Introduction to Fluid Mechanics and Fluid Engineering
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Lecture – 24
Integral Forms of Control Volume Conservation Equations (Reynolds Transport Theorem) (Contd.)

Last time we were discussing about the linear momentum conservation and we were looking into some example to illustrate the use the Reynolds Transport Theorem for working out problems related to that. We will continue with some more examples.

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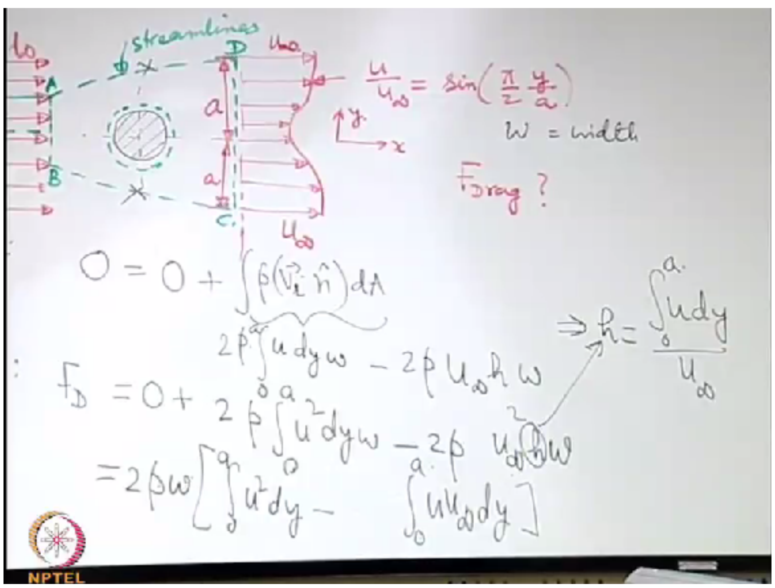

Let us say you have a solid body say a solid body of whatever shape may be circular shape if you want it to be so. And fluid is flowing it is coming from a free stream with a uniform velocity say u infinity and because of the presence of the solid the velocity is disturbed. And if you go a little bit away from the solid and if you draw a velocity profile say the velocity profile is obtained something like let us make a sketch of how the velocity profile is there.

Let us say the velocity profile varies in this way. In one of our later chapters we will see that what are the factors that we will determine that what should be these velocity profile or what should be the nature of variation of this velocity profile. But for the time being let us say that this is a qualitative sketch of how the velocity profiles varies. Assume that it is totally symmetric with respect to the center line and the velocity profile is such that like at the middle.

If you draw it totally in a symmetric manner. At the middle it is like a minimum and then it increases in both sides comes to almost u infinity at a given height. Let us say that this height at which it comes to almost u infinity is a. And let us say this velocity profile is given in terms of the x and y coordinates. Let us say that x is the axial direction and y is the transverse direction. And the velocity profile say is given by u/u infinity say =sin this is given.

Not that it has to be like this. This is just an example we are trying to satisfy the condition that when y =a u=u infinity that is how this velocity profile is there. So, the question is that what is the total drag force on the solid body exerted by the water or the fluid? Let us assume that the density of the fluid is rho and we have to find out based on these dimensions. So, now do we go about this?

We worked out a very similar problem when we were considering flow over a flat plate. This is not flow over a flat plate this is flow past some body of arbitrary contour. But the policy or the philosophy remains the same. So, we have to basically find out or we have to identify the control volume and see what is the net force on the control volume? So, identify a control volume will have some inlet and some outlet.

So, one inlet is this one which is straight forward that the flow is entering. Outlet this is straight forward and you can see that outlet is interesting only up to y =a or y= -a because beyond that the velocity is uniform. So, if we take a control volume say something like this where we consider one inflow boundary, one outflow boundary and across other boundaries we do not want any flow. So, what should be the edges of the other boundaries?

They should be streamlines so that there is no flow across those. So, let us say that we consider a streamline like this these are not horizontal lines these are inclined ones. Let us just magnify those a little bit and represent that. Let us say that this is one extreme streamline this is another extreme stream line. These are streamline keep in mind that it is not necessary to choose a control volume which contains streamlines.

But only elegance it gives to us is that we do not have to bother about the cross flows. But if we just take say some horizontal lines at the top and the bottom and constitutes a rectangular piece as a control volume. Then there will be flow across that and one have to make calculations related to that. It is just a matter of convenience for choosing the control volume. May be you consider only the water in the control volume so you exclude the solid part.

Now, let us write the expressions for the mass and the linear momentum conservation for the control volume. One thing that remains unknown that what is this height h? Because we have constructed a streamline from the edge of this layer where u becomes u infinity this is not a boundary layer. We will discuss later that what is the difference between this and the boundary layer as such.

So, when we have the streamlines starting from the edge the streamline will end up here at some arbitrary point which is not known to us. So, we have to find that out. So, let us say that we mark the edges as A, B, C, D and try to identify that what are the expressions for the conservation of mass. So, if you write the conservation of mass you have the emdt for the system I am just writing it straight away without going for the explanation of different terms.

Because we have already encountered that. So, dmdt for the system in the left hand side is 0 then we assume that it is a steady flow not changing with time. So, the right hands side you have first term 0 and then the term integral of rho vr dot n ba. So, that term we have to basically write. So, what will be the corresponding expression here? So, you have one outflow and one inflow. So, there is no flow across these 2 that is advantage of taking the streamline.

So, what will be these terms for the out flow? Say rho what is dA? dA you can take as dy* the width. Let us say w is the width perpendicular to the plain of the figure that is the width of these bodies. So, rho*u dy*w that is the total when integrated from say y = - a to + a that means 2*y=02a that is the outflow. And then the inflow that is there that is with the –sign because velocity is along the positive x outward normally is along the negative x.

So, what will be that – again 2 rho u infinity * h* w. So, from here you can get what is h? Because that is their unknown that you have to find out mass conservation tells us how to find out that. So, h= integral udy, 0 to a divided by u infinity. Next conservation of linear momentum. Conservation of linear momentum what it will tell us? The resultant force which is acting on the control volume. So, what is the resultant force that acts on the control volume?

There is a pressure distribution but because it is open to that ambient the pressure is same all around. So, the net effect of pressure distribution may be 0. But in reality it may be so that here the pressure is not same as the atmospheric pressure but something which is different from that. But let us assume that there is a uniform pressure distribution just for simplicity. Then the only force that remains along x.

So, let us say we are bothered about the linear momentum transport along x. The only force that remains important is the drag force = again the unsteady term is not there in the right hand side first and then. Basically these things will be multiplied by another u. So, it will be +2 rho integral of u square dy w then -. So, you can substitute the value of h here. 2 rho we have already taken as common so that will be the corresponding expression.

So, what we have done is we have replaced h with this expression. Now, what does this force represent? This is the force exerted by what on what? This is the force exerted by the solid body on the fluid, control volume. And you can easily see it say you do not know it. But the mathematical sign will tell you see u square is less then u infinity. So, this integral when it is evaluated it will be negative.

So, that means you have a negative force that means force which is along the negative x direction. So, what force is there along the negative x direction? It is definitely, force exerted by the solid on the fluid because it is trying to resist the motion of the fluid. On the other hand, the force exerted by the fluid on the solid is equal in magnitude to this but opposite in sense that is along the positive x direction.

That we also call as drag force on the solid object. There are certain interesting thing that we can observe from this problem one interesting thing is it appears as if this force does not depend on the shape of this object but that is an illusion why? Where the effect of the shape of the object comes into the picture. The velocity profile so the velocity profile will very much depend on what is the shape of the object.

So, we have assumed the velocity profile but this is like it not that it is comes just arbitrarily. Whatever is the velocity profile that velocity profile should come from the shape of the body and therefor that is where the shape of the body becomes critical. Now, the other thing is that it will also appear as if the force does not depend on the viscosity of the fluid. It appears so. So, it is a kind of like not an intuitive thing that you expect the force due to viscosity.

Because if there is no viscous effect perhaps there would be no drag but there is no viscosity here. There is no presence of the parameter viscosity. So, what is the viscosity doing here? Or the question may be posed in this way that is it always necessary that viscosity will directly come into the picture for the drag force calculation. See, viscous effect is there. There are 2 important effects which are prevalent here.

One is the viscous effect another is the contour of the body. As the fluid is flowing over the contour of the body there is a change in pressure. So, one is the geometrical effect another is the viscous effect and this velocity profile is the combine consequence of what has taken place. So, once you are given a velocity profile you may be abstracted of anything else but where from the velocity profile has originated for that viscosity may be important.

But once you get a velocity profile that is what the integral balance is giving. Integral balance is the net effect. It is not microscopically looking into what has happened at individual points. But it has got a gross consequence what is the consequence? Some velocity profile at the outlet and this kind of gross consequence is important because you can measure it experimentally. Experimentally, point to point measurement is difficult.

It is not impossible but it is always more expensive to do that. But experimentally, you can at least find out velocity profile on a given section. You can have different probes or say may not be as simple as fitter tube prob but you may have any velocity measurement along this section that is not difficult. And uniform velocity which is the free stream velocity that you know. So, from the experimental understanding of what is the velocity profile at the inlet and at the outlet.

You may be in a position to experimentally calculate or rather to calculate from the experimental data what is the drag force and the limitation of that it does not pin point that how the flow field varied from one point to another point to give rise to that drag force. But it gives the total effect in an integral sense. Now, how it varies from one point to another point for that we have to look into the corresponding differential equation for viscous flows that we will do in our next chapter.

Now, let us look into another problem. Let us say that there is a pipe like this.

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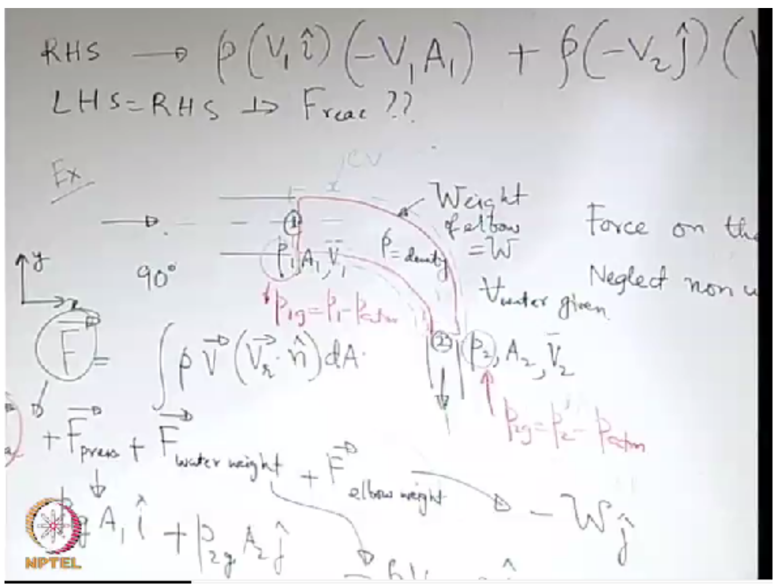

It has to be fitted with another pipe which is like this. I am trying to go you an industrial perspective of the problem rather than just stating the problem as it is. So, the problem is you have to connect these pipelines. It is very common that no matter whatever plant you visit you will see that there are lots of pipelines. And pipelines are not always straight because they have to connect different systems and there are space constraints and so on.

So, the pipeline has to be bent many times. So, there must be some fitting which connect this pipe to this pipe. There are 2 things which have happened one is the direction has changed may be the axis of these are oriented at an angle 90 degree. It is not required, not necessary that it has to be 90 degree but let us say that the angle between these is 90 degree + there is a reduction in cross sectional area that also is not a mass sometimes cross sectional area.

May remain the same or may even increase. But one has to just have a fitting to fit that. And that fitting in industry is known as elbow. So, what an elbow does? It basically tries to have a fitting like this to fit or match with pipelines of different orientations and different sections. If the angle between these 2 axis is 90 degree it is called as a 90 degree elbow. So, if we assume that this angle is 90 degree we call it 90 degree.

Now, this elbow it cannot be free in air because we will see that there is lot of force that is being exerted on this elbow because of the change of linear momentum of the water that is entering and leaving. And because of that there is a force on the elbow and it has to be supported. So, it must be supported with a support that proves some necessary reaction force which balances those forces exerted by the water on the elbow so that it is in equilibrium.

Otherwise, it might have a tendency to get deflected from its equilibrium configuration. And that will disturb the entire stability of the structure. So, when you design a structure you have to be careful of what are the support forces that need to be sustained by that structure. For that the support force has to basically balance the force exerted by the fluid on the structure. So, we have to know what is the force exerted by the water on the pipe so that is our objective of solving this problem.

That is we want to know that what is the force exerted on the elbow. What are the data given let us try to release that force on the elbow? Let us say that you have points 1 and 2 or let us call this as sections 1 and 2. So, at the section 1 say there is an equivalent pressure p1 which is given, area of cross section is given at the section 2 you have p2 and A2 these are given. Let us say that the velocity profile is uniform if it is not uniform at least you know what is the average velocity.

So, you know the average velocity at section 1. You know the average velocity at section 2. Where from you know the average velocities? Experimentally, what you can always find out what is the flow rate? And if you know the area of cross section say it is a circular one you know the diameter so you know the area of cross section. So, A into the average velocity is the air flow rate from that you can find out the average velocity.

If you neglect the viscous effects than the average velocity is the same as the velocity at any point. So, you may neglect that let us say that neglect non-uniformity in velocity profile. What else is given that what is the weight of the elbow say it is equal to w. Weight of the elbow it is solid so it has its own weight. So, we are considering that weight and we are given the density of the water which is there inside rho is the density and what else we require?

Let us say the angle between these inlets and the outlet so the water is entering like this. It is leaving like this. The angle is 90 degree. If it is not 90 degree then also like if it inclined it will have its horizontal and vertical components of the flux velocity and so on. Now, we are interested to write the expression for the force component along x and force component along y. Sometimes you see that the pressure at 1 is given but pressure at 2 may not be given.

But if you assume an inviscid flow and you connect a streamline say from the center of the section 1 to center of section 2 you can use the Bernoulli Equation to find out what is the pressure and if you assume that the pressure is uniformly distributed over the section then that will be the pressure throughout the section 2. You have to keep in mind that what can create a non- uniformity in pressure. So, if you have pressure at the center line say at p1.

What can make it deviate if you go to a different location in the cross section? Curvature of the streamlines so if streamlines are almost parallel to each other than the change in pressure is very, very small or negligible. So, then we are assuming that streamlines here and here are almost parallel to each other see if you take that on the bend that is not valid so we are considering the section 2 which has actually crossed the curvature part of the elbow.

This is an assumption. In reality the piece may be short so that may not be a very good assumption but this is what we are assuming otherwise you have to also consider a non-uniformity in pressure across the section which itself adds with the complexity. We are not going into the complexities but I am trying to highlight the complexity because these are important, may be important in some realistic conditions.

So, 2 important complexities may be non-uniformity of velocity over each section and non-uniformity of pressure over each section. And when non-uniformity of velocity over each section is occurring then that means and it is always there until or unless it a highly turbulent flow at the velocity profile due to high mixing almost uniform. Otherwise if there is a velocity profile it gives an important understand that yes viscous effects are important.

And when viscous effects are important you cannot apply Bernoulli Equation along a streamline between 1 and 2. Still you can use A1, B1 average =A2, B2 average that is the conservation of mass that does not depend on how viscous forces are occurring or not. But you cannot really relate the pressure at 1 with pressure at 2 using the Bernoulli Equation. One has to solve the viscous flow equation to find out that.

Now, when you write the resultant force along x. So, let us try to write the resultant force in a vector form. So, we are using the Reynolds Transport Theorem the right hand side the first term due to unsteadiness that is 0. Next term is integral of rho v. When you are writing this force F what is this force F? Let us now write what are the constituents of this. One force is the force exerted by the elbow on the water. So, that is F reaction then what other force is there?

Force due to pressure is there + force due to 2 weights, 1 is the weight of the elbow itself another is the weight of the water which is instantaneously there within the elbow. So, f due to water weight and + F due to elbow weight. So, let us try to write this expression of these forces of course this is what you are interested to find out F reaction so this is an unknown. Force due to pressure. How do you find out what is the force due to pressure?

What is the force due to pressure, resultant force due to pressure on the control volume? P1 A1 for section 1 along x P2 A2 for section 2 along y like that. It is not like that. I have mentioned it earlier why when you have a pressure distribution on a surface you have to consider the force due to gauge pressure only because atmospheric pressure is there from all sides and that is nullifying the total force when it is integrated over a closed contour.

So, when you are writing the force due to pressure it should be the net force because of the pressure over and above the atmospheric pressure. So, to calculate the force P1 has to be converted into the gauge pressure. These are subtle but very important things. These are places where like the most cases students will make mistakes. Of course if you practice enough problems you will never make such a mistake.

But general tendency is like before the exam you just look into worked out examples so when these things are not highlighted you just look into the gross formula. But these are very important things that you have to keep in mind. Do not just take it as a formula keep in mind that why it should be so that why you have to take the gauge pressure for evaluation of the force. So, force due to the pressure what should be the corresponding expression?

p1 gauge pressure*A1 that is a net force due to pressure at section 1 so in a vector notation we can call it this i gap then + p2 gauge *A2 j cap you have to keep in mind that pressure is always into the surface. In whatever phase you are considering pressure is always acting towards that. Then force due to the water weight, what is that? It is not impossible to calculate what is the volume of this given this contour.

Let us say the volume of the water is given that is the volume of the elbow basically. So, if the volume of the water is given then what is the mass? Rho * volume of the water that *g is the weight. g is acting along negative y then this is – of this j and the elbow weight what is the elbow weight? – w j and the right hand side the integral of when you are considering this integral first is what are the surfaces across which fluid is flowing 1 and 2.

What is the control volume that we are taken? Since we have represented the elbow weight we have considered the elbow also as part of the control volume. Let us draw the control volume so till you express explicitly that what is the resultant force that you are having? It may not be so straight forward to say that what is control volume that you have taken? So, if you say if you take this as the control volume then that means this excludes the elbow.

And then if elbow weight is not there. But once if elbow weight is there you have basically taken including the elbow. So, it is elbow + the water that you have taken as the control volume. So, the question is then when you have taken this as the control volume. The outer one is the control volume say which includes both the elbow and the water. The question is then what is this F reaction?

It is not now not provided by elbow by the water so what is this? So, if considered there is a support which is there outside which exerts the force on this elbow + water system. So, there is some support which is there which is not draw in the figure but it is highlighting that support. So, now for that particular control volume we are having how many inlets and how many outlets? We have one inlet and one outlet and let us write that.

So, the right hand side first let us write for the section 1. For the section 1 if you assume a uniform velocity profile then like this entire integral will be based on the velocity say v1 which is uniform over the section 1 so rho then for v1 it is v1 i cap and then *v1 a1. So, with what sign + or -? You can clearly see that if there is velocity variation along y then this expression is not valid.

Then you have to integrate the velocity profile and that would give net momentum flux. This is like a momentum flux. So, the net momentum flux will be different and if one assumes the uniform velocity profile and it is really not so then that is an error and one is to adjust that error with some momentum correction factor may be. But here because of uniform profile assumption that such a correction is not necessary.

Otherwise if the velocity profile is given to you, you can integrate it to get this expression then there is no correction factor necessary. Then for the surface 2 + rho what is the velocity at the section 2 let us say, v2 it is directed along which direction –y. So, - v2 j* this is +. So, the left hand side is equal to the right hand side. And that will give you what are the components of the reaction force? This is the force exerted by the support on this system.

So, it should provide an equal and opposite force on the support and the support must be good enough to sustain that force. So, if it is unsupported then because of some resultant force the elbow may start moving. Let us look into one more example.

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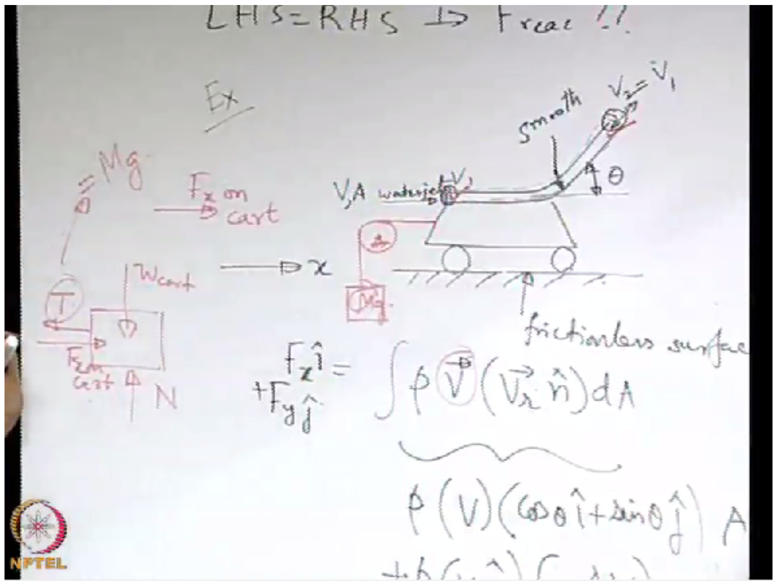

Let us say there is a cart like this and water jet is striking on the cart and it is changing its direction. Let us say this angle is theta. Let us say that the velocity of the water jet is v and the corresponding area over which the jet is moving here is A, cross section A. And let us assume that this is smooth. So if this is smooth that means there is no friction that the fluid is encountering as it is moving along the cart.

Only its direction is getting changed. The first question that we will like to answer is that will it be possible to keep the cart stationary if such a water jet falls on it and changes its direction. For simplicity let us assume that this is a frictionless surface and may be assume that the cart is having a particular weight but that is not of great concern for us. Because we are interested to consider the motion along x whether there will be any motion along x or not.

First of all, let us say that what do you expect to be the velocity at which the water leaves the cart? Say it enters the cart at 1 and leaves the cart at 2. What is the velocity that you expect? If the velocity here is say v1 which =v. What is the velocity at 2? First of all, if you consider a streamline that connects some point at the inlet with some point at the out let then could be apply the Bernoulli Equation along that streamline?

The first question is if it is inviscid flow then that is the first question that you would like to ask. So, assume that it is an inviscid flow. If it is an inviscid flow, provided other conditions are satisfied. What are those? You have density as constant and steady flow obviously although unsteady version of Bernoulli Equation is also there but let us assume that it is a steady flow. So, if this is a smooth one and the water this is quite thin.

And because of the smoothness there is no such wall roughness effect that is propagated into the fluid. So, it is as if like a frictionless flow. Although this is a great idealization in reality the effect of the solid boundary will always be propagated into the fluid and in all cases it is likely to give viscous resistance. But here we are just idealizing it by too much and assuming that effect is not there. If that effect is not there if the velocity here is v1.

The velocity v2 should be =v1 provided that the difference in height between one and 2 is neglected. So, we ae neglecting the z2-z1 that is neglected. It is really a very small height and the corresponding potential energy change is insignificant as compared to the kinetic energy of the jets. See in engineering when we say that we are neglecting something there is a very important thing that we should keep in mind we are not actually neglecting potentially energy.

We are neglecting the change in potential energy and that change itself may not be negligible in the absolute sense what we are banking on is that the jet is falling on with a very high kinetic energy with respect to those kinetic energies the potential energy effect is negligible. Not that it is always in an absolute sense negligible. And regarding the pressure both are exposed to atmospheric pressure. So, the pressure is like p1 and p2 are same.

So, if the z2-z1 is neglected and if we may apply the Bernoulli Equation with all the assumptions satisfied then you have v1=v2. In general, if there is a friction here v2 will be somewhat less then v1 but because the friction less nature v2 is v1 and then you can apply the continuity equation then A2 also must be same as A1. Now, let us say that we are interested to find out what is the resultant force along x because that is what is going to make it move may be.

So, the resultant force along x what is that? So, you have 2 sections basically. So, you have one section like this where the fluid is entering and you have another section at 2 where the fluid is leaving these are only the 2 flow sections. Where do you choose your control volume? See since there is no friction on the ground it will not be any different if you include that to the like all the structural part of the cart and exclude the structural part of that cart.

For obtaining the force along x. Definitely for force along y it will be mattering but not force along x. So, for force along x the right hand side first the unsteady term is 0 and then integral of rho v vr dot ndA. So, this is like Fzi + Fyj because this is in a vector form. Now, let us try to write it in terms of its scaler components at the section 2 what is v? V has a magnitude v what is the direction?

Cos theta i + sin theta j so that is the v in the vector form and then the remaining is AV that is the integral of v dot ndA. What were the section 1? For the section 1 what is the velocity? Vi and – AV. So, what is the force along x? You can find out only the x component of that. That is rho AV square *cos theta -1. This is the force exerted by the solid structure on the fluid. So, if you consider say a control volume like this we just encompass the fluid jet.

So, this is the force exerted by the cart on the fluid. The fluid exerts an equal and opposite force on the cart. So, this force is positive or negative. This is negative this is along –x. So, force exerted by water on the cart is along + x so you have a + effect that is there on the cart. And that is quite obvious even if you do not go through the mathematics if there is a jet striking like this it should exert a force along the x direction. So, if there is an Fx on the cart.

So, the cart under this force may try to move and if this is a frictionless surface it will move always. If there is friction, the static friction may just balance it and keep it in equilibrium. But if it is not then it has to move. If it moves the question is this consideration valid? That is here we are having to use the relative velocity but we do not know what is the velocity of the cart. So, how you should go about it that is the first question.

Second is whether this velocity then we have to use the absolute velocity or the relative velocity? So, these are the questions that we will like to address in a subsequent theoretical development where we consider also the moving reference frames. Till now we have considered only the stationary reference frames but in the jurisdiction of stationary reference frame if you have to consider it. You have to consider somehow that this is stationary.

Now, how can you design a system such that this remains stationary there could be many ways. Let me give one alternative and you say whether it is good alternative or not? Let us say we have a pulley like this and let us say there is a weight MG which is there. This pulley is hinge is supported like this. Is it acceptable? Will it work? No or yes? It depends on what is this weight and that you can design exactly because you know what is the exact magnitude of the force.

So, if you draw the free body diagram of the cart what are the forces that you will see? You will see a tension in the string and you will see a force effect exerted by the water on cart. So, when you have these 2 of course the other y component is there. So, y component you have the weight of the cart then you have a normal reaction like that. But for us interesting is the x component and if you want to keep it in equilibrium you have to balance T with Fx on the cart.

And if you consider it to be all those idealistic situations that it is a frictionless pulley and then what you get is that you get this tension same as the Mg so this in turn mass pulley system =Mg. So, you know that what has to balance what? So, you can put the correct mass here to keep it in equilibrium. But you can clearly see that this is a forceful arrangement to keep it in equilibrium but in general because of these forces it will not be in equilibrium.

And when it is not in equilibrium with these forces it might have a velocity that velocity itself might change with time. So, it might have a situation when the reference frame which may be attached to cart itself is moving and moving with arbitrary velocity or arbitrary acceleration. So, we have to also be equipped with an analytical ability by which we can encounter such situations that is situations where you can encounter accelerating reference frames in general.

When we say accelerating reference frame we mean accelerating frame, reference frame in all respects that means it could be linearly accelerating it might have an angular velocity because of which it has its original acceleration. So, we have to next go for an analysis for accelerating reference frames.

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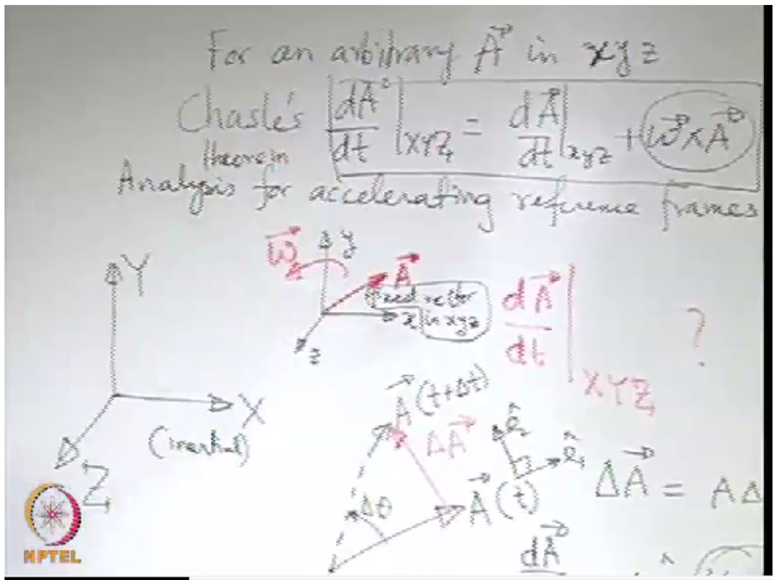

So, we will use certain nomenclature we will consider an axis say X, Y, Z for an inertial reference frame and small x, y, z reference frame as an arbitrary it may be inertial may be non – inertial but it is a moving reference frame. If it is moving with an acceleration, then it is non-inertial. If it is moving with a uniform velocity it is still inertial. What we are interested to find out is that if we have a vector say A here in this reference frame.

And let us say that this reference frame is moving with an arbitrary angular velocity omega then what is the derivative of this vector with respect to the inertial reference frame? Can you show it, if I ask you to show it? How do you show it? Say you have a vector A which is there in reference frame that is rotating with an angular velocity omega. Let us say that the angular velocity is such that the rotation is taking place in the plain of the board.

The rotation will take place in some pain. What is that plain? The plain is perpendicular to the axis of rotation. It might not xy plain or yz plain like that but it is some plain. So, in that plain this vector A is rotating. So, when it is rotating it comes to what state? It comes to a new location say this is at time t, it is at time t+ delta t. What we are keeping in mind? We are keeping in mind it is a fix vector in a moving reference frame.

So, there is a fix vector in zyz that we have to keep in mind. It is not any arbitrary vector. That means if you are sitting on this you do not see any change at least in the length and if you are outside although it is same in length but because the reference frame is rotation this also rotates. Let us say that it traverses an angle delta theta over this time delta t. So, what is the change in the vector? The change in the vector is this delta A. So, what is this delta A?

For small delta t the delta theta is small so this is just like an arc of a circle. So, delta A in term of magnitude is what? A delta theta, in terms of a vector you have to give it a proper direction and sense. So, if let us say this is A is in the direction of e1 then it should be a direction which is normal to e1, say e2. If you want to find out what is dA/dT if we are not mentioning any subscript XYZ that means, we are talking about inertial.

Then it is basically we are dividing this by delta t and taking the limit as delta t tends to 0. So, Ae2*limit as delta t tends to 0. Delta theta/delta t which is nothing but the magnitude of the angular velocity. And what is omega cross A, what is omega vector? It is omega scaler time a unite vector e3 which is perpendicular to the plain of the board so you may take e1 like x e2 like y and e3 like just like that.

This cross A is A e1 so it is omega Ae2 e1, e2, e3 form orthogonal basis just like xijk. So, you can write that dA/dT XYZ =omega cross A but this is only for a vector A which is fixed in that xyz reference frame. If it is moving in a xyz reference, then that velocity also has to be added with this. So, in general for an arbitrary vector A in xyz you have dA/dt XYZ =dA/dt xyz + omega cross A.

So, this change is felt even if A is fixed. But A is moving relative to zyz this is an additional change. So, that is the total change and this you know from your earlier studies that this is known as Chasles' theorem. So, we will take up from this and try to write an equation of linear momentum conservation for a control volume which is having arbitrary motion. It may have angular motion. It may have linear motion.

It may be a non-accelerating reference. It may be accelerating reference frame in general. We will take that up in the next class. Thank you.