Introduction to Fluid Mechanics and Fluid Engineering

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Lecture – 23

Integral Forms of Control Volume Conservation Equations (Reynolds Transport Theorem) (Contd.)

In this lecture we will look into the integral form of the conservation of linear moment. To do that we will start with the Reynolds Transport Theorem general expression which is valid for any conservation. Now, if we want to conserve the linear momentum then what should be this N.

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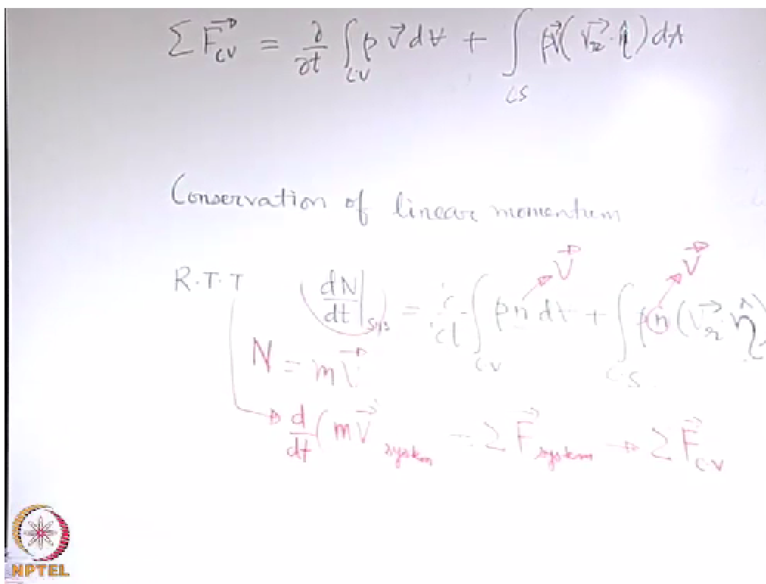

N is the mass* the velocity that is the linear momentum. So, we can write the left hand side as d/dt *mv of the system. So, when we write d/dt *mv for the system by Newton second low of motion is the resultant force which is acting on the system. We have to keep in mind that we have derived this expression with the limit as delta t tends to 0. In the limit as delta t tends to 0 system tends to the control volume that we have to keep in mind.

And therefor in that limit this also tends to the resultant force acting on the control volume. So, the left hand side essentially becomes the resultant force which is acting on the control volume. And the right hand side we can express by writing what is n, N per unit mass. So, this v and this is also v. Very important observation this v in a reference frame which is stationary we have considered a stationary reference frame here.

We will see later on that it is not necessary to have a stationary reference frame we may have a moving reference frame not only that reference frame moving arbitrary with arbitrary rotation as well. So, a non-inertial reference frame what happens to the statement of the Reynolds Transport Theorem in a non-inertial reference frame that we will also see. But here we are considering inertial and the special case of inertial which is stationary.

So, here this v is the absolute velocity but even in this particular term we are not trying to disturb this particular term because this particular term is going to always this irrespective of the control volume moving or stationary or whatever. Because this is something which is giving rise to a net flow of mass across the control surface and that depends only on the relative velocity not the absolute velocity.

So, here it will be absolute velocity here it will be relative velocity and that we will keep in mind. But when it is a moving reference frame this will also change to relative velocity that we will see. So, the statement of the Newton second law here boils down to like an expression which is the resultant force acting on the control volume. And see the power of the Reynolds Transport Theorem we never committed that N should be a scalar vector or whatever.

So we have applied it for a vector. And there is no restriction towards that. With this understanding on the form of the conservation of the linear momentum for a stationary reference frame let us try to work out a problem. Let us say that you have a water jet.

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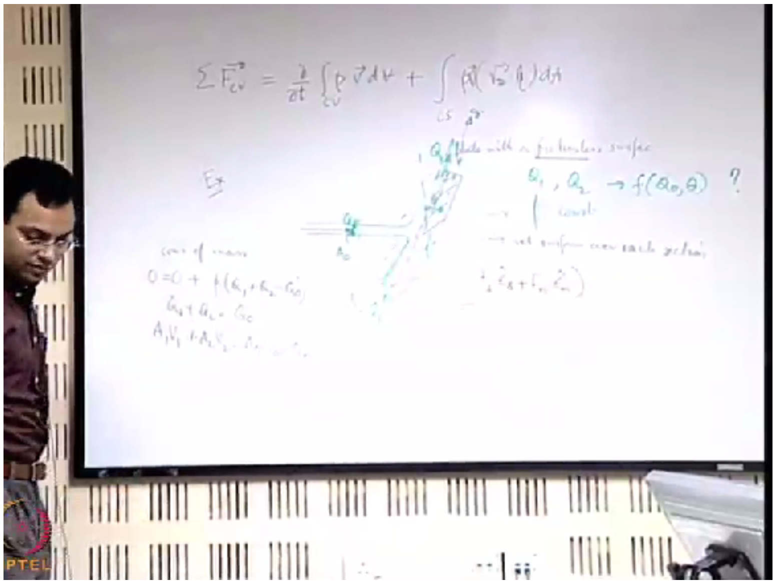

Which comes out of a tube or may be a small pipe and say there is a plate which is frictionless and oriented inclined to the jet this is a plate with the frictionless surface. So, what happens to the water? The water extreme stream lines come like that and water leaves across the ends of the plate this way. We have to find out if the volume flow rate here is Q0 and here the volume flow rate is Q1 and if the volume flow rate Q2.

And let us say that the angle made by the plate with the vertical is theta. It is the angle between the plate and the vertical when we say vertical let us neglect the change in height between various points just to simplify this equation. Let us say that it is like vertical means the vertical line draw in this plain. The entire thing may be located in the horizontal. Or even if it is a vertical arrangement the plate height is so small that the change in elevation.

The change in potential energy between various points is insignificant as compared to the other forms of the energy like the kinetic energy. Now, let us say that this area of cross section of these extreme streamline so these are like extreme tubes these are called. So, if you see that this is not a pipe basically. This is a free water jet exposed to the atmosphere but the water jet takes the form like a tube. Its outer periphery takes the form of a tube.

And out once are the extreme streamline so the figure that is drawn here the lines represent the envelop of all the streamlines. So, this is called as extreme tube which engulfs all the streamlines. The same is there for the (()) (07:41). So, what are the considerations that you now could apply for finding out so what is our objective. The objective is to find out Q1 and Q2 as a function of Q0 and theta that is our objective.

To do that one important thing that we should not forget that mass conservation is always applicable and we should try to apply that not that whenever we are trying to apply a momentum conservation, mass conservation mass (()) (08:37) we did like that. So, no matter whatever conservation we are applying again first identify a control volume. Let us say that this is the boundary or the surface of the control volume that we are looking for.

You will see that like whenever we draw the boundary of a control volume in a controlled surface we just draw arbitrary dotted line which not coincident with the surface. This is just for the clarity representation. This is very, very symbolic. It is not that if I have drawn it a bit far that means the air here is involved as a part of the control volume. So, do not take it literally. Now, when this control volume is drawn we are interested to find or write an expression first for the mass balance.

Conservation of mass. Assume that the density of the fluid is a constant. Let us say that the areas of flow here are like here it is A2 and here it is A1. These are the parameters that we do not know because these are not within our direct control? What is in our direct control may be this area because this depends on the ejected jet from the tube or the pipe whatever nozzle. But when it comes on the plate and moves it depends on the many other things.

Directly we cannot say. But one thing we can say is that the mass will be conserved. So, if you write the conservation of mass by this time we have got habituated in writing for different cases so we will not write the full Reynolds Transport Theorem form. We know which terms are important and which terms are not. Left hand side first term is 0 always because the system has a fixed mass.

The second term if you have a control volume which is of a fixed volume and density as a constant that will be 0. So, it will be only the rate of mass outflow –inflow. So, what is the rate of mass outflow? So, that is rho *Q1+Q2 is outflow –Q0 so that is the outflow –inflow. So, you can straight away write Q1+ Q2 =Q0. Let us make another assumption that the velocity is uniform over each section.

So, you can write this as good as A1, V1+ A2+V2 =A0V0 or Q0 whatever is (()) (11:49). Now, there is a relationship between V1, V2 and V0. It is possible to have a relationship between V1, V2 and V0 but before coming into that relationship let us complete the exercise of looking into the linear momentum conservation. What linear momentum conservation gives us? See one very important thing is it is considered to be a frictionless surface.

That means there is no shear force between the fluid and the plate in the direction tangential to the plain. So, if you write a linear momentum conservation with the component say along x, x is like say tangential to the plain or may be let us call it is to indicate that it is a tangential to the plate and may be a direction n which is like normal to plate. But we are bothered about for the tangential to the plate. So, tangential to the plate what is the situation?

The situation is that there should not be any force on the water which is there on the counter valve because it is a frictionless thing. No, matter whether it is tangential or normal. So, let us write this F for the control volume as, let us write it in the general vector form. So, let us write that conservation of linear momentum if as Fs* unite vector along s + Fn * unite vector along n. This is the resultant force on the counter volume.

General form - we know that Fs is 0 from the physical considerations of a frictionless plate. This is equal to for the first term we are assuming that it is a steady flow velocity is not a function of time. We are assuming a constant density, density is not a function of time and the volume of counter volume is not also a function of time. So, all these 3 consideration leads to the fact that these terms have to be 0.

So, when these terms are 0 then you come to the second term so what will be the second term. So, for the second term you have so the integral is not important because the velocities are uniform over each area. So, we have to write this for the 3 surfaces 0, 1 and 2 just like that individually. Integral will eventually not have to be evaluated because of the uniformity of the velocity. So, let us write this for the section 0.

So, for the section 0 you see what is v. So, may be let us again make a chart what we did for solving one of our earlier problem just for your convenience to begin with.

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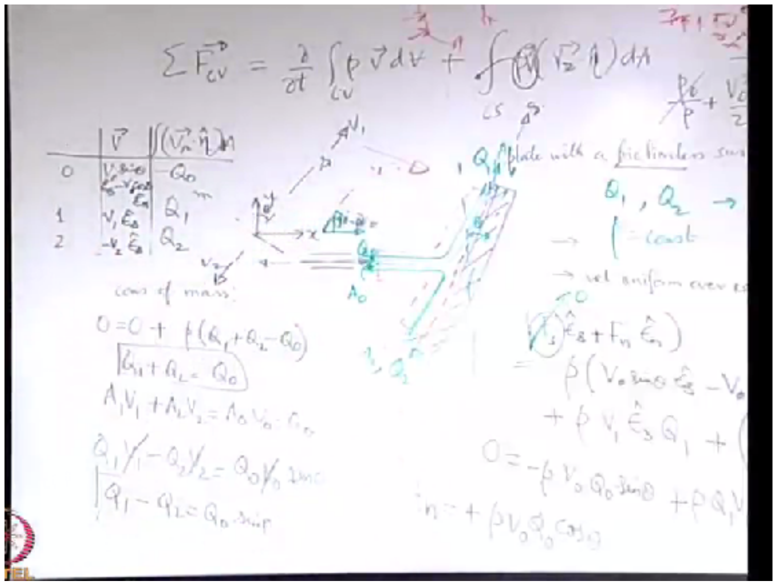

So we have a section say 0 at which we are interested to find out what is v and what is the normal vector. The normal vector will be insignificant we will see because eventually we are dealing with the flow rates. So, if you consider v to be uniform and rho to be uniform you can see rho*v you can take out of the integral. If you take rho * v out of integral, then what remains within the integral is integral is Vr dot ndA that is what?

That is the total volume flow rate over the section that you are considering. So, it is as good as rho * velocity vector * the volume flow rate over the section with a proper sign + or -. So, keeping that in mind we will not write this vector with not that it is not existing but it is not important for solving our problem. It will just give a redundant exercise we will put too much of effort which is not necessary. But the velocity vector is important.

So, the velocity vector for this one like let us say that we have our original axis like this is x and this y and the x axis is like this which makes an angle theta with y. May be you can also choose n axis which is normal to that. Now, so what is v for this one. v is a v0i cap may be instead of writing this normal vector let us write what is Vr dot nA or more fundamentally integral of vr, ndA. So, what is there that for the surface 0.

See Vr or Vn is unite vector they are oriented in the same sense or opposite sense? This is the direction of v. This is the direction of unite vector of this surface so this will be what? -*QD then for the surface 1 so you are having a flow for the surface 1 in this direction which is having a velocity of v1 so that may be resolved along x and y. So, V1 will have component what? V1 sin theta I+ D1 cos theta j. What is v dot nda here?

It is Q1 because the load direction and the unit vector out of the area they are oriented in the same sense. For the surface 2 what is v? Magnitude is V2 so the velocity is like this now. This is V2. It will have some component along x and some component along y. What is the component along x? –v2 sin theta I and –v2 cos theta j. What is this one? Q2. See this is very straight forward.

But I want to emphasis on this because this is a place where students make mistake many times. Because by default it is a common thing that the outflow boundary give rise to a positive flow rate but the velocity that comes with it, it has nothing to do with the flow rate. So, that you have to write in proper vector sense. So, this term + or – has no conflict with the flow rate term that you keep in mind.

Otherwise just by inertia of writing you might write this in a similar sense that written here. So, let us now substitute that so if you substitute that let us say that we are interested about writing the expression in terms of the tangential and the normal components. Why we want to do that? Because we are interested about equating this tangential force to 0 so that we want to exploit in that case these coordinates are not good coordinates for us.

This I just wrote for a practice of showing that how to write different terms. But more convenient will be to use the S and the n coordinate. So, in terms of the s and n coordinates if you want to write, the good thing is the flow rate does not change it is independent of the coordinate type.

But what changes is now the expression for the velocity vector that you are looking. So let us write these expressions now in terms of the s and n vector which will be useful.

For solving this problem. So, in terms of the velocity at section 0 so you have a velocity at section 0 like this you have V0. You have a coordinate s like this and a coordinate perpendicular to that as N and with this one it makes an angle 90 degree –theta. So, you can resolve V0 into components along s and n. Typically, how the resolution will look like may be you have these along s, these n so resultant is this one. So, what will be the component along s?

So, v0 sin theta n-v0 cos theta epsilon n. This is for the surface 1 v1 epsilon s for the surface 2 – v2 epsilon s. Let us put that here. So if you put that then it will become first we put for the surface 1 so rho *v p is v0 sin theta epsilon s – v0 cos theta epsilon n *-q0. Then for the surface 1 + rho v1 epsilon s * +Q1 for the next one –rho pv2 epsilon s multiplied with (()) (23:39). So, if you extract the tangential component and keep in mind.

That because of the friction less case this is =0 you are left with that if we equate the epsilon s components you will get 0 = - rho v0 Q0 sin theta + rho Q1 v1 – rho Q2 V2. So, you again see the deceiving thing as if this minus sign is giving inflow but this is a – but this is outflow. So, the origin of the + and – has to be clear. It is not because of outflow or inflow, the + or – that has come as a combination of outflow.

And the direction of the velocity wavelength. So, rho being a constant you can write like from that equation Q1P1-Q2V2 =Q0V0 sin. Now, the question is what is the relationship between say v0v1 and v2. Let us say somehow you could measure both A1 and A2 somehow you could measure. Now, let us try to see that how that measurement could give us a good picture? Let us say that you know what is v0 you are interested to find out what is v1.

So, what you will be tempted to do? If you know what is v0 your objective is to know what is v1 from that what intuitively you try to use. Let us consider a streamline that goes from the section 0 to the section 1. Because it is a uniform velocity like if you take a point 0 here and a point 1 here these are representatives of the velocities as good entire section 0 and entire section 1.

Because it is a uniform velocity at each section that we have assumed.

Now, if you apply the Bernoulli's equation between any 2 points on the streamline. So, let us try to apply that and see that what new information we get. So, the first information that we get is first of write the information here. We assume that both 0 and 1 are atmospheric and it is true it is not an assumption because it is free jet in the atmosphere. It is also a free liquid till being passed on to that atmosphere.

So, in both places section 0 and 1 it is exposed to atmospheric pressure. And because the height difference is not that large atmospheric pressure will not change substantially. So, these 2 are equal and because the height difference is not appreciable as we assumed earlier so the z0 and z1 are approximately the same or their difference is negligible. So, from this we can conclude that at least in an approximate sense v0 is same as v1.

And with a similar consideration similarly you can say v0 is approximately same as v2. So, in this expression where you have v1, v2, and v0 you can cancel that from the both sides. So, what you get? You get Q1-Q2 =Q0 sin theta. So, let us say we get 2 expressions involving Q1 and Q2 in terms of Q0 and theta. So, you can solve for Q1 and Q2 individually as a function of theta from this very simple.

And important thing is it does not require any information on what are the areas of these liquid (()) (29:14) at 1 and 2. So, it does not require what is A1 and what is A2? There could be a very interesting observation now you find out what is the normal component of the force? Normal component of the force definitely is not 0 and the normal component of force will come out if you see that because of the terms rho v0 q0 cos theta.

So, because of the normal component of the force this is the force on what? This we have to keep in mind as right (()) (29:52). So, fn + rho v0 q0 cos theta. Q0 you can write in term of A0 and B0 so it is like rho A0 B0 square cos theta. Question is this is the force on what? These are very subtle points. Always it is not here the answer ends. We have found out the normal component of force. We must ask ourself that is force on what? Force exerted by whom on whom?

If we are not clear about that then what is the good in finding out a mathematical expression. So, this is you have to keep in mind that this is a force on the control volume. So, if you had drawn a free body diagram of the control volume if you would have done then what would have been the forces acting on that control volume? That we would have shown in the free body diagram. Let us say that this could be your control volume.

So, you could have a force in the normal direction. In the tangential direction Fs and in the normal direction Fx, assume that the same thing is occurring almost in the horizontal plain so the weight you are not considering in this plain. Not that weight is not there but in this plain, forces weight is not important. Then any other force on this? Think of the fundamental considerations of forces in continuum mechanics that we discussed. You have surface force and body force.

So, body force we have considered the weight just not shown in this plain because it is not significantly there in this plain. Surface force pressure distribution is there. So, there is a surface force from all sides normal to the areas whatever. Have we considered the contribution of this? We have not considered but magically we are expecting that it is giving us the correct information on forces.

And the magical thing is possible because if you have a close contour over which you have a uniform pressure distribution that irrespective of the shape of the contour the resultant force is 0. Because of a very special nature that pressure local is always normal to the direction of the surface. So, no matter how bad the shape of the contour of the surface is so long as it is closed and if you have a uniform pressure throughout then the resultant force due to that will be 0.

It is a very important thing and you and you could be able to derive it by yourself. If you are not able to do that you let me know later on I will try to help you out but you should be able to do. So, if that is the case then if there is a variation of pressure only that will matter to a resultant force but throughout it is atmospheric then it does not matter so therefore our negligence of the pressure in this case has not mattered because it is a uniform pressure.

And that is why you will see that important for us is not what is the atmospheric pressure but the deviation from the atmospheric pressure that only can give rise to a local force. So, always when we consider forces here due to pressure we consider the gauge pressures at different sections at different section. Because the deviation from atmospheric pressure is what is interesting for us. Atmospheric pressure if it was the sole feature it would have existed throughout equally.

And it would have given rise to a 0 net force. So, in terms of giving rise to a net force any deviation from atmospheric is important so the gauge pressure is the important quantity. Now, here of course the pressure contribution does not come so FS and FN these 2 are there. Question is that on which this FS is acting and it is exerted by whom on whom and on which FN is acting? The FS is 0 we have seen and that gives a clue that it is between whom and whom?

When we say FS =0 we were thinking about an interacting between the plate and the water. So, FN also should be consequence of interaction between the plate and the water. So, it has 2 possibilities what? Force exerted by plate on the water or exerted by water on the plate? **“Professor - student conversation starts”** Yes. The should be same but same in magnitude not same in sense. So, whatever answers we have got here this has a particular sense.

So, it represents only a unique thing. So, when we have said Fn epsilon n it has a proper sense of the vector epsilon n. So, this is force exerted by water on the plate or plate on the water? Plate on the water. It is force exerted by the plate on the water. **“Professor - student conversation ends”**. Because it is force exerted on the control volume and what is there in the control volume is not plate but water.

So, this is force exerted by the plate on the water present in the control volume. By Newton third law you will be having the force exerted by the water on the plate as minus of this. So, the force exerted by water on the plate will be opposite to the positive N direction. So, force exerted by water on the plate will be in this direction. This is you see this is in the positive any direction that we have considered. So, opposite to that is the force exerted by the water on the plate.

And that is quite obvious like logically if you see if the jet strikes on the plate like this it should have a force in this direction not the opposite direction. That is a resultant force which is exerted on the plate in its normal direction. So, this force might be good enough to make the plate move and the plate should be adequately supported to prevent such movement. In general, if that jet is fallowing on the plate the plate will also move because of this normal force.

And later on we will encounter such problems to try to see that if the object on which the jet is striking or which is interacting with the jet in any form if that itself is moving then how we adopt our analysis of momentum conservation. So, this is like a very simple problem but it gives a lot of important concepts that we should keep in mind. Let us try to solve another problem. So, in the next problem we will revisit the case of a boundary layer on a flat plate.

That we discussed quite some time back when we were discussing about viscosity. So, what the situation that you have a flat plate like this.

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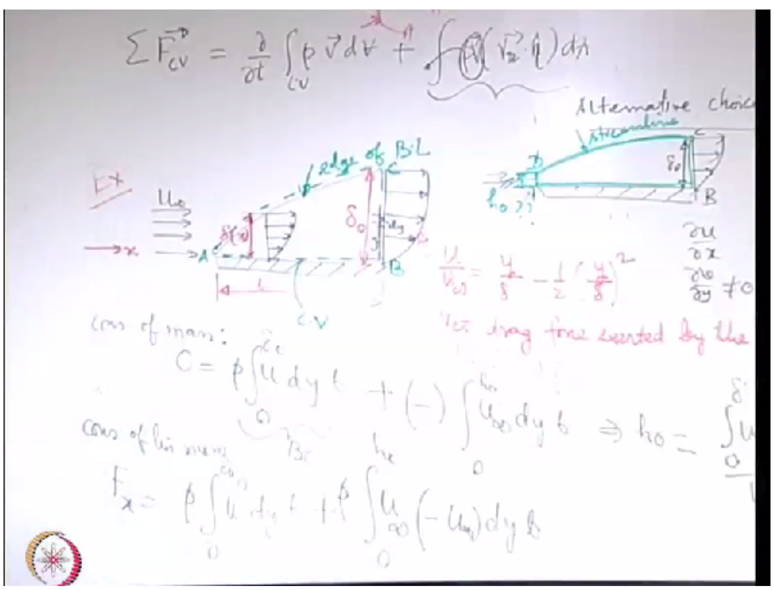

Fluid is coming from height stream with a uniform velocity say u infinity. This fluid is falling on the plate and because of its viscous interactions the effects of viscosity within the fluid. The first thing is the plate tries to slow down the fluid which is in contact with that and that effect is propagated to the outer fluid through the viscosity of the fluid. There is a thin layer close to the wall where this effect of the plate is felt and the velocity gradient is created.

And when you go outside that layer the velocity gradient is no more there. So, outside that layer the fluid does not actually fill rest of the plate so to say and that thin layer is known as the boundary layer. So, if you have a boundary layer growing on the plate so you have a velocity profile at different layers like this. Since we have discussed in some physical details without going into some mathematics.

But at least the physical details when we were discussing about the no slip boundary conditions and the viscosity. So, let us say that at the edge of the plate you are given the velocity profile. What is the velocity profile that is given? Let us say that locally the thickness of the boundary layer is given by delta which is a function of x. So, here you have let us say the value of this delta is the boundary layer thickness at this location.

Where the x is l. x-axis is oriented along the free stream direction. Let us say this value of delta is delta 0. And the velocity profile is given. Let us say it is given by u/u infinity =y/delta -1/2. Let us say this is given then what is great sanctity of these types of velocity profiles or whether it would be different or not that we will discuss in details later on in one of our chapters known as boundary layer cube. But right now let us say that this is something which is given.

Now, what is your interest? Your interest is to find out what is the net drag force exerted by the plate on the fluid? This is a physically interesting parameter because of the viscous effect there is a drag force that is there that is the plate tries to slow down the fluid. And our objective is to find out what is that net force acting over this length l. So, to find out a force we may understand that like we might require a linear momentum conservation.

Because the force is involved and when you have a linear momentum conservation the mass part of that should also be conserved so we should also be consistent with the mass conservation. So, it depends like we should choose a control volume. Let us first choose a control volume which is intuitive and try to solve the problem and then see we will try to investigate whether that intuitive choice of the counter volume is good or bad.

So, let us say that we choose a, so this black line what is there? It is the edge of the boundary layer. What is the edge of the boundary layer? We know that within this layer only the velocity profile is there. Beyond this the velocity is uniform =u infinity almost. So, if you want to take a control volume let us say we choose a control volume like this just engulfing the edge of the boundary layer.

With respect of this control volume let us write the first the conservation of mass. Let us give some names of the edges of the control volume. Let us say A, B, C. If you write the conservation of mass what we are getting integral form left hand side first term is 0 because we are talking about a identified system of fixed mass. Right hand side first term let us assume again it is rho =constant and this is not a deformable control volume.

So, right hand side first term is 0 then the next thing is basically what? Basically the net mass flow rate so you have 3 surfaces here clearly across AV there is no flow because of what no penetration. So, because across AV there is no flow but across BC there is some flow. What is the flow across BC? So, rho is coming out of the integral now we will directly write the integral. Because we have experience how to write that in earlier example.

So, rho this is the velocity profile so the velocity and the outward normal is in the same direction so product will give something positive. So, we just write the scalar form. So, if we take at a distance y from the bottom we will take of (()) (44:32) with dy what is the da? da is dy* the width of the plate. Let us say b is the width of the plate. So, rho* q*dy*b. Y is integrated from 0 to delta 0 and mu is function of y. And then there is a flow through AC.

So, when there is flow through AC let us not try to write it in a complicated way. Let us call it m.ac. So, m dot Ac this is algebraic so it may be + or – here you can see that m dot Ac = - so what it appears is that see this is such a control volume there is nothing in the left so to say across one surface something is living to make a balance of that as if some mass is coming from outside across the edge of the boundary layer to compensate for that.

And that is given by this one. So, it is like a m. entering the control volume. Now, simply conservation of linear momentum till now we are have only talked about (()) (46:17). So, let us write what is the x component of the force because see the drag force should be oriented along the direction of the relative velocity between the free stream and the plate. So, that is oriented along x so force might have some components but let us write only the x components.

So, Fx = the right hand side the time derivative term is 0. We are writing the Reynolds transport theorem for linear momentum conservation. Then next term looks into that next term, basically we are writing this. So, the next term that has contributions again for BC and AC. So, for BC let us first write what is for BC. So, for BC you see we are writing only the x component so we will just keep in mind that we will write only the x component of the velocity vector.

So, when we write the x component of the velocity vector what should be here? First we are writing for the surface BC so integral of rho. See this V (()) (47:46) this is like a scaler it does not give the directionality. So, the directionality the direction for which we are looking for the force should come from the component of the velocity vector that you are taking. So, we should take u and here u is anyway the only component.

So, rho not that u is the only component u is the only important component because you can clearly see that this delta is a function of x. This delta is varying with x so if you differentiate u partially with respect to x you will get that as != 0. Because it appears not to be a function of x but implicitly it is a function of x because delta is a function of x. This being != 0, continuity equation for incompressible flow.

The corresponding the other term this is also != 0. Because v is 0 at y =0 its gradient is != to 0 so you have a v not that you do not have a y component of velocity. From the continuity equation even this you can find out what is the y component of the velocity how it is varying? But it is much smaller than the x component of the velocity. But here when you are writing the x component of force only the x component of velocity anyway is important.

So, rho u and then again u v dy from y=0 to delta 0 then you also have for AC. So, for AC it will be like if you take this v as uniform and take out of the integral then the remaining term inside over AC is the mass flow rate across AC. So, it is nothing but +u over AC*mass flow rate over

AC. What is u over AC? u infinity because AC is the age of the boundary layer at which u becomes =u infinity. So, you can substitute it a Fx now m.AC you can substitute.

So, it becomes rho, rho is a constant you can take it out so 0 to delta 0 again p is a constant u square dy –u infinity *udy. You may write it in a more compact form that is in a single integral u square –u infinity dy from 0 to delta 0. Now, let us just look into that what would be an alternative choice of the control volume may be to solve the problem a bit more elegantly. Not that it is too dirty but one could even like do it in bit more elegant manner.

One interesting thing you observed from here that there is only x component of velocity here and y component of velocity the free stream is 0 but there is some flow across the surface KC. Now, would be choose a surface AC or surface of similar type such that there is no flow across it. So, let us try to draw a separate sketch with an alternative choice of control volume. So, for the alternative choice of the control volume.

Let us say that again do we have the velocity profile and everything. You have this A, B we want to choose a line from here C such that there is no flow across that line. We have seen there is a flow across the edge of the boundary layer. So, what should be that line that we choose here so that there is no flow across it. Through what type of line, you have no flow across? Through a stream line so if you choose a streamline which is passing through the point C.

This is the stream line then we know that there is no flow across it but it gives rise to a new unknown because we do not know that where that streamline is intersecting with this one. Let us say h0 this we do not know. So, now let us make a new control volume say A, B, C, D where we have tried to get rid of a problem by considering the streamline when there is no flow across it. But we have a new flow boundary AD.

So, for the new control volume let us write the conservation of mass. So, let us quickly write the conservation of mass and conservation of linear momentum for the new control volume and let us try to see whether it makes us converse to the same answer. So, conservation of mass first you let us for AB it is 0 mass flow, for BC we have seen what is that integral of rho*dy from 0 to delta 0 this is for BC for CD it is 0 because it is a streamline.

There is no flow across it but there is something for AD. So, what is that for AD? Whether it is + or – so the normal direction is opposite to the flow direction so with the – integral of u infinity dy *v from 0 to h0. U infinity is a constant so from here you can clearly find out what is h0. So, h0 =integral of udy/from 0 to delta/ u infinity. Then conservation of linear momentum along x, Fx= again only the last term will be there.

So, for Vc flow integral of square dy*v for CD, you do not have a mass flow rate. So, the linear moment flow is 0. Because you do not have any mass flow rate across that so if you take this v out of the integral because it is uniform you see the remaining is the mass flow rate over the area that you are considering. And the mass flow rate is 0 across the streamline. So, for CD it will not come but again for DA it will come.

So, for DA what will be that for DA? Last integral of rho you take out first x component of velocity that is u infinity then v dot nda that is –u infinity dy, 0 to h0. So, what will be our Fx let us just write it clearly.

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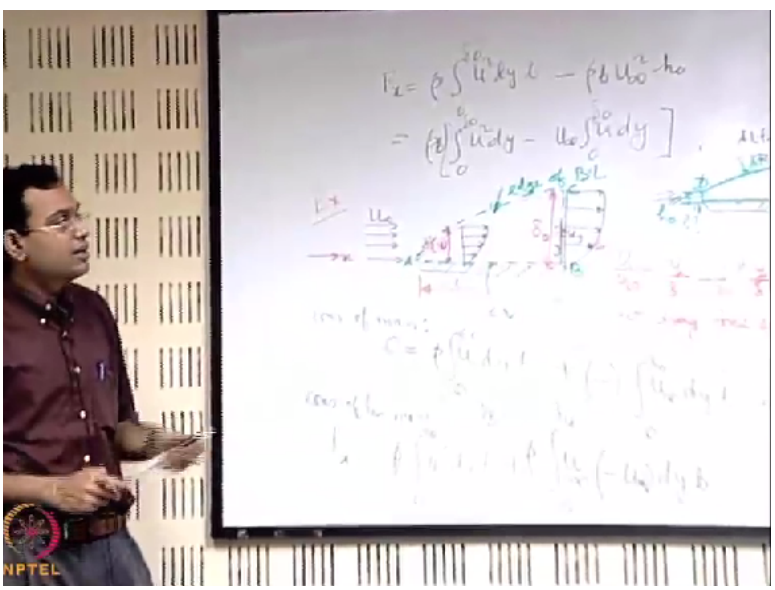

So, Fx = rho integral of square dy t from 0 to delta 0- rho b u infinity square *h0. That if you substitute the expression for h0 it is becoming u square dy, rho b you can take this common –u infinity integral. You can clearly see that you get the same expression back. So, 2 different choices of the control volumes are giving back the same expression and to me the alternative choice of control volume is not bad.

Because it gives a better visualization of what is happening. Because this gives a direct visualization that something is entering here and something is leaving and these 2 are not participating. For the case of the age of the boundary layer it is physically not that intuitive. Mathematically it is not that difficult or straight forward but the greater physical picture is being provided by this counter volume but both are find and the remaining part is easy.

You may substitute u as a function of y and integrate because u as a function of y is given to find out the expression. Again this is the force exerted by what on what? This is the force exerted by the plate on the control volume and can you make you that whether it should be positive or negative? You can clearly see that here you have u square and u is less than u infinity and here is like u*u infinity. So, here u is multiplied with a number which is > u itself.

So, this term must be < the second term. So, intuitively this should come as negative and that is what is obvious because it is a drag force it tries to slow down the motion of the fluid. So, this is something which is a physical understanding that is important once you solve a problem you get a sign out of the problem and you should develop an intuition of what that sign implies. Let us stop here today and we will continue again in the next class. Thank you.