Introduction to Fluid Mechanics and Fluid Engineering
Prof. Suman Chakraborty
Department of Mechanical Engineering
Indian Institute of Technology - Kharagpur

Lecture – 25
Integral Forms of Control Volume Conservation Equations (Reynolds Transport Theorem) (Contd.)

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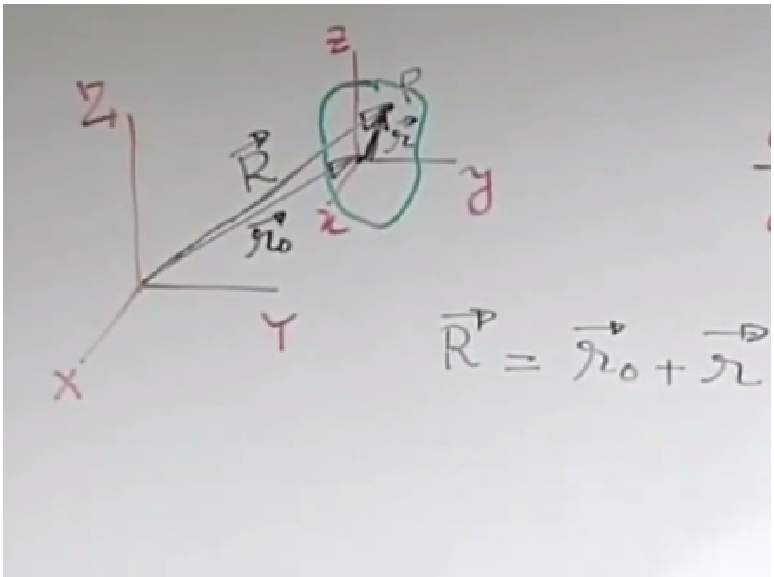

We will go ahead with the description of the Reynolds transport theorem in an arbitrarily moving reference frame. So, in the last class we were discussing about the derivative of a vector in a arbitrarily moving reference frame and the result that we saw that if you have 2 reference frames; one is capital X, Y, Z another reference frame is small x, y, z, where small x, y, z may be having an arbitrary motion in terms of an angular velocity.

In terms of a linear velocity, which may even vary with time and so on, so if you have the vector A, which is there in the small x, y, z reference frame, the derivative of that with respect to the capital X, Y, Z which is like a stationary reference frame is this + omega cross A. Clearly, if this small x, y, z reference frame is having no omega then the derivatives are identical.

Now, what we will do is; we will try to utilize this to figure out that what happens with respect to the velocities and accelerations of fluid elements located in a control volume because of the movement of the control volume. So, now we are going to consider that the control volume is not stationary, it is moving and when we say that the control volume is moving let us say that we consider the control volume to be such that small x, y, z is attached to the control volume.

So, the way in which the control volume moves small x, y, z motion represents that so, the small x, y, z motion has 2 important aspects; one is the translatory motion, another is the rotational motion. So, the translatory motion is like if you have the origin of the small x, y, z as this one then if you have the position vector say, let us call it r0, the rate of change of these r0 with respect to time gives the translatory velocity of the small x, y, z reference frame.

On the top of that small x, y, z reference frame is having a rotational velocity in general; there may be special cases when rotational velocity is not there. Now, if you consider say a point in the control volume let us say, a point P, this point in general represents a point where the fluid has a velocity, acceleration and so on so it is a point in the flow field, so this point in terms of its position vector is described by this position vector, which we say call as capital R.

But if we are trying to analyse everything with respect to the small x, y, z reference frame for us the important quantity or the important vector is the position vector of the point P relative to the origin of the small x, y, z reference frame and let us say that is small r. So, capital R is the position vector of the same point P relative to capital X, Y, Z origin and small r with respect to small x, y, z origin.

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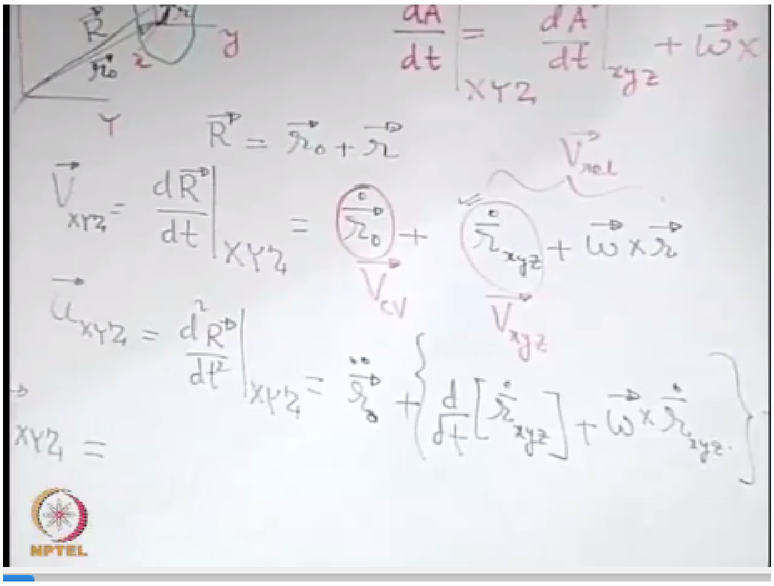

So, directly from vector addition, we can write capital R is = r0 + small r, okay. Now, what we will do in the next step is; we will try to find out the velocity, so the velocity of the point P in an absolute sense is dR/dt that is d capital R dt with respect to capital X, Y, Z, this is the velocity with respect to capital X, Y, Z, okay. So, this is just in a shorthand notation, we may not always write d/dt but write a dot on the top to indicate that it is a time derivative.

So, this is r0 dot + it is d/dt of small r relative to capital X, Y, Z, d/dt of small r with relative to capital X, Y, Z is d/dt of small r relative to small x, y, z + omega cross small r from this theorem, so that means this is as good as r dot small x, y, z + omega cross r, right. So, if you try to figure out that what are the implications of these different terms, let us just try to understand.

This is nothing but the velocity of the control volume, the control volume is having a translational velocity, this is the translational velocity of the control volume because this is the time rate of change of position vector of the origin of the control volume of the origin of the reference frame describing the control volume and this is what; this is the velocity as visualized from the reference frame small x, y, z and this is of course, omega cross r.

So, these 2 terms together may be sort of thought of as a relative velocity, so this is the absolute velocity, this is the velocity of the control volume, this is the velocity relative to the control volume like that. So, the relative velocity has one component because of the translation, another component because of the rotation and if we want to find out the acceleration; acceleration is what the sole important quantity is for us.

Because by that we can relate with the Newton's second law of motion, so acceleration with respect to capital X, Y, Z that means you have to differentiate this again with respect to time, so this is; so, let us write different; let us identify the different terms and write, the first term it will become r0 double dot right, next r dot small x, y, z just substitute that with A in this expression.

So, the d/dt of this one is what we are looking for, so what is d/dt of this one? That is d/dt of r dot small x, y, z + omega cross r dot small x, y, z, right that is for the second term, for the third term again, the same formula we can apply capital A replaced by omega cross r, so that will be d/dt of omega cross r relative to small x, y, z + omega cross omega cross r, okay. So, next stage is just the simplification and collection of the relevant terms.

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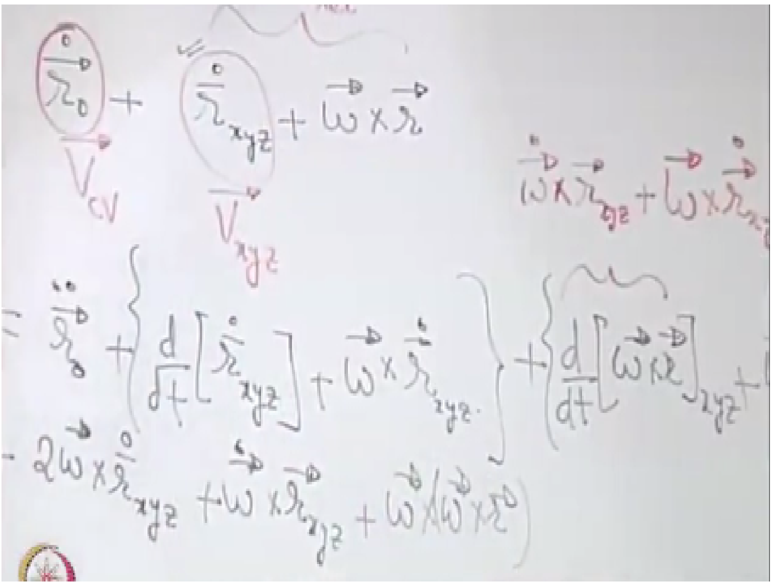

So, the first term r0 double dot + r small x, y, z double dot + you have omega cross r dot but let us simplify another term before writing that let us simplify this one, this you can differentiate just as if it is like a product rule but maintaining the order because for the cross product the order is important, so it is omega dot cross r small x, y, z + omega cross r dot small x, y, z, just like the product rule.

So, if you collect now omega cross r dot small x, y, z and this Omega dot; omega cross r dot small x, y, z, so it will become 2 omega cross r dot small x, y, z, then you have omega dot cross r small x, y, z + omega cross omega cross r. So, if we just give the interpretation as we did for the velocity for the different terms what is r0 double dot; it is the acceleration of the control volume; linear acceleration, so this is the acceleration of the control volume.

Then, what is r double dot relative to small x, y, z, this is the acceleration of the particle under consideration relative to the control volume and then if you consider these 2 terms, these are angular acceleration sort of that is; if you have an angular acceleration effect, so this is not directly angular acceleration effect but because of the time rate of change of the even a fixed vector because of the angular motion within the element.

So, if you have an even a fixed vector because of the rotation that vector is getting change with respect to time and this is directly because of the angular acceleration of the reference frame, so these are directly as a consequence of the angular motion of the control volume, so the angular velocity and angular acceleration. These are direct consequences of the linear; this is a direct consequence of the linear acceleration of the control volume.

This is because of the linear acceleration of the particle relative to the control volume and this is a combination of the linear and angular motion effect, so as you all this is called the Coriolis component of acceleration. So, where from this Coriolis component of acceleration comes?

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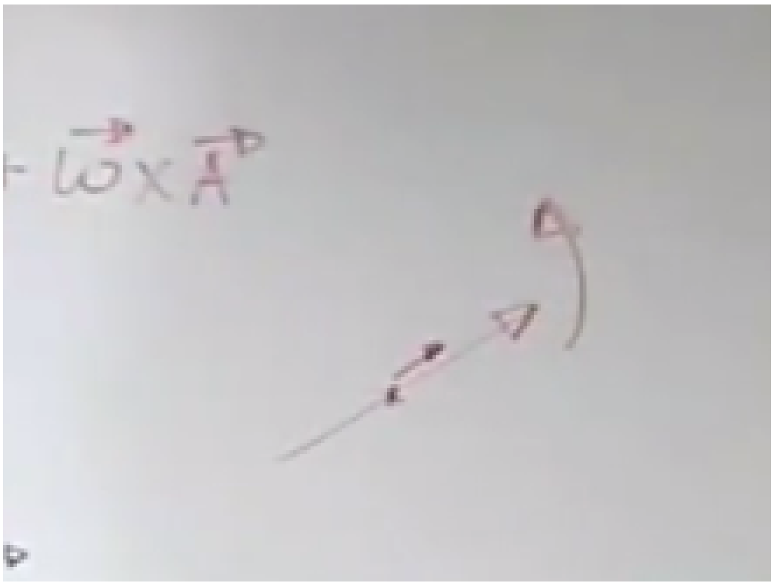

So, if you have a reference frame that rotates and relative to that reference frame something translates, so the combination of that gives rise to an acceleration. So, the two things which are necessary for this; one is the reference frame should be a rotating reference frame, another thing is that there should be a translatory motion relative to the reference frame and then it will experience a sort of acceleration.

And that acceleration will try to deflect the particle from its original locus and it is very common in particle mechanics also in fluid mechanics just think about ocean currents, so arc is like a rotating object and on the top of the; over the arc, the ocean currents are moving, so you have the water moving in a sea with a particular velocity on the top of a reference frame, which is rotating and that is why and the rotational sense is different at different places.

So, you will see that there will be a certain deflection of the ocean current in the Northern hemisphere and in the southern hemisphere these things you have studied in junior classes of geography but these are these are very important examples of the implications of Coriolis effects in fluid mechanics. Now, what we can do is; we can just write it in a bit of a more compact way.

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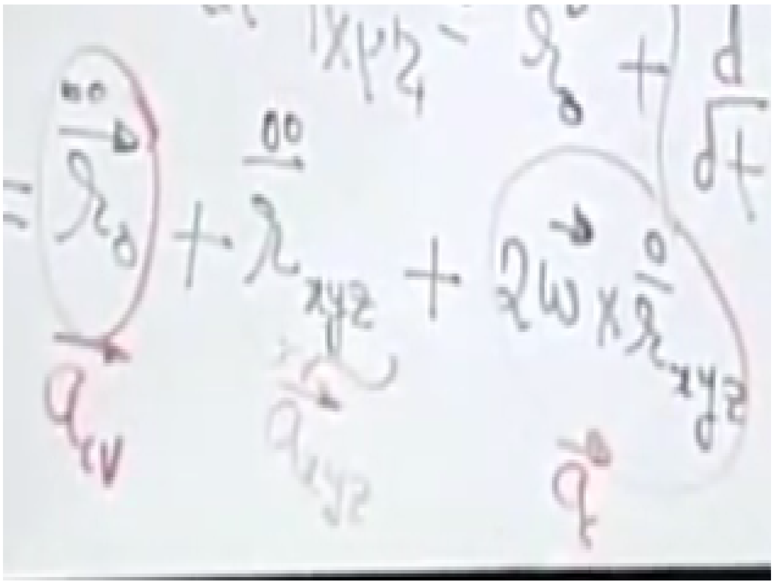

We can write that; so, this term we will write as A small x, y, z and all the remaining terms like this this we know, this is like a, Coriolis and this is the angular velocity and angular acceleration effect.

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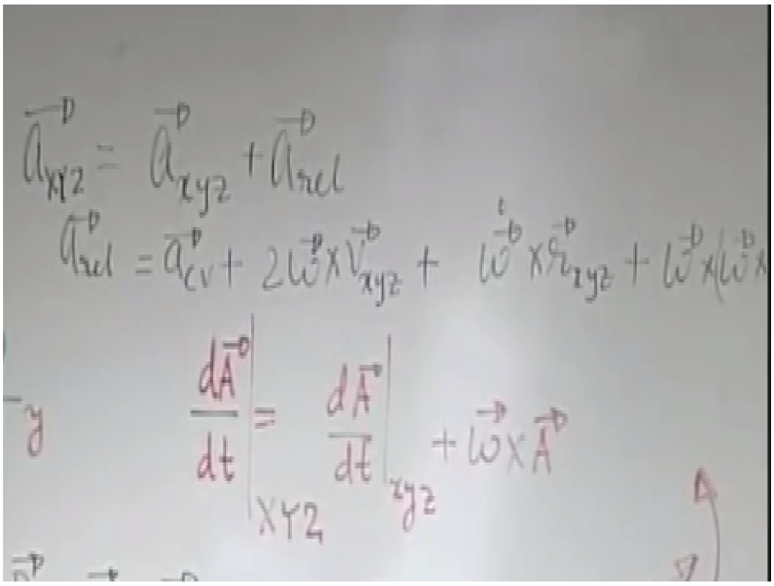

So, we will just write a capital X, Y, Z as if is = a small x, y, z +; let us give the all other terms and name a relative, this is just a name that we are giving, so in the a relative, we have a control volume + 2 omega cross v small x, y, z + omega dot cross r small x, y, z + omega cross omega cross r; r is again small xyz. So, you can clearly see that this type o; this is basically a transformation from reference frame, which is stationary to are arbitrarily moving reference frame.

And when you do that the transformation terms appear in the form of the quantities, which you are visualizing relative to the moving reference frame because when you are writing your equations of motion related to the moving reference frame, all quantities you are measuring relative to the moving reference frame, so that position vector, the velocity vector all those things you are measuring.

So, you can clearly see that when the transformation is made in the right hand side, everything is written relative to small xyz, so small xyz has become your reference. Now, how will this be important in the context of use of the Reynolds transport theorem? Let us try to go back to that let us say, that we are interested to write the Reynolds transport theorem for linear momentum conservation.

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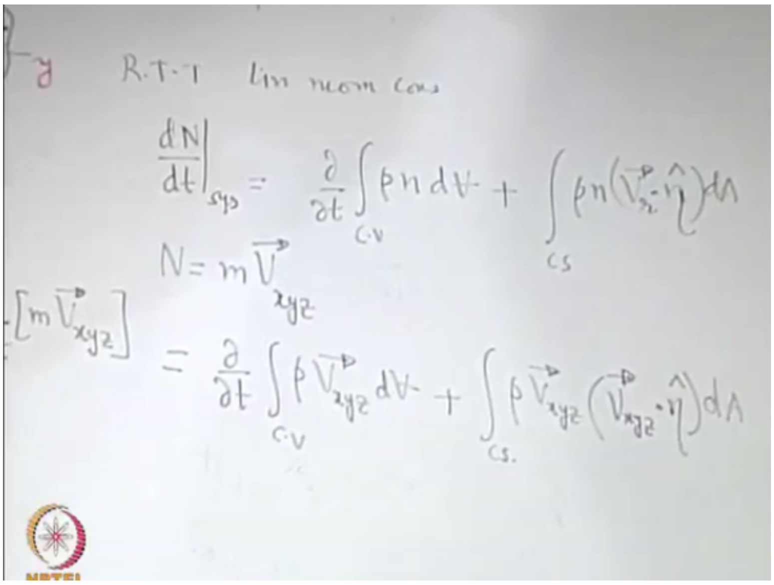

So, Reynolds transport theorem applied for linear momentum conservation, let us say that we are interested to write the theorem relative to this control volume, this green coloured contour is the surface; control surface of the control volume and that is our basis for which we are writing the theorem. Let us first write the general theorem dN/dt system, we recall that capital N is the quantity that we are interested to conserve.

It may be a scalar, it may be a vector, small n is capital n per unit mass and this eta is the unit vector normal to the area that elemental area that is considered for writing the area flux term.

Now, one important thing is that we have no restriction on the choice of this capital N and small n that is we are not restricted to write this only as quantities relative to an inertial reference frame.

We may even write this relative to quantities in a non-inertial reference frame because when we derived this theorem, we never had any consideration for a specific inertial or non-inertial choice of the reference frame, so we can as well write say, capital N as say m * V small xyz as an example nothing restricts us from doing that because it is just some quantity that we were looking for, it may be linear momentum relative to any reference frame.

So, when we write that left hand side becomes d/dt of m, let us complete, first write the right hand side and then we will see that whether the way in which we have written the left hand side is the proper way or not. We have to keep in mind that this V relative is nothing but V small xyz that is the velocity of the fluid relative to the control volume, so this is V small xyz dot, now let us devote our concentration on the left hand side.

When you look into the left hand side see, when we write this what is the assumption; the assumption is that the entire system is having the velocity V small xyz, relative to the small xyz but that is how we have written that this is the mass into this one because this is the total linear momentum of the system, so total linear momentum of the system, if we write that as mass of the system times this velocity.

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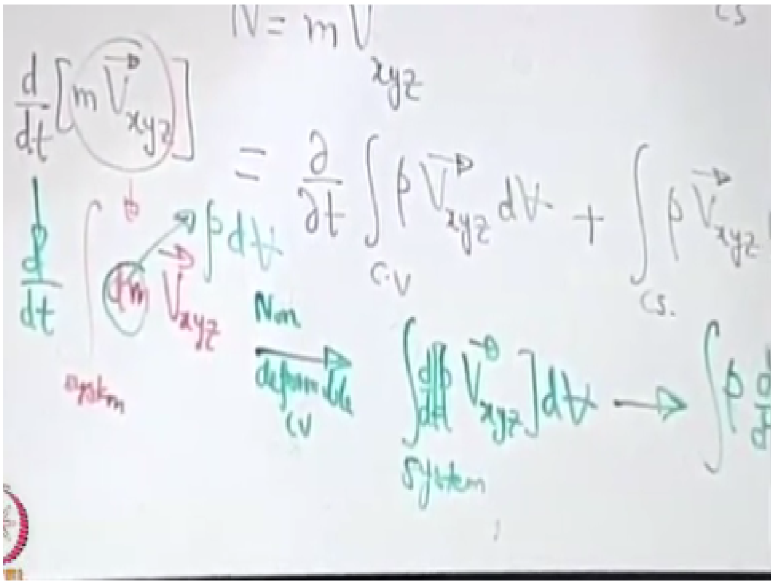

That means we are implicitly assuming that the entire mass of the system is having this velocity, which is not correct because fluid is a deformable medium and in general you can have different velocities at different points, so it is not appropriate to write it in this way but write it as an integral form of like integral of dmV small xyz over the system. The reason is quite clear that at different points in the system, it might have a different velocity.

So, when you want to represent that total linear momentum physically, we are representing the total linear momentum, the linear momentum consideration should keep in mind that there could be different velocities at different points in the system and then we may write this as dm as; how do we replace dm in terms of the elemental volume? So, this is the rho * an elemental volume dV, okay.

Next step, so if we put the d/dt outside, then you have the d/dt also, next step; can we put the d/dt inside, yes or no? We have discussed about these things earlier, see what is the variable with respect to which integration is done; volume, so if that volume is not a function of time then we may put it inside the integral without requiring to put any correction terms, so we are assuming that it is a non-deformable control volume.

So, with a non-deformable control volume, we may write it as integral of say, rho V small xyz d/dt of this one, dV, see this assumption is not a very restricted assumption because for most of the problems that we are considering this, it is not so common to have a control volume, which is arbitrarily accelerating plus deforming but I can give you a nice example, if you are fascinated with mechanics try with this example.

Say, you have a balloon, in the balloon you fill it up with water, just throw it with a spin and make the water come out of that and try to figure out how the velocity of the water coming out is changing with time, so you require everything, a deformable control volume arbitrarily accelerating because the balloon; when the water is coming out the balloon might be having arbitrary rotation and arbitrary linear motion.

And in an elementary level, I mean this problem experimentally one can do but theoretically I would say it is one of the very tough problems in mechanics which will require the combination of the understanding of the fluid mechanics and solid mechanics and deformable control volume, arbitrarily accelerating control volume, it will test your understanding of mechanics to the best of abilities.

Let us not go into that complication here, so we assume that this is what we are writing, next step is that we will assume that rho is not changing with time, so if rho is not changing with time, we will just write this as integral of rho d/dt of V small xyz dV, the other thing is that see magically, we were; as if we transformed it from the system to the control volume that we will not do immediately.

But we should keep in mind that when we are writing it in a limiting sense that is we are dividing it by delta t and in the limit as delta t tends to 0, we can write it in the limiting sense tending to a control volume because in the limit as delta t tends to 0, system tends to control volume that is what we have experienced earlier but that we will do in a bit of a later stage till now, we will preserve it as a system sense.

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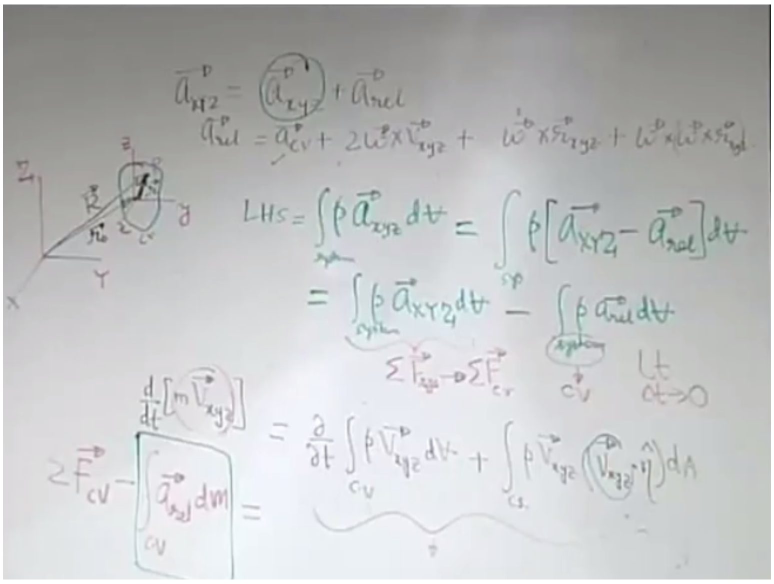

Why? The next step will be giving you a clue of why we are trying to do that. So, in the next step what we will do; we will write the left hand side simplification this is again, system you have to keep in mind that what are the assumptions that we made, so you have the rho and what is d/dt of V small xyz? This is the acceleration, small xyz, what is appearing here, so this is rho acceleration small xyz dV system.

What is our trouble? The trouble is that when we are having acceleration relative to small xyz, we cannot directly apply the Newton's second law of motion for that because Newton's second law of motion is valid for an inertial reference frame, small xyz in a special case when it is moving with a uniform linear velocity still an inertial reference frame but in a general case not, so to address the generality, we have to keep in mind that we need to first transform it to capital XYZ.

So that for that, we can use the Newton's second law of motion, so we will write this as integral of rho in place of a small xyz, we will write a capital XYZ - a relative, okay. So, when you take this integral of rho a capital XYZ dV for the system and minus; then what are the implications of these 2 terms? The implications are very straightforward; the first term represents the mass into acceleration of the system as an effect the integrated; the total effect mass into acceleration of the system as viewed from an inertial reference frame.

So, by Newton's second law of motion, this you can write as resultant force acting on a system and since we consider the limit as delta t tends to 0, this is as good as resultant force on the control volume and again in that same limit, you have in that limit, this also the system tends to control volume keeping in mind the limit that we are considering for these time derivatives that automatically implies that the delta t is tending to 0.

So, what is the final form of the equation that we are getting? Let us write it in the final form, so the final form is; the left hand side becomes what? The left hand side becomes resultant force acting on the control volume -; let us say integral of rho dV, you can say as dm elemental mass, so that is just like here also, so it is like integral of a relative dm is = the right hand side, whatever is there.

So, if you just look into it from a user viewpoint or a formula user viewpoint, what is the correction that has occurred because of the transformation? The correction has occurred mainly with one important thing that you have this correction term and this correction term is because of the acceleration of the control volume because if you see all the terms are related to acceleration, this is directly related to acceleration of the control volume.

These; wherever there is angular velocity you expect that it is related to that, so angular velocity, angular acceleration and again angular velocity, so these are all related to the nature of an accelerating reference frame. So, if the reference frame is not of accelerating nature then or if the reference frame is not having an angular velocity also because having an angular velocity automatically makes it of accelerating nature at least, the last term is important.

So, when none of these terms are important then this will be 0, we will see when and in the right hand side, you see that nothing great has happened, we have replaced in all the terms the absolute velocity by the relative velocity. So, when you are writing the Reynolds transport theorem for linear momentum conservation, then basically all the absolute velocities are replaced with the relative velocities, this was anyway always relative velocity.

But the other terms, these are now replaced by absolute velocity; relative velocity and in the left hand side, there is a correction because of the non-inertial effect of the reference frame just like minus mass into acceleration, this is just like a pseudo force, okay, so with that understanding, let us try to solve a few problems to understand that what we do for a moving reference frame in reality.

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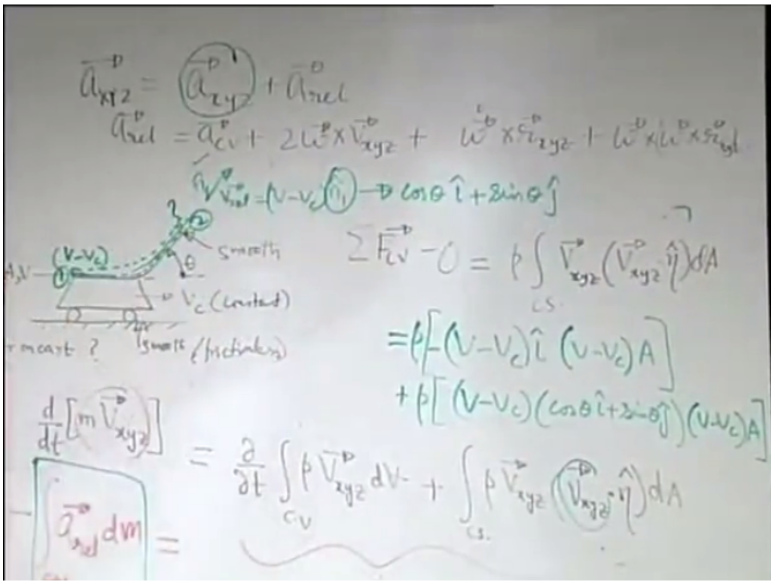

Let us start with the problem of a similar type, which we attempted in the previous lecture, so you have a cart like this on a frictionless ground and there is a water jet, which comes on it and which leaves the cart, the angle change is theta relative to the horizontal and the surface over which it is changing its direction the water that is considered to be a smooth surface and this is also considered to be smooth, the friction is 0, frictionless.

Now, what we have seen earlier, we have seen that because of the impingement of the water jet on the system and because of its change in direction, there will be a force and because of the force, the cart will have a tendency of moving. So, we saw that one way of like restraining it from moving is like maybe making a pulley mass arrangement, so that it is restraining from its motion but in general that will not be the case.

So, in general it will be free to move and therefore it will start moving, so we will consider a first case when it is moving but moving with a uniform velocity just as a simple case, we will then move on to more and more complicated cases when it may move with an acceleration. So, first let us consider that it is moving with a uniform velocity, so this cart is moving with a velocity VC towards the right.

The water jet comes out with a from an area A with a velocity V and it gets deflected like this and we are interested to find out what is the resultant force exerted by the water on the cart; water by water on the cart, what is that; that is the question okay. So, only difference from the previous problem that we considered is that now it is moving but this is constant, it is not moving with a variable velocity.

So, let us try to write it relative to the moving reference frame, so the left hand side resultant force on the control volume - the relative acceleration see, what are the terms which are there; first is acceleration of the control volume here, the control volume we attached the control volume on the cart, so or that is we attach the sorry; we attach the reference frame on the cart small xyz reference frame on the cart.

So, small xyz is moving with a uniform velocity VC, so it is not having any acceleration; linear acceleration, there is no angular motion of small xyz that means the, a relative term is totally 0, so that is 0 and the right hand side, if you consider it to be a steady situation where the velocity is not changing with time anyway, the density we are considering to be constant, so this term will go away and you are left with the last term that is rho integral of V small xyz * V small xyz dot eta A, okay.

So, if you consider say, the flow boundaries, say this is the flow liquid film that we are considering, so in this liquid film, we know what is the velocity here V because that is what is the velocity on which it is say impinging from a nozzle or someplace, question is what is the velocity here. Again, you may neglect the height here by considering that the kinetic energy effects are much more important.

Question is and a very big question is; that if all the assumptions of the Bernoulli's equation are valid, still can we apply the Bernoulli's equation from here to here directly, remember it is a moving reference frame; yes, or no? See, more and more these types of questions you answer yourself your fundamental understanding of that is clear, many times it is commonly understood that Bernoulli's equation is one of the easiest things in fluid mechanics.

To me, it is one of the toughest things in fluid mechanics, it is very easy formula at then but its restrictions are not well understood in many times. So, the question is can you directly use it keeping in view that all the assumptions that we have learnt otherwise are valid except we are now in a dilemma, why we are now in a dilemma because this is itself moving. See, although this is moving, we are saved with one important thing this is moving with a uniform velocity.

So, this is still an inertial frame, so we have seen that in the Bernoulli's equation all the quantities are relative like pressure is relative, you may express it relative to something, potential energy is relative you may express it relative to some reference or datum, the height, similarly kinetic energy is also relative provided you are still using an inertial reference frame.

So, here since this reference frame is inertial, you may have the velocity like the kinetic energy, the relative velocity that is what is only of importance, so what is the relative velocity here? Relative velocity here is V - Vc and that relative velocity will be preserved because the kinetic energy is preserved even in a reference frame, which is moving but non accelerating. So, if you apply the Bernoulli's equation here then that V should be replaced by V – Vc.

Because you are writing it relative to the reference frame, so the thing is that when you write this expression here, what will come out? So, there are 2 boundaries; flow boundaries 1 and 2; first let us write for 1, so for the flow boundary 1; - rho okay, first let us write plus minus sign will we will see inside, rho then what is V small xyz here? Let us write it in a vector form, so that is V - Vc i cap.

What is that remaining term again, what assumption we are making that it is a uniform velocity profile over the thickness that we are considering, so this is as good as some relative velocity into area with a proper sign. So, that means relative velocity is V – Vc, the area is A, the proper sign is negative, so we put a minus here then for the outflow boundary; for the outflow boundary, what you have?

V small xyz, it is V - Vc in magnitude because kinetic energy only bothers about the magnitude, so that is preserved means V - Vc is preserved but it has change its direction. So, when it is coming out, the relative velocity is V - Vc with this normal direction say n1 and what is this n1? Cos theta i + sin theta j, so this is cos theta i + sin theta j, so keeping that in mind it will become V - Vc cos theta i + sin theta j.

And the; what about the V relative into a that term, positive A*V – Vc, okay remaining work is easy, you can separate the x and y components by taking the coefficients of the i cap and j cap and write what are the components of the forces, these are forces exerted by the cart on the fluid by Newton's third law, the force exerted by fluid on the cart will be equal in magnitude of this one but opposite in sense, right okay.

So, we have seen an example where the control volume is moving, it will not make us very happy because the expression that we have derived we have not able to exploit that fully, the entire relative acceleration term vanishes. So, let us look into some example, where it does not get entirely vanished and then what happens.

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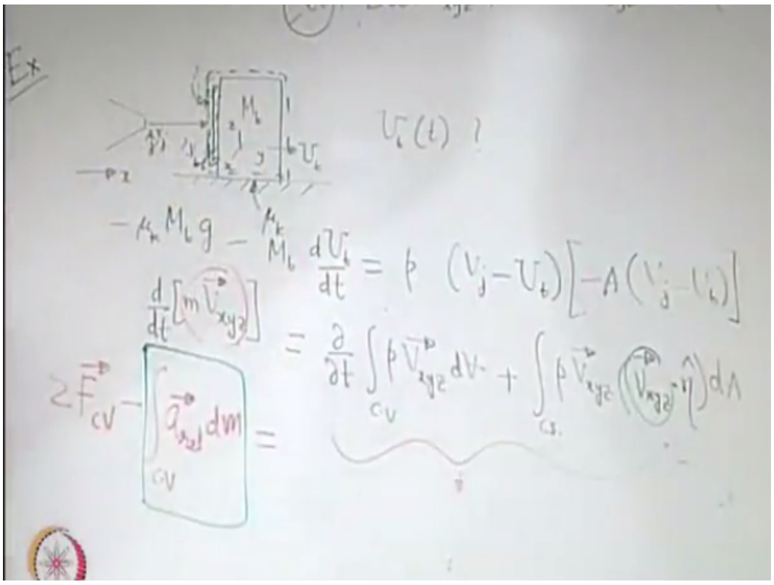

Let us say that there is a block, which is moving on a surface or which may move on a surface from a nozzle a water jet comes strikes this one and gets deflected in the two sides okay, the area of the velocity jets; area of the jet that comes out is given the velocity of the jet that comes out Vj that is also given; Aj, Vj; area of the jet and velocity of the jet. The coefficient of kinetic friction between this surface and the block that is given okay.

And our objective is to find out an expression of; if let us say U is the velocity of this block or let us give it a different name, maybe Ub for the U block, then how this U block is changing with time, okay. The mass of the block is given, let us say Mb is the mass of the block, let us try to look into this problem with the framework of an arbitrarily moving reference frame, now this is also a simple form of arbitrarily moving reference frame.

Because this cannot have angular motion, I mean if this may have angular motion again that becomes a very interesting problem, if you consider the extent the dimensionality of this and consider the toppling possibilities because of the force, which is acting on it but if you just idealize it like a point mass type of a thing then it is having just a translation but it may have a variable translational velocity in general, it should be like that.

Because before the jet is striking on it, it is stationary, when the jet is striking on it, it will try to make it move, so its motion from zero velocity will come to a non-zero velocity over a finite time, so there will be a finite acceleration. So, it is expected in reality to be an accelerating reference frame if you choose your reference frame attached with the block that is your small xyz attach with the block.

Now, let us consider the control volume may be let us say that we have the control volume like this, which contains the block and whatever water is adhering to this vertical plate. One important thing we should understand, what is that important thing? The thing is that; now let us say, we now at least, we are confident in one thing that we should be bothered about the relative velocity when we are writing with respect to the reference frame small xyz, say which is moving.

So, all the expressions that we write in the Reynolds transport theorem should correspond to the relative velocity that much we have seen. So, when we are bothered about the relative velocity, now if you say that velocity here is Vj, there should be some relative velocity here as it is the water jet deflected into 2 parts; one will go to the top, let us say it goes with the top as Vt; V top and another goes to the bottom with a velocity V bottom, okay just two parts of the jet.

Question is; is it so that V top in a relative sense is Vj - Ub assuming that other assumptions, other restrictions of the Bernoulli's equation of somehow satisfied, answer is no because it is now an accelerating reference frame. So, kinetic energy in an accelerating reference frame cannot be in a straightforward way brought out from the Bernoulli's equation as preserved, so we do not know, what are these?

In the previous problem, we could exploit the situation that the control volume was moving with a uniform velocity, now it is not moving with a uniform velocity. So, we do not know actually what these are, even in a relative sense but the good thing is that since these are vertical, these will not have any consequence on the horizontal momentum transfer. So, our ignorance or lack of knowledge on these will not be magnified.

This is a good thing for solving the problem but it is important to know that this is a very important ignorance; it is not so easy to tell; what are these velocities because this is an accelerating reference frame. Now, let us try to apply this equation for the resultant force on the control volume, so for the resultant force on the control volume let us write it only the x component because that will give rise to the variation of velocity.

Because we do not expect that this block will be lifted from the ground because of this, so now if you write the equation, the force then what are the forces which are acting on this; so if you draw the free body diagram of the control volume or maybe free body diagram of the block plus water whatever is contained there, so you have one frictional force. So, in the vertical direction, there is equilibrium of forces because the block does not get out of the surface.

So, you have n = Mb * g now, you have to keep in mind that you are not considering it just the block; block plus some water but the water mass may be neglected, it is very very small as compared to the block mass, so whatever mass of water is just like flowing in contact with this plate that water mass it is so small as compared to this Capital Mb that for weight or mass calculation aspect, we will not consider the water weight that is a very; that is an assumption that we have to keep in mind.

Otherwise, whenever we write the total weight, it should be the weight of the water in the control volume plus the weight of the block, which is there okay but we are neglecting that weight of the water in the control volume, which is a very valid assumption because it is a very small amount of water, which is there that is very practical. So, then the normal reaction will be capital Mb * g because of the equilibrium along the y.

So, the friction force will be - Mu k * Mb * g, then there is no other force along x, then what is this correction term? This correction term here is important because you do not have the omega related things but you have acceleration of the control volume and here this integral will just become a relative into the mass because it is just moving like a rigid body, so all points have same acceleration.

So, this will be - mass of the block times acceleration; acceleration is d U block dt, right. So, it is clear why we could take this a relative out of the integral because all points on the block are moving with same acceleration and we could have been in trouble, if there was lot of water in the control volume because then for water, we cannot have such a rigid body type of assumption, then at each and every point, the acceleration would be different.

But we are neglecting the mass of water present in the control volume for writing these expressions, the right hand side again; there is a very important thing. The unsteady term is fundamentally not 0 but we can approximate it by 0 because again if you see the volume of the water that is present in the control volume is small, so it is not because that V small xyz is small but because of the volume of water which is there in the control volume is small.

Then its time rate of change with respect to time is also very small, okay, so these are certain important things. See, we drop this term for solving the problem in exam fine, when you drop the term and solve the problem in the exam, if you solve it correctly, we have no way to deny you from the credit but your conceptual understanding is not satisfied there, you must be convinced that why this term is dropped despite the fact it is an unsteady flow.

It is not because that this velocity is 0 that is why you are doing it clearly, it is coming out with some velocity but just because you neglect the effect of that volume of water, which is there inside and the time dependence associated and the final term will be, what? See here only the x direction is important because in the y direction whatever it goes the out flows, so there are outflow boundaries but those are not along x.

So, this velocity; if it has component along x, then only that will remain for force calculation along x, so that will be rho A into what; you can straight away write it, what is V small xyz? So, Vj - Ub and then - Vj or; if you want to put the minus sign properly then this times minus of A into; that is the flow rate, Vj – Ub, okay. So, once you have that expression, now it is quite convenient because it is a simple differential equation with variation of Ub as a function of time.

What are the constants in this expression? The constants in this expression are this Vj, so maybe let us write one more step, so what you have here is Mb d Ub dt, so if you cancel all the minus signs in all both sides, - rho A Vj - Ub square + Mu k mg that is = 0, keeping in mind that at time = 0, you have Ub = 0, so it is a straightforward differential equation. Even if you do not solve it, you can find out that when it attains a steady state, what should be Ub?

When it attains a steady state see, that initially the velocity is 0 now because of the striking of the jet, it will be pushed to a velocity and it will come to a standstill, it will come to a uniform velocity that change in velocity will not be there anymore, when the d Ub dt term will be 0, so it will come to an limiting uniform velocity, which then will be considered by equating these 2 terms, okay.

So, it is possible to find out that what should be that equilibrium velocity and that equilibrium velocity is quite obvious, it is a situation that occurs when the driving force on the block is just sufficient to overcome the kinetic friction and then it is in equilibrium but it takes some time for the driving force to be in equilibrium with the kinetic friction till the time the velocity will be changing with time and that how it changes with time is governed by this equation.

Now, when we have considered this problem, one important thing that; when we have considered this theory as such, one important assumption that we have made is that the fluid in the control volume is of constant mass but it might so happen that the mass itself is a variable within the control volume. So, what we will do; if the control volume is accelerating plus the mass within the control volume is a variable, it is not a constant.

So, we will see that in the next class, in the next class we will take up the cases of variable mass in the control volume. Thank you.