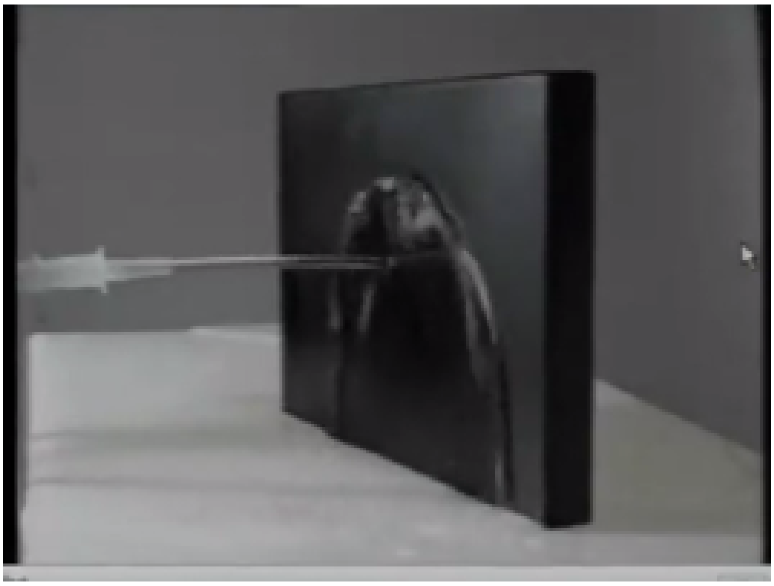
Introduction to Fluid Mechanics and Fluid Engineering
Prof. Suman Chakraborty
Department of Mechanical Engineering
Indian Institute of Technology - Kharagpur

Lecture – 27
Integral Forms of Control Volume Conservation Equations (Reynolds Transport Theorem) (Contd.)

In the previous lecture, we were discussing about like how to write the basic principle of angular momentum conservation for a control volume and we will look into some examples related to that but just to physically appreciate that what kind of examples are there may be let us look into one or two cases, we will see that what are the differences in like practical cases, when you have a linear momentum and angular momentum conservation.

(Refer Slide Time: 00:51)

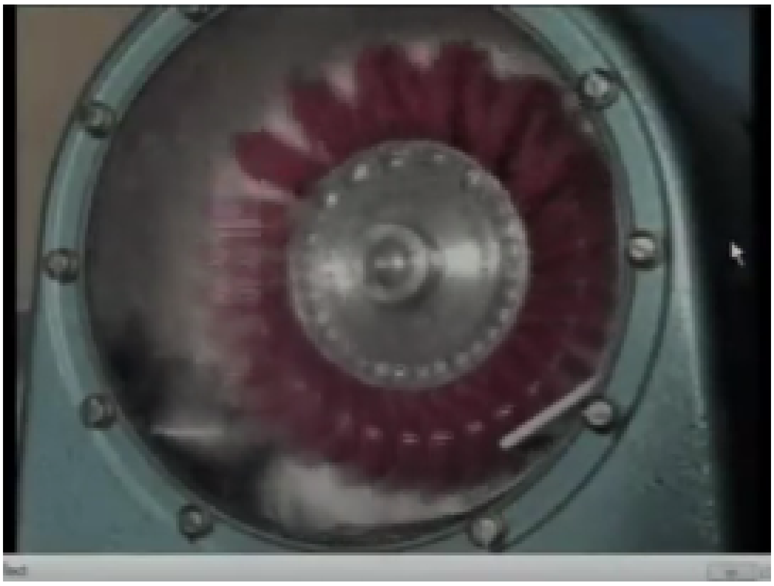


So, first we look into an example which we discussed it in some way that basically you have a flat object on which there is water which is falling.

(Refer Slide Time: 01:09)

And you can see that the reality is so different from what we considered with the water jet may not be thin; the water jet may be having different thicknesses at different sections.

(Refer Slide Time: 01:17)



So, it is a gross simplification of what we could see, now think about a; look into a situation when the water jet is falling on something, which is mounted on with respect to one axis and when it is doing like that it creates a moment of the linear momentum or the angular momentum and that mounted wheel starts rotating and that is the basic principle of a turbine that we will see later on when we will be discussing about the hydraulic machines.

(Refer Slide Time: 01:51)

But now, before going into such complicated things of fluid machines, see this example this also gives a very important example of angular momentum conservation, this is a lawn sprinkler, so this is commonly used for watering the gardens and if you see that water is coming out of the limbs of the sprinkler and when water is coming out that water comes out with a velocity.

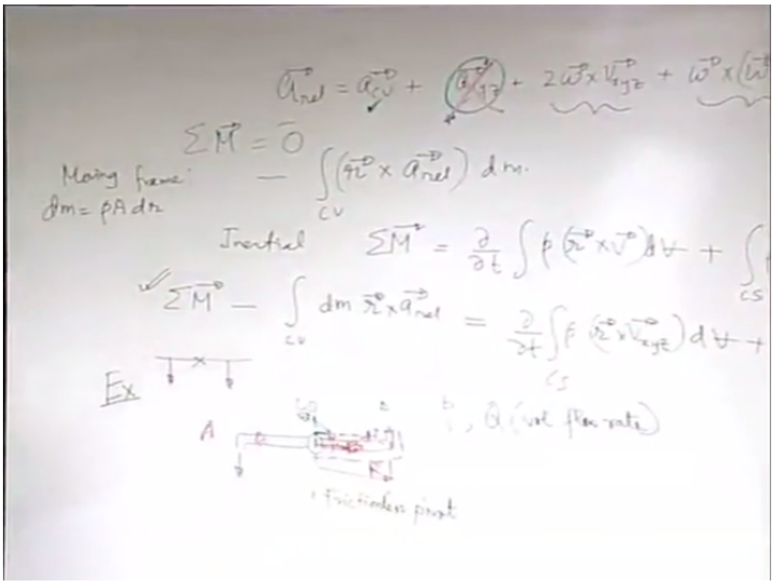
Because water is supplied at a given velocity and when water is coming out with that velocity, what is happening? Basically, it is having a linear momentum that linear momentum has moment with respect to the axis of the sprinkler, so there is an angular momentum and there is a rate of change of angular momentum that makes it rotate, even if it is like; if there is no frictional resistance that will make it rotate very fast.

Because of some frictional resistance against rotation in the pivot which is there, it will not rotate as easily as we might think but still you have seen in this example that it might rotate quite easily. So, if you see that; see, initially there is no rotation but when the water is coming out see, we are trying to develop an understanding that it is actually an unsteady problem, so that is the first thing to appreciate.

It is not a steady problem because you see that the entire physical behaviour what we are observing here is a function of time and our important objective will be to see that if it develops an angular velocity from say, time = 0 to sometime = t, then how that angular velocity evolves with time? So, this lawn sprinkler example is a very simple example but it can demonstrate a good use of the angular momentum conservation principle.

And this principle in a more elaborate way is used for analysing those fluid machines, which are basically of rotating nature like there are 2 very common devices, which we will be discussing later on; one is a centrifugal pump and another is a rotating turbine. So, we will be discussing about these things in more details that will probably be our last chapter in this course of fluid mechanics that is the fundamentals of fluid machines.

(Refer Slide Time: 04:19)



Right now, we will not go into that application but we will concentrate more on a simple example, so we will use this for a case, so let us take that example, when we consider that example we will simplify whatever thing which we saw in the practicality, we will consider that there is only one limb like this of the sprinkler and this entire thing is in a horizontal plane; rotating in a horizontal plane, just think that it is one arm.

There could be many such arms but we are just concentrating on one arm, so the water is moving at uniform velocity relative to the rotating arm, so the arm is rotating, if you had looked into the examples very carefully no matter wherever the arms are there, the water related to the arm was coming out with a fixed velocity because that is the rate at which it is being provided.

So, the water it comes to the center and then gets distributed along the arm moves along the arm with a uniform or a constant velocity relative to the arm and eventually, it goes out like what shown in the figure and let us say that it starts rotating with an angular velocity omega which itself may be a function of time and our objective is to find out what is this omega as a function of time.

Let us say that we know that rho is the density of the water and Q is the total rate of supply of water; volume flow rate; so this is volume flow rate like meter cube per second. So, we will try to analyse this problem in 2 ways; one is with an inertial reference frame, another is with a moving reference frame and we will try to see whether we converge to the same conclusion or not.

We start with the moving reference frame that is we; as if we attach the reference frame with the sprinkler on which is rotating with a given angular velocity with not a given; with some angular velocity, which is a function of time and therefore it is subjected also to some angular acceleration but it is not translating. So, now we are having a control volume, which is a rotating control volume but not a translating one.

See, this problem is a symmetric problem with respect to the center of the arm, so we may take say a control volume say, half of the entire thing, when you have a problem which has symmetry in certain ways and the physics is also symmetrical, geometry is also symmetrical then you can reduce your task or burden by taking a symmetrical part of the problem. So, we have taken one symmetrical half of the problem.

In this problem, we have neglected one thing, we have assumed that the vertical part of the limb is very very small, so it is mainly a horizontal one and the vertical is very, very small, remember that this is actually in a horizontal plane, so thus because we have to draw it in the plane of the board, it appears to be like it is upwards but it is like in a plane, it is a bit bend like that and that bend part is quite small that we are assuming.

It is just to give it a sufficient change in direction but not enough length that is what is the assumption and let us say that the radius is capital R, which is the radius of each limb and that is symmetry. So, we will write first the equation for the moving reference frame, so when we write the equation for the moving reference frame, we will use this form whatever we discussed in the previous class.

One important correction which one of your friends has mentioned and it is quite correct that this term is not going to be there in a relative expression because this like; when we wrote a relative that was basically a capital XYZ - a small xyz and that is how this actually got nullified in that expression. So, we need not duplicate it, so this term is not there in a relative one, just keep in mind that was a mistake and you please correct that.

Now, next is; so let us try to apply this one with respect to this expression, so the resultant; again we are considering that it is a frictionless pivot, so what it tells is that there is no frictional resistance moment, when this is rotating, so this is rotating like freely, so to say, so when this is rotating freely, you have the resultant moment of all external forces because of what; one may be because of the frictional torque, which is present.

So, frictional torque we are assuming to be 0 but there could be torque because of the weight distribution and because it is symmetrically distributed, the resultant torque due to weight, it is there or not there? So, let us consider a small element in there, so you have a symmetrically located thing, a small element in this side will always be counter balance by an equivalent small element in the other side, in terms of the rotational thing.

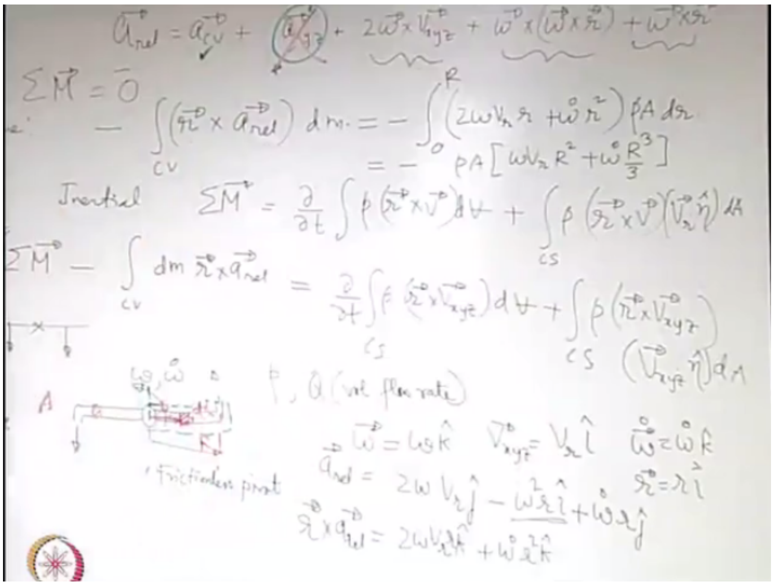
So, there is actually no resultant moment of external forces because only 2 external forces could be that rotational resistance plus the moment of the distributed weight, okay, so that is not there, so that is a null vector. Then, let us; if you consider the moving reference frame, so in the moving reference frame, we have this correction term, so a relative. So, in a relative first term is acceleration of the control volume that is a linear acceleration of the control volume, here that is 0.

Next are the 3 rotational terms that we need to consider, so what are the rotational terms? 2 omega cross V small xyz, so V small xyz is what? It is the velocity of the fluid relative to the control valve, we are assuming that it is moving at a constant velocity relative to the control volume, so V small xyz is like a constant but if you want to write this term properly, let us write - integral of r cross a relative * dm for the control volume.

So, what we will do; we will let us say, we will split this, a relative in different parts instead of just writing it in a cumbersome single expression, so we will find out the contribution of the 3 different terms, which are there. So, 2; before doing that what is this dm; that you have to keep in mind, dm is some small element at a distance; at a radial distance of small r, you can take a strip of width dr.

And let us say that capital A is the area of cross section of the sprinkler, this area of cross section, so dm is what? If dm is the elemental mass of the shaded volume, then what is that? Dm is rho A dr, so basically when you are writing the expression, this will be integral of whatever you write as a relative with this entire thing that multiplied by rho r dr, integrated from small r = 0 to small r = capital R that will be that correction term in the left hand side.

(Refer Slide Time: 13:11)



So, before applying that correction term, let us write these things in a vector form, so what is omega in a vector form? Let us say this is xy plane, yes, omega K; what is V small xyz? No, this is by positive sign convention, if it comes out to be minus something then that will be like that. So, then what is V small xyz? So, it is moving; no, no V small xyz is what; Velocity of the water relative to the sprinkler.

So, if say it is moving at a velocity Vr relative to the sprinkler in a magnitude sense, vector sense yes, with these 2 options, i or j, yes only 2 options are there, it cannot be K, so only i and I, yes. I or J? You are not very sure it should be i, see when you are considering these, this will be a volume integral, this part is negligible in comparison to this part, so it should be ideally, for this part i and for this part j.

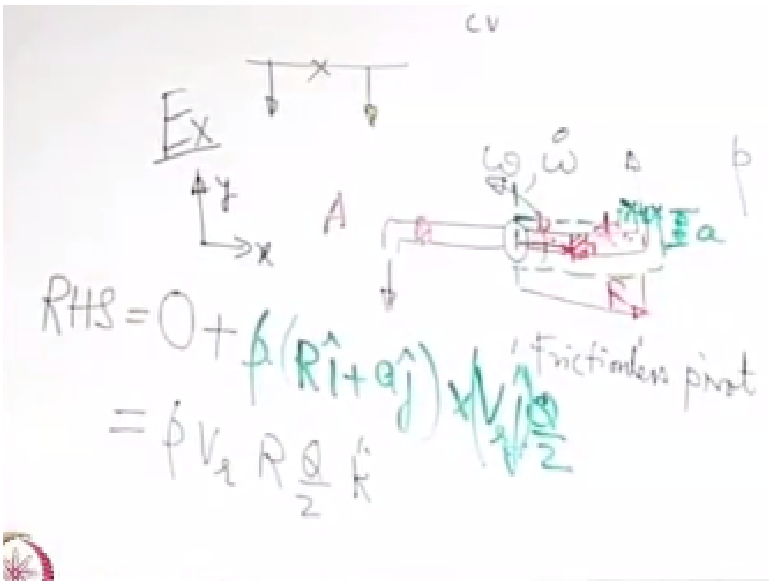
But because I have already mentioned that this is like of negligible length, in the volume integral only this horizontal part is mattering and there it is moving radially, so it is like Vri okay, then omega dot that is the only other thing that is remaining, let us say that is omega dot K, let us say omega dot also has a positive sense like that. So, a relative; first 2 omega cross V small xyz, so these 2 cross product; so, 2 omega Vrj then, omega cross omega cross r.

What is omega cross r, what is r in a vector form, ri? So, omega cross r is omega rj and omega cross omega cross r is - omega square ri, so it is just like centripetal acceleration, so - omega square ri, then next is omega dot rj. Now, if you make r cross a relative, so you will clearly see that the centripetal term will not be there because it is directed along r, so its moment will be 0 but other terms will be there.

So, 2 omega Vr K + omega dot r K with r multiplied, right, so let us write r, r square. So, when you write this particular term, now we are in a position to write it in a proper integral form, so that will be - of 2 omega Vr r + omega dot r square * r dA sorry, rho A dr that is dv, 0 to capital R, right, so that will become - rho A omega Vr capital r square + omega dot r cube/ 3, okay.

Then the right hand side, let us write the right hand side; now you tell, the left hand side we have completed, right hand side what will be the first term? You have to see what are the things which are functions of time, V small xyz is not a function of time because we are assuming that relative to the sprinkler arm, the water is moving at a constant velocity, so no matter how the arm is moving but relative to that.

(Refer Slide Time: 18:56)



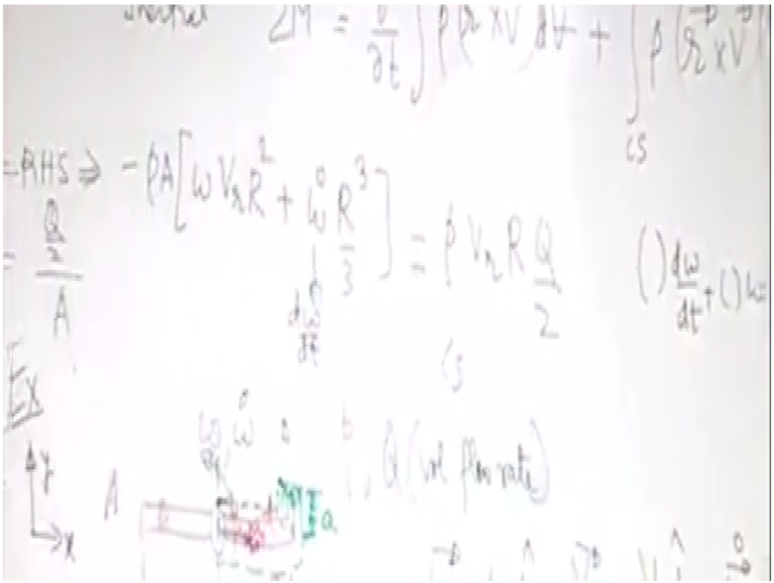
And we are writing it only relative to that it is not changing with time neither the volume of the control volume is changing with time, so that this term will be 0, so in the right hand side the first term will be 0 and what will be the second term? Second term is the flow boundary; it represents what happens at the flow boundary, so for the second term when you substitute that so let us assume that these velocities are again uniformly distributed over the flow boundary.

So, V dot eta dA, this term if you just keep it inside, now r cross V small xyz, can you take it to be independent of the area and bring it out of the integral, yes or no, what is this r now? These r where; this is; some r located at the flow boundary that means at these location somewhere because this is very thin, it does not really matter exactly that where is the variation might be somewhere here.

So, that is basically capital Ri + something j + some little something j that is the radius outside, so let us say that it is capital R, let us give it a name say, a keeping in mind that a is small, so ri + aj that is r cross V small xyz is a constant, so that also we can take it out of the integral provided with also consider it to be constant over the area; uniform over the area. So, if you assume it to be uniform over the area, we just take it out and that is V relative.

And then rho is there of course, rho let us, write the remaining thing which is there in the integral is the volume flow rate over that section, so what is the volume flow rate over that section? Yes, it is Q/2, see we have only taken the half of this; that half of the Q flows through these, half of the Q flows through these, there is no exception because it is perfectly symmetrical, so this into Q/2 that is the right hand side.

(Refer Slide Time: 22:25)

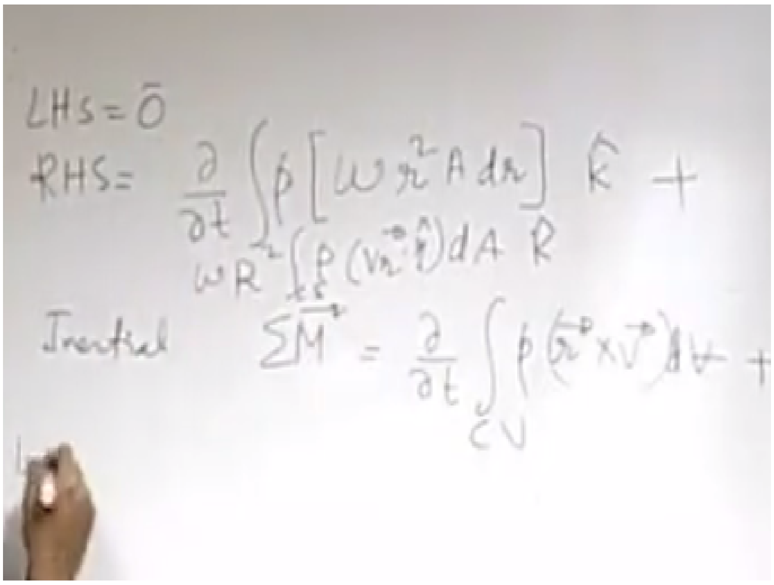


And the question is what will you put this in a vector form, so Vr what; i or j? This one, V rj right, so you will see that this little part a will not matter because of the cross product, so it will become; let us write it completely, it is = rho Vr r Q/2 K, since the left hand side is equal to the right hand side; so the left hand side equal to the right hand side, which implies what? -rho A omega Vr r square + omega dot r cube/ 3 is = rho Vr r Q/2.

This we miss that K cap with the vector sign that was there, so this omega dot is nothing but d omega dt, Vr you know because what is Vr; Vr is the flow rate divided by the area, so Q/2 divided by A, so given the flow rate, you should be in a position to substitute Vr, which is a constant, so only the variables are d omega dt and omega, so it is a differential equation of the form something * d omega dt + something * omega is some constant, right.

So, it is; you may separate the variables easily to solve it because these are constant coefficients here. Now, let us try to look into the problem from an inertial reference viewpoint and see that whether we can solve the same problem in that way. So, now we will get rid of the non-inertial corrections and we will just consider that we are looking for a control volume, which is maybe schematically, you draw it like this but that is not moving with the; that is not moving with the arm.

(Refer Slide Time: 25:05)

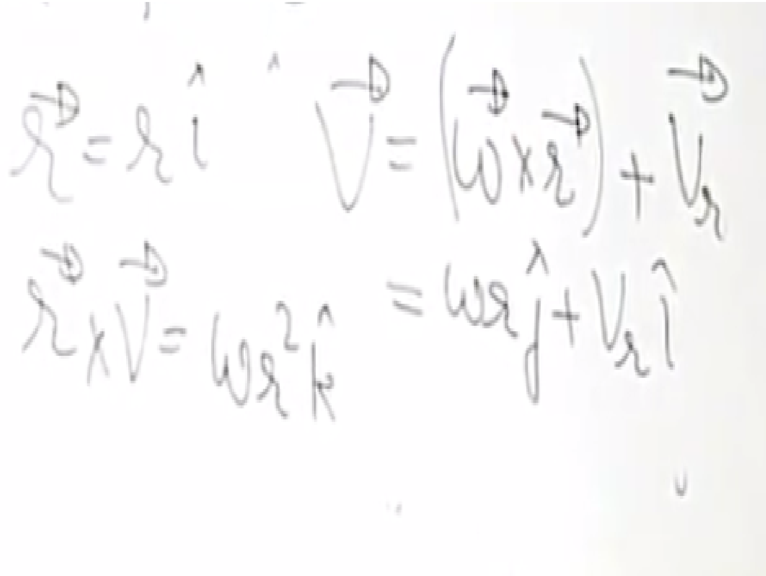


So, even if the arm goes to a different place, the control volume is fixed, so at that instant, we are trying to; we are we are considering one particularly instant, when the control volume and the arm they are coinciding and at that instant, we are trying to figure out what is happening. Now, if you consider the inertial problem that is the inertial reference frame way of looking into it, the left hand side is 0, right; the left hand side is 0 or null vector.

The right hand side; now, you see that with respect to an inertial reference frame, it will be unsteady, right. If it was moving at a constant velocity relative to the reference frame then sitting on the reference frame, you look it as a constant but standing from outside, which is inertial you see that velocity is changing with time because the frame velocity itself is changing with time.

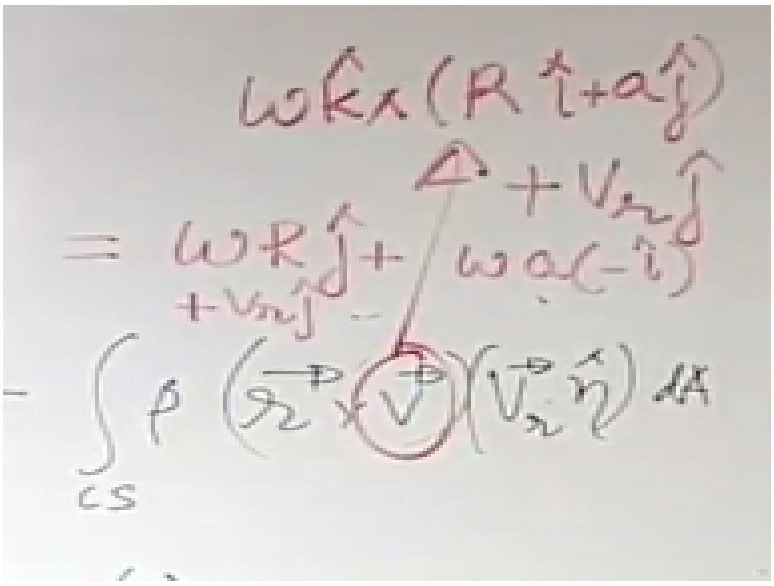
So, this time derivative term with respect to an inertial frame will not go to 0, so the right hand side therefore, you have to write this time derivative term properly. So, when you write it, then let us write r again, r you write as ri cap because the volume, which is involved for the control volume integral; this is a control volume integral, so their major part is the horizontal part, then what is this V; what is this V?

(Refer Slide Time: 26:32)



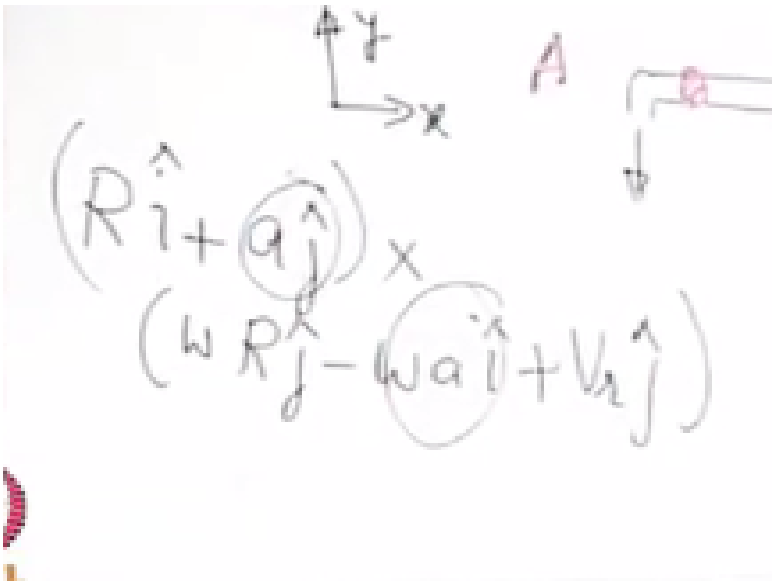
This is velocity of the solid plus velocity of the fluid relative to the solid, so what is the velocity of the solid at the coincident point at a radial location r, it is omega cross r, so this is the velocity of the solid plus the velocity of the fluid or V relative or Vr, so this is omega rj+ Vri, right. So, we substitute that but before substituting maybe it is useful to find out how it is r cross V.

(Refer Slide Time: 28:35)



So, that is omega r square K then, so you write this as omega r square then in place of dV, A dr this thing K, right then plus, again you assume uniform velocity distribution, so when you write the next term, then what should you substitute for this V? This one; yes, remember this V has to be inertial reference frame V, so what is that? So, omega K cross with capital ri + aj + Vrj, right, so what will be that? Omega rj + omega a - i + Vrj.

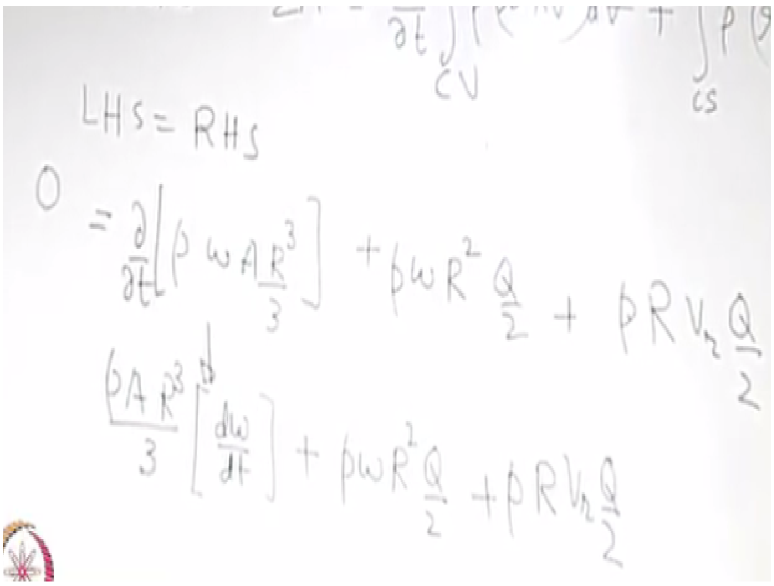
(Refer Slide Time: 29:54)



So, if you make another r cross with that then, what is that r? That r is again this capital Ri + aj, so what is that R cross; cross with that one, omega Rj - omega ai + Vrj, right. So, here if you neglect small a, then like that is what is given in the statement of the problem, so just neglect this small a part, so it is omega rR + Vr * capital R K, so the next term we will write; yes, this is omega cross r is what?

So, if you consider a point here, the point first how do you consider; a coincident point in the solid plus velocity of the fluid relative to that coincident point, okay that is the absolute velocity of the fluid, Vr is V relative, yes, so the same Vr that we used in the; even the previous method. So, the next term is omega R square then integral of rho Vr dot eta dA over the control surface with a K cap.

(Refer Slide Time: 32:03)



So, now what we do; we simplify the different terms just like what we did in the previous case or in the previous method, rho is a constant that we are considering for this problem, so we are taking that rho here out of the left hand side and right hand side. So, left hand side equal to right hand side when we write, so 0 =; now you see, first of all we integrate with respect to position.

So, when you integrate with respect to position, it is from 0 to capital R, so you have basically, first let us complete the integral, so integral will be rho A omega capital R cube/3, now a partial derivative of that with respect to time and then the remaining one + omega R square * Rho, if the remaining term is Q/2 because remember no matter whether it is inertial or non-inertial, these velocity is always V relative for flow rate part calculation okay.

Now, rho being constant A being constant, R being constant but 0mega is not a constant, so you have rho A omega maybe by divided by 3 also sorry, rho A R cube/ 3 then you have d omega dt +; now there has been a point, where I have omitted one term, where is that term? This is your work to identify, you may get a clue by comparing with the previous method and this method, they should at the end give the same result.

No matter, what is the control volume, so some term is omitted somewhere, **"Professor – student conversation starts"** which term? Vr *; where is that term? R cross v, so what is there, no; you tell, I will just write; R * Vr * Q/ t, so one term because of multiplication of this with R because of this with r that was omitted and not omitted just somehow not written here, so that is equal to 0 **"Professor – student conversation ends."**

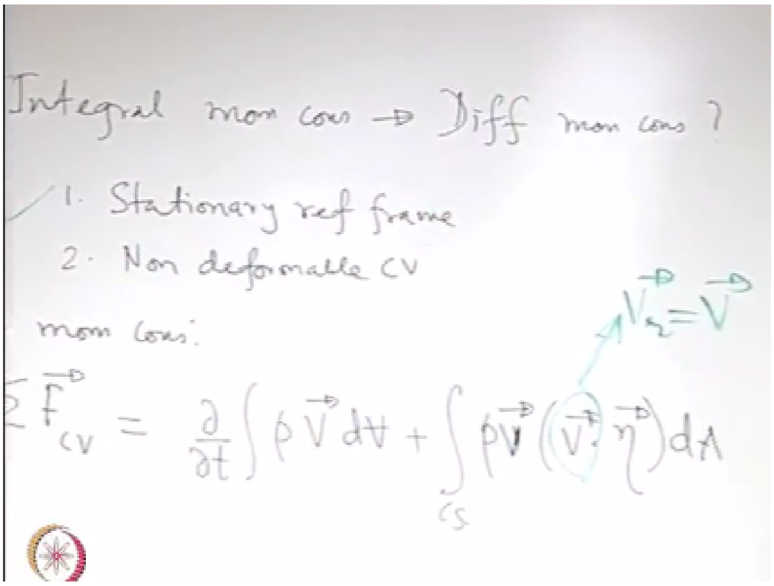
And this should give the same differential equation as what we obtained with a non-inertial reference frame, it should not be fundamentally different, okay. So, what we have understood is; that you may use different reference frames but when you go to your final analysis, the final analysis should converge to the same conclusion. Now, what you may do a little bit of modification of these that when you take the part small a, not very small that is a; it is of substantial length.

Then neglecting it and not neglecting it will matter more for approach 1 or approach 2 that is will matter more for the inertial frame based analysis or non-inertial frame based analysis that is a bit of more thing that you can add with this one but again the principle will be very similar then you do not neglect the terms with small a and just do the algebra and then you will easily find it out.

Now, what we have seen here; so in this particular chapter, what we have discussed, in this particular chapter we have found out how or we have learnt how to write the integral forms of conservation equations for mass conservation, linear momentum conservation and angular momentum conservation. In one case, we have at least shown that you may convert the integral form to a differential form and that is the mass conservation example which gave back the continuity equation.

Let us try to see that whether we may do it for other cases or not, so now our objective will be let us say, we take an example of linear momentum conservation, can we write a differential equation for linear momentum conservation starting from the integral form, so that we will do.

(Refer Slide Time: 37:40)

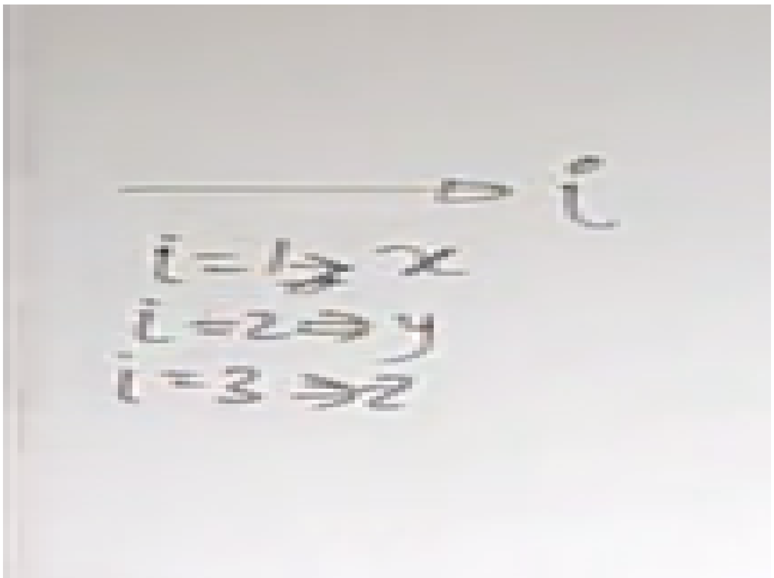


So, the objective of the subsequent analysis is to figure out that is there a route from integral form of momentum conservation to differential momentum conservation. The differential equations are important because in many cases, you want to get a point by point variation of the velocity field or the pressure field like that so then you have to solve the differential form not the integral form.

So, let us say; let us write, let us make certain assumptions; let us make first assumption is that we have a stationary reference frame and number 2; that non deformable control volume.

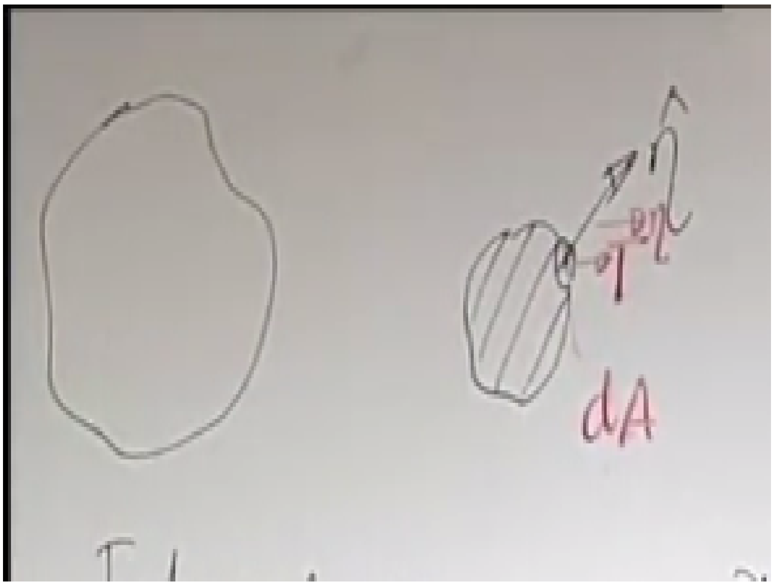
Based on these, let us write the linear momentum conservation, so if you write the linear momentum conservation, integral form, resultant force on the control volume is =; so we have assumed straight away that it is a stationary reference frame.

(Refer Slide Time: 39:53)



So, that Vr is = V that is the first assumption that we add made and regarding the second assumption, it will be possible for us to take this inside the integral. Now, we will concentrate more on the left hand side and see this is a vector equation, so it has its different components, so it may be convenient to express it in terms of a component, say in the direction i, we will again start using the index notation somewhat.

(Refer Slide Time: 40:26)



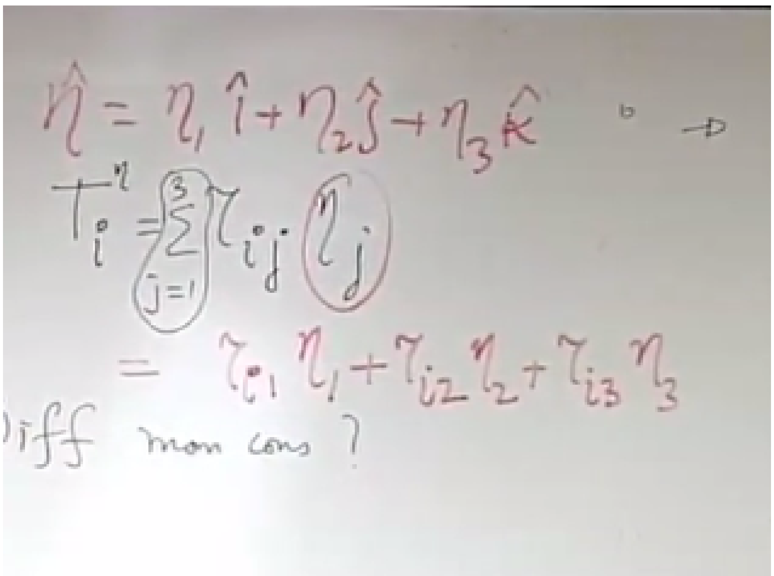
So, i = 1 will mean x, i = 2 will mean y and i = 3 will mean z, okay, so let us write this for the direction i, so we are interested to find out what is the resultant force acting on the control volume in the direction i. Let us make a sketch and try to see, if you recall our very basic discussions that we classified the force in continuum mechanics in 2 categories; one is a body force, another is the surface force.

So, what is the body force? The body force is the force that is distributed over the extent of the volume of the body, so let us call this as F surface for surface force plus F body, so when you have the surface force that is distributed over the area of the surface, so when you have area of the body, so when the body is like a control volume it is a force distributed on the surface of the control volume.

So, now if you want to find out that what is the body force or what is the surface force first, so if you consider small chunk of an element like this with a direction normal of eta. So, when you have the direction normal as eta and how do you express the surface force at a particular location? You represent it in terms of the traction vector that is the force per unit area but it is dependent on the choice of orientation of the area.

So, let us say that Ti, let us say that T with superscript eta is a traction vector at this point based on the chosen area say dA, right. So, this traction vector we may write in terms of what; we may write in terms of the stress tensor components that we have discussed and provided we know the direction normal of the area that is under consideration and which theorem gives this? Clausius theorem.

(Refer Slide Time: 42:36)



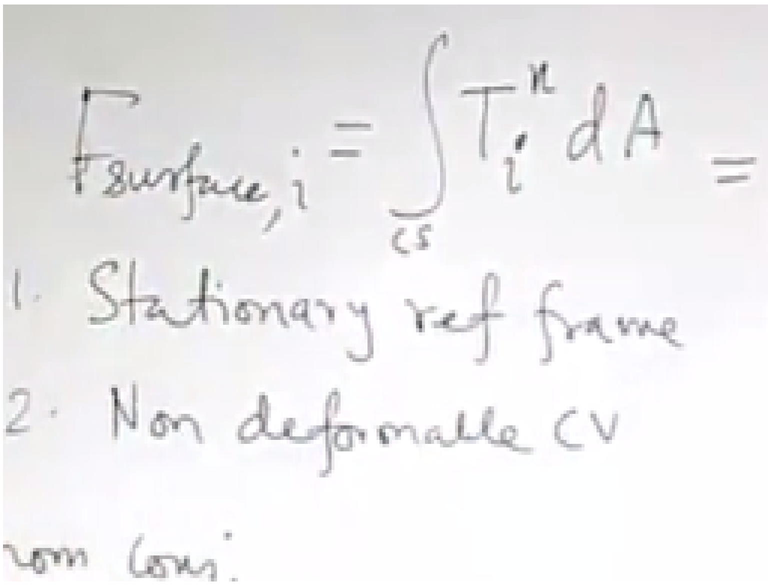
That is you have the traction vector; the ith component of the traction vector is given by tau ij * this, remember that invisible summation sign for j = 1, 2, 3, this is actually not usually put in the sign convention; in the representation convention. Fundamentally, it was tau ji * nj but because tau ij = tau ji, how we have got tau ij = tau ji from angular momentum conservation. So, usually you see that for fluids, we do not separately write many times angular momentum conservation in a differential form.

because it is already inbuilt with some of the considerations in the linear momentum conservation, whenever we write tau ij = tau ji, so that is already inbuilt here and that is how it becomes like this, so if you just expand it, so what is this; this is like if you have the unit vector, it has its components along xyz as say, n1, n2, n3 are the direction cosines. Now, tau; therefore, this one we can write; see i is an index which is there in the left hand side that should remain also in the right hand side.

So, tau i1 n1 + tau i2 n2 + tau i3 n3, okay, can you express this as a dot product of some vector with the n vector, what is your objective? The objective is; we will replace the area integral in terms of the volume integral as we did for the continuity equation and because we know the divergence theorem that relates these 2. For that we have to know; we have to get something of a form of like some vector function dot with n dA.

So, that is why we are looking for a vector which dot with n gives the same thing, okay, so the objective should be clear. Whenever we are doing an analysis, it is not that out of nothing we are doing this manipulation that is the objective of the manipulation that we should keep in mind, so whenever you convert from an integral to a differential form, the main way in which you always do is by converting some of the area integrals into the volume integrals and that by using the divergence theorem.

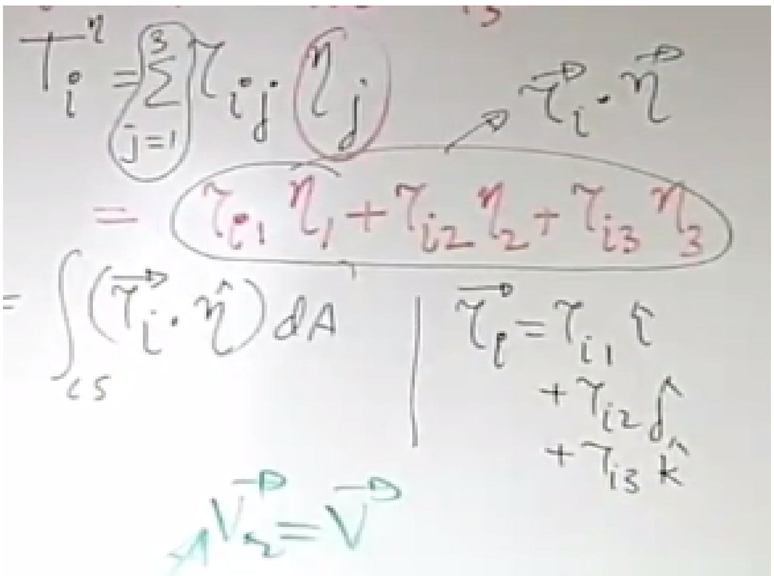
(Refer Slide Time: 45:42)



So, you should make the all the terms compatible for use of the divergence theorem, all the area integral terms at least. So, when you are trying to represent the surface force, what is the surface force, F surface is? F surface i, so let us just write the i component, x component, so when we call the components will not put a vector sign because the components are just the scalar components.

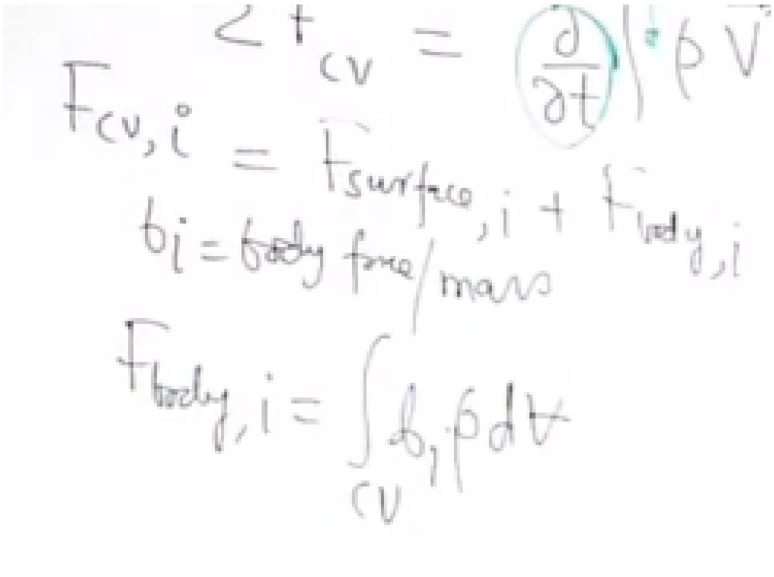
So, F surface i will be what; traction vector is the force per unit area, so it is the traction vector ith component times dA integral over the control surface; the surface of the control volume. So, that will represent what; see, the control volume surface, if the control volume is like a free body in mechanics, so if you have the internal action reaction forces within the control volume they get cancelled out, only the surface forces at the outer boundary remains.

(Refer Slide Time: 47:00)



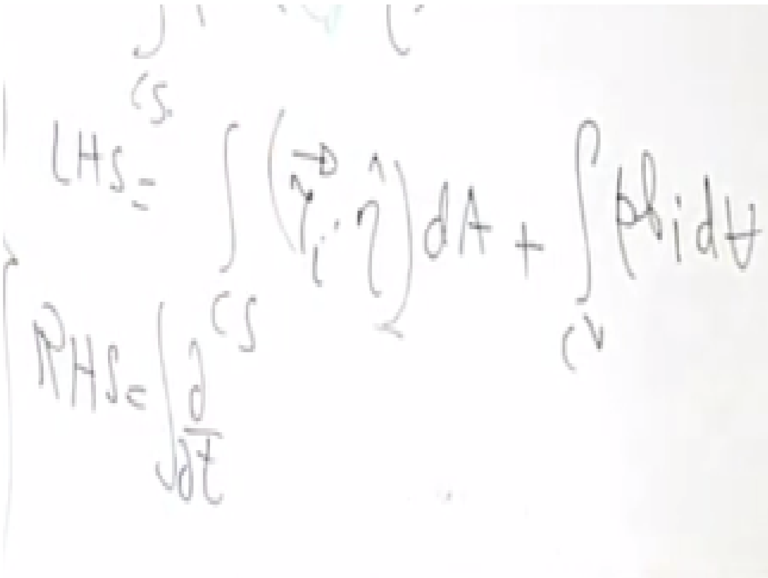
So, it is just considered equivalent to a free body, so this; now, our objective is to write it as some vector dot with n, so clearly if you see that if you write a vector say, tau i just give a name of a vector like that tau i1 i cap + tau i to j cap + tau i3 K cap, then this term, is what? This term is nothing but this tau i dot with n that you can clearly see because when you make it a dot within, you see tau i1 is coming with n1, tau i2 is coming with n2 and tau i3 is coming with n3, right.

(Refer Slide Time: 48:10)



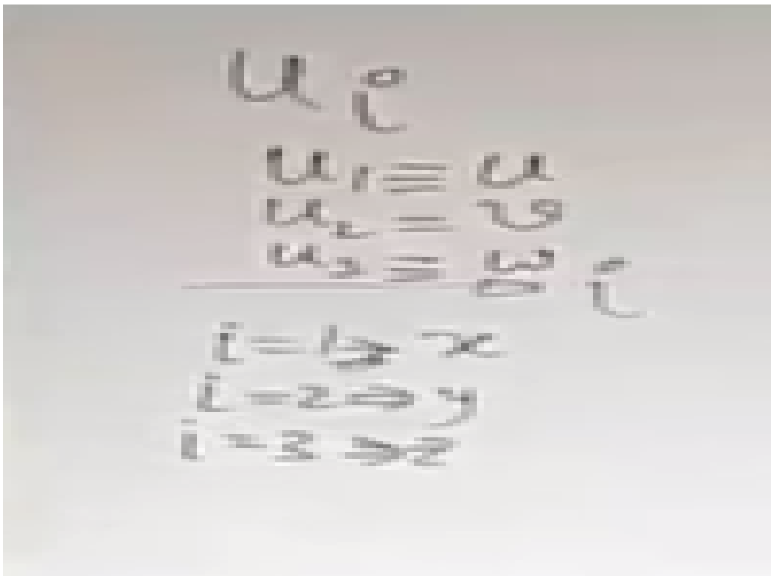
So, we have formed an artificial vector, just to make use of the divergence theorem. So, we may write this as integral of this new vector say, tau i dot n dA, what about the body force? Let us say that Vi is the body force per unit mass, so Vi is body force per unit mass. So, how do you express the F body? So, basically you take a small volume inside of dV, the mass of that is rho * dV and the corresponding body force along i is Vi * rho * dV.

(Refer Slide Time: 48:51)



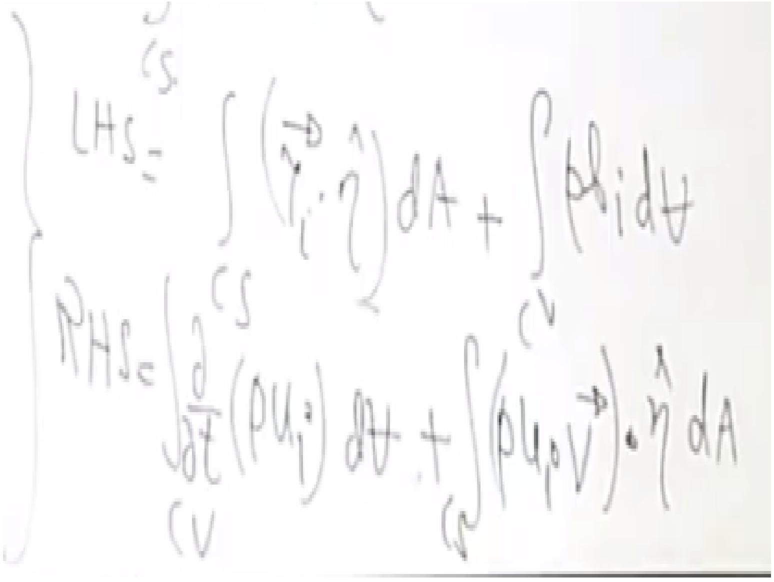
If you integrate it over the control volume, it will give the total body force along i, okay, so we have been successful in writing the left hand side in a certain way for force components along i, so the left hand side will become integral of over the control surface tau i dot with eta dA + control volume rho Vi dV, what about the right hand side? We will take the time derivative term inside the integral because of non-deformable control volume.

(Refer Slide Time: 49:32)



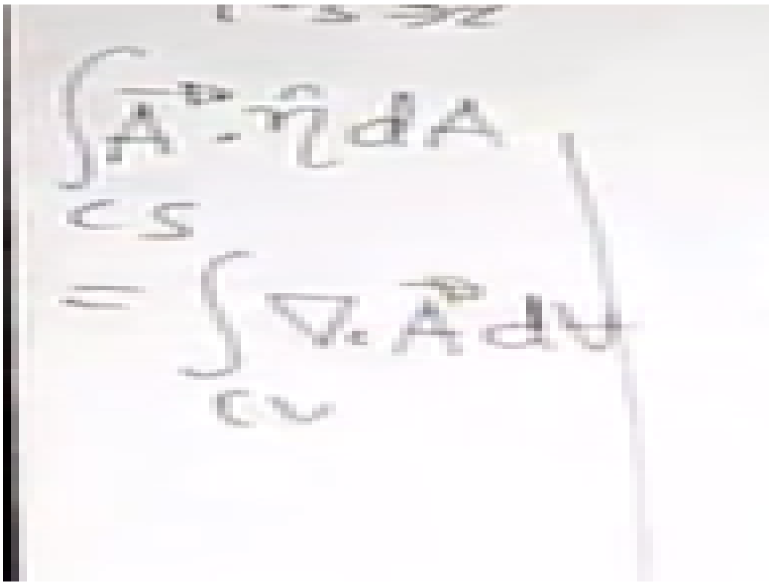
Now, see we are writing the scalar components, so in place of V we will write Vi or ui usually, the notations for velocities, we call ui, so u1 is like u, what we have learnt, u2 is like v and u3 is like w, the usual notation for the velocity components. So, this in general when you consider the ith direction, it is ui; i = 1 means x, i = 2 means y, i = 3 means z.

(Refer Slide Time: 49:59)



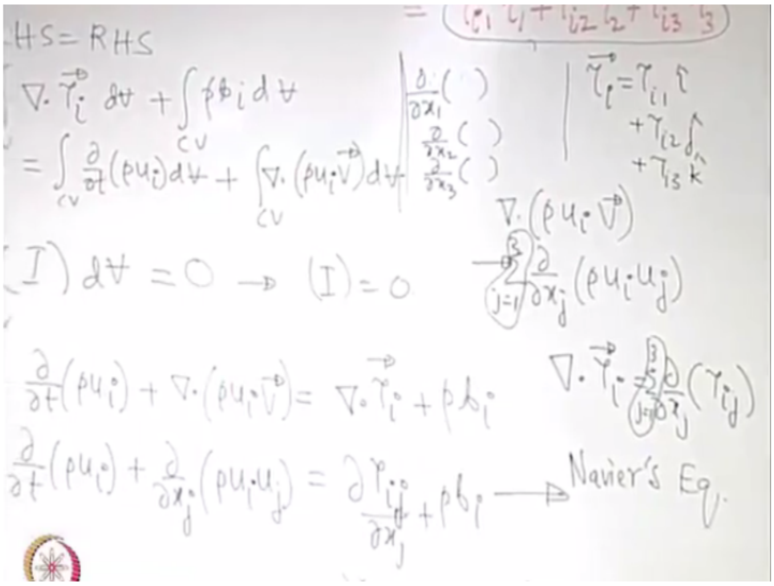
So, this of rho ui, this V is replaced by its component along i and then the remaining integral of rho, this V is replaced by ui, okay, the next steps are quite simple, the next steps are converting the area integrals into the volume integral, so that we may convert all the area integrals into the corresponding volume integrals.

(Refer Slide Time: 50:47)



So, we should keep in mind that if you have a vector functions say, A, so A dot n dA over the control surface is the divergence of A over the control volume, so this surface has to entirely bounding the control volume.

(Refer Slide Time: 51:09)



So, if you consider that and if you consider the left hand side is equal to the right hand side, then what you get? So, the first term, this; so here this tau i is like the vector function that we are talking about, so then what will be the left hand side? First term, if you convert into a control volume term, divergence of tau i dV then integral of rho bi dV = the right hand side, first term is already control volume term plus next term divergence of rho ui V, right.

So, we now collect all the terms and if we collect all the terms you will get therefore, some integral of some function dV = 0 right because all terms are of integral dV nature. And we have seen that when we discussed about the continuity equation, same thing applies that this volume that we have selected is arbitrary, so for an arbitrary elemental volume, if this has to be satisfied, the integrand has to be 0. So, that means you have this integrand, let us say this as i, so this i has to be 0. So, if i is = 0 then you are left with this particular form, okay.

Now, it is possible to express, see this is partially written in index form, partially written in vector form but it is possible to write it in a proper fully indexed form, so if you have just write divergence of rho uiV, this in a proper index notation is written in this way. See, divergence is what; divergence is what you will have a term with partial derivative with respect to x, then partial derivative with respect to y and partial derivative with respect to z.

So, just like x1, x2, x3, some of these 3, so in this; so you can see that when you have a partial derivative, so when you have the divergence here, the partial derivative with respect to x1 should be involving x component of this V. When it is with respect to x2, it should involve x2 component of V. So, in general, when it is with respect to xj, it should involve the j component of the velocity.

So, actually it is a invisible summation from j = 1, 2, 3, okay, so you can clearly understand the notation, it is very important and similarly how do you write this one? Now, you tell; remember that what is this one, what is this tau i? Tau i is; it is; so you have the partial derivative with respect to that component, so for j = 1, it is one component for j = 2 that is 2, j = 3 that is 3.

So, in general tau ij, again there is an invisible summation, j = 1, 2, 3, right, so you can see from this that is how their tau was defined from that. So, in that index notation, you can write it like this, so this is an equation of motion, equation of dynamic equilibrium in a differential form and this is known as Navier's equation of equilibrium or simply Navier's equation. So, we have been successful in writing an equation of motion along the direction i.

If you put i = 1, it is x, if you put i = 2, it is y, if you put i = 3, it is z, now you see what is the complication of this equation? The complication of the equation is as follows, you do not know this tau ij that is you do not know the stress tensor components, so you must know to make its closed system of equations, you must know the stress tensor components as a function of the velocities or their gradients or pressure on all those quantities.

So, it should be expressed in terms of these primary variables, which are velocity and pressure or their gradients and that varies from one fluid to the other, till now we have not assumed that it is a Newtonian fluid or whatever, so we will see in the next class that now we will make some special assumptions of the type of fluid because based on the constitutive behaviour of the fluid this tau ij will be dependent on the velocities, pressures or maybe the gradients of velocities.

And we will assume a special case of a fluid known as Newtonian and Stokesian fluid and in that special case, this equation will be simplified to a form known as Navier Stokes equation, so that we will do from a next class. Thank you.