to basically understand that what will be the average quantity of this after doing the differentiation. We had earlier discussed that if you have say for example u prime and the average of u prime is 0, that we have shown very easily.

The question is, what is the average of the derivative of u prime and to do that basically we have to just use the limiting definition, the definition of the derivative in terms of limit. So you can say that what we are looking for is that uj prime average at times, or say here is the special average, so at xj+delta xj-uj prime averaged at xj/delta xj in the limit as delta xj tends to 0 and see individual constituents of the limit are the fluctuations and the averages are 0.

No matter the averages are evaluated where. So this therefore will become 0 and hence we come up with the average continuity equation and once this is satisfied, if we substitute it back to the original continuity equation, then what we are left with is that the fluctuation components also satisfy the continuity equation. So that is one of the important things that we will remember. With this technique of averaging, we will next look into the averaging of the Navier-Stokes equation.

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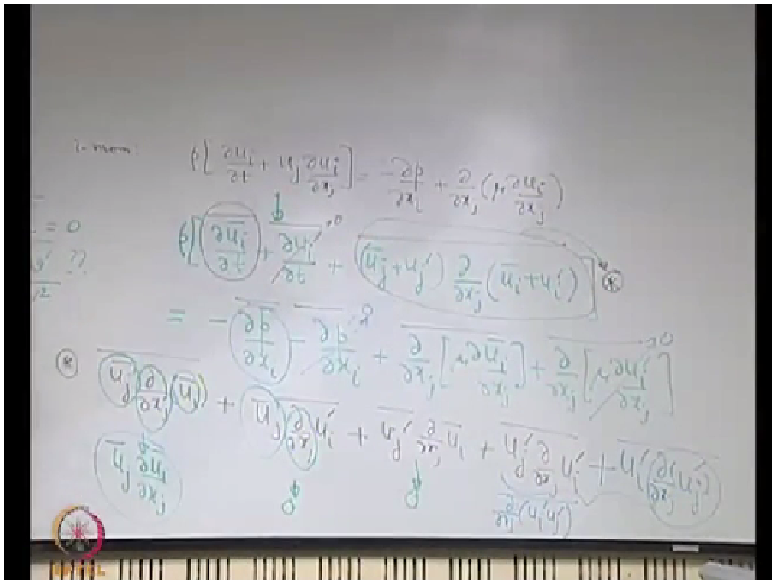

Let us say we first write the i momentum equation that is the linear momentum conservation or general momentum conservation in the ith direction. So we write rho…, we have not written the body force but if there is some body force, you can always add beyond this. So what we will do? We will do the same thing as we did for the continuity equation. So substitute each term in terms of its average + the fluctuation.

When we do that, what we get in the next step? So this is what we get as the left-hand side. This = the right-hand side. Again we decompose all those terms in terms of the mean and the fluctuation. Just as what we did for the continuity equation the same thing we will do that is will now do an averaging of each of these terms. So when we do an averaging over each of the terms, some of the terms come out to be very straightforward, for example, this first term, it becomes just itself.

There is no change. The second term, we have seen that the derivative operations commute with the averaging operation. So we can take as if we find the time average of ui prime or maybe ensemble average of ui prime and find the derivative with respect to time. One of the important thing is that that averaging operation is not conflicting with this t. Why? Because the timescales over which we are averaging are different for the timescale for turbulent fluctuations and the timescale for the system level fluctuations.

So although you have a sort of a timescale involved with the averaging of ui prime and the timescale involved with the differentiation with respect to time. These are over 2 different scales and ranges. So they do not conflict with each other. So here when this is averaged, this averaging will give that this town will become 0. When you come to the third term, so let us look into the simplification of this third term a bit more carefully.

So if we consider this third term which is this particular term. So it is uj del del xj of ui bar averaging of that+uj… So we have just split it into 4 terms and once we have split it into 4 terms, it is possible to simplify it quite conveniently. First of all, when you do the averaging operation. This is like a constant for the averaging operation. It remains as it is. The derivative commutes with respect to the averaging and the average quantity is there.

So that means it just remains the same as it is. So this becomes uj bar del ui bar del xj. When you come to the next term, you see that you have one huge average which remains as it is after averaging, then the derivative commutes and it is taken to be independent of the averaging. So it is basically dependent on the averaging of ui prime which is 0 and therefore this term will become 0.

By similar argument, the next term will be 0 but the fourth term, there is nothing to believe that the fourth term will be 0. It might not appear to be so intuitive that why it may be non-0 but we just have to keep in mind that the average of u prime is 0, maybe average of u prime is 0 but what is the average of u prime v prime and that in general, there is no basis for us to conclude that it will be 0 always.

There are certain special cases in which this will be 0 but there are certain cases when this will be not 0. On the other hand, like if you are talking about the same variable u prime square with an average. So ui prime uj prime with j=I, that is anyway never 0 and we will see soon why. Now when you come the last term, see we may manipulate with the last term by adding one extra term.

What is that extra term? Let us say we add the following term. Let us say we add ui prime, this one. Adding this extra term is as good as adding 0 because from the continuity equation, we concluded that the fluctuation component of the velocity satisfies the continuity equation. That means this term is itself 0. So whatever is multiplied with that and averaged, that is also 0. The advantage that we have gained now is that you may combine these 2 terms and write it as nothing.

But this one, by the product rule of differentiation. So to summarise when we have simplified the left-hand side, we have got a term which is exactly of the same form as that what we have got even without averaging plus we have got an extra term here and this extra term is something that we have to keep in mind while simplifying it further. The right-hand side is something which is again much more straightforward.

So if you have the first term, the first term is same as itself because it is already averaged. The second term will be 0 because it will deal with averaging of single fluctuation. The third term will remain as it is and the fourth term will be 0 because it again involves averaging of a single fluctuation. So whatever we have got from the simplification after averaging, if we try to cast it in the form of the Navier-Stokes equation but now expressed in terms of a statistically averaged description, let us see how it looks.

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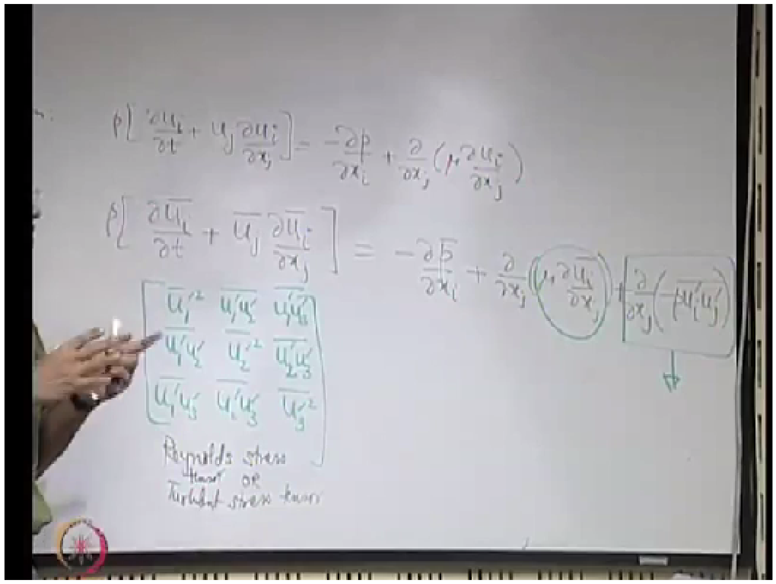

So we have the left-hand side, if we just keep it just as in the same form as the original Navier-Stokes equation, there was one extra term in the left-hand side because of averaging and that extra term we will bring to the right-hand side. Here since we have considered a problem where rho is a constant, so we can just take it inside and outside the derivative without any problem. The left-hand side you see, if the time dependence of the average velocity or the mean velocity is considered to be 0, that means it is considered to be a stationary turbulence or a steady turbulence.

It does not mean that the flow is turbulent, only statistically averaged behaviour is, it does not mean that the flow is steady. It just means that the statistically averaged behaviour is steady. So here this term, existence of this term or nonexistence of this term in either way it is unsteady. The unsteadiness is always there with the fluctuations but when you have averaged it out, then if the average quantity is independent of time, that if this term is 0, then we say that the turbulence is a stationary turbulence or sort of steady turbulence.

Steady turbulence is a misnomer, of course because turbulence cannot be steady. So sometimes stationary maybe a more convenient term but just it is a matter of terminology. Now if you see that what is the term in which this equation differs in form from the non-averaged Navier-Stokes equation and that is with this last term which actually was the extra term that was there in the left-hand side and we will try to understand first of all that what is the mathematical nature of the term.

And then we will try to get a physical feel of what is the physical origin of this term or what could be the physical origin of this term. So mathematically if you see that the term here if you consider this particular town term, this term has originated from the shear stress in a Newtonian fluid. On the other hand, the term which is there inside, we cannot conclude that it has originated from a shear stress physically.

But just by looking into the dimensions of this terms, we can say that this term also has a dimension of stress and that is one of the important mathematical characteristics of this term. Not only that, mathematically it has almost perfect resemblance with the stress because this also requires like the stress tensor 2 indices for its specifications. So the description of this ui prime uj prime average is given by a stress tensor which is known as a turbulence stress tensor or a Reynolds stress tensor.

It is a tensor of order 2 and the name stress tensor comes from the physical resemblance of the stress tensor that we get for a case when we are writing either the averaged quantities in this equation or the non-average form of the Navier-Stokes equation. So if we want to write the tensor in the form of a matrix with all its components, so you have say -rho if you take as common, so basically you have u1 prime square u1 prime u2 prime u1 prime u3 prime…, we can see that there are 6 independent components.

And this is a symmetric tensor, just like the usual stress tensor. So this is known as the Reynolds stress tensor or the turbulence stress tensor. Why turbulence stress? Because like this stress tensor has originated because of the turbulence in the flow, because of the turbulent fluctuations in the flow. The important question is, that how will you treat this components of the tensor mathematically and it is actually a very involved problem.

So to understand what is actually the involved problem, let us look into the equation. We started with the Navier-Stokes equation. We knew that the Navier-Stokes equations are very much valid for turbulent flow. Of course, if the other assumptions like Newtonian fluid and Stokesian fluid, they are satisfied but it is not so easy to deterministically obtain the variables from the original form of the Navier-Stokes equation because of the uncertainties in maybe boundary conditions or say initial conditions like that.

So what we concluded is that they are statistically averaged forms are so-called deterministic and therefore we statistically average them. One good thing of the statistically non-average form of that, the system of equations was closed. So number of equations and number of unknowns were matching with each other. Now we have new sets of equations where the variables are very much deterministic, the statistically averaged ones, but the equations are not closed.

Because you have now come up with many new unknowns through this term. So you have 6 extra unknowns and there is no magical way by which you can have 6 extra equations for this 6 unknowns. Of course, you may try to write 6 extra equations but that might give rise to another new set of unknowns. So closing the number of equations with number of unknowns, is one of the very toughest things in statistical averaging.

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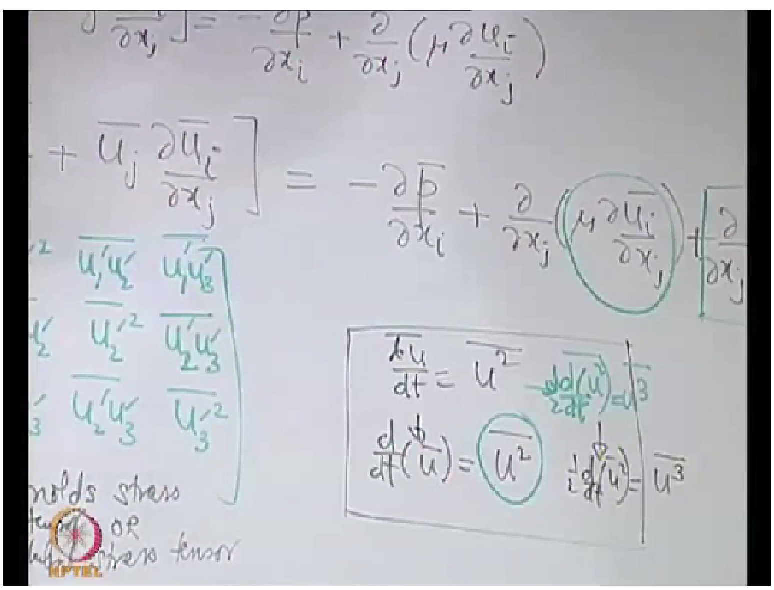

To get a fair idea of this problem, let us consider a very simple type of equation which is this one but just for mathematical analogy. Say you have a equation dudt=u square, okay. Of course, this equation has a very simple solution, if you give the initial condition at time=t0, what is u and you can very easily integrate it, one of the simplest differential equations. But let us say, you do not want to solve it in this way, you want to solve it with statistical averaging.

So then what you do, you average the left-hand side, you average the right-hand side. So what you get out of this, you get the statistically averaged from of the equation as ddt of u average=u average square. The problem is that with this averaging, you have given rise to a new variable, u square average. Of course, you may think that well I will solve it by introducing a new equation and that new equation you may introduce say by multiplying both the sides with u square.

So then what you get? Say you have dudt=u square. So if you multiply both the side with u, you will get ddt 1/2 of ddt of u square=u cube. Just multiplying both sides by u, non-average form. Then if you average it, yes you get a governing equation on average u square but you get a new variable average u cube. So every stage, you try to do statistical averaging, you are finding it tremendously difficult to close the system of equations and that is what is precisely happening here.

So you have after Reynolds averaging in the equations certain terms for which you do not have automatically explicit governing equations and therefore, number of equations and number of unknowns or number of independent equations and number of unknowns are not matching and the system of equations is not closed. So that is known as the closer problem in turbulence. This is one of the biggest nightmares that one has to deal with.

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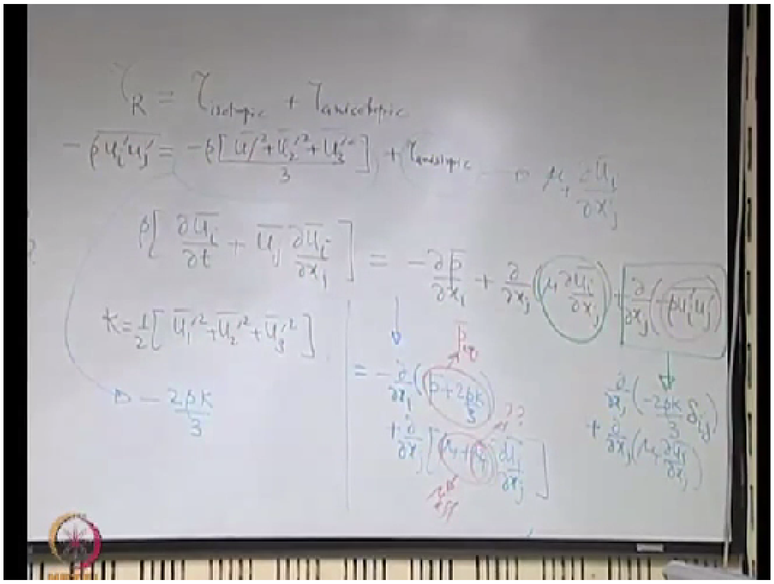

So the closer problem in turbulence is that basically you have certain additional terms in the governing equation after Reynolds averaging and these additional terms do not have explicit governing differential equations. So you have a mismatch between the number of independent equations and number of unknowns.

So only way in which, you may patch up this gap is to make a mathematical or a physical model by which you postulate this term as an equivalent of some other term which you may know or which you may pretend to know by some sort of modelling and that modelling will not be something which is exactly the physical reality. But it might be an approximation of the physical reality and that is the job of the modellers in turbulence flow analysis and that is the job of turbulence modelling.

We will not go into the details of turbulence modelling, that is not within the scope of this discussion but what we will try to do is we will try to see that what is the approximate way in which these term may be modelled. To do that, we will have a deeper look into this Reynolds stress tensor and try to draw an analogy of this with our usual stress tensor.

So if you recall that in our usual stress tensor, we had the diagonal terms and off-diagonal terms and the diagonal terms were representative of the so-called the normal components and the off-diagonal, the sheer components of the stress and when we derive the constitutive relationships, we decompose the stress tensor into 2 parts.

One is the hydrostatic component which was obtained by sort of averaging the diagonal terms and that was manifested in the form of a pressure which is acting equally from all directions and a deviatory component or the deviation from the hydrostatic component and deviation from the hydrostatic component is something which is important for modelling the constitutive behaviour because that is something which gives rise to the relationship between the stress and the deformation.

Because that is what is responsible for the constitutive behaviour in terms of the stress deformation relationship. So similarly here also you may decompose this stress tensor into 2 parts. One part is obtained from the diagonals of the terms, of the diagonals of the stress tensor, the diagonal of the stress tensor and that is like the equivalent to the hydrostatic stress tensor component.

Here we do not call it hydrostatic because it does not have the same meaning. We call it isotropic component of the stress tensor and whatever is remaining contribution from the stress tensor, we call it the anisotropic component of the stress tensor. So if we give the stress tensor a name say tau, say Reynolds, so it has 2 parts, isotropic and anisotropic. it is just equivalent to the hydrostatic and deviatory and the anisotropic part should be responsible for the description of the equivalent constitutive behaviour in terms of the average quantities.

So when you write the isotropic part, isotropic part is basically -rho ui prime square+, sorry u1 prime square+u2 prime square+u3 prime square averaged/3, that is the mean of the terms in the diagonal and the remaining is the anisotropic. So the Reynolds stress tensor is -rho ui prime uj prime averaged, it has the isotropic part and an anisotropic part. The isotropic part, let us see how we write the isotropic part.

We may write the isotropic part through a definition known as the turbulent kinetic energy. So turbulent kinetic energy is sort of represents the kinetic energy because of the fluctuation velocities in the flow. So that is defined as just 1/2 of u1 prime square+u2 prime square+u3 prime square. Therefore, this term which is there in the brackets, it may be written in terms of what?

It may be written as, you can write this one as -2rho K/3. What about the anisotropic part? Now this anisotropic part, we may write or we may describe in terms of what? We may describe in terms of an equivalent constitutive behaviour and just by analogy of the form, let us say we want to write it as some equivalent viscosity which is not actually a molecular viscosity but some equivalent characteristic*the average rate of deformation.

Once that is done, you may substitute this extra term as, this is one part which is the isotropic part. So that is why this delta ij is there. This is only there if j=I and plus… So if we combine these terms, we can write the right-hand side and the right-hand side becomes… Cosmetically this is a greatly relieving form of the governing equation because it assumes a form virtually the same as the non-average form of the Navier-Stokes equation.

What are the important changes? One important change is instead of the pressure, you have the average pressure+a term dependent on the turbulent kinetic energy and let us say we call it p average equivalent. So if we were solving it in the same form as that of the Navier-Stokes equation, no problem but whatever p we get, we have to understand that it is not p average but equivalent with some term because of the kinetic energy.

The other important observation is that, the mu in the original Navier-Stokes equation is replaced by some equivalent mu which is mu+mu t or you may call it mu effective and this mu t which is known as the turbulent viscosity is contributing to this mu effective and just this form, this form was originally introduced by (()) (40:40) and of course this is just like a hypothesis. It is not that this is exact and we have discussed that what is the philosophy.

The philosophies are disparate act of closing the system of equations and then of course it might be written in this form because it is an acceptable constitutive form, you have seen that it satisfies the basic requirements of continuum mechanics at least in form but the big question is now the mu t is something which is not known. So apparently it looks as closed but it is not a closed system of equations because you have a mu t which is not a fluid property which depends on certain conditions in the flow in terms of the statistical averaging of the fluctuations.

And therefore there might be an additional description or there must be an additional description or modelling of this mu t which is one of the important jobs for turbulent flow modelling. So the big understanding is that while we were trying to close the system of equations, we came up with certain extra terms and we are seeing one of the approaches by which this extra term maybe cast in the form of the original stress tensor form of the Navier-Stokes equation.

But it gives rise to some extra term in the equivalent viscosity definition and that extra term comes out of turbulent fluctuations and their averaging and therefore, it is not so straightforward to have a description of it without having more involved considerations. Now keeping this a background, we will now see that what are the consequences of this fluctuation velocities and the big question that we did not answer till now, is that why this quantity will be non-0.

Because if this quantity is 0, one of the big problems is resolved and so we have to understand that what is happening which is not making it 0 and clearly we have to distinguish between the terms which are like the isotropic components and the anisotropic components. In this context, we will just remember that sometimes this fluctuation kinetic energy is expressed as a non-dimensional parameter in terms of the kinetic energy or the mean flow at the inlet of a pipe for example.

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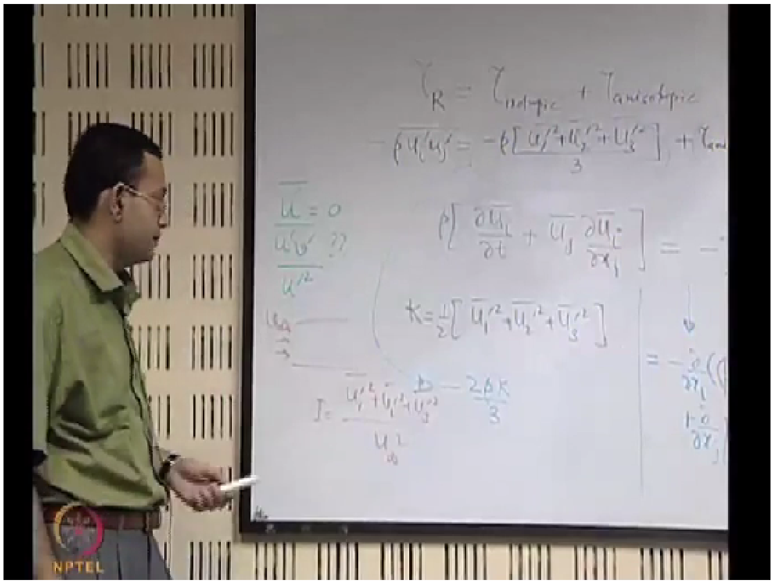

So if you have at the inlet of a pipe some flow with the velocity say u infinitely, then you may have likes u1 prime square+u2 prime square+u3 prime square/u infinity square as a non-dimensional description of the or non-dimensional measure of the inlet turbulent kinetic energy or may be at anywhere and this is known as turbulence intensity and many times, it is considered to be as a fitting parameter for different mathematical models, may be 5%, 10% that is what is commonly taken.

Now the other important point is, the physical description of this ui prime uj prime, and to do that, we have to keep in mind that we are now having to deal with situations which have some isotropic component and which have some anisotropic component depending on whether i=j and whether i != j. So let us try to revisit the important definitions of isotropic turbulence and a related quantity, homogeneous turbulence.

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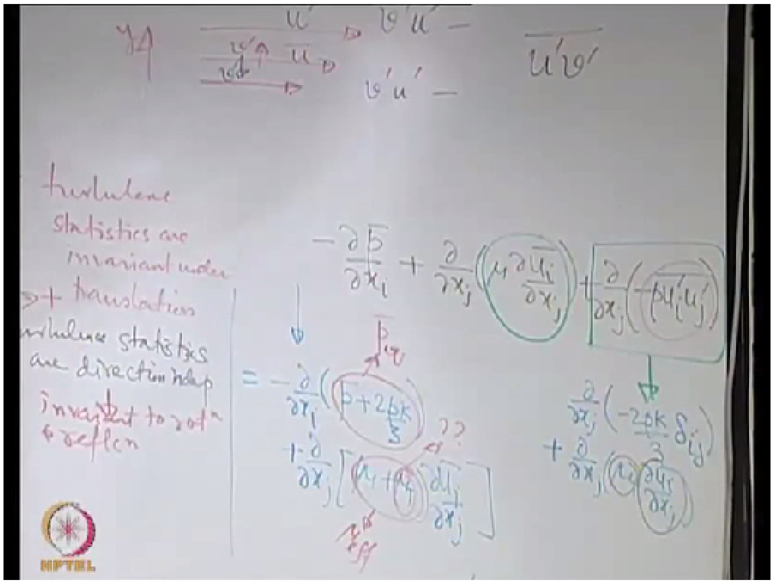

So when we discussed about homogenous turbulence, this terminology at least we introduced in the previous lecture and what we remarked is that, if you have a homogeneous turbulence, it means that the turbulence statistics are independent of position. So that means if you have say u1 prime square at a point, then if you go at different points, you will get the same turbulence statistic.

That means turbulence statistics are invariant under translation. When we say invariant under translation, questioning is translation of what? Obviously translation of the coordinate system by which the turbulence statistics are described. So when it is invalid under translation, that means no matter you translate the coordinate axis to a different location and find out the same quantity in a statistically average sense, they are the same. It does not mean that u1 prime square is same.

It means the statistical average of u1 prime square is the same at all locations. Now when you go to isotropic turbulence, isotropic means as we discussed earlier, direction independent. that Means the turbulence statistics are direction independent and when we say they are direction independent, it means what? Direction independent means that if you have a set of coordinate axis say x1 x2 x3 and you have say u1 prime square u2 prime square u3 prime square like that some statistically average quantities, there could be many such quantities.

Now when you have a rotated coordinate system with x1 nu x1 like this, nu x2 like this and may be nu x3 like this. Of course, orthogonality of the axis that is preserved. Then if you have the u1 prime square with respect to nu x1 u2 prime square with respect to nu x2 and maybe u3 prime square with respect to this nu x3, they are the same. So the statistical description is rotation invalid.

Again it does not mean that u2 prime square is the same. It just means u2 prime square when statistically average remains the same. Just like that. So not only rotation, also reflection. So turbulence statistics are direction independent means invariant to rotation and reflection of coordinate axis. Now another important catchword is that for when we are describing isotropic turbulence, in the special case, we are also having a constraint that it must also be invariant under translation.

So that is an additional constraint over and above this requirement. So this plus it has to be invariant to translation and from this we may answer the question that we asked in the previous lecture, we asked to ourselves that does it mean that homogeneous turbulence has to be isotropic, number 1 or does it mean isotropic turbulence has to be homogenous. We can clearly see that isotropic turbulence must be homogenous because it also has to be translational invariant, the statistics.

On the other hand, there is no necessity, there is no guarantee that homogeneous turbulence will be isotropic. So that is one important thing we need to remember. So with this understanding of isotropic and anisotropic, now what we will do, we will consider that how this ui prime uj prime terms are coming. So let us say we want to describe this with an analogy of the origin of shear stress that we discussed when we are talking about viscosity in one of our earlier lectures.

If you recall, we introduced about 2 important physical origins of viscosity. One is the transfer of molecular momentum between different fluids layers and the other is intermolecular forces of interaction. Now we are trying to draw an analogy with the transfer of molecular momentum and here instead of molecular momentum, you may just consider the exchange of momentum between Eddies present in the turbulent flow.

So let us say that you have 3 different layers with a velocity gradient along a particular direction. Say you have an increasing velocity gradient in the y direction. Let us say that this layer has a velocity along x and u average along the x direction. Now there is a fluctuation velocity component which is like v prime of that layer. The average flow may be one directional or 1-dimensional but the turbulent flow is always 3-dimensional and unsteady.

Only the average quantity may be 1-dimensional or 2-dimensional or steady. So because of these fluctuations, what is happening? The fluid element, there is some fluid element which is joining the top layer. What it will do with the top layer? It will try to reduce the velocity of the top layer. Why? Because it is coming from a layer which is having a reduced velocity. So it will try to exchange that reduced, exchange that momentum with the top layer and reduce the moment of the top layer in the process.

That means because of a positive v prime, you have a negative u prime. That is the fluctuation in u that is being created because of a v prime from a slower moving layer to the faster moving layer at the top, the act is to have a reduction in u. So therefore, calculate u prime v prime or v prime u prime, whatever, this product will be negative for this case. Positively v prime negative u prime.

Similarly, you may have a v prime along the negative y direction. That is a fluid element try to exchange momentum with the bottom fluid layer. So when it tries to do that, that it will do? With a negatively v prime, you will have associated positive u prime. So again v prime u prime is negative. So in both cases v prime u prime is negative. When it is moving to the top and when it is moving to the bottom.

Therefore, we can say that when you statistically average this u prime v prime, all the negatives add together and it is not the sum is, the statistical average is not 0 in general. So that is why see when we wrote this -ui prime uj prime where i != j, when i != j means the deviatory component, we wrote it in the form of mu t* this velocity gradient.

Here velocity gradient along y is positive and therefore mu t has to be positive because ui prime uj prime when average when i != j, is itself negative, that negative with the minus becomes plus. Here del u del y in the average sense is positive and therefore mu t has to be positive. However, the situation may change dramatically if you have this type of a situation.

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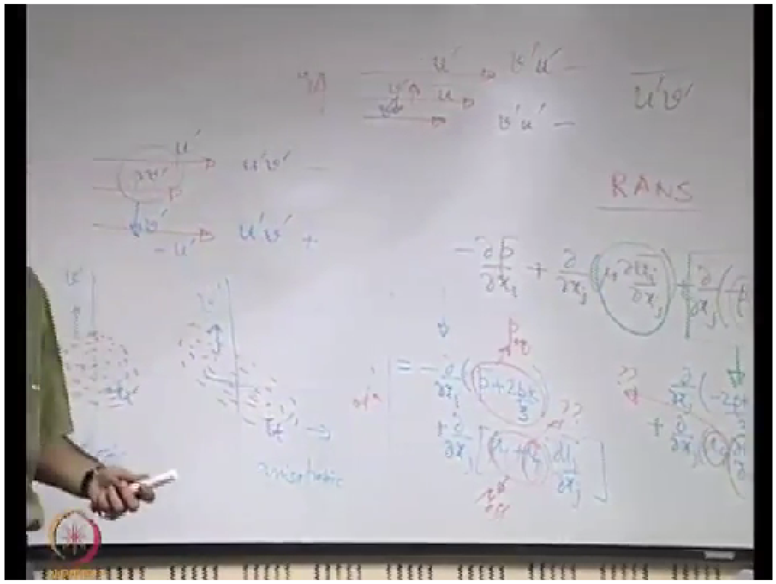

Let us say that you have a situation where you are having the central fluid layer like this. The upper fluid layer like this, the lower fluid layer like this, perfectly symmetrical. This is like as if y=0. Now in this case if you have a positive v prime, it will associate it with the negative u prime. So u prime v prime is negative for this interaction. Let us consider a similar interaction when it comes to a negative v prime and it interacts with the lower layer.

So here a negative v prime is associated with what? A negative v prime is associated with a negative u prime because it comes from a slower moving layer, so it tries to slow it down. So negative v prime and you have a negative u prime, so u prime v prime, product is positive and if it has a sort of a symmetry in the distribution, then these 2 effects may nullify each other for such pairs so that the sum total is 0 and that when it becomes a case that is ui prime uj prime average becomes 0 if i is != j, then that is what is isotropic turbulence.

Because then in the turbulence stress tensor or the Reynolds stress tensor, you only have the diagonal terms. All the off diagonal terms are 0 and the diagonal terms are because of the isotropy and the off diagonal terms are because of the anisotropy. So we can clearly see that what are the possibilities? The possibilities are that when this ui prime uj prime they are averaged, either they may be 0 when it is isotropic for i != j or the average maybe negative and if you just try to draw this in terms of a scatter diagram and try to represent it.

Say we are plotting v prime versus u prime in 2 cases. One is isotropic, another is anisotropic. So when you have isotropic, you see let us consider one case where you have say a deviation from with the considerations say u prime=0. So if you have a case with v prime as some positive quantity, you also have an equally probable case with v prime as negative of the same because the average of u prime v prime taken over all possible value should be 0.

So if you draw a scatter diagram with such cases of points, so these points are what? These points are scattered data. This scatter diagram in the form will look like a circular description and the reason is straightforward that it does not have any bias towards coordinate axis. So you rotate the coordinate axis, you will have a new u prime, you have a new v prime, still orthogonal to each other, it is perfectly circularly symmetric.

So this will be the scatter of the data. So on an average, u prime v prime average will be 0. However, when it is an isotropic, the scatter diagram, the scatter of points maybe like this and why it is scattered like this? Because if you calculate the correlation between the 2, we have seen that u prime v prime average has a negative correlation and that is why if you fit a regression line, it will have a negative slope and that is why this will be the sort of scattered diagram for the anisotropic one.

So by looking into the scattered diagram of u prime v prime or the correlation between u prime v prime, u prime v prime was just 2 examples of ui prime not equal or i != j, ui prime uj prime and by looking into this type of diagram, it is possible to have a clear picture on the extent of isotropy or extent of anisotropy in the description of the turbulence in the flow. No matter whether it has a whole lot of anisotropy or a whole lot of isotropy.

The big question will remain is that, how we reflect that within the description of this new unknown that we have introduced in the Navier-Stokes equation and the average when we say the Navier-Stokes equation, we mean the averaged Navier-Stokes equation or the Reynolds average Navier-Stokes equation. So the equation that we are talking about now is known as RANS or Reynolds Average Navier-Stokes equation.

So in the Reynolds average Navier-Stokes equation, now we have a situation where we have an extra term, the extra term is because of what? It is because of the momentum exchange between Eddies with different fluctuations. It is no more molecular momentum exchange but turbulent momentum exchange but has a sort of analogy with the molecular momentum exchange and as we discussed earlier, the molecular momentum exchange has also a sought of similarity with the kinetic theory of gases, that is exchange of momentum between gas molecules.

And therefore by drawing analogy with the exchange of momentum amongst the gas molecules and the exchange of momentum between the fluctuation components of Eddies, it is possible to have some simplistic description of how to go about to describe this turbulent viscosity and that may be achieved by a very simple but phenomenal engineering model known as Prandtl's mixing length model, that we will do in the next class. Thank you.