Introduction to Fluid Mechanics and Fluid Engineering
Prof. Suman Chakraborty
Department of Mechanical Engineering
Indian Institute of Technology - Kharagpur

Lecture – 33
Introduction to Turbulence

Today, we will start with an introduction to turbulence and before having a sort of formal introduction to turbulence, let us try to look into some very classical experiment that Osborne Reynolds performed and we will first have a brief look into one such experiment.

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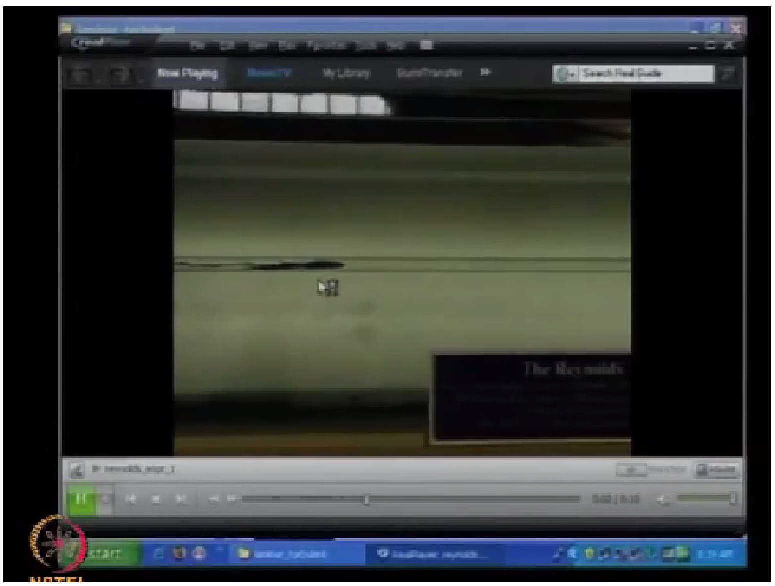

So have a careful look. If you see in this experiment, we are having a tube. So Reynolds took a tube, a glass tube and what he did is let us play it again. He injected some colour dye into the fluid and when the velocity of the flow was changed keeping the fluid as same, keeping the diameter of the tube as same, it was found that the behaviour or the characteristics of the colour dye started displaying interesting features. So let us look into those features in a bit more detailed manner.

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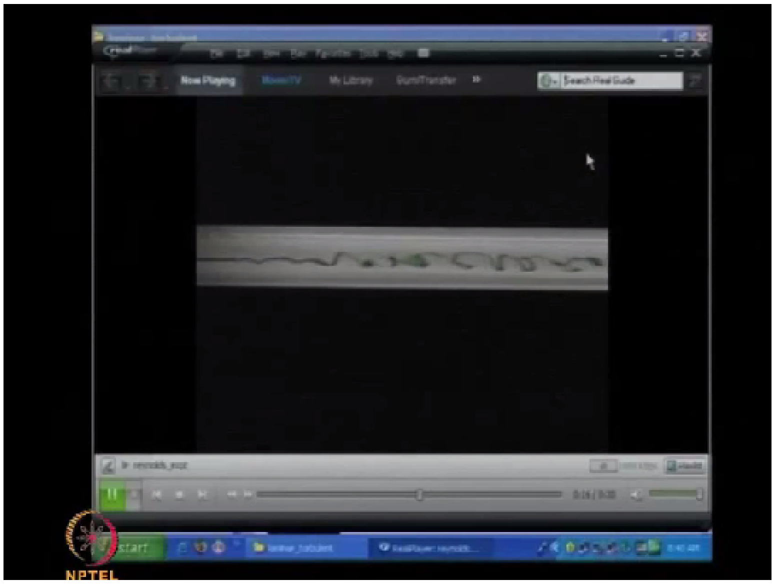

So this line what you see is the initial coloured line representing the dye. So it is just like a streak line in a flow and with increasing flow velocity, see the characteristics are changing and it will come to a different state altogether as the velocity is increased further and further and you can say that the general change is as follows. So initially it was like a regular orderly motion. Once the velocity is increased, so let us play it again and see that what happens when the velocity is increased.

So here it is a regular orderly motion. Now you see that that regular orderly motion is disturbed somewhat but it is disturbed somewhat, you see that the dye streak is no more straight and parallel to the axis of the tube but it is getting defused and at the end, you see that this is totally random and chaotic motion and there is a whole lot of mixing within the flow. So something is happening which is changing the regular orderly motion in to some random or apparently erratic motion.

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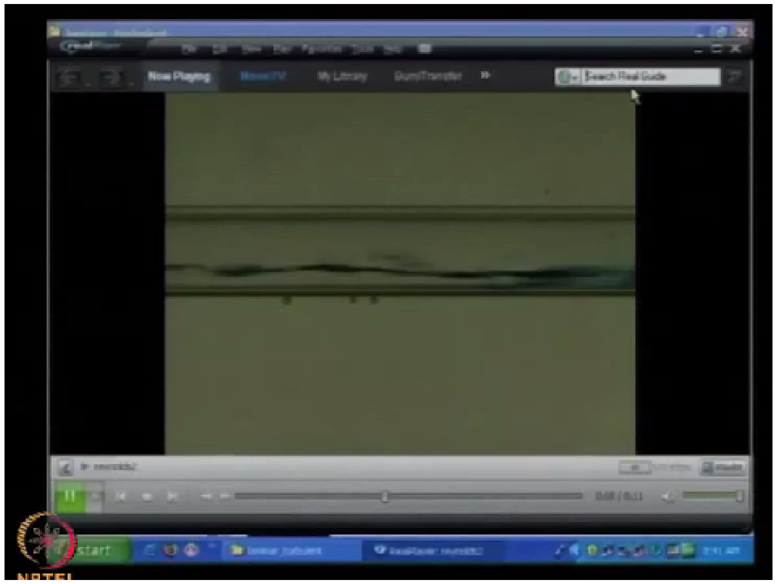

If you want to have a more detailed look into the sort of erratic motion possible, I mean one may have these types of cases where you may have even alternative regular and irregular motions. So this is also a sort of structure, flow structure possible. So all sorts of complications in the flow structure may be possible when you are increasing the velocity or say keeping the diameter of the tube unaltered and the fluid property unaltered.

So that might give an indication that increasing the velocity does something and the big question is, is it only the increase of velocity that does something or there could be other properties also influencing the same physical phenomenon? to understand that, Reynolds performed experiments with tubes of different diameter now because he also wanted to see that does the diameter of the tube have any role to play or even does the fluid property have any role to play.

And then he figured out that this transition from the so-called regular behaviour to the apparently irregular behaviour takes place around a particular transition. And the transition is not just dependent on the velocity but also the diameter of the tube and the viscosity and also the density. So to come up with the condition under which that sort of transition from a regular behaviour to an irregular behaviour that was occurring, he tried to have an estimate of what were the competing forces under these conditions.

So if you have, say a system where you are increasing the velocity, say or you are trying to accelerate the fluid. Now when you are trying to accelerate the fluid, the fluid will have a sort of a force which is like an inertia force.

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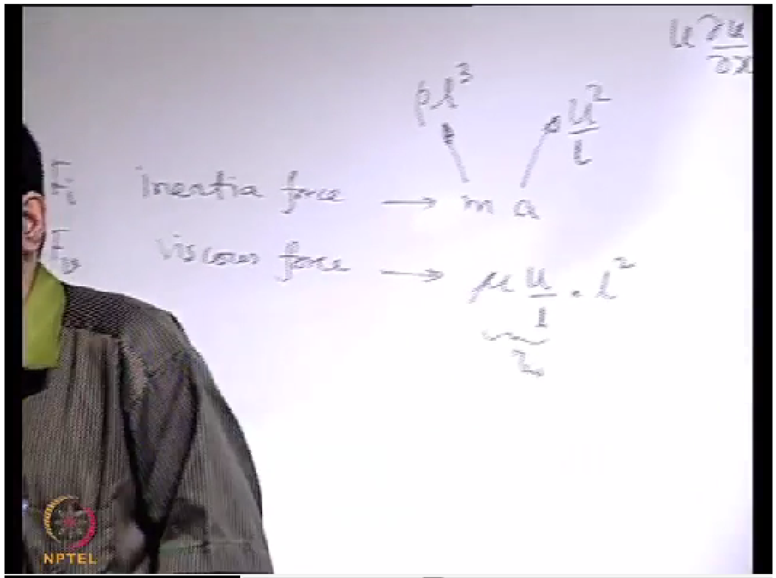

So if you want to characterise what is the inertia force? So it is something like the mass*acceleration. So mass*acceleration and let us just try to write in a dimensional way, that means if you have l as the length scale of a system, so we have discussed earlier what is the length scale of a system. These are characteristic lengths over which characteristic gradients of variables are present or characteristic changes take place.

For example, for flow through a tube or a pipe which is the diameter of the tube or the pipe. So the mass is like of the order of rho*l cube. So l cube is dimension of volume and rho is the density, so this is proportional to the mass of a fluid element. Then acceleration, if you recall that the acceleration of a fluid element is like of this form, similar terms for the different components of acceleration, the temporal component and the convective component and so on.

But just one term is good enough to give a fair idea that it is like u*sum u/the characteristic length. So it is like u square/l, right. Ududx is like u square/l. Now when the inertia forces is trying to have a driving effect, there may be something which is a sort of trying to have a resistive effect and viscous forces are sort of competing with the inertia forces in terms of having a balance.

and the viscous forces for a Newtonian fluid may be given by the viscosity expression that is if you say mu as the viscosity * the velocity gradient. So velocity gradient is u/l, so this is the shear stress, order of shear stress, * the area that is the shear force. So area is of the order of l square. So this is just pure dimensional arguments not that area=l square, that one has to understand.

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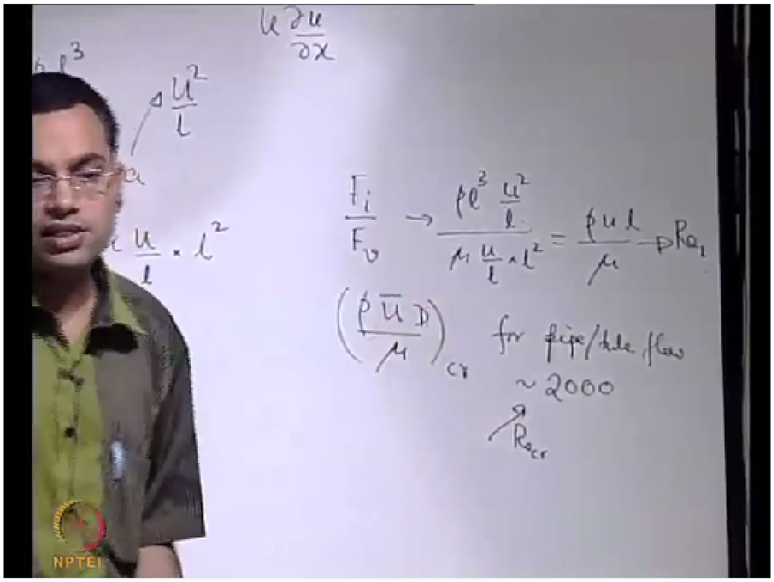

So if you want to have a sort of the relative importance of these 2, so you have the inertia force/viscous force, that is rho l cube*u square/l/mu u/l*l square. So rho ul/mu which is known as the Reynolds number based on the length scale l. Of course when you are considering the flow through the tube, then the length scale l is the diameter of the tube and u is the average velocity because u is varying over the cross-section.

So depending on the situation you must have a fair idea that what velocity you put and what length you put for having an estimate on the Reynolds number of flow. Of course, when Reynolds did the experiment, he did not call it Reynolds number, right and he cannot call a number by his name but the entire number was given in the honour of his name because if you see that Reynolds identified a very remarkable thing.

He identified it is not the flow velocity that just matters but if this collection of parameters in terms of this non-dimensional number was kept unaltered, then the physical behaviour of the system was unaltered, that means the physical behaviour of the system was dictated by a combination of these parameters in terms of this non-dimensional number. So what Reynolds figured out is that like for flow through a tube or a pipe, if there is a critical value of this number, then beyond that critical value of the number, there is a transition of behaviour from that regular motion to the apparently disordered motion.

Whereas if the non-dimensional number was less than that then that never happened and that range of non-dimensional members over which this transition took place, was sort of like, was within a small range and that range for flow through a tube or a pipe, so if you consider the average velocity and the diameter of the pipe and this one, so this the value of the critical parameter of this for say pipe flow or a tube flow was roughly of the order of 2000.

Does not mean that it is exactly 2000, it maybe 1700, it may be 2300. So roughly within the range of 1700 to 2300, this was the range it was observed. So that means something happened at this. So this later on is termed as a critical Reynolds number. Not critical Reynolds number for all cases but flow-through pipes and tubes and this critical Reynolds number is different for different types of flows.

This is just an example where we are talking about flow-through a tube or a pipe because in engineering flows that has lot of relevance. Now what is happening below it? To understand that, we have to understand that what are the characteristics of a low Reynold's number flow or a low velocity flow. Why we are talking about the low velocity flow because see in an experiment, it is easy to vary the velocity, vary the flow rate by keeping other things unaltered.

Say you are doing experiment with water. So you have a particular density and a particular viscosity at a given temperature. You have a tube or a pipe of a given diameter. So these things are like parts of your setup the working fluid and the setup but the velocity of flow by varying the flow rate may be you can have higher or lower average velocities and therefore, it is not a bad idea to consider first what happens for very low velocities.

So if you have very low velocities, obviously the Reynolds number will be low and let us see what happens to that. So let us look into some cases where we are looking into the low Reynolds number possibilities.

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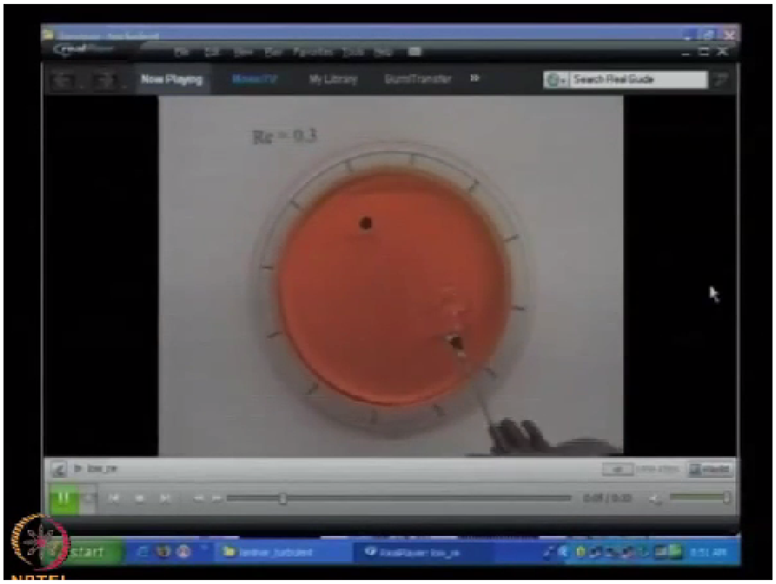

So this is an example with Reynolds number=0.3. See there is a red dot marker which is there in the flow and you see that initially if you were careful, you saw that there was a disturbance, like it was stirred a little bit but that disturbance has no influence on the flow as you can see that it is having this marker which is sort, this marker is important because it is a sort of indicator or visualiser of the flow.

And you see that that marker is having a very regular type of motion unperturbed by the disturbance that was created initially and this disturbance was forcefully created but we have to keep in mind that even if you do not create a forceful disturbance, there is always a perturbation or disturbance in the flow in the practical case because you cannot have an experiment in a quiescent addition.

So in an experiment, you are always having some perturbations. So these disturbances are called as perturbations. So when you are perturbing the flow, may be one of the reasons is the presence of the roughness of wall of the tube or the pipe. So those are perturbing the flow. Also like your initial conditions, the entry conditions, those might fluctuate, those might not have very well determined repeatable values.

So you will always have perturbations with respect to some average or expected condition. So these perturbations are always there. In these experiments, these perturbations are triggered additionally by certain mechanisms but it does not mean that it has to be triggered artificially. In a practical condition, these perturbations are there and you can see that for this low Reynolds number case, no matter whatever is the perturbation, the effect is not there, that means the perturbation is dying down.

So there is something which is happening which is allowing the perturbation to die down. Now let us look into a different case.

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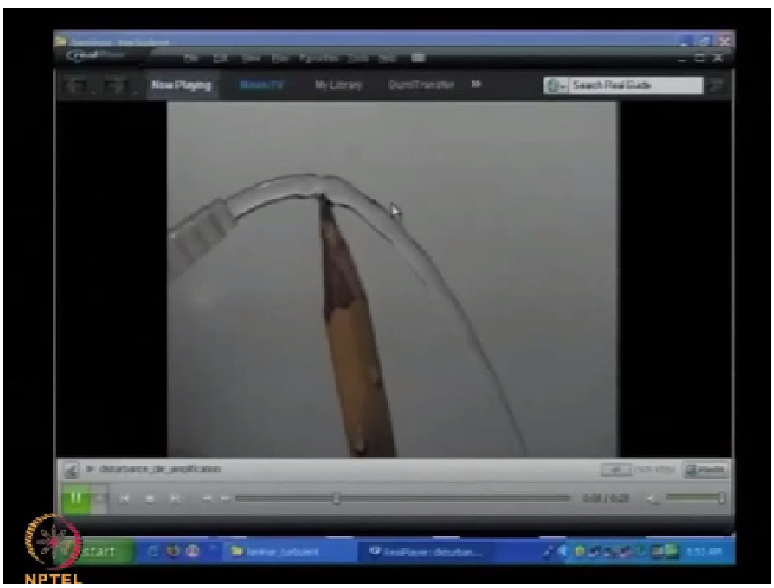

Again if you see, so this is a water outlet. Now, you see with a pencil, the flow in the water is disturbed and after it is disturbed, see it has no influence. So once the pencil is withdrawn, you see that it is just like it.

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Now here look at the situation. So here a perturbation is created by an arbitrary rotation and here if you see, look at the motion of the coloured balls. It is totally like sort of apparently erratic. So if you see that we have now come to a situation where we are encountering 2 types of examples. One type of example is where you have a perturbation and the perturbation is not getting amplified.

It is dying down and in certain case, there is a perturbation which is getting amplified. So the question is when the perturbation is going to get dying down and when the perturbation is going to get amplified. So when the perturbation dies down, one of the things we may say is that. First of all, what was our observation? The observation was that for a low Reynolds number flow, the perturbation apparently died down.

So for the low Reynolds number flow, what happened? For the low Reynolds number flow, possibly from the scaling of the Reynolds number, it is the viscous force that somehow dominated and made sure that whatever was the perturbation that was diffused throughout. So the perturbation at a point could not get amplified because viscosity is a sort of a characteristic or more importantly, it is not the viscosity but viscosity/density or the kinematic viscosity.

So the kinematic viscosity is such a property which tends to defuse a disturbance in momentum in the flow and if that diffusion is very successful, then what happens? Then the perturbation cannot grow in amplitude at a particular point and it sort of gets destabilised. On the other hand, if you have a high Reynolds number flow, then the inertia forces are sort of important and when you have perturbations, the perturbations will try to get amplified.

And because of the amplification of the perturbation, you have these types of situations. So these 2 qualitative pictures are sort of hallmarks or representatives of laminar flow and turbulent flow. Laminar flow is something where like for very low Reynolds number, we have seen that you have a regular orderly motion of the fluid elements. Let us look into some Reynolds number, low Reynolds number experiment where you have laminar flow.

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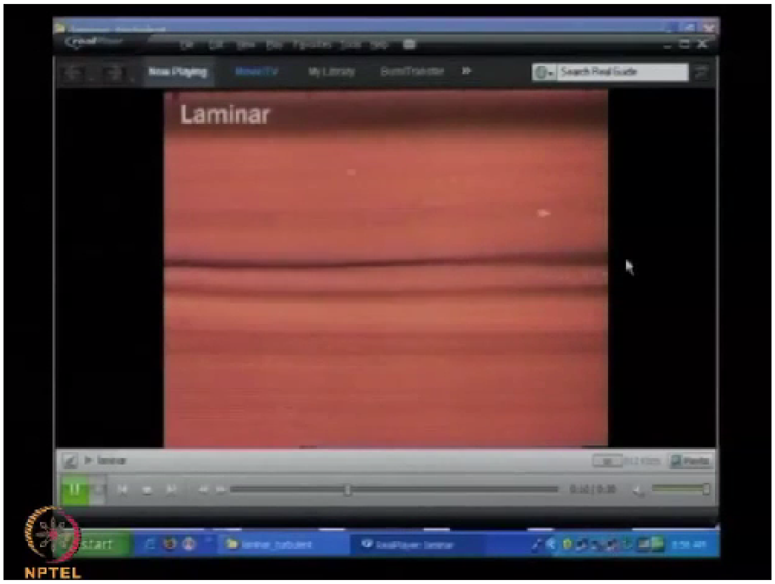

So in these experiments, some coloured dye was there and you see that it is a layer by layer motion of fluid elements. Say this is also a tube, flow-through a tube experiment. So it is a laminar flow. So there was a coloured dye that was injected and the coloured dye is just following a parallel sort of motion.

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And we may look into an overlay of this laminar flow with a sort of turbulent flow that we are talking about. So our whole thing is that see we will get into some mathematical description of turbulence but what we are trying now is to develop a sort of qualitative description, at least of the laminar flow or the turbulent flow because that is what is important at the end.

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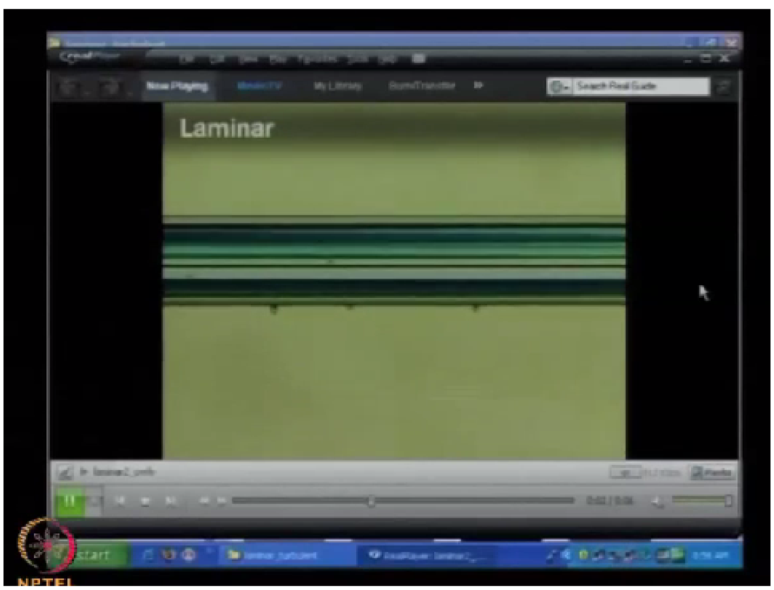

So if you see this example, this will like if you have here just different coloured dyes were used and you can see that it is just like one moving on the top of the other in a very regular and orderly manner. So the thing is that if you laminar flow and if you have a turbulent flow, the characteristics are somewhat different.

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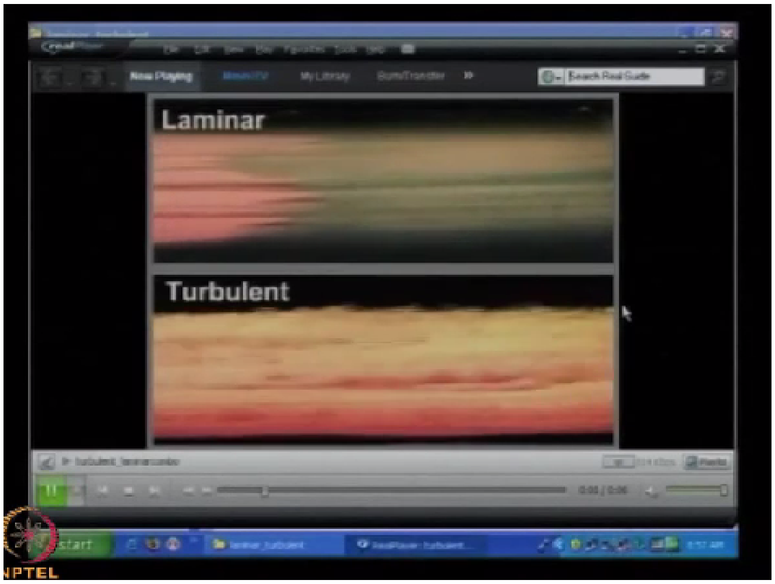

And let us look into an overlay of the laminar and turbulent flow to appreciate that. So just try to develop this qualitative picture because any mathematical description should not be devoid of the physical and qualitative picture. So let us just play it again. See the difference between the laminar and the turbulence. So the turbulent flow itself looks like a sort of flow which has a lot of randomness with respect to time and also like lack of order also with respect to space and that is one of the hallmarks of a turbulent flow.

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So if you just have a look into the turbulent flow as it is, you can see that there are vigorously rotating structures in the flow which we call as Eddies and these are randomly fluctuating with respect to time as well as with respect to space. So with this understanding, what we will try to do is we will try to develop some sort of feel that what are the important characteristics of a turbulent flow. So to do that, we will first keep in mind that whenever we are having a laminar flow, the laminar flow means basically a regular orderly motion of the fluid elements.

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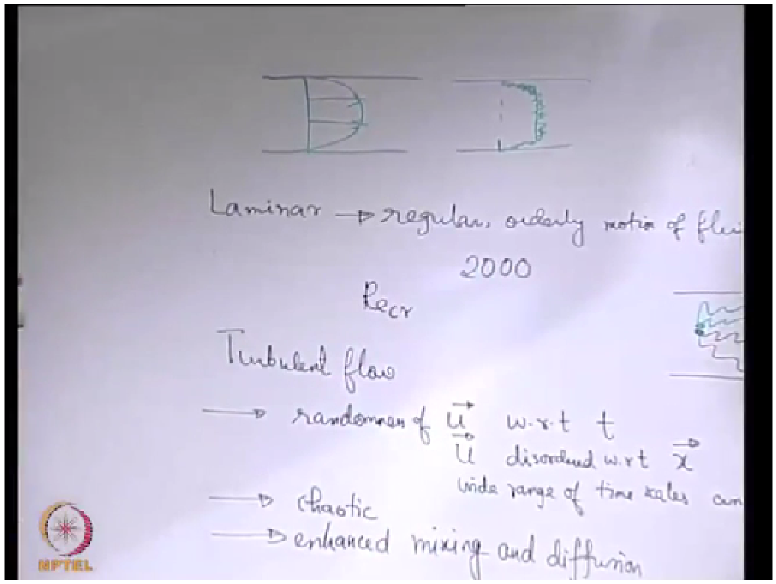

So let us just identify it that when you have laminar that means basically a regular and orderly motion of fluid elements. This is very qualitative but in a laminar flow, what is happening at least is that if you have a disturbance or perturbation in the flow, the perturbation is not able to get amplified irrespective of the level of disturbance, that is very important that means if you have a slight disturbance, then even in a very high Reynolds number, the disturbance may get absorbed.

But if you have a very high disturbance or a moderate disturbance even, then in a high Reynolds number case, it will be a turbulent flow whereas if the Reynolds number is somewhat low or very low, then no matter how much, how large the disturbance is, the disturbance will not be amplified. It will be sort of going to a stabilised situation where the disturbance dies down. So the critical Reynolds number in a pipe flow experiment, say if we say 2000.

Does it mean that beyond 2000, Reynolds number if you do experiment, it will be a turbulent flow? Yes, if you do experiment in a sort of like undergraduate laboratory where the sophistication of the experiments is not that much, you cannot keep a very undisturbed condition, yes it will be like that but say if you are doing experiment in a very controlled research environment, it is even possible for flow-through a pipe to have sort of laminar structure even for a Reynolds number of 50,000.

I mean there is nothing which denies that because it all depends on the level of disturbance. If you have an infinitesimal small disturbance, then you may make sure that disturbance is not going to get amplified, even for high Reynolds numbers but that requires a very careful set of experiments. On the other hand, in the normal case, the normal experimental conditions have enough perturbation to have transition to turbulence around these Reynolds number.

But the other way around is more important, that below this sort of Reynolds number, no matter how much the disturbance is, it will always die down. So the critical Reynolds number does not mean that beyond the critical Reynolds number, the flow will always be turbulent. Whether it will be turbulent or laminar, will depend on the level of disturbance in the flow but below this, the flow will always remain laminar irrespective of the level of distance.

So that is the significance of critical Reynolds number. Significance of critical Reynolds number is not that beyond that it will become turbulent. Yes, in a normal disturbance condition, beyond that it will become turbulent but below that, no matter how much the disturbance is, that will die down and therefore the flow will remain laminar. That is the understanding of a critical Reynolds number.

So that means in a laminar flow, the disturbances die down quite easily and viscous forces play a role in making the disturbances die down. Whereas when you are talking about a turbulent flow, so it is not so easy to have a single formal definition of turbulent flow and we will not try for that. We will go for a qualitative physical feel of the turbulent flow with the important characteristics of a turbulent flow.

So let us identify some of the important characteristics of the turbulent flow. So one of the characteristics we could identify by visualising these apparently simple but very much thought-provoking and interesting experiments. The first one is that, there is a randomness of the velocity. Let us say u is the velocity with respect to time and now not only that, this velocity is also disordered with respect to space, say if we call this position vector x.

So it is also disordered with respect to space and the other important characteristics which we will come subsequently when we describe that what is physically happening here but we just note it down here that you have a wide range of timescales and length scales over which activities are going on. So what are these activities? We will see what are these activities which are going on.

So at least we can see that in a turbulent flow, the activities are quite strong, that is there is some vigorous thing that is taking place and these activities are not characterised by a single length scale that is we cannot characterise these activities by just the diameter of the pipe. So there are ranges of length scales over these activities are taking place and ranges of timescales from a small time to a large time, these activities are taking place.

So we will try to identify what are these scales but at least, we can understand qualitatively that it is not a single length scale over which some activity is taking place. It is a wide range of length scales and timescales. The important consideration which is related to this that the motion is chaotic. So what do you mean by this? Let us say that you have a system in which you have some fluid and the fluid is flowing and let us say that we introduce some particles.

Let us say we introduce a particle at some location, say we introduce a particle here and say we introduce another particle here. So these 2 particles, introduction of these 2 particles, is what? Introduction of these 2 particles is like say one way of visualising the flow, not just to but many such particles are there but we are focusing our attention on 2 particles. Say 2 particles representing the particles of colour dyes.

Now how these particles will evolve with time, that is if you want to describe the motion of this particles with time maybe the green particle, it has some motion with respect to time and maybe the black particle it has some other motion with respect to time. Now if you slightly disturb or if you slightly alter the initial locations of these particles, only slightly, only infinitesimally. It might be possible that these particles are going to a distance like or going to the sort of trajectory which are somewhat a large distance apart from what was there when they were like a bit closer.

And in the limit when they were coincident, they will follow the same trajectory but when you were taking the particles one away from the other only slightly and perturb it only slightly, you see that the resultant output is that the distance between the 2 particles are arbitrarily changing and therefore it is a sort of like it is deterministic in a strict sense if you exactly know what is the location of the particle but it is probabilistic in an experimental sense because it is very difficult to exactly have the particles located at the location that you want.

And therefore it is considered to be a sort of chaotic advection, the terminology means in simple terms like this that it is highly sensitive to initial conditions, that means with a slight perturbation in the initial condition, the distance between the 2 particles will arbitrarily vary with time and that is like it might arbitrarily increase or get amplified. So one of the good things of this is like, now if you want the flow to mix well, see 2 particles which were very very close, now are getting diffused into the flow at different locations.

So the effect of whatever was there here, is getting quickly mixed or transmitted to other places and if this is the property, then we must be assured that the turbulent flow ensures a very good mixing in the flow which may be important for many of the engineering application. So when you have a chaotic motion but one has to understand that any chaotic type of motion is not turbulence because one may have local patches of chaotic motion, not the chaotic motion throughout the system.

So locally it might be chaotic at patches but at other locations, it might not be chaotic. So that is not a turbulent flow. So turbulent flow will first be triggered like this. You have a base type of flow and you have a disturbance on that. The disturbance gets amplified, so it goes to a unstable state and that unstable state again interacts with the mean flow and takes it to a further different unstable state because these are nonlinear interactions and at the end, it will go to a completely chaotic state throughout the domain and that is what is so called the turbulent state.

So the turbulent state will have sort of enhanced mixing and diffusion and enhanced mixing and diffusion is important because of many reasons. In some cases, it is desirable that is why augmenting the possibility of a turbulent flow by creating a turbulence or forcefully inducing a turbulence by good mixing or by increasing the flow velocities, if you introduce that, then you might have a very good mixing and may be let us look into one such example where we see that how the turbulent flow is creating a good mixing.

So let us look into may be one of the video demonstrations.

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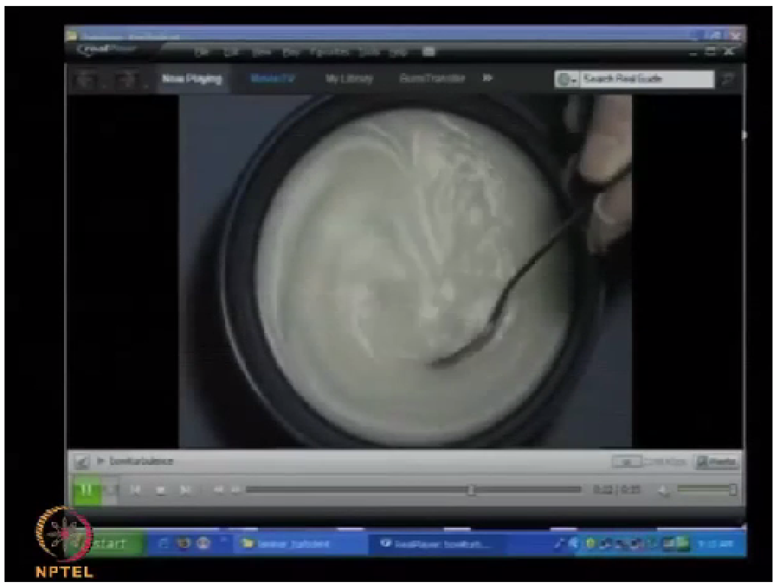

So this is a very simple way in which like if you have something to drink and you want to mix something with it, this is what you do is stir and stirring is like creating a sort of forceful velocity. So this velocity is a rotational velocity and the whole idea is that you want to increase the mixing. See when the disturbance has died down, this is a forceful way of having the disturbance propagated.

So we again stir it and you see that nicely rotating structures are visible in the flow and these rotating structures have certain characteristics and we will soon learn that one of the hallmark of this rotating structures is having something known as Eddies. Now before that, what we will see is we will look into the consequence of this good mixing in terms of a velocity profile.

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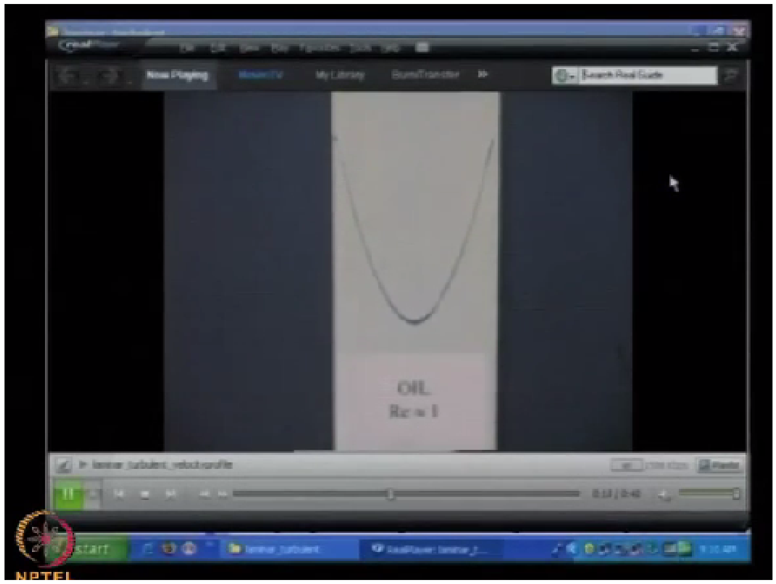

So let us look into this experiment where we are doing experiment with oil with the Reynolds number of 1, very low Reynolds number. So there is a coloured dye and the colour dye moves according to the velocity profile. You see that with the low Reynolds number, you see there is a lot of so-called dispersion in the velocity profiles. So the centreline velocity is very large, wall velocity of course is 0 by no slip boundary condition and it is a parabolic sort of velocity profile that we have seen like a fully developed flow through a pipe.

So whatever fully developed flow we have analysed analytically in a very nice way, that is a fully developed laminar flow.

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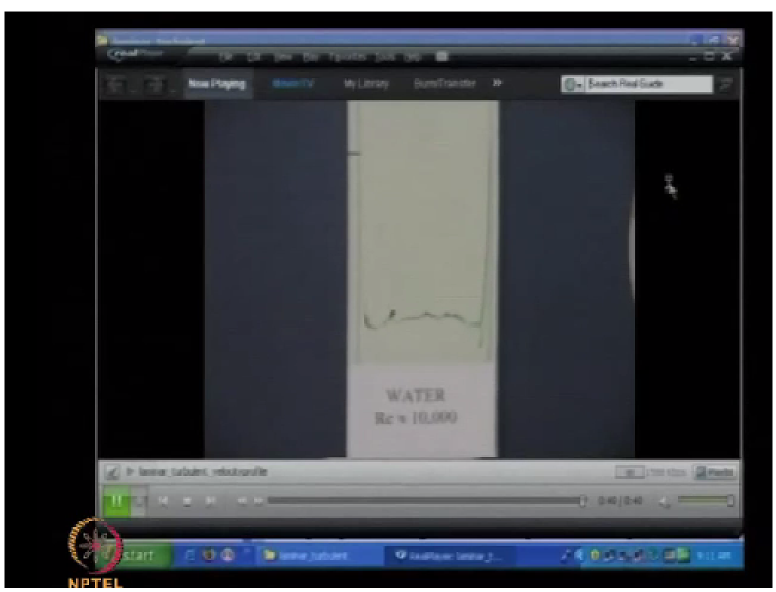

Now you look into the experiments with a high Reynolds number, may be let us just play it again to look into the experiments a bit more carefully. So the first experiment is with a very low Reynolds number and if you see that what is happening with this experiments, you have sort of regular orderly motions, so layer by layer the motion is there and because the effects of the wall are propagating into the fluid towards the centreline, you have that parabolic sort of distribution.

Now if you look into the Reynolds number with the same apparatus with 10,000 with water as a fluid and colour dye is injected. See what sort of velocity profile will be apparent? So let us look into the velocity profile. First of all, you will see there is a lot of fluctuation in the velocity profile and not only that, the velocity profile is virtually uniform, that means that parabolic distribution, sort of distribution has got vanished.

So if you have a sort of 2 cases where you have 1 fully developed laminar flow through a pipe. This may be the velocity profile but if you are talking about a turbulent flow, may be the sort of velocity that we are looking for is something like on an average it is something like this where you have may be lot of fluctuation of the velocities. That is what you could see with the experiments.

So it is more and more uniform. If you look it on an average, it is more and more uniform. So where from this uniformity has come? It has come from a very good mixing of different fluid layers. So uniformity is more when you have less gradient and more mixing ensures that there is less gradient and therefore it is a sort of almost uniform on an average but on the top of the average, there is some fluctuation but only close to the wall, it has to deviate or depart from the uniformity because it has to satisfy the no slip boundary condition.

So this is a qualitative difference in the physical picture of the velocity profile for may be flow through a, laminar flow through a pipe and turbulent flow through a pipe. So next we will try to understand that see we have now realised that in a turbulent flow, you have some random fluctuations and these random fluctuations are such fluctuations which are very difficult to repeat or record with experiments in repeatable manner, that means these random fluctuations are literally random.

In a different experiment with the same sort of initial conditions and boundary conditions apparently, these will be different. Therefore, it is very difficult to have exactly the same reproducible experimental behaviour with keeping the same initial and boundary conditions. So one of the important things is that what is more repeatable or what is more predictable and that is the statistical property of turbulence, that means you do not just look into the instantaneous properties because instantaneous properties will fluctuate.

And therefore if you repeat the same experiment again and again, because of a slight change in disturbance, those fluctuations will be different but if you make a statistical averaging of that, then the statistically average property might have a much better predictability and that is what is very important and that gives one a motivation of going into or looking into the statistical properties of turbulence.

And most of the mathematical description of turbulence is based on the statistical characteristics of turbulence. Before going into the statistical characteristics of turbulence, we will try to understand that what are the instrumentalists who play a big role in sustaining and creating the turbulence and to do that, we will look into the characteristics of these creatures known as Eddies.

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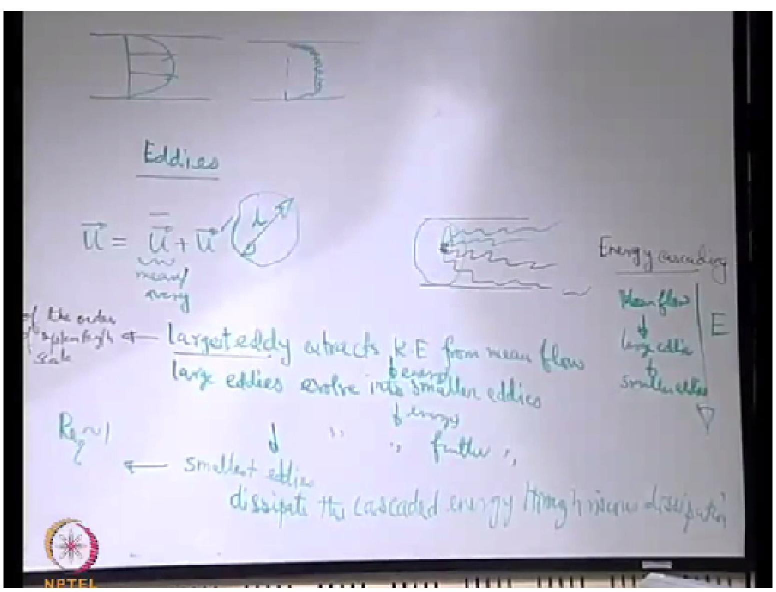

So what are these. These are basically lumps of fluids or strongly rotating structures and how big or how small these are, this is a very interesting question because the largest Eddies may be as big as that of a system and the smallest Eddies may be as small as the molecular dimensions or may be close to that. If not exactly the molecular dimensions that may be an exaggeration but it may be very close to very small length scale.

So that we understand that when we mention that there is a wide range of length scales, we are talking about like some mechanisms or some media through which the turbulence is participating. It is having its activity and that is through such Eddies which are of widely varying length scales. So what is happening with this Eddies? To understand that, what we will do is, we will say that if you have a velocity of the flow, if we somehow try to write it in terms of some average velocity+some fluctuation over the average velocity.

So this is mean. See mean is again not a complete definition because what sort of mean, what sort of average, that we will see that when you go to the statistics of turbulence, there are many ways in which you may do an averaging but right now, so we are going from more qualitative feel to a more quantitative feel. So right now we just consider it as some kind of averaging. What kind of averaging, we will see and important thing is that over and above that average, there is some random fluctuation.

So where from this random fluctuation is there? These are there because you have some disturbance in the flow. So the disturbance in the flow may be instigated by the roughness elements of the wall or maybe slight changes in the inlet conditions of flow through a pipe or a tube and that is how this fluctuation is there. So when this fluctuation is there, then what happens?

First of all, these fluctuations create a rotating structure or structures of rotating elements of different length scales in the flow and such rotating elements are called as Eddies. So when you have the largest Eddy? So the largest Eddy has a length scale which is of the length scale of the system. So in the largest Eddy scale, what is happening? The mean flow has some kinetic energy.

So the large Eddy, what it does? So large Eddy extracts some kinetic energy from the mean flow. So when it extracts some kinetic energy from the mean flow, then what is happening? Now the large Eddy has some activity. It has sort of like a rotational kinetic energy and then with that activity, it might get evolve into smaller Eddies. So the next step is that the large Eddy or we should say the largest Eddy because large is again a comparable term.

So largest, whatever is the largest in the system and the largest Eddy we should keep in mind that it is of the order of the system length scale. Then the large Eddy, these large Eddies, such large Eddies, what they are doing? They are getting evolved into smaller Eddies. So therefore in a system, you have large Eddy and then smaller Eddy and smaller Eddy but there are important characteristic differences between the large Eddy and the small Eddies, what are these?

The large Eddy has the order of its dimension as that of the system length scale. So if the flow over the system length scale is such that the flow is of high Reynolds number, that means with respect to the length scale of the large Eddy. Let us call that some length l. With respect to this length scale of the large Eddy, the Reynolds number is high. If with respect to the system length scale, the Reynolds number is high and we have seen that the turbulent flow has characteristic that it is for a high Reynolds number.

So the Reynolds number is high and when the Reynolds number is high, that means for a large Eddy, inertia forces dominate over viscous forces. So much much more significantly dominate, not just dominate, much much more. So if the Reynolds number is say 10,000, that means roughly in a 10,000 times mode becomes the inertia force. So for the large Eddies, these Eddies, you have the Reynolds number based on the Eddy length scale is very large which means the inertia force is much much more significantly dominate, then the viscous forces.

Now large Eddies evolve into smaller Eddies and energy is transferred or extract from the large Eddy to the smaller Eddies. So there is a transfer of energy, first the energy, so if you look for the transfer of energy, first there was a mean flow. We are basically talking about kinetic energy. So from mean flow to large Eddies, then to smaller Eddies. So this is how energy is being transferred from large Eddies and the large Eddies is evolved into smaller Eddies.

So you have the transfer of energy and these evolve into further smaller Eddies. So again the energy is transferred like that. So this is known as cascading of energy. This phenomenon is known as energy cascading. What is important is that what happens at the end, what is the smallest length scale that we are looking for? That is where will this energy cascading stop? What will be the smallest Eddy size?

Because that will give us an idea of the range of length scales that we are having. So in this way, the energy will finally reach the smallest Eddies. What are the characteristics of the smallest Eddies? See as you are reducing the sizes of the Eddies which are of your concern, you see the large Eddies have the largest characteristic length scales but if you go to smaller and smaller Eddies, the length scales of those Eddies are smaller and smaller.

That means the Reynolds number based on the length scale of the Eddy become smaller and smaller. See we are talking about the length scale of the medium through which the turbulence is being generated and sustained and that is the Eddy. So that is of widely varying length scale because the largest one is of system length scale but you also have smaller and smaller Eddies and in this way, you will come down to situations where now the viscous forces are tending to get more and more important.

Because the Reynolds number based on the length scale of the Eddies is getting progressively smaller as we are thinking of smaller and smaller Eddies. So where we will stop? We will come to a stage when you have smallest Eddies and in the scale of the smallest Eddies, you have viscous forces at least equally important as compared to the inertia forces. That means when we talk about the smallest Eddies.

We talk about the case when the Reynolds numbers is of the order of 1. That is the limiting case. Why it is the limiting case because now viscous forces will take over and when the viscous forces will take over, whatever energy that has been extracted from the mean flow by the largest eddies and it has been cascaded to the smallest Eddies, that will be dissipated through viscous diffusion.

So the energy extracted from the mean flow comes to the smallest Eddies and these dissipate the cascaded energy through viscous dissipation. Seems the length scales of the large and the small Eddies are different. To exemplify that, we will consider that the smallest Eddy length scale, we give it different names. So we will call into eta. So eta is different from l, l is the largest Eddy length scale and eta is the smallest Eddy length scale.

And let us try to develop a sort of qualitative understanding of these length scales. So to do that, let us try to figure out that what is the kinetic energy that is extracted from the mean flow and what is dissipation of kinetic energy that is there through the smallest Eddies by this energy cascading mechanism.

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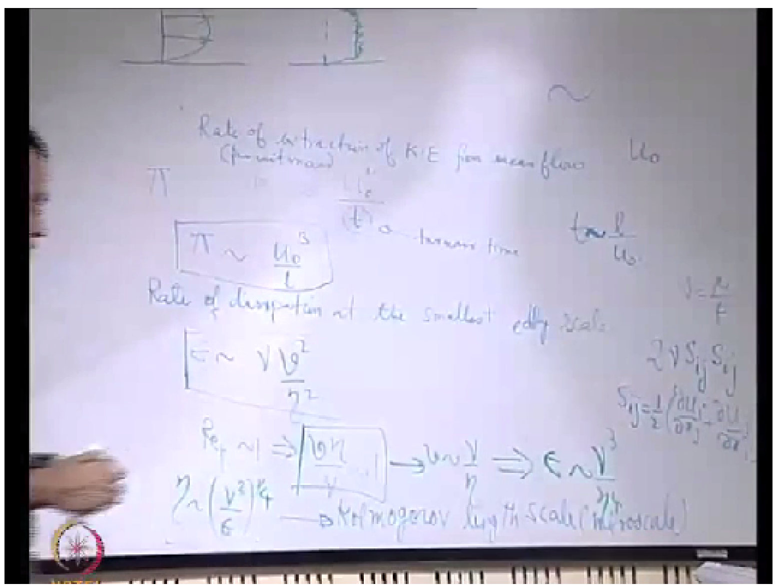

So to do that, we understand that if you have the rate of extraction of kinetic energy from the mean flow. This is through the fluctuation, so if we know what is the characteristic velocity scale for the fluctuation, let us say that u0 is the characteristic velocity scale for fluctuation in the largest scale. So there is a whole lot of fluctuation and because of this fluctuation, you have the kinetic energy extracted from the mean flow by the largest eddies.

So largest eddies are getting energised because these fluctuations are getting amplified. In a laminar flow, such Eddies cannot be sustained because this perturbations or fluctuations cannot get amplified. So Eddies can sustain only when they have sufficient energy because they are rotating lumps of fluids so to say. So there must be something which helps them in sustaining their motion and that is extraction, the key is in the large scale, it should be able to extract kinetic energy from the mean flow and that is only by the fluctuations.

So if you have fluctuations dying down, then that is not possible. Therefore, you cannot have Eddies in a laminar flow. Now if you find out the rate of extraction of kinetic energy from the mean flow, then what is happening? Again this, basically we writing the kinetic energy per unit time. So kinetic energy is like if you write it in terms of per unit mass, so let us write everything per unit mass.

So we are not mentioning it explicitly. So it is just like 1/2mv square. So per unit mass if you write, it is just like 1/2v square but that 1/2v is not important for us, we are just writing the order. So it is just like say u0 square/the time. This time in the large Eddy scale is known as turnover time. What is this turnover time? This is a time that is necessary for the large Eddy to be energised by extracting energy from the mean flow.

So what is the time that it takes? Characteristic timescale that it takes to be energised by extracting the energy from the mean flow and that will depend on the velocity of the flow. So that we can write as l/u0. I mean always when we are writing these, we should give a better symbol as this one ~ which is meaning the scale. That means it is not that t is exactly = l/u0 but order of magnitude of that is dictated by the length scale and the velocity scale at that condition.

So that means, so if you give it a name let us say pi, so this is per unit mass we have to understand. So pi is of the order of u0 cube/l, that is the rate of extraction of kinetic energy from the mean flow. So let us write the scale. So let us say that you have mean flow or let us say we have a large Eddy and a smallest, largest and smallest Eddy. So largest Eddy and smallest Eddy. So the length scale of the largest Eddy is l, the velocity scale is u0 and the timescale is t.

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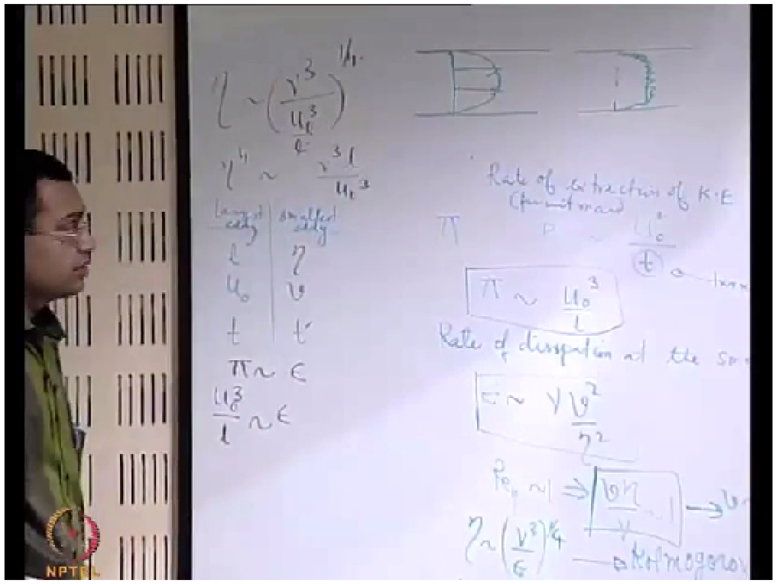

So we will have some scales for the smallest Eddy also and our objective will be to compare these scales to see what is the total range of scales over which the activity is going on. So for the smallest Eddy, we have let us say eta that is the symbol that we have given as the length scale. Let us say v is the velocity scale and maybe say t prime is the timescale, just some names. So in the smallest Eddy scale, what is happening?

There is a viscous diffusion that is taking place. So whatever is the kinetic energy that has been extracted and being transmitted to the smaller Eddies, the smallest Eddies, that it is now dissipating through viscous mechanism to its sounding fluids. So entire energy so it is not able to sustain its rotationality any more by going to smaller scales because it is dissipating entire energy.

Because of the dominance of the viscous force, now the dissipating mechanism is very strong. So the rate of dissipation is something what is important. Rate of dissipation at the smallest Eddy scale. So what is the rate of dissipation at the smallest Eddy scale? So if you have say let us call it epsilon. So the rate of dissipation is given by like if you again write it as per unit mass, it is given by 2*nu*the rate of deformation.

If you call it Sij as the rate of deformation, then Sij is like 1/2del ui del xj+del uj del xi, that is the rate of deformation and the rate of dissipation at the smallest Eddy scale is, I mean this is not just for smallest Eddy scale but any scale but here we are talking about the rate of dissipation. So the rate of dissipation at the smallest Eddy scale will also be governed by these rule, only the velocity of length scale we have to identify properly.

So what is that? So 2, again the factor 2 we forgot. We just write nu which is nu is the mu/rho, the kinematic viscosity. So nu*the rate of deformation square basically, into the rate of deformation square. So the rate of deformation is what? Rate of deformation is some velocity/its length scale. So velocity is v and length scale is eta for the smallest Eddy. So nu v square/eta square, order of magnitude.

That is one of the important things. The other important constant is that over the smallest Eddy length scale, the Reynolds number should be of the order of 1. So the third constant, so let us just keep these scaling expressions in mind, that in the smallest Eddy length scale, you have the Reynolds number with respect to the length scale eta is of the order of 1. That means v*eta/mu=1 or of the order of 1 again.

So this length scale you may write in terms of this epsilon, the rate of dissipation at the smallest scale. So it is possible to write V is of the order of mu/eta and therefore, you have epsilon is of the order of mu*v square/eta square, that is nu*nu square/eta square. That means nu cube/eta square. That means what is eta? Eta is of the order of nu cube/epsilon to the power… yes sorry, so v is of the order of nu/eta, so again, sorry, this will be v to the power 4 right.

So nu*v square/eta square that means mu*, let us just write it bit properly. So epsilon is of the order of nu*v square, sorry, yes, v square is nu square/eta square/another eta square. So nu cube/eta to the power 4. So this will be nu cube/epsilon to the power 1/4, right. So when it is nu cube/epsilon to the power 1/4th, as if if you know what is epsilon, you know what is this eta, the smaller scale but this is not complete because you say that we do not know what is epsilon.

So to know what is epsilon or to estimate what is epsilon, we will go through maybe one simple step but it is important to understand that if we know epsilon, this gives the length scale of the smallest Eddy and that is known as Kolmogorov length scale. Kolmogorov length scale are microscale. So how do you estimate the Kolmogorov length scale, that is quite straightforward because at the end, you must have whatever energy that has been extracted with a certain rate from the mean flow, the energy at the same rate has got dissipated from the smallest Eddies.

Otherwise, there will be accumulation of energy at the intermediate Eddy scales and that will disturb the structure of this energy cascading. So you must have the order of pi same as the order of epsilon. So whatever is the rate at which the energy has been extracted from the mean flow, that has been the same rate at which the energy is dissipated.

So that means you have u0 cube/l of the order of epsilon, right. So you can write epsilon from the characteristics of the system. So that means you can now write the eta as of the order of nu cube/epsilon is like u0 cube/l to the power of 1/4th. So what it means is now you can write this eta, so let us take a 4th power of all the sides, so you have eta to the power 4 is of the order of nu cube*l/u0 cube, right.

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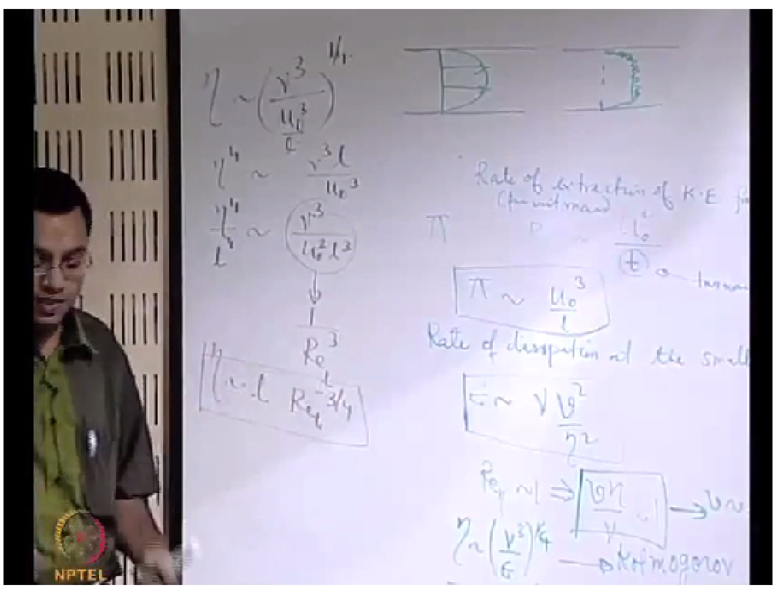

So if you simplify it one more step, if you write eta/l, that is if you divide by l to the power 4. So what it will bring rise to…, okay. So this is nothing but 1/Reynolds number with respect to the system scale cube. So eta is of the order of l*Reynolds number to the power -3/4th. This Reynolds so we are relating the smallest Eddy length scale with the system scales. So let us stop for this lecture now and in the next lecture, we will see that what are the approximate magnitudes through some practical numbers that what are the approximate magnitudes of these length scales and the velocity scales, okay. Thank you.