Introduction to Fluid Mechanics and Fluid Engineering
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Lecture – 34
Introduction to Turbulence (Contd.)

In the last lecture, we were trying to assess the different lengths scales of the different size Eddies. So before going to a formal assessment using the scaling relationship that we have established, let us now look into some visual demonstrations of how these Eddies might look like.

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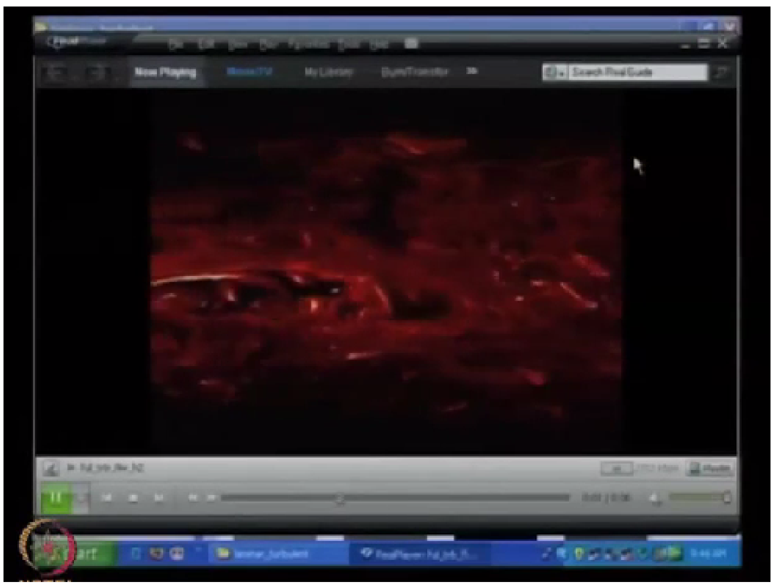

So just look into these. Some of these are simulated flows but just look into the rotating structures and you will see that these rotating structures are having a wide range of length scales.

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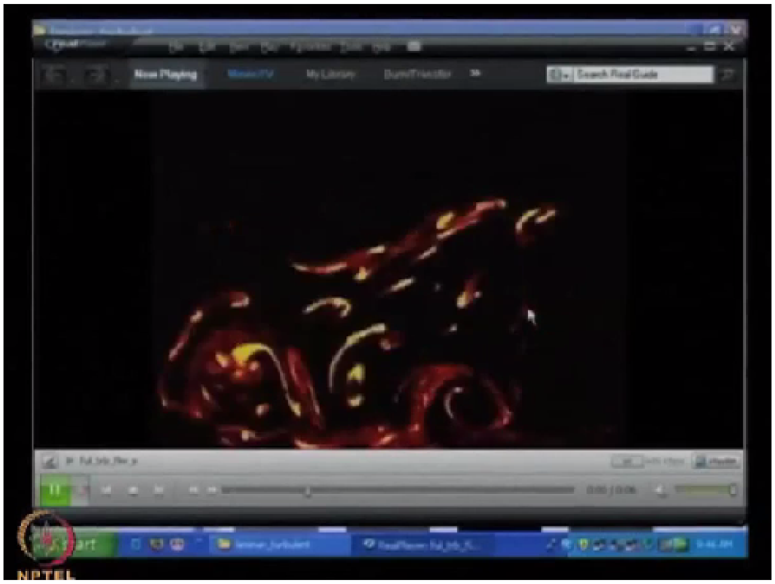

So if you see that they are really having a wide range of length scales and they fluctuate over a wide range of time scales.

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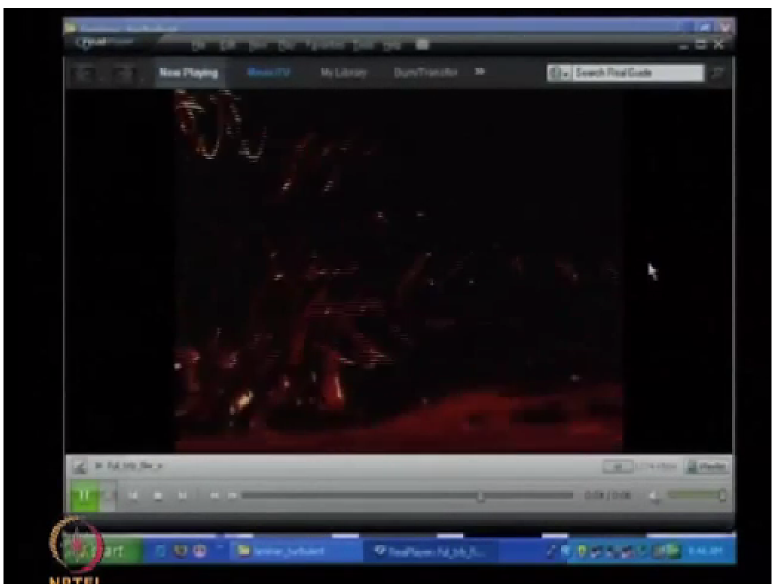

So we will just go on looking into some of these types of visual demonstrations to figure out the roles played by the Eddies.

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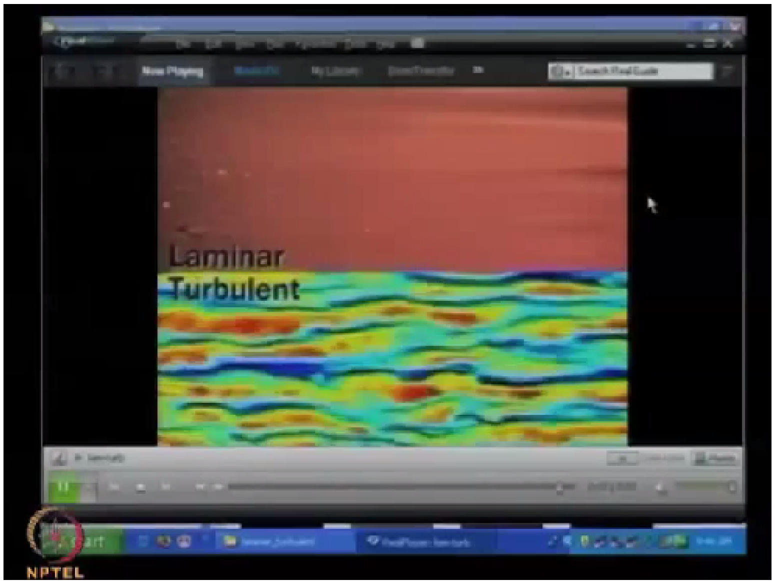

So just see the roles played by the Eddies which are not there in the laminar flows. So these Eddies are turbulent Eddies. May be couple of more ones.

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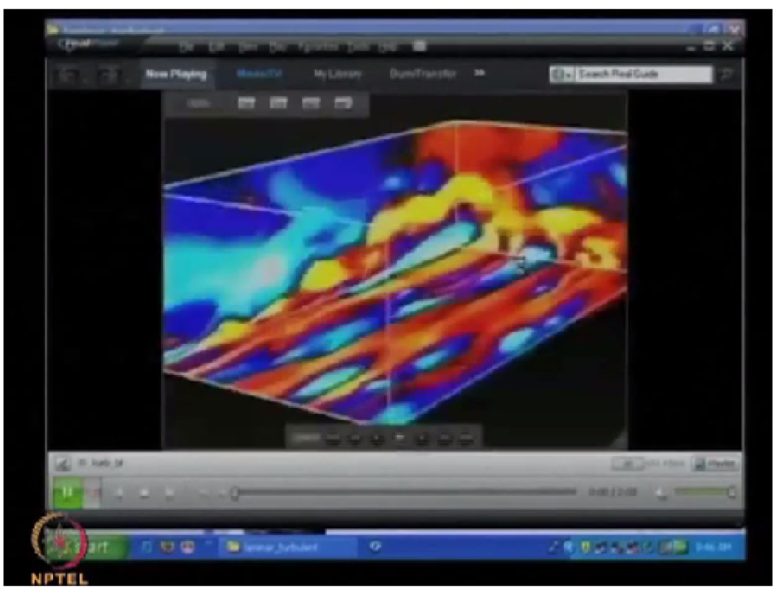

So these are 3-dimensional visualisations. So all these have been generated by computer simulation. So you can see, visualise the structure of these Eddies. We will just pass it a bit fast and try to figure out. If you want to see the Eddies in different planes, so you can see that the structure in Eddy, structure of the Eddy is changing from one plane to the other. So it clearly gives us an indication that there is nothing called a 2-dimensional turbulent flow.

Turbulent flow is always 3-dimensional and unsteady fundamentally. So that is the first understanding that we develop out of this. So at all different sections and at all different planes, you see these different characteristics of this rotating structures and these rotating structures are continuously evolving with time. That is also one of the important thing. So you have not only a wide range of length scales but a wide range of timescales and we will try to have an estimate of these ranges of length scales and timescales.

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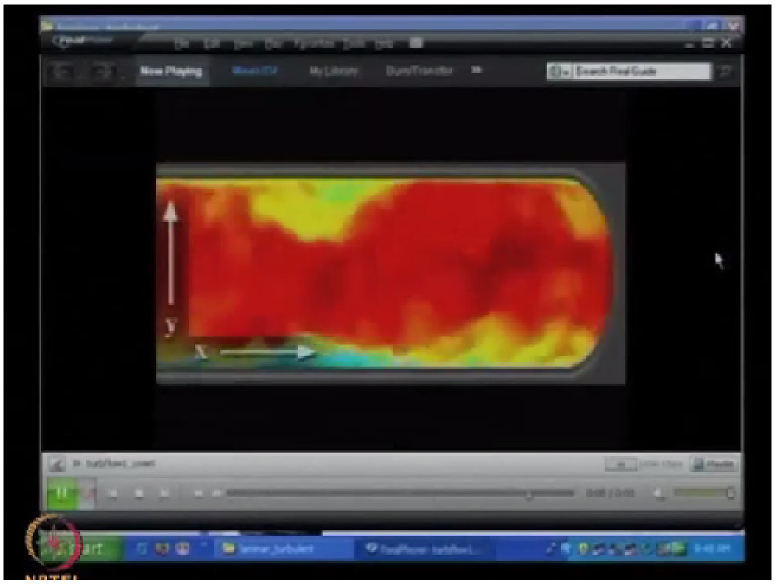

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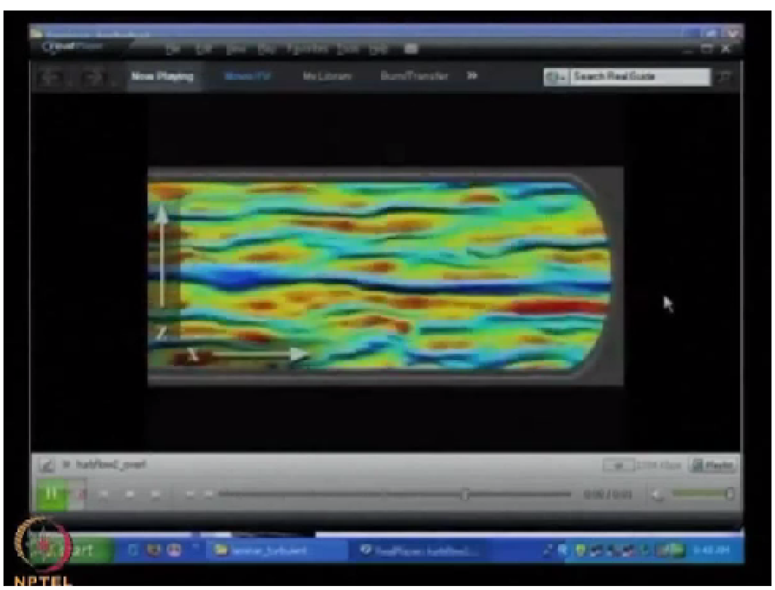

So the whole idea of this understanding was to have an appreciation that you may have wide range length scales of the Eddies and to quantify that, let us say that we are now interested to get a feel of the difference between the system length scale and the Kolmogorov length scales or the smallest Eddies length scale.

So the system length scale or the largest Eddy length scale sometimes known as integral length scale. So let us see that what is this. Let us say that l is of the order of 1 meter, this is an example. Just we are trying to take good number so that we come up with easy estimates. So the system length scales say you have 1-meter system length scale, the largest Eddy is also of that length scale.

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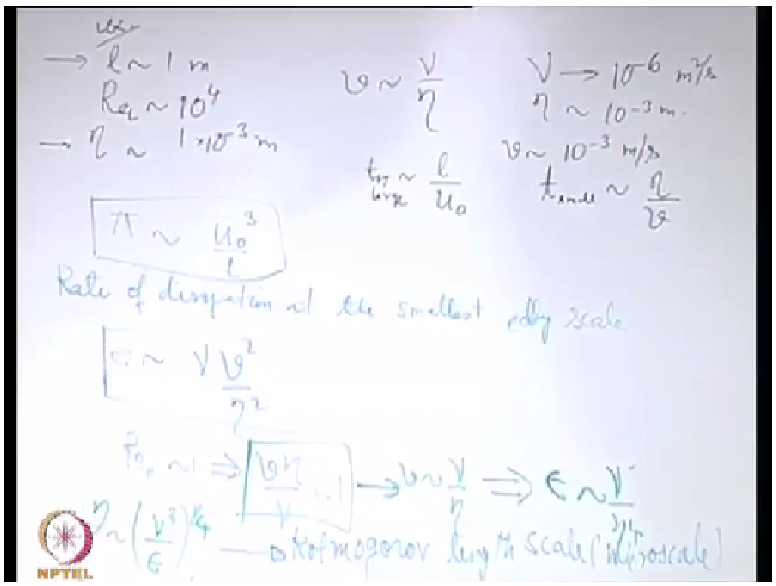

Let us say that the Reynolds number is in that is 10 to the power 4. So then what will be eta? So 1 meter*Reynolds number to the power -3/4th. So 1*10 to the power -3 meter, right. So if you make the Reynolds number larger and larger, the disparity between l and eta becomes more and more. 10,000 is not a very large Reynolds number. It is just like a moderately large. So if you make the Reynolds number really very very large, this disparity will be more and more and you have Eddies at almost all intermediate length scales between these.

So that is what we say that the existence of multiple length scales. Not only multiple, a wide range of length scales differing in order of magnitude by at least 1000. You see the order of magnitude difference. Similarly, if you look into the timescales and the velocity scales, so the velocity scale in the smallest Eddy. So velocity scale in the smallest Eddy, how do you estimate? The velocity scale is of the order of nu/eta from the Reynolds number scale=1.

So the kinematic viscosity, it is roughly like say 10 to the power -6-meter square per second for water, mu/rho, 10 to the power -3/10 to the power (()) (05:39) and if you take eta as say 10 to the power -3 meter, then you come up with a V of the order of 10 to the power -3 meter per second. These are small velocities and not only that, if you look into the system scale velocity, that is quite large.

So the system scale velocity that is u0, that is governed by the system scale Reynolds number and that is quite large. The timescale, so the timescale for the system, it is l/u0, or the large Eddy. For the large Eddy, is of the order of l/u0 and for the small Eddy, that is the Kolmogorov timescale, so this V is Kolmogorov velocity scale. So if you consider the timescale for the small Eddies, that is sort of eta/V.

So it is possible to have an estimate of the timescales and the length scales and the velocity scales. The other important aspect of the large Eddy and the small Eddies of the distinctive aspect is that; the large Eddies have a sort of directionality or a directional preference. Because they are large and they have some preferred directions over which they have their activities. On the other hand, smallest Eddies have no directional preference.

And the distinction therefore is that the largest Eddies are very much anisotropic. So they do not have like isotropy or a direction independence type of behaviour. On the other hand, if you go to smallest Eddies, they are virtually isotropic. So it is not that they are actually isotropic but they are approximately very much isotropic. So the transition of paradigms from the largest Eddy to smallest Eddy is also in the form of big anisotropy to a reasonably good state of isotropy.

And that is possible because the Eddies will tend to become more and more isotropic as and when they are able to dissipate whatever energy is being transferred to them through viscous effects because viscous effects sort of tries to equilibrate it in all possible directions. So viscous effects are stronger and stronger for smaller and smaller Eddies and that is why as you go towards smaller and smaller Eddies, the dissipation effect makes it more and more isotropic or direction independent.

Now we will try to understand another important thing that see these Eddies are having rotations and when they have rotations, they must have vorticities. So we will try to see that how these vorticities evolve for these Eddies. We will try to develop a sort of governing equation for vorticity and we will try to understand that qualitatively by understanding the relative interaction between the large Eddies and small Eddies and so on. So let us say that we start with the vector form of the Navier-Stokes equation.

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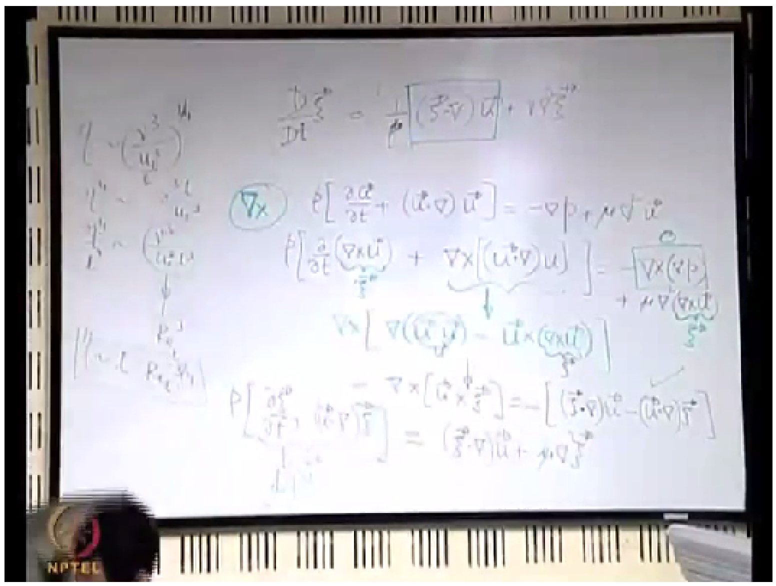

So let us say that we have rho… So this is like the momentum question in the vector form which we derived. Now what we are interested to do is to get expressions for vorticity out of that. So we know that vorticity is the curl of the velocity vector. So let us take curl on both sides of this equation so that we have a chance of coming up with a vorticity. So just take curl on both sides. So if you take the curl on both sides, then what happens?

First term, so curl is a vector operator with respect to the spatial gradients. So with respect to time, you may just take it inside outside without any problem. Then…, okay. So now let us try to simplify this. So for simplifying this clearly, we understand that this is equal to the vorticity vector, let us call it zeta. There is another term which we can, of course this is also zeta. There is another term which we can clearly simplify.

What is this? This is 0, right. This is the curl of gradient of a scalar. So curl of gradient of a scalar by a vector identity, this is 0. So we have now to concentrate on this term. So it is curl of, now let us write u.del u, so what is this? This is by the vector identity…, this one, right. So this is just we have written a vector identity. So when we have written this vector identity, so let us first consider the first term.

So this is like, this is a scalar, u square/2 or just u square. So when you write, so this is a dot product of a vector with a vector, right. So when you write this, one important thing that you are getting out of this is whatever it is, it is a scalar. So you have curl gradient of a scalar. So the first term becomes 0. Therefore, this falls down to -curl of this one because this is the vorticity vector and this is as good as… so the left-hand sides become what?

Rho… one of the terms that is this term will retain in the left-hand side, the other term, we bring in right-hand side. So the right-hand side becomes… So can you identify what is the term which is there in the square bracket in the left-hand side? This is the capital DDT of zeta, the total derivative of zeta. So we have got a transport equation of vorticity by starting with the Navier-stokes equation.

And let us just write it in terms of the kinematic viscosity if you divide both the sides by the density, then you have this one is equal to…So you can clearly see that this is what if you have a vortex element, that is an element within which there are elements of vorticities, then this vortex element may have a change in vorticity, so the total derivative is representing what? This is a change in vorticity of an element because of a combine effect of change in time and change in position in going to a different place where the velocity field is different.

Therefore, it is subjected to a different velocity gradient at a new location and with respect to time also, there has been a change. The total effect is a combination. **“Professor - student conversation starts**” Yes. (()) (15:53) sorry 1/rho. **“Professor - student conversation ends”** So the total derivative of the vorticity is what? You have one term in the right-hand side. This is quite straightforward to understand, that is the second term. The second term represents what?

It represents the viscous dissipation of vorticity so to say. So there is sum total rate of change of vorticity. It is related to something which is viscous dissipation but something else also which is not viscous dissipation and we will try to understand that what is this something else or what is the implication of this term and that we will do in a very simple and qualitative manner. So when we do it in a simple and qualitative manner, we will again go back to the picture of the large Eddies and the small Eddies.

So when you have a large Eddy, for a large Eddy, the Reynolds number is large, right. So for a large Eddy, the inertia force is much much greater than the viscous force. For smaller Eddies, the viscous forces are also there. Now if you just simply in a rough way model an Eddy, you can say that a vortex element, the rate of change of angular momentum of a vortex element, let us say I omega.

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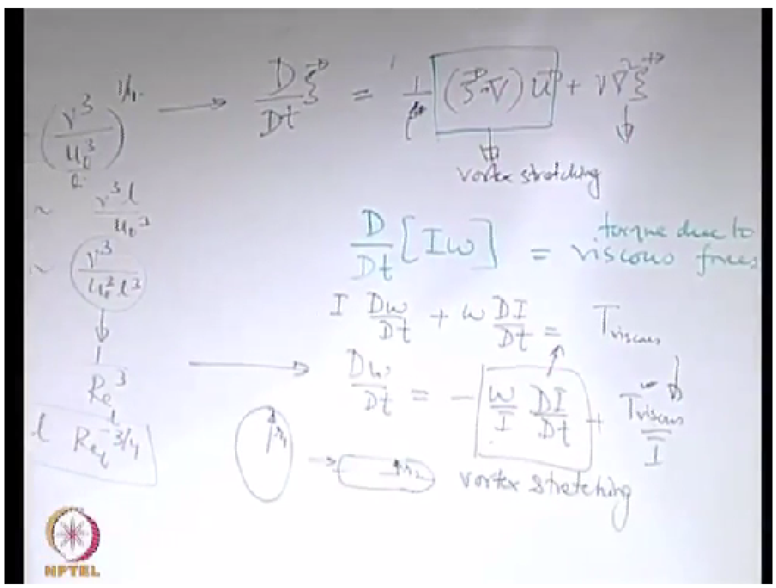

When we say the rate of change, it is like we are writing the total derivative because the state of change may be because of many things. So this is as if we are tracking a fluid element which is going from one place to the other because of change in time and change in position, combine effect is the sum net rate of change of angular momentum and that must be equal to the viscous, the torque due to viscous forces.

So here because whatever force what we are seeing is the torque, is the viscous force that is what is a forcing parameter. So this you can write as ID omega Dt+omega DIDt=the viscous torque, I am just writing it symbolically. Therefore, D omega Dt=-omega/IDIDt+viscous torque/I, okay. Let us consider the large Eddies. So what happens for the large Eddies? Those are very special cases when the viscous effects are very very negligible because for the large Eddies, the Reynolds number is very large.

So for the large Eddies, the angular momentum is conserved. Now when the large Eddies are extracting energy from the mean flow, what is going to happen? There angular velocity will increase. So omega will increase. But if I omega has to be conserved, then I should decrease. So I should decrease means, there sort of radial length scale should decrease and therefore if they were more or less, if the large Eddies were more or less like this, the small Eddies, I mean their subsequent transformation to preserve the angular momentum will be of a shape which is if you consider this as a radial dimension, this is r2, say this is r1.

So r1 will come down to a lower r2 but if it is a same large Eddy, the volume has to be conserved. So if the radial length scale has decreased, the lateral length scale should increase and therefore the vortex element or the Eddy has got so-called stretched. This is known as vortex stretching. So vortex stretching, vortex stretching is, one should not create a misunderstanding that vortex stretching is only for the largest Eddies, not like that.

But we are giving an example of a very large Eddy where viscous effects are negligible just to give a clear relationship between what is the change of the moment of inertia, how it is related to the angular velocity. So if you have now Eddies which are rotating at a higher speed, they must sacrifice it in terms of having a lower moment of inertia so that I*omega I is conserved and when you have that, that means it is sort of stretched to have the same volume.

But now with a lower radial scale so that the moment of inertia is now less. Now what happens for a case when viscous effects are present? Qualitatively same. Because if you see that when this viscous effects were not there, you see D omega Dt. So D omega Dt is the rate of change of the angular velocity and you see that is related to -DIDt. So on increment is another's decrement and now the viscous torque will also play an additional role.

Now if you come back to this equation, see there is a lot of similarity between this equation and this. This is a differential equation, this is a very qualitative, this is also like some sort of differential equation but not vigorously derived. So it is just by putting terms qualitatively and what we see here is that if you consider the left-hand side, see this is the total derivative of vorticity, it is almost very closely related with the total derivative of the angular velocity because the angular velocity is half of the vorticity vector.

So these 2 are related, the viscous terms, these are related; therefore, whatever is this term and whatever is this term, these 2 must be carrying the same meaning. That means, so what does this term indicate? This term represents the effect of vortex stretching. With a larger omega, you have a decrement in I. Therefore, this term in the vorticity the storming transport represents a vortex stretching.

So vortex stretching is one of the very important activities that is taking place in a turbulent flow structure and it can be shown that this effect is important there only for a flow with the 3-dimensional structure and the turbulent flow has only a 3-dimensional structure. On an average, it might be 2-dimensional or 1-dimensional, or whatever but fluctuations are there in all possible directions. So even if the mean flow is 0 in a particular direction, but you still have fluctuations in all possible directions.

The question is how we quantify these means and the fluctuations? The next thing where we will go to is the statistical description of turbulence.

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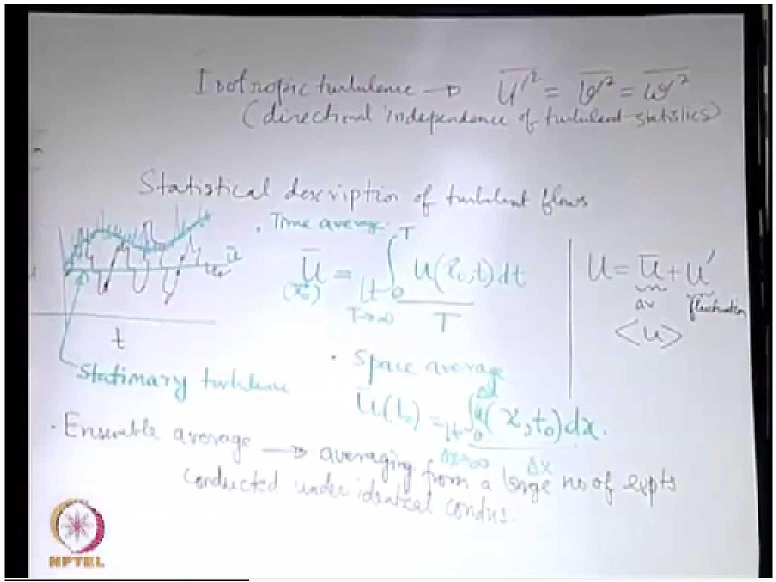

So when we say statistical description, that means there is some uncertainty which we want to show, which we want to express by some sort of averaging or finding the standard deviation, these types of parameters. So one of the most important or most fundamental and sometimes considered to be the easiest statistical description is averaging. So we are now going to discuss about some concepts of averaging in a turbulent flow.

So to have a qualitative picture, let us say that you are plotting a velocity as a function of time. So if you are having a turbulent flow, may be you are having this type of random fluctuation in velocity as a function of time. It might so happen that you are not interested about this randomness, you are interested to see that how it is on an average but the question is if you want to find out average over a given period of time, what is that time?

So the average over a given period of time say if you want to find out u average, so u average is what? You must indicate u with respect to time over a time interval, say a time interval of, say time=0 to time=capital T/t and take some limit. Formally it is written as limit capital T tends to infinity. What is the meaning of this infinity, that we have to understand? So this is a formal definition of something known as time average.

So time average at a given location. So time average at a given location, that means this u is a function of a given position, say x0 and this u what we are writing inside is a function of both position and time. So u as a function of x0 and time. The time effect has got nullified by integration with respect to time. So this is only at a fixed position. So now this timescale, what is this timescale?

See the turbulence fluctuations have very small timescales. So you can see that over very short time, it is having a very rapid fluctuation. So if you want to average it out, you must take a timescale which is much larger than the characteristic timescales over which the turbulent fluctuations are there so that turbulent fluctuations are averaged out (()) (26:54) and that means that with respect to this turbulent fluctuations, these timescale is like infinity, very large.

So this infinity is not in the literal sense. It is with respect to the locally fluctuating timescales but this should also be much less than the system length scale, the system time scale. The system timescales set out from here to here. There is some change. So now if you consider this timescale as the entire system timescale, then you will not be able to capture the transiences in the average sense.

So that means you will lose all the information and come up with a single value so if you average over the entire time. So the time period over which you are averaging is very critical. It should not be too small so that still it is within the range of the individual turbulent fluctuations but it should not be too large so that you can resolve the transiences on an average. So it should be something in between.

It should not be as large as the system timescale bit it should not be as small as the individual turbulent fluctuation time scale. So if you now make a sort of this type of averaging and plot u bar, then it is possible say you get the u bar like this and u bar maybe in this example, u bar is not changing with time. So this type of case where the actual thing is time-dependent but on a stylistically average sense, the average is not a function of time.

This is known as stationary turbulence or steady turbulence. Usually we use the term stationary turbulence because steady is a misnomer. Turbulent flow is steady but the meaning is like, stationery is like time-independent. So this means time independent average behaviour. It does not mean time independent actual behaviours, only on a statistically average sense, this is not a function of time.

But it is also possible to have it a different way. Let us say that you half an average like this and on the top of that, say you have fluctuations like this. So the line which is drawn with a blue is an indicator that you may have a turbulent flow which may be having a time average which is not a constant, which is varying with time. So the solid line which is going through the middle is an indicator or may be let us just mark it with a different colour.

So if we mark it with this particular colour, so this example is a case where it is not a stationary turbulence. So this is a stationary turbulence. But the other is not. So after time averaging, you may still get time dependence but that time dependence is over a timescale which is important for the system and that is something we need to keep in mind. So this is called as time averaging. Similarly, you may also go for a space averaging.

So what is space averaging? Very very similar. So spare averaging is averaging with respect to space or position at a given instant of time. So very similar, just swap the space and the time variables. So now what we are doing, at a given time, we are taking the data at different spatial locations and finding an average out of that. In experiments, we usually are not very careful about time averaging and space averaging.

But in experiments whatever averaging we do intuitively is known as ensemble averaging. So let us see what is an ensemble average. What is an ensemble? So if you do a large number of experiments with identical conditions, so that is called as an ensemble. So let us say that we are doing an experiment where we want to find out the velocity variation as a function of time and position.

So what we do, so let us say that we have a pipe at a given location. So there to be many locations. Let us say that this is one location. At this location, we want to measure velocity as a function of time. So we are doing one experiment where we are doing it. Again we are doing another experiment, then we are getting a reading at this point. These experiments are all done at identical conditions but because of the randomness, the output is not identical because there could be slight variations in the experimental conditions and those got amplified, that is like an instability that is there in a turbulent flow.

So therefore if you repeat such experiments, for each experiment you will get some sort of data or information at your identified points at a given instant of time and you can make an average prediction of out of all experiments and that average prediction out of all experiments is known in ensemble averaging. You have to keep in mind that these experiments must be performed under identical conditions.

So identical conditions, but the irony is, you are believing it is identical condition but there is always a slight perturbation or slight difference from one condition to the other which is making it deviated from the exactly identical condition. So when you have ensemble average, it means averaging from a large number of experiments conducted under identical conditions, okay. So when you say averaging from a large number of experiments conducted under identical conditions, we will also try to understand the implications of this averaging and the relationships with time averaging and space averaging for certain special cases.

We will come to those special cases subsequently. Now when you have average, you also have a deviation from the average. In a statistical sense that is represented by the RMS or the standard deviation. So now if you write say the x component of velocity as an example, say as u average+u prime. So this is the average. So when you are saying this is average, we are not committing what sort of average.

It could be time average, space average, ensemble average, whatever but with respect to average, there is always a fluctuation. So this is average and this is fluctuation. In most of the textbooks, the average is written by an upper case and the fluctuation is written by a lower case or something like that but like when we write in the board, it is very difficult to distinguish between the uppercase and the lowercase.

So we are going to use this, the prime for the fluctuation and the bar for the average. In some cases, for the average, these type of braced symbol is also used. So this is a typical symbol for ensemble average but again I mean one may use either these sort of symbol or the over bar, I mean either case, fine as a notation for average. So if you want to find out what is the RMS. So first of all, you want to find out, so what is RMS.

So root mean square deviation. So first is the deviation. So deviation from what? Deviation from the mean. So deviation from the mean is u-u bar that is u prime. So you have to find out the summation of this and squares, basically squares of the individuals and the summations and then so you sum it up, divide it by the number of data and that will give you the square of the standard deviation or the variance and the square root of that is the standard deviation.

So basically you are making an averaging of this. So all those expressions we are representing by this. So basically summing up of these data over a number of data and dividing by the same number of data is averaging but why we are using this symbol is because the averaging may not always be on a discrete data. It may be on a statistical sense with a probability distribution function.

So we do not know that whether it is on a discrete set of data that you are doing it or you are fitting the data with a probability function and then finding out an average with that probability function. So we are not committing with any specific definition of averaging but just marking it with this one. So when you do that, it is like the mean square deviation and when you make a square root, it is the root mean square deviation.

So the RMS of u is like this one, okay. Similarly, we will have RMS of E and RMS of w for the velocity components along the 3 directions. There is some important terminology called as isotropic turbulence. What is isotropic turbulence. Isotropic turbulence means that the turbulence statistics are independent of direction. That means the RMS values of the velocities should be direction independent.

That means whatever is RMS of u, same should be RMS of V and same should be RMS of W. So that means you must have… so this is directional independence of turbulence statistics. Why the directional independence of turbulence statistics is going to be important? Because it may enable us in simplifying certain considerations when we are mathematically modelling a turbulent flow.

So we have to keep in mind that when you consider the averaging, so if you consider the averaging of say u prime, what is this? Say you want to find out the average of u prime. Let us say time average. So what will you do.

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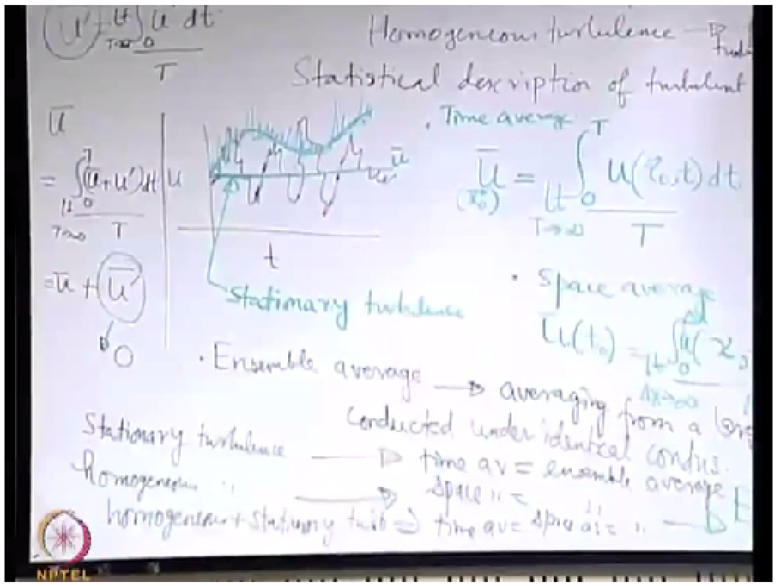

This one, right but what is its value. Let us say we want to find out what is the average of u. So average of the u is limit as, with the limit and other things, we will write later on. So u is u bar+u prime with whatever limit. So the first term say u bar is like an average which is not the varying with this time. So the first term becomes u bar. So u bar integral of Dt from 0 to capital T/T. So u bar*T/T that gets cancelled out.

So +this one. That means the fluctuation of this, this must be 0 average. So average of the fluctuation is 0. That is why it is a fluctuation, random fluctuation. Its average over that time must be 0. But the average of u prime square or V prime square or W prime square is not 0. So the average of product of the 2 fluctuations that will not be 0. That we have to keep in mind. So we have introduced the terminology of isotropic turbulence and similarly we may introduce a terminology of homogeneous turbulence.

So what is homogeneous turbulence? Just by the name it is clear that the turbulence statistics are independent of position. Just like the isotropic, it is independent of direction. So this is position independent turbulence statistics. It is better to say turbulence statistics, okay. Now let us consider a stationary turbulence. So that is also another terminology and stationary turbulence means you have the mean is independent of time or the average value is independent of time, that is stationary turbulence.

So for a stationary turbulence, what we may say is that. If you are doing a time averaging. If you are doing a time averaging, then the times average value is what? The time average value if you repeat many number of experiments, then the time average value is as good as ensemble averaging. That means at a given point, if you record the data and average the data at different times, so doing the data at different times is as good as doing the identical experiment at different conditions because the average should not vary with time for a stationary turbulence.

So for a stationary turbulence if you are doing the experiment at different time, only you are allowing the random fluctuations to be there as a function of time. Average is independent of time and the random fluctuations of those which make one experiment different from the other at a point. Therefore, if you do identical experiments and if you get an average of that at a given location with respect to number of experiments, that is as good as time averaging for a stationary turbulence.

That means for a stationary turbulence, you have the time averaging same as ensemble averaging. So stationary turbulence will have a conclusion that the time average=ensemble average. If you have a homogeneous turbulence, then… Homogenous turbulence is where you have the turbulence statistics position independent. So when you have turbulence statistics position independent, that means at a given time, if you do the experiment at different positions.

So if you do the experiment at different positions then what happens? So at a given time, you are doing the experiment just at different positions but different positions have the same behaviour in terms of the average. That means doing the experiment at a time at different positions is just like doing different simulated experiments at different positions.

So that means for homogenous turbulence, you must have the space average=ensemble average because varying over space is just like having identical experiments in terms of the mean characteristics. The mean characteristics should not be function of position for homogenous turbulence. Only fluctuations are functions of position. So if you have a stationary plus homogeneous turbulence, then you must have time average=space average-ensemble average, right.

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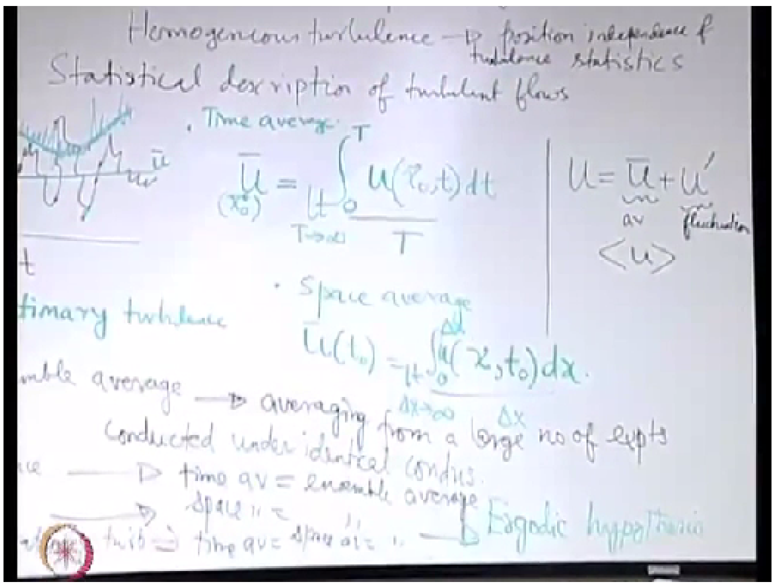

So homogeneous+stationary turbulence will imply that you must have the time average=space average=ensemble average and this is known as Ergodic hypothesis, okay. So we have looked into the averaging and finding out the RMS of different quantities and of course one may do certain other statistical operations using the features of turbulent flow and one of those important features that we will consider is by developing a correlation in the turbulent flow. So a correlation and correlation coefficient for turbulent flow.

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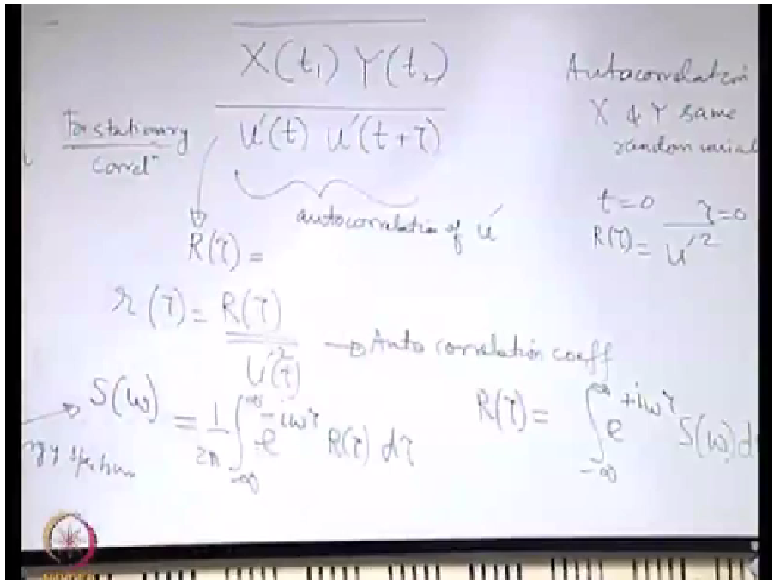

Let us say that you have a random variable X. So when you have a random variable, we say, what is a random variable? The outcome of a random experiment which depends on chance. So if there is a variable which has its outcome dependent on chance or probability, that is a random variable. Now if you have a random variable, at a time say t1 and another random variable say y at a time t2.

Then if you just make a product of these and find the average, it sorts of represents the average correlation between the random variables, x and y at times t1 and t2. If x and y are the same random variables, then it is called as autocorrelation. So autocorrelation when you have x and y same random variables. So what it basically tries to represent? So if you have the feature of a random variable at a particular time and if you have the feature of the same random variable at a different instant of time.

You are trying to see that how these 2 features are correlated. That means say what is happening now and maybe say what is happening after 20 seconds or 10 seconds or 100 seconds. So if those outcomes are there and if you make a product of those outcomes and average it over all possible datasets, then what average information you get is that on an average how the events are correlated and if the same event is there, that means if the random variable x and y are same, then how the outcome of the random experiment at time t1 is related to the outcome or correlated with the outcome of the random experiment at t2.

If it is a stationary turbulence, then it does not matter what is the origin of this t1. So you may have say X at t and say X at t+an additional time tau. So this t is immaterial, the origin of this t is immaterial if it is a stationary turbulence. That means the turbulence statistics are not function of time. Then for stationary turbulence, for stationary case, you are able to write the correlation in this way.

Now what is this random variable? The random variable that we are looking for here is mostly the velocity. So let us say that we are looking for the velocity or maybe one of the velocity fluctuations. So this is known as the autocorrelation of u prime, okay. So when you have the autocorrelation of u prime, the important thing is that this may be normalised or this may be expressed in terms of a so-called non-dimensional manner because it is like velocity square.

So if you normalise it, the normalisation is with respect to the mean square deviation. So that means if you want to write it in terms of a coefficient, so this if you call as say, you give it a namesake capital R which is a function of tau, then you may have small r which is again a function of tau which is capital R/maybe this one, it is known as autocorrelation coefficient. So if you have an autocorrelation coefficient, this is just like having a normalised way of writing the correlation coefficient, then it may be shown that its magnitude is always between 0 to 1 by the Schwarz inequality which is commonly used in statistics.

So this will be normalised always between 0 to 1. Now it is also possible to have a sort of Fourier transformation of the autocorrelation coefficient into some frequency domain and that is possible. Like if you have, say for example if you consider a transformation like this. So this is known as energy spectrum of the turbulence and we will see that why it is so? And with the inverse Fourier transform, it is also possible to recover the R, okay.

So when you have this one, so if you, sorry, this is minus and this is plus, okay. Now when you consider the special case of tau=0. So if you consider tau=0 and say since it is stationary turbulence, you do not care what is this t. So you may also consider t=0 because the statistics will not be function of time. So when you have t=0 and also say tau=0, then your R tau will become u prime square. Mean of u prime square. So u prime*u prime at 0. So then what does this represent? S omega at tau=0 or S0.

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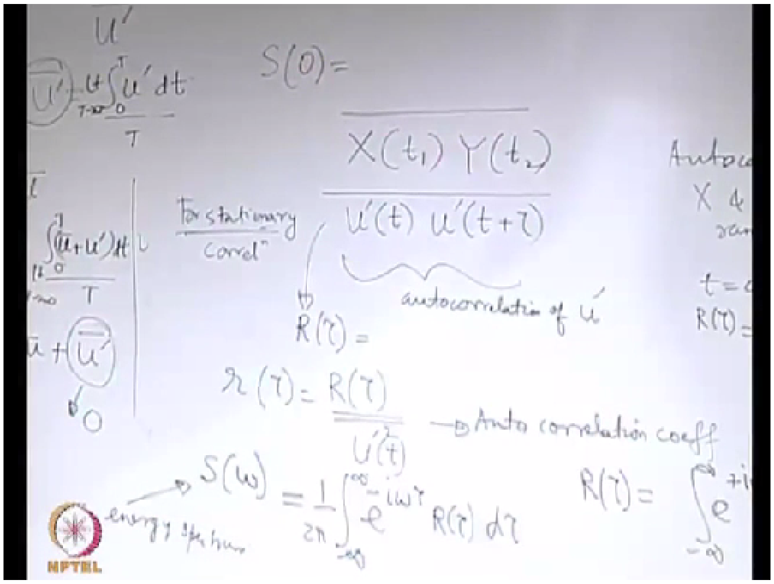

So S0 will be nothing but a representative or let us say you put omega=0, okay. So just put omega=0. So if you have S0, what does it represents? To understand that, let us first put what is r equal to, what is r at tau=0 that will be easier for you to follow first.

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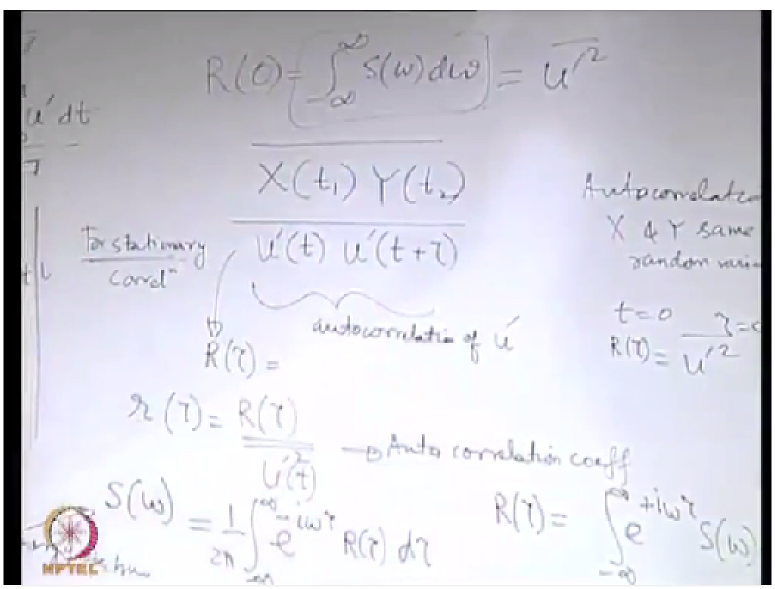

So first let us say what is R0, so what is R0? -infinity to infinity S omega d omega, right and that from the definition of R is this one, right. Therefore, what we may conclude out of this that whatever we consider as energy spectrum, the integral of the energy spectrum over all possible frequencies is an indicator of the RMS of the velocity fluctuations, right. So that is how, so the Fourier series analysis is important because you may mathematically get this easily if you know the power energy spectrum distribution and from that directly, you get a statistical behaviour of the RMS of the correlations.

Now the big question is that why we are going for all the statistics? Why it is necessary? So that question is the ultimate question for analysing turbulence. See the turbulent flow may be approached or solved by the Navier-Stokes equation. So if somebody asks you a question, are the Navier-Stokes equation valid for turbulent flow, very much valid. But what makes it almost intractable for solving the turbulent flow problems.

One of the important things is a wide range of length scales and timescales. So if you want to resolve such as big range of length scales and timescale by a solution strategy that is very very tedious. Not only that, there is a whole lot of uncertainty about the sensible dependence, the sensitive dependence on the initial conditions or the experimental conditions of the boundary conditions.

So if there is a slight perturbation, the perturbation will get amplified and therefore if you exactly know what is the initial condition and boundary condition, your Navier-Stokes equation will still give the exact solution but if you do not know, then the Navier-Stokes equation will not give the solution based on the experimental condition that you are simulating. No matter whether you are resolving whatever scales.

So invincible equations are applicable but because of the uncertainties and strongly sensitive dependence to slight perturbations from the initial and boundary conditions, the outcome is not something which is acceptable and that acceptability becomes more and more vulnerable as you are not able to resolve all these length scales and timescales. So what is the alternative. Alternative is you go for a statistical description but the thing is that, when you go for a statistical description, we will see that in terms of statistical behaviour, turbulence will be deterministic.

That is you may have randomly fluctuating variables but their statistical descriptions are not random. They are deterministic but we will see that a problem will come that the governing equations will not be closed. So this is an irony that in terms of the statistical description, you have the governing equations which are sort of deterministic but they are not apparently closed. On the other hand, in terms of the actual variables, the governing equations are perfect, they are closed but if they are not deterministically solvable.

So these type of dilemma is there and that is why understanding turbulence through mathematics is one of the very difficult things and it has been, till now, not solved and in classical physics, this is considered to be the last unsolved problem in classical physics, that is understanding of proper mathematical description of turbulence. So whatever discretion of turbulence we will be having, will be very very elementary just to give you a qualitative picture and that we will do in the next class. Thank you.