Introduction to Fluid Mechanics and Fluid Engineering
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Lecture – 26
Integral Forms of Control Volume Conservation Equations (Reynolds Transport Theorem) (Contd.)

We will look into some cases of linear momentum conservation, where this system that we are considering is of a very special type, where the mass that is there in the control volume may change with time, so we will see that how we may take care of that. So, let us take an example.

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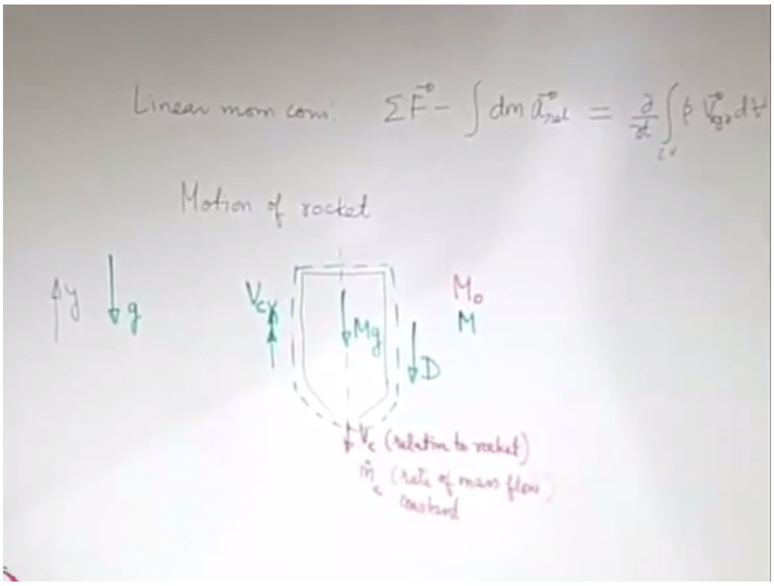

Let us take an example of something, which is quite common say, motion of a rocket, we will have a simplified picture of what it is; the whole idea is to get the essential feel of how you can use the Reynolds transport theorem to analyse the motion. Let us say that you have a rocket shell of some shape and products of combustion, they leave the rocket with a velocity say Ve relative to the rocket.

And let us say that m dot e is the rate of mass flow of this one, so essentially what happens; the rocket has a structure plus there is some fuel inside, which undergoes a combustion process, the products of combustion come out and these create a so called thrust on the rocket, which takes it to a great height. So, the basic understanding is well, rocket motion has been studied from maybe junior high school level.

And you see the level of explanation changes, so when you talk about that how a rocket moves maybe when the Newton's laws of motion were introduced the question if it was asked to you, your answer was by Newton's third law of motion. Now, after that maybe if you; when you went to a bit of a higher level school studies, if it was asked maybe you could have said by conservation of linear momentum.

And now we will see that precisely what it is about and well, it involves conservation of linear momentum, it involves the Newton's third law of motion but those are very qualitative statements, we will see that quantitatively how you can describe that what should be the parameters, which would dictate the velocity or acceleration of the rocket. Now, what we are assuming is that whatever the mass flow rate that is coming out; that is coming out at a constant rate, so this is a constant that is what we are assuming.

It is not difficult to maintain it as a constant because if the reactions are occurring at a constant rate, it is possible that the products also come out at a constant rate. Now, what about mass of the rocket? The mass of the rocket is like when you have an initial mass say, M0 is the initial mass, it is the mass of the structure + fuel + whatever was there initially, now this would decrease with time because some mass is leaving the rocket.

So, if you consider a control volume, which includes the rocket then that control volume has mass, which is variable, which is not a constant. Now, let us try to write the expression for integral form of the linear momentum conservation for this rocket. So, we identify a control volume let us say, we identify a control volume something like this and we try to identify all the forces, which are acting on this control volume.

So, what are the forces which are acting? Let us say that at any instant M is the mass, which is there inside the control volume, so M is a function of time and gravity is acting in this direction, so what are the forces which are acting on this? So, you have the weight, it includes the weight of the structure and the weight of the other materials, fuel etc., whatever is inside then there is nothing called as thrust force.

See, in the free body diagram; in a free body diagram, what forces you show, what are the external forces which are acting on it? So, thrust force is something, which is like; which is a;

which is something as a consequence of this ejection. So, when you show this is ejection, you cannot show a duplicated thrust force, so it is basically there is nothing called such a thrust force.

So, what is the external force, which is there? There is an air resistance, right or at whatever atmospheric resistance, so we call it a drag force. So, there is a drag force, so if the rocket is moving in this direction, let us say that the rocket is moving in this direction and let us call it Vcv that is the velocity of the control volume because we will assume our control volume to be this moving control volume and which may be accelerating that is the velocity may change with time.

So, if this is the direction of the velocity then what should be the direction of the resistance or the drag force? It should be downwards, let us say that capital D is the drag force, so when you have such forces, the rocket seems to have some sort of resistance, one is this weight, another is the drag force and it has to overcome that resistance in moving upwards, the reaction provided by this flux; momentum flux that comes out that should be the prime driving force for moving it upwards against these resistance forces.

Now, let us write the linear momentum conservation for this moving control volume, so the linear momentum conservation; so what we have is resultant force - integral of say, rho dV is like dm * a relative that is over the entire mass, which is there inside is equal to; that is the unsteady term and finally the flux term. So, if we write the scalar form of this, we may write it in a scalar form because here everything is occurring in the y direction, if that is the y direction say, then we can write the y components to have the corresponding scalar forms.

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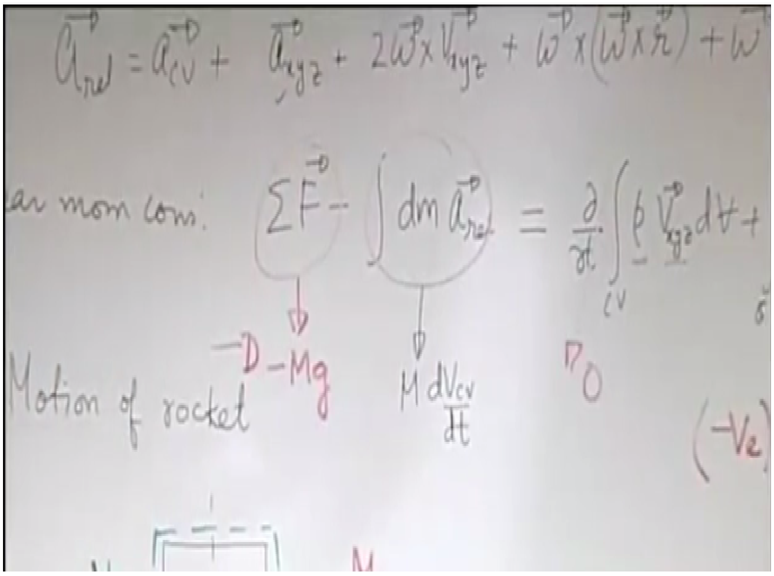

When we do that let us try to see that what are the different contributions of different terms forces, we have identified, what should be this one. So, what is a relative? A relative was acv + there is 1 a small xyz + the terms, which are related to the rotation. So, let us write those, so these are the terms, which are there, right. So, we have to see that here since there is no rotation the last 3 terms are not important.

What does the second term represent, a small xyz? It is the acceleration of the fluid with respect to the small xyz reference frame, which is the rocket here, so what will be that? See, the products of combustion whatever these are coming out at a constant velocity, so there is no rate of change of that velocity; time rate of change of that velocity as viewed from small xyz reference frame that means, this is 0, okay.

So, only acceleration of the control volume; linear acceleration of the control volume that is there in this term and because it is assumed to be like a rigid body, so all points are having the same linear acceleration and that acceleration is nothing but dVcv dt, so this becomes as good as so, a relative is a constant, which is equal to dVcv dt that comes out of the integral; integral of dm becomes m, so this becomes M dVcv dt.

We are writing it in a scalar form as in the y direction, so I am not writing it in a vector sense only plus and minus will indicate whether it is upwards or downwards, so keeping that in mind, we have this M * dVcv dt. Now, this M * dVcv dt, the important thing is this M is not a constant, it is a function of time that we have to keep in mind. Coming to the right hand side, you see that there are 3 different things, which could potentially be function of time.

But here, rho is; we are assuming rho is not a function of time in reality, because of some compressibility effects rho could be function of time, V small xyz, this is definitely; this is what; this is velocity of the fuel, the products of combustion with respect to the control volume and that is we are assuming to be some constant, so that means that is not changing with time.

And V is the volume of the control volume that also we are assuming to be the fixed one that is also not changing with time. With those assumptions this term will be 0, then the last term; so the last term again it depends on how this velocity is distributed over the area through which it is coming out, let us assume there is a uniform velocity distribution. If it is a uniform velocity distribution, then V small xyz is uniform over this dA.

So, then how this may be simplified, so you may take out V small xyz out of the integral first and that is - Ve because it is in the negative y direction, then integral of V small xyz dot eta dA * rho that is the mass flow rate, so that is m dot; m dot e, okay. So, we have isolated this one first out of the integral, the remaining is the mass flow rate and the proper dot product is positive because it is the outflow.

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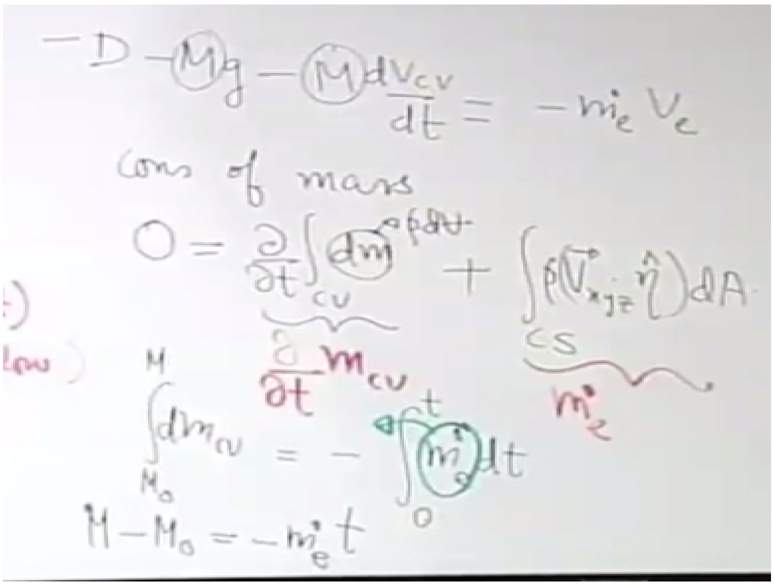

So, if we now write what are the forces for the force contribution, you have - D – Mg, right, these are the forces which are acting. So, keeping that in mind, we may write the equation of motion for the rocket as – D - Mg - M dVcv dt is = - m dot e Ve. Of course, m dot e may be written in terms of Ve, if you know the area of the cross section through which this jet is coming out.

So, if that is Ae, then rho e; rho * Ae * Ve is the m dot e okay, now this equation is allowed not sufficient to solve for the velocity of the control volume as a function of time. The reason is; that you do not know the mass as a function of time, so which fundamental principle should give you the mass as a function of time? The conservation of mass, so let us write the integral form of the conservation of mass.

So, conservation of mass again, left hand side is DM Dt of the system, right hand side is the partial derivative, this is basically this is nothing but what; this is rho DV; rho DV is like Dm, just because of writing convenience, we are writing in place of rho DV Dm that is all the same that is elemental volume expressed in terms of elemental mass nothing very special about that.

Then, + integral rho V, so what is this; let us write the first term that is what is this first term; let us call it partial derivative with respect to time of the mass of the control volume, right that is what it is mass; within the control volume and the next term is, what is this, so how many flow boundaries are there? Only one flow boundaries there through which fluid is flowing and what does it become, this is nothing but m dot e, this outflow; rate of outflow.

So, we can say that mcv at time; so if you just integrate it like this, say like dm cv from say at time = 0, m = m0 to a given time when it is M is = - integral of m dot e dt from time = 0 to time = t like that this is just like ordinary differentiation because this could be function of time and nothing else because the special effect has already been integrated; averaged out, so only the effect that is remaining is the time effect.

So, what do we get from this equation? We get M – M0 is = what; now it all depends on whether m dot e is a constant or is a variable, so here in the problem it is given that it is a constant, so if this is a constant it comes out of the integral. So, it becomes - m dot e * t, so now we have 2 equations; one for the mass conservation, another for the linear momentum conservation and these 2 equations may be together solved.

So, you have one equation for the mass conservation, which is at the bottom and the one for the linear momentum at the top. So, many times just for solution convenience let us say if you neglect the drag force, if you neglect the drag force, it becomes a little bit simpler differential equation to solve but even if you do not neglect it, it is something which you may easily write.

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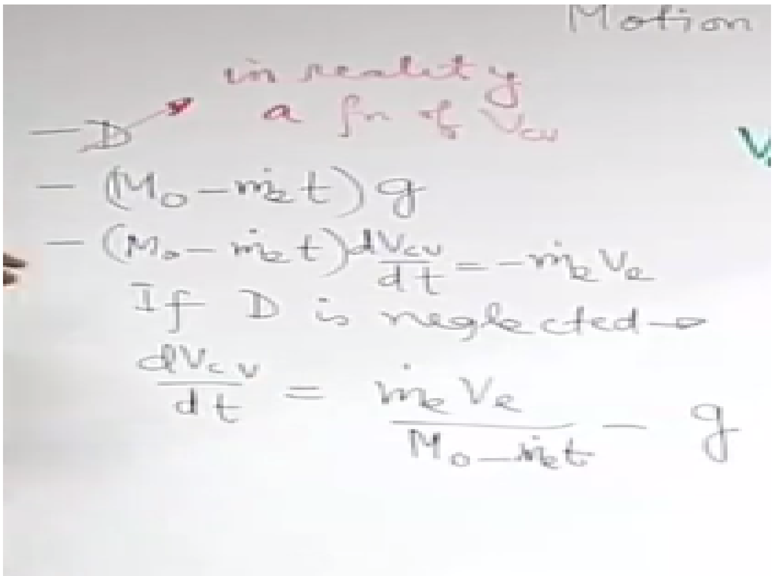

Only thing is that one has to keep in mind that drag force is not a constant; drag force in reality is a function of the relative velocity between the rocket and its surroundings, so you cannot just take it like a constant, if you want to represent or model it properly but like if you just write it in a symbolic form – D, then, minus now, you write M0 - m dot et – M0 - m dot e t dVcv dt is = - m dot e Ve.

Now, when you have the drag force, so we should keep in mind that in reality this is a function of Vcv, at least that we should keep in mind, this in reality is function of Vcv that function may be quite complicated because it really undergoes a wide range of variation of speeds, so that air resistance is something which also varies quite significantly but at least, we know that what are the parameters on which it should depend mostly.

Now, if you as an example if you neglect the drag force not that neglecting the drag force is something very realistic but it just tends to give you a first-hand feel of what it is, so if you neglect the drag force, if D is neglected then what do you get at the end? You get DVcv dt is = m dot eVe, right. We have divided both the sides by M0 - m dot e *t, so this gives a sort of clear picture that what are the driving parameters.

So, if DVcv dt is positive that means the rocket is accelerating upwards and what is helping it to accelerate upwards is basically the rate of ejection of the products of combustion. So, if that is at a very high rate, you can see that it becomes a helping term and more and more the products of combustion come out, the denominator becomes what; the denominator as time increases, the denominator becomes smaller and smaller.

And you have the rocket itself not like existing as if, if this becomes 0 but this will not become 0, this will this; I mean when you are considering this, you are thinking that products of combustion are leaving but the structure has some weight, which is there so but in the limit its a lot of weight is there because of the products; because of the fuel which is inside and once that is combusted and that is living, it is actually a lot of weight leaving from the system.

Because it requires a huge amount of or huge rate of momentum that requires to be generated to give it a force, so misnomally this is sometimes known as a thrust force, keep in mind is not an external force, this is just because of the action of the fuel that the products of combustion, which come out. So, you may give it a name thrust again like, just like there is nothing called as a centrifugal force, when you consider it to be an externally applied force, this is also like that.

So, it is an action because of some motion of the system, so it is not just like you have applied this external force, so this external force tends to become more important as the products of combustion come out with higher and higher velocities and the mass inside gets decreased, so this combination is quite good because that is what it is helping and weight is of course not helping it and if drag force was there, it would have also not helped it that would also be against the motion.

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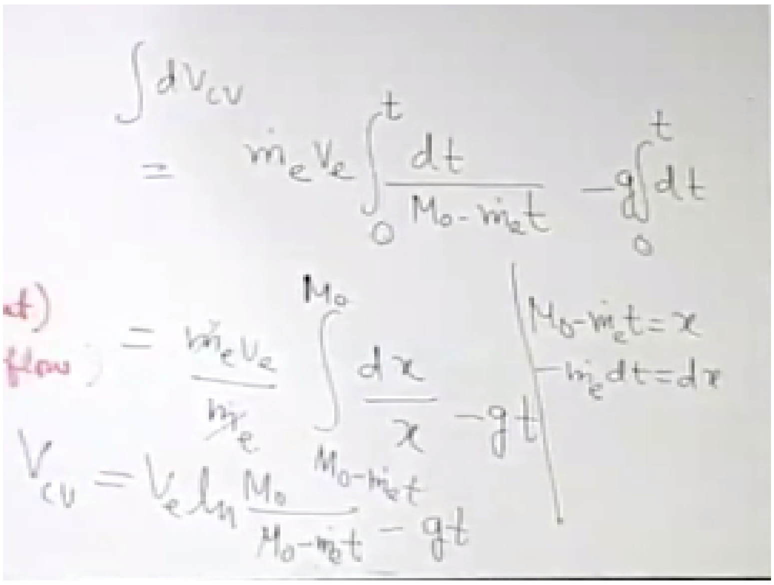

So, now if you want to find out that how this the velocity changes with time, you may simply integrate it, so integral of DVcv is = m dot eVe integral of dt y, we are integrating it from time = 0 to time = t, you can complete the integration quite easily, this is very simple integration, let us just substitute may be M0 - m dot e t = x, so - m dot e dt is = dx, so this integral becomes; the limits of integration, okay so this becomes Ve ln of M0 / M0 - m dot et – gt.

This is say, the velocity of the control volume at time t, assuming that it is 0 at time = 0, so it is possible to see that how this velocity is changing with time, like it is useful to get some physical picture of what it is. So, I will advise you that you please try to make a plot of this as a function of time and see the behaviour at different times; you will see that the behaviour at different times will be interestingly quite different.

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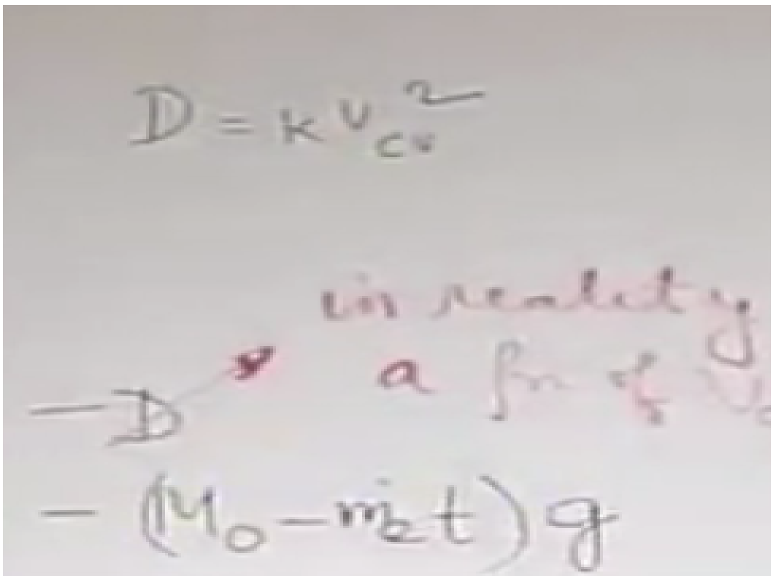

So, at least this analysis gives us a picture of like, if you want to find out the velocity of the rocket, what should be the methodology? Now, if you have a complicated drag force say that the drag force is given as sum K * V cv square, say as an example that the drag force is there and it is = K * V cv square, so you just substitute in place of that KV cv square, only thing in the analysis will be that there will be another complication in the integration.

So, it will maybe a bit more difficult for integrating but the principle remains the same, so the important thing is what is the differential equation of motion and we could realize that what are the basic principles on which we can write the differential equation of motion. So, this illustrates an example of a case when the mass inside the control volume is a variable. Let us take another example to where we may encounter such a case.

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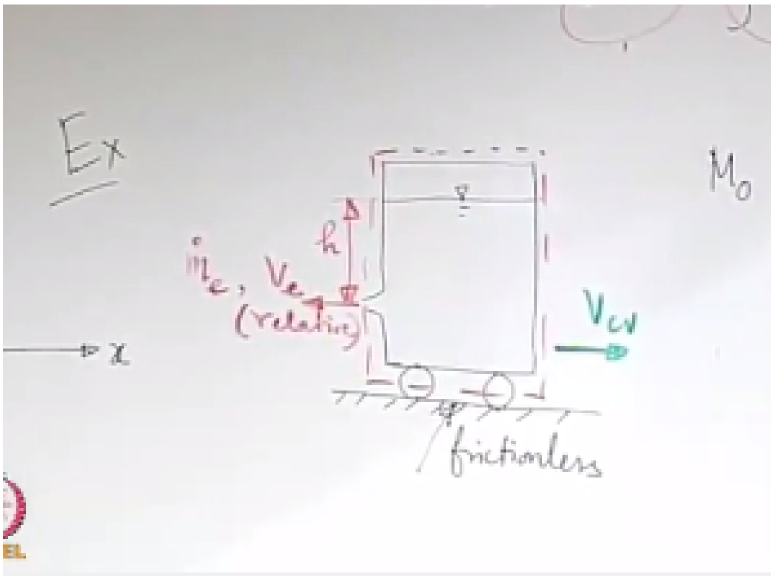

Let us say that you have a tank filled with water or maybe partially filled with water whatever, say filled with water up to this much and there are some wheels which are there with the tank, water comes out of the tank like this say with velocity V relative to the tank, this is the relative velocity just like the rocket case, this is also a relative velocity. The initial mass inside it is M0; this includes the mass of the water plus the mass of the tank.

And again it is the water is assumed to come out at a rate of m dot e, I am trying to draw an analogy between this problem and the rocket problem that we have done, so some mass is coming out, let us say that it is mounted on a frictionless surface not that that is the necessity. If there is some friction, you have to figure out that how the friction is influencing its motion.

So, now you realize that because of the water coming out of this, there is some sort of force or thrust so to say.

But it is not an external thing, it is just an effect of these water coming out, there will be a force in the; towards the right on the tank, which will try to make it move and it is basically a fundamental principle of a propulsion system. So, it will try to make a system propel with a particular velocity that itself might change with time. Let us say that you have this height of water at the instant that you are considering is h.

The question is that how far this situation is similar to the example of the rocket that we have considered and how far it could be different? Let us try to find the analogy and try to address a simplified picture such that it matches with the simplified picture of the rocket, so what is the simplified picture? Again, let us say we have considered the control volume like this one, which is a moving control volume.

So, we have the velocity of the control volume let us say, it is positive towards the right and it changes it is a function of time. The similarity is clear that the mass in the control volume is a function of time, so if you intend to write the equation of motion, what you see? You see that the left hand side resultant force acting on it along let us say, now we write it for the x direction because x direction is or the horizontal direction is the direction of motion that will give us a picture of how Vcv changes with time.

And since this is a control volume, which in general may be accelerating, so we have to write in terms of a relative velocity just in the general form of the moving reference frame. So, what is the resultant force along x on this control volume? 0, right, so you have 0, appears to be even more simple than the rocket, then there is no rotational motion and let us say that this fluid is coming out with a velocity V.

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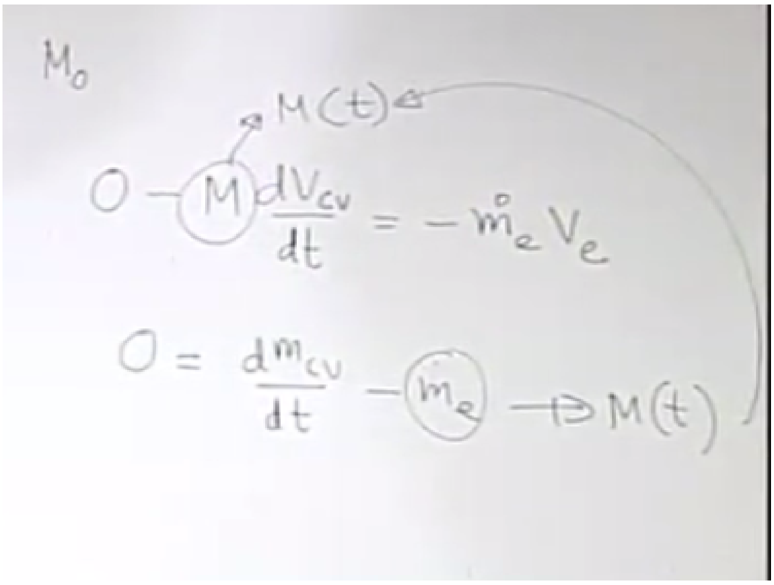

So, if you just consider it like a case of a rocket, then it is like, you write - M * a cv or a cv is dv cv dt, let us first write it like a equation of a rocket and see that how it should be different from that. So, for the rocket whatever we wrote, let us just write it, the right hand side is like what; it will become - m dot e * Ve square, right, sorry V not V square, okay. The assumptions are that you have a uniform velocity profile at the exit.

So, that like you do not consider the variation of V small xyz over the area through which the water is coming out and this M we know is a function of time and what equation tells us that is the conservation of mass, so you have; so this should dictate that what should be M as a function of time, which you need to substitute here, the velocity of the control volume as a function of time, right.

So, in that way it should not be greatly different from that of the analysis of the rocket that we have presented. Now, can you tell that like other than considering the frictionless, what are the flaws in this? First, here V small xyz that is Ve, this is not a constant, right in reality, this is not a constant, when can you assume this to be a constant, see there are certain things which many times we do which is not exact.

But we sometimes make an approximation and say that although, it is not exact, we are following that and that approximation sometimes is justified, sometimes it is not, always approximations are not bad, there are situations when approximations may be justified. So, in general, you are very correct that this Ve is not a constant but when can we assume that this Ve is roughly a constant, area of the tank is so large as compared to the area of this one.

So, that the rate at which these drops is very small may be at a very high velocity this is coming out but the area is small, so area into velocity whatever that is same as the area of the tank times the rate at which it is coming down, if you consider it to be approximately described by that form a1 V1 = a2 V2, so then this V1 is very, very small, so the rate at which this drops is so small, let us say we are considering a time range, which is not very large.

So, if you consider a time range, which is not very large and the area of cross section of the tank is quite large as compared to the area of the nozzle through which or the orifice through which the jet is coming out, then it is not a very bad approximation that Ve is a constant because if this height is a constant clearly, Ve will be a constant. If this height is a variable, it will be a variable.

But in general, we know that Ve is not a constant, so if Ve is not a constant, how will you treat that? See, it becomes a quite a complicated problem then because now, if you say that these area is somehow; is not somehow very, very small as compared to the area of cross section of the tank and let us say that if you take a stream line, which connects any two points between these 2 and write the Bernoulli's equation for say, Ve that Ve, you may write in terms of an unsteady Bernoulli's equation.

But again, it will not be as straightforward as the unsteady form of Bernoulli's equation that we have discussed earlier because now, the reference frame is accelerating, okay. So, I am just trying to give you the complication of the problem not that you have to just solve the full complication of the problem with full rigour but at least you have to appreciate that what are the rigours, which are involved in even such a simple case.

Like, this may be so simply you can have a tank and you have water like, we have seen many times that like if you go to a city, where the road is being somehow washed by water, you will see that there are big tanks like which are maintained by the municipality or corporation and they just have some mechanisms by which water is coming out and so it is like a primitive way of like cleaning the road and it is not so advanced in terms of technology.

But see the science is very advanced actually, if they want to precisely get a feel of how the velocity of the tank will change with time and you know if the person who is driving that cart is very optimistic will say that no I will shut off my motor because with this thrust, I hope that the tank will go on moving but you know the road has lot of friction and we live in a country where optimism is not very strong.

So, I mean he will be in full gear and trying to drive that but the reality is that it is quite a complicated scenario, it is not as simple as even like unsteady Bernoulli's equation use itself will give a complication because of the variation of Ve with time and variation of Ve with time, will give the variation of m dot e with time. Let us say somehow that approximately the unsteady Bernoulli's equation is valid.

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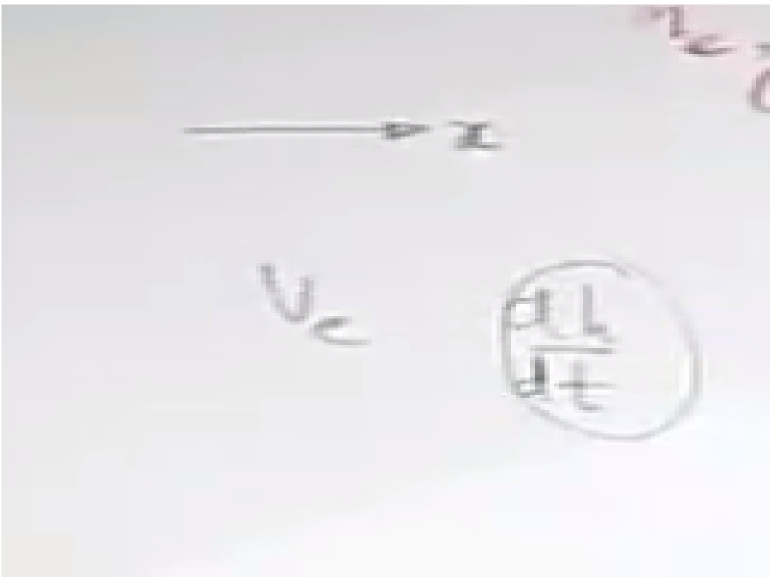

If it were valid, then what would have happened? If it were valid then this m dot e; we would have been able to find out as a function of time because m dot e is a function of Ve and Ve as a function of time, we could write in a differential form by writing the unsteady Bernoulli's equation between 1 and 2 but that will involve, what are the parameters that we involve; that will involve Ve and dh/dt, these are the parameters it will involve.

So, then Ve as a function of time will be a function of dh/dt, so Ve will be an implicit function of time because Ve is a function of dh/dt and h is a function of time and that function of time will appear in this m dot e because m dot e is rho e ae * Ve, so this will become a complicated function of time but not a constant that we have assumed just now. So, then in principle same equation but when you integrate it, it is not a constant that is going to be there.

So, it will be again a complicated function of time that function of time, you have to substitute here, the other thing is that what is the other like; if you still do that say by doing it correctly, then what is the other approximation, which is there, which is not being taken care of here. So, one approximation which may be properly taken care of is by properly describing what should be the rate of; rate at which the mass comes out as a function of time by whatever analysis.

Now, if you substitute here that as correctly says somehow, is it complete or still there is some approximation that is already inbuilt here; yes, which integral? This one, forget about the other things we are not considering that the tank is rotating with respect to certain axis and all, I mean if you want that complications, one may add, you may have a rotating reference frame in which a rocket is moving or something like that.

Those are very good for academic purpose problem solving, you will see that in some books like in exercise, those types of problems are given, it is not bad, if you practice it at least, you will have a skill of using all the terms in this equation but it is not very common to have a rocket rotating in a reference frame and not trying to move up in some way or when it is trying to move up, it is always rotating with respect to some axis, it is not very common.

So, for that is; that is just like a hypothetical way of like treating the complexity of a problem but even in; what I am trying to convey is even in the simple form, it is so complex, forget about a complicated form when like it is mounted on say, a rotating platform or whatever but even like in this case, when you have this a small xyz that was not considered in the a relative only this term was considered.

So, a small xyz is something which is the time rate of change of the velocity of the fluid relative to small xyz and that is basically related to; so it will be like V dot e, so that will be a function of time and that will come along with this one, so with this plus there will be some term, from a small xyz with proper vector sign plus minus whatever, so because you are writing along x, so the vector sign will be given by the scalar with a plus or minus.

So, that is; so we have seen that is a complicated scenario, where you treat all that rigour, now again like engineers, may not be so happy in dealing with such a complicated scenario but such over simplification also one might not like, so if you want to make a compromise now, which is like; which is not as good as the reality but it is not as bad as the worst assumptions. So, then what could give you a guideline of that?

See, if you neglect the unsteady type of the Bernoulli's equation then, what is this Ve, then this Ve roughly is root 2 gh, again the assumption is the velocity at 1 is very small 0, so even if the tank area is very large, still here you will have this as root 2 gh. There are other assumptions but we will not go into the detailing that we have already discussed that the root 2 gh formula whatever the; what are the assumptions which are involved.

So, when you have this as root 2 gh, so you can put that in the calculations approximately Ve is = root 2 gh and h as a function of time is like a tank emptying problem that we have discussed in the context of Bernoulli's equations; use of Bernoulli's equations that how do you find out that how the height of the tank changes with time. So, if you have Ve = root 2 gh and if you with an approximate analysis know that how h is changing with time then that can like simplify some of the expression that we are looking for here, okay.

So, I would encourage you to set up the proper differential equations for at least, taking such an approximation, you need not solve it because you will not be able to solve it but at least you set up the proper equation. See, when we solve a problem, there are 2 parts associated with the problem; one is a problem formulation that means, whenever you are being described of a physical problem, how can you convert it into an equivalent model problem; model mathematical problem, this is known as mathematical modelling.

So, when you go for a mathematical modelling of a physical problem, you clearly should be having a picture of the assumptions that you are making, justifications of the assumptions and if the assumptions are partially justified, then how certain things are approximated, with that approximation, you should be able to come up with some set of equations governing the physical behaviour and those set of equations should be in principle closed.

That means if now; if you were a champion in solving a differential equation, you could have been able to solve it, so these problems are like called as initial value problems that are time = 0, you know that what is the picture, now there is a differential equation, which gives the variation as a function of time, where everything is in principle is known as a function of time, now you have to integrate just it to get the variable as function of time.

So, that is the problem formulation, now, how will you integrate to get it as a function of time, how will you solve the differential equation that is a problem solution part, so formulation and solution need not be treated together, I mean for analysis formulation is what is very important because these days even if you are not very good in solving differential equations, you have many software tools which will do it for you.

But no software tool will formulate the problem for you because it requires an effort of understanding of the physics of the problem to formulate it, so getting the answer is very important but you will invariably come up with a wrong answer, if you do not formulate the problem correctly. So, problem formulation is the heart of the problem solution in these cases.

So, the examples that we have seen; we have seen that if there is a variable mass, then the variable mass system is something, which may be treated in the same framework as that of like as if like a constant mass back but what is the problem? The problem is that you are basically putting an instantaneous mass within the control volume, which is changing with time and many times the complication arises.

Because you are not confident about how these mass changes with time, so that complication arises because of the variable relative velocity of the fluid which is coming out, which may not be in the control of the designer because whatever, here the height is changing with time in certain cases, it might be something else. Now, next what we will do; we will try to see some other examples, where we do not consider the linear momentum conservation.

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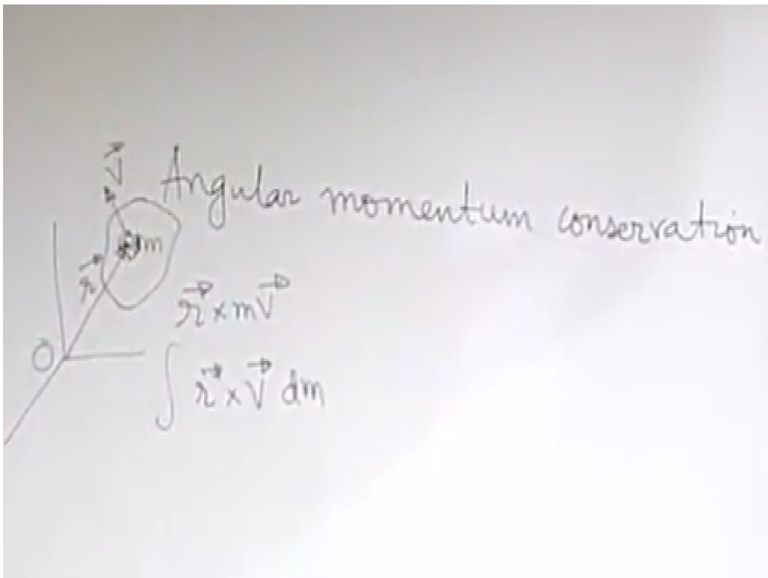

But the angular momentum conservation but before that let us write the statement of angular momentum conservation. So, angular momentum, if it was a point mass of say, mass m and let us consider a picture like this where you have the origin of the coordinate system, you have a point mass m, it has a velocity V and position vector is r, then the angular momentum is given by r cross mV that is the point mass or particle picture.

And see, if you recall the basic classical mechanics that you have learnt, see always when we; when you have learnt rigid body mechanics, the natural way to learn it is that you have a particle from that you extend to a system of particles and then from system of particles to rigid body because rigid body is a special case of system of particles, so up to system of particles before you consider it to be a rigid body, it is as good as a unified concept for a fluid and a solid.

When you come to a rigid body, then obviously you cannot have a fluid except for a very special case we have seen that there are certain cases when fluid moves like a rigid body but that is not very common. So, up to the system of particles viewpoint like if you consider one mass like this and if you have a system of particles with a continuous mass, then this will be just replaced by r cross V; r cross dm V basically integrated over the entire mass.

So, as if you have an elemental mass dm and so this now is replaced by an elemental mass dm, which is moving with a particular velocity at a certain position vector and that is integrated over the entire mass, which is there, which is a collection of particles. If it is a continuous distribution, then it is integration, if these are discrete particles that are replaced by summation that is the obvious thing but in principle, understanding is same.

Now, how do you write the angular momentum conservation here? So, let us say that we are interested to write the angular momentum conservation in this arbitrarily moving reference frame, it is better to do that because of what; because many times when you are talking about angular momentum, there is something which might be a rotating component in the entire arrangement and when there is a rotating component and if you look into the phenomena by by sitting on the rotating component, you are yourself located on a non-inertial reference frame.

So, therefore non inertial reference frame becomes or may become a natural choice for analysing angular momentum conservation cases but what we will do is; we will write the 2 forms separately; one for the case which is with respect to an inertial reference frame and another with respect to a non-inertial reference frame to see that what are the distinctive forms in the two.

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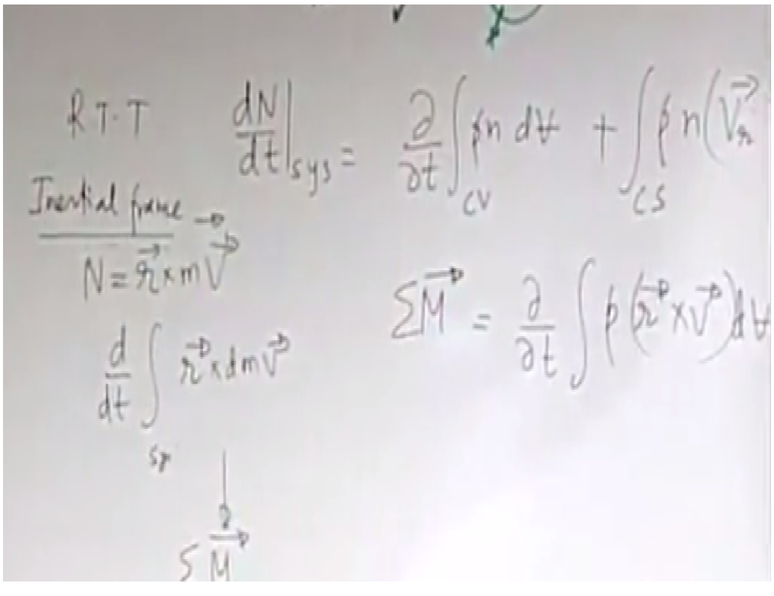

So, if you consider the Reynolds transport theorem again, dN/dt of the system, let us say that we first want to do it for an inertial frame, first is inertial frame, objective is angular momentum conservation, so what should we replace capital N with? Yes, so capital N is something like r cross mV, now you have to keep in mind one thing. See, loosely we write it as r cross mV or whenever we wrote linear momentum mV but you have to keep in mind that the velocity of different particles in the system is different.

So, when we wrote the linear momentum conservation actually, it was an integrated effect so that V was the velocity of the center of mass, so here also it is an equivalent V of the system for which it may be represented by an equivalent r cross mV. If you do not want to do that then simply you can think about it has like d/dt of integral of r cross dmV of the system. Then you do not care that whether V varies from one point to the other.

But like this; so this is the left hand side, so what is this equal to? So, this is basically like; so these are elemental angular momentums, this is the rate of change of elemental angular momentum and that is summed up over the entire mass, so that is the sum of the rate of change of angular momentum of individual elements, so that is the sum of the resultant moment of all forces, which are acting on the system maybe couple moment or force moment but resultant moment.

So, this is like the resultant moment of whatever forces couples whatever, acting on the system. The right hand side, small n what should you replace with; r cross V, so you have integral of rho r R V dV + integral of rho r cross V, V relative dot eta dA and since the left hand side is equal to the right hand side, this is the resultant moment of all external forces. Remember these all external forces, the action reaction does not come here.

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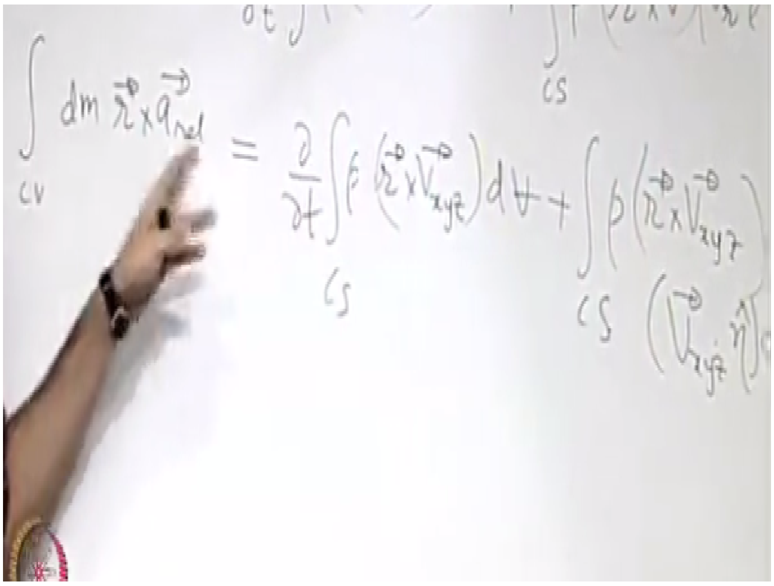

So, the resultant moment of all external forces, which is there, for a moving reference frame, so this we have written for inertial frame, when we say moving again; moving maybe inertial but general moving reference frame. So, for general moving reference frame we must have a correction term with these, what should be the correction term? See, what was done for the linear momentum conservation.

In the linear momentum conservation, capital N was substituted as V small xyz; m * V small xyz for the moving reference frame, now we will replace that with r cross mV small xyz, so all the terms are basically crossed with another r, nothing more than that. So when you have that see the first term is like; it becomes the resultant r cross mV capital XYZ d/dt of that, so that is the resultant moment of all forces.

Because in that inertial frame you can write the equivalent rotational form of the Newton's second law minus a correction integral of dm r cross a relative, this is for the mass of the control volume, which is instantaneously there and the right hand side you replace V with V small xyz with another r cross, so r cross we have already taken, dV + integral of rho r cross V small xyz * V small xyz dot eta, dA the control surface.

So, you can see clearly the form is same as the linear momentum as if only the thing is that every term has been now made r cross; pre r cross that is r cross is there before all the terms because it is r cross mV type, so linear momentum had mV, now it is r cross with that that is the only change, so this is a general moving reference frame. We will stop here and in the next lecture, we will see some examples illustrating this. Thank you.