Introduction to Fluid Mechanics and Fluid Engineering

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Lecture – 21
Integral Forms of Control Volume Conservation Equations (Reynolds Transport Theorem)

Today, we will start with the new chapter which is the integral forms of the conservation equations?

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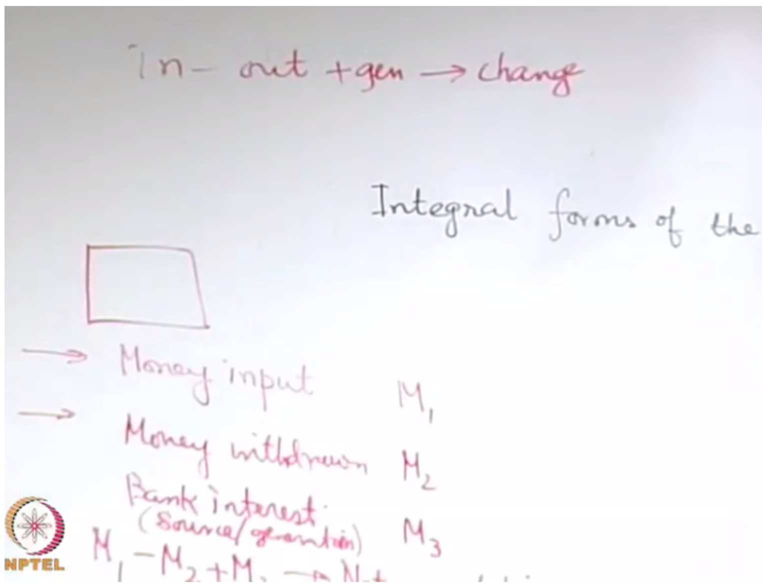

This is our logical extension of some of our previous discussions where we were discussing about the differential forms of the conservation equations and some of the differential forms of the conservation equations were in terms of the conservation of mass, like we discussed about the continuity equation and conservation of the linear momentum may be expressed to the Newton second law of motion that was the Euler's equation of motion for an inviscid flow.

And we say that subsequently it could give rise to a form of mechanical energy conservation also, but most of those forms that we discussed were of differential in nature and we will now see we will try to look into the feature of integral forms of this conservation equations, but before going into that we must try to develop a feel of what we mean by these conservation equations and how they are important in fluid mechanics. So when we talk about conservation equations.

We are talking about certain mathematical form, which represents the physical meaning of conservation of something and we will try to see that what is the basic physical principal that gives rise to the sense of conservation of whatever may be mass, momentum, energy or whatever and how we can express that in an equivalent mathematical form. So first of all we will try to get a feel of what is the conservation principal.

So when we talk about a conservation principal, it may not be a bad idea to discuss about it in a bit of a more general frame work that is not very specific to may be mass or momentum or whatever. Let us take an example which is totally deviated from fluid mechanics so that it is not as boring as many of the issues of fluid mechanics. Let us talk about conservation of money. So money is something which excites most of you and so I hope that you will listen to whatever example that we are talking about.

Let us say that you have gone to a bank to open a bank account. So your bank account starts at a given instant, where let us say this is like a symbolic representation of your bank account. So you have put some initial money in your bank account and the bank account operates from then onwards. Now say you are working in a job earning a fabulous salary, but you are having such a great amount of money otherwise that you do not care about what is going on in your bank account.

One fine morning you feel that well you need to see what is there. Let us think about traditional way of banking of course these day you go to like you log on to your computer to see what is there in your bank account and do you all sorts of transactions. Let us say you physically go to the bank, so we physically go to the bank and you put a query that like how much is there in your bank account, because you want to withdraw some money also, not that you need that money, but you want to just make your account, keep your account in a regular form.

So you have seen that what is there in the bank. What you see initially there was some money that was put in the bank. Now when you have gone to the bank and even if you have not enquired of what is the total amount of money that is there may be you have an estimate of what is there and you have withdrawn some money. So you have some money withdrawn and if the bank is good enough it depends on the economy.

But poorer the economy the bank is better for you to give you a good interest and then you will find that whatever money that was originally input or may be was being input month after month by your employee as your salary and the money that you have withdrawn is not they are just the 2 important constituents you also have a bank interest. So there is a bank interest. So if you have originally put a money input of say M1 not originally, but M1 first may be month after month after it has got accumulated total money amount input says M1.

Money withdrawn if it is M2 say the bank interest is M3 then at the end of transaction and everything when you are coming out of the bank may be you are happy or not so happy whatever that remains in the bank it is what? M1 was the original input. You had withdrawn M2. There is a bank interest which is in favour of you, so this is the net accumulation. So this is the change in your bank account that you see before visiting the bank and after visiting the bank.

So this net accumulation is a sort of like coming out a conservation of money here. So some money was being put. A part of that was withdrawn. Something has been so called bank interest is like a generation of source, source of generation. It does not mean that money is automatically generated bank of course invest it in different ways and that is how you get the interest, but from your point of view it is like as if there is a source of money or generation of money.

So you have an input. You have something which has been withdrawn and there is an interest so the net effect of that taken algebraically with plus or minus is the net accumulation that is there in your bank account. So this talks about principal of conservation of say money in a bank account say the bank account is like a control volume so across which there is something which is entering and something which is leaving we have talked about mass entering, momentum entering.

But here is like money entering and money leaving, but if you now abstract yourself a bit from money to something which is a bit generic one then it is like something has entered so something has entered minus something has left so in - out. So in is like input, out is like withdrawn, then something is generated. This is like interest is leading to a net change.

So this is very simple, you cannot even think of simpler way of looking into conservation and believe me or not whatever equations of integral or differential form that we write in fluid mechanics representing conservation of various quantities fundamentally follow these principal. So we will try to see that how from this fundamental principal, we can develop we can derive some of the very important conservation equations in fluid mechanics.

When we want to do that we need to keep in mind that there is a general way of looking into this conservation as represented by the simple statement. So our objective will be to express this simple statement in a somewhat mathematical sense and how we do that there is a very important theorem that tells us that how to go about that expression and that theorem is known as Reynolds transport theorem.

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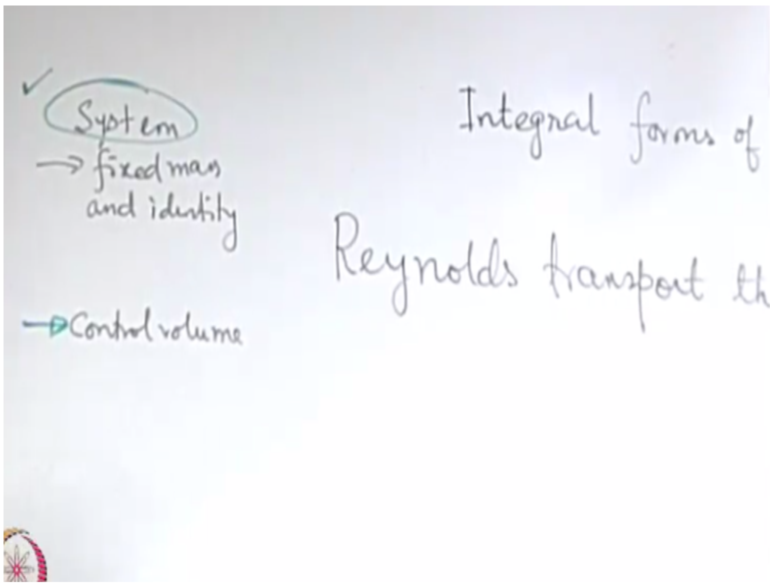

Reynolds transport theorem is so powerful that if you know how to make use of this theorem you can derive any conservation equation in fluid mechanics by using this theorem. So this is very very, very powerful exploiting this, it is really possible to derive any equation in fluid mechanics. So one we will be tempted to look into the perspective of this theorem that is what this theorem is essentially trying to do.

So before going into the theorem, let us try to understand the motivation behind that theorem in particular the Reynold’s transport theorem. So if you recall we have discussed many times that the difference between a traditional way of looking into mechanics, is particle mechanics and fluid mechanics is the reference frame mostly like in particle mechanics you are looking for a Lagrangian reference frame and in fluid mechanics mostly we are discussing in the context of a Eulerian reference frame.

The reason is obvious that fluids are continuously deforming and it is actually impossible or very difficult to track individual fluid particles and see that how they are revolving. So the approach that fits mostly with fluid mechanics is a control volume approach. On the other hand, all the basic equations which have been classically developed in mechanics have been not based on a control volume, but based on something which is of a fixed mass and identity and that we call as a system that also we have discussed earlier.

So we have something as a system which is a fixed mass and identity and we have something as a control volume which is an identified region in space across which any mass, energy or whatever can flow and the apparent travel is that all our conservation equations classically have been developed for a system, but in fluid mechanics tried to express that in terms of the corresponding phenomenon over a control volume.

So we require a transformation from a system approach to a control volume approach, so that the same type of equation let us say Newton's second law which is their originally defined for a system, we want to express an equivalent form for a control volume. So we require a theorem that gives us a kind of transformation law in perspective of what happens across a system with respect to what happens across the control volumes.

So in very brief transformation from system to control volume or may be vice versa from control volume to system and that is given by this Reynold’s transport theorem. So the motivation is this is to get general formulation which will try to give us a guideline of how to have a transformation from a system approach to a control volume approach. So to do that, we will start with like very simple way of looking into the theorem and maybe we start with a sketch of what happens to a system and what happens to a control volume.

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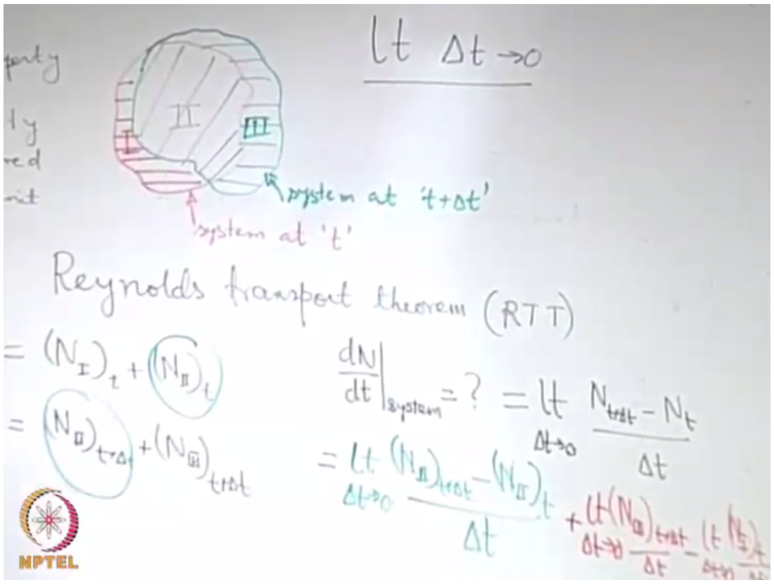

Let us say that we define a system with its system boundary like this. So this is an arbitrary way of just sketching a system. What it means? That there is something may be some fluid inside this. This is the boundary and the fluid particle, which are there inside this these are identified. So that is the system. Let us say that this is the system boundary at time t. Now let us consider a small time interval of delta t.

Over this small time interval of delta t this system boundary has now occupied a different configuration. In reality if the delta t is small these configurations which is there with the green colour is almost merging with that of what was the configuration at time t drawn with a red colour. Just for distinguishing these 2, we have really amplified the change here in the figure, but keep in mind that these 2 envelops of the system which are the so called system boundaries will be actually almost merging one on the top of the other as the delta t time interval tends to 0.

So this we call us system boundary at time t + delta t. Keeping in mind, delta t is very small. So this is just to isolate the 2 system boundaries that we have drawn in a magnified way, but please keep in mind that these 2 are almost coincident not that they are coincident, but they are actually falling one on the top of the other. Now when we have drawn a sketch like this, you can clearly see that there are 3 important regions in this sketch and one important region straight away is the common intersection between the 2 configurations.

So let us identify that mark that in this way. Then there are other parts like you have, this is one part where it is solely belonging to the system which was there at time t and there is a third part which solely belongs to the system at time t + delta t. So these 3 zones just for our own convenience let us give some name. Let us say the name of the first zone which is marked with the red one is I, the next zone is II and the third one that is to the right is III.

These are just 3 different volumes that we identify for our own convenience of demarcation. Now what we can say let us say that we want to write a conservation law for some quantity. So we are not really committing what is the quantity. Keep in mind the quantity that we are interested to conserve may be mass, may be momentum, may be energy or whatever or let us say that we are interested to conserve some quantity.

So let us say N is the quantity that is conserved. Quantity to be conserved means that it is satisfying the basic conservation principal that we discussed through an example before this and let us say that n is N per unit mass. So N is like it is the total and n is that expressed per unit mass and that total may be any property. So when you say quantity it is better to like call it a property and what property that we will see.

Now when we say a property that we need to conserve and when we express the property also as per unit mass that means we are trying to implicitly make a statement that N depends on the total mass of the system. So it is not any property, but some property which depends on the total mass and that we call us the sort of extensive property that is a property that depends on the extent of the system and when we express it per unit mass we call it as specific property that is the property expressed per unit mass.

So when we look for such a property we will look into examples to see that what could be such properties, but right now we are very, very abstract in a mathematical way of looking into this. So when we say that N is the property that is there so when we say N at time t. So N at time t is nothing but whatever is N at which is being occupied which is because of the volume occupied by the region marked as 1 at time t + N for the region which is occupied by the marked II at time t. So we can say that N at t is N of I at t + N of II at t.

So when we are describing the N we are describing the N in terms of the system that you can clearly see. The reason is that when we are calling it that it is dependent on the total mass of the system, so we must have an identified mass and so this fundamental is originating from a system concept that you have a system with identified mass. There is some property which depends on the total mass and the total property is the sum of the property over the regions I and II, very straight forward.

Now what happens to N at time t + delta t. This at t + delta t it occupies the regions II and III. So it is N at II at time t + delta t + N at III at time t + delta t. What is our interest? Our interest is what is the rate of change of n? So we are interested to find out what is the dN/dt of the system. This is our objective and for that we can write it in the fundamental definition of a derivative as limit as delta t tends to 0, N at t + delta t - n at t by delta t.

So the basic definition of the derivative becomes applicable here also. There is no reason that it should not be applicable. Now we are interested to write these term in a bit of more explicit way. So when we write this n at t + delta t and n at t you can clearly see that there are again different types of terms. So when we subtract Nt + Nt from Nt + delta t you have to keep in mind that this term N at II it is a very special term, because it refers to the same region, but different times.

So these 2 terms may be grouped together to give a very special meaning of what is the change over an identified region. On the other hand, the other terms which are remaining they are having a different interpretation and we will see what is the interpretation. So mathematically we will write it or we will express this or break it up into 2 different terms. So the first term will be limit as delta t tends to 0, N at II at time t + delta t - N at II at time t/delta t.

Then plus the other terms N III at time t plus delta t/delta t - limit as delta t tends to 0, N1 at time t/delta t sort of that. So the next step that what we will be doing is very important conceptual interpretation of what is there as the limit as delta t tends to 0. So let us try to figure out a case when you have the limit as delta t tends to 0.

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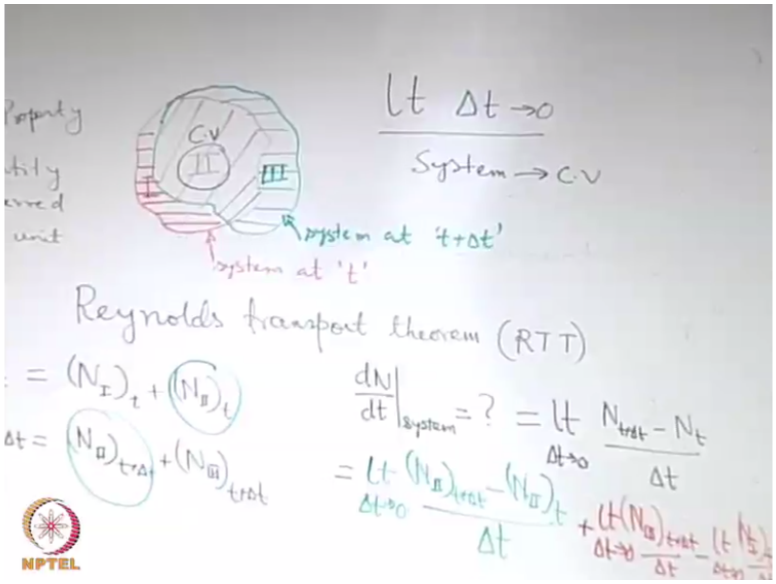

So this a very important conceptual paradigm. So limit as delta t tends to 0 is not just a mechanical way of like evaluating a limit and therefore getting a derivative. when in the limit as delta t tends to 0 we have already hinted that then the system boundaries at the 2 time instants are almost coincident that means in such a case if you call the region II as a control volume this becomes identified. So you may focus control volume is what?

It is an identified region in space over which you may focus your attention and see what is the change that is taking place? So in the limit as delta t tends to 0 the uncertainties in the configuration of the system at the different time instance is not that strong, because the time interval is so small that it is almost like identified at where the things are and let us say that somehow you have identified this region II as your control volume.

So if you identify the region II as your control volume in the limit as delta t tends to 0 your system tends to the control volume that you have to keep in mind. So always not a system is a control volume, but in the limit as delta t tends to 0 again it is not equal to control volume. The system tends to the control volume, because the configurations are merging on the top of the other. The whole idea is that.

Using that limit we will be able to express what happens in a system in terms of what happens across the control volume. So in the limit as delta t tends to 0 system tends to control volume and what is the control volume that we have identified. This region II is the control volume that we have identified. Keeping that in mind, let us try to simplify the expression for dN/dt for the system further.

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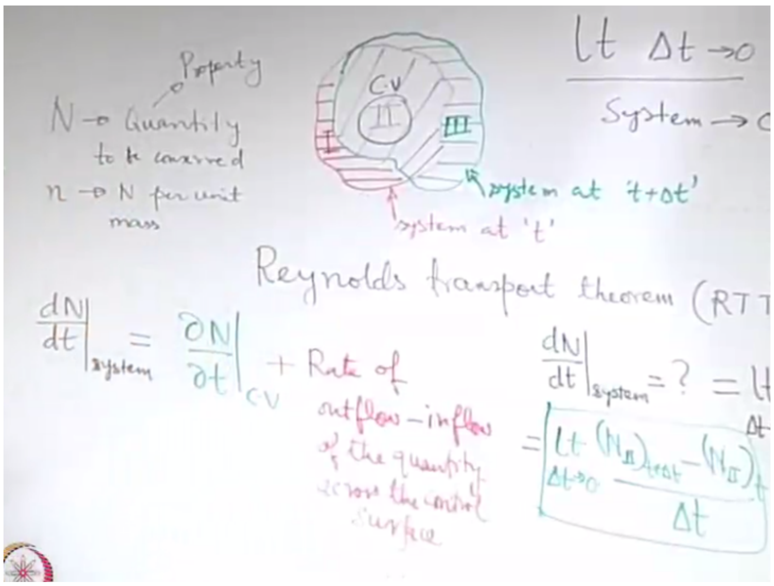

So dN/dt for the system = now let us try to write the different terms mathematically. So let us first concentrate on the first group of terms. So what does it represent? It represents the time rate of change of N in the control volume because II is the control volume and we will write it as the partial derivative of N with respect to time within the control volume.

Why partial derivative with respect to time? N is in general a function of position and time. You have fixed up the position so at a given position which is like II given symbolically by II you are trying to see that what is the rate of change with respect to time? That is why a partial derivative. Now this is not the only term. There are 2 other terms which are remaining and instead of writing those 2 mathematically at this stage let us try to understand physically what they are.

So that physical understanding we will try to write in some sense here. So physically in the limit as delta t tends to 0 what happens then what happens then what does the fluid with which has been located at I at time t the rate of change of that what does it represent? In the limit as delta t tends to 0 then this is a very small region so this represents some fluid which has been located on the surface of the control volume.

So in the limit as t tend to 0 these volumes will shrink to almost like located on the surface of the control volume. So at the surface of the control volume there is some fluid and in the limit that you have delta t tends to 0 then the term corresponding to 1 at t what does it represent? It represents a rate process because you are having division by delta t in the limit as delta t tends to 0. So per unit time, it is representing something.

So per unit time is a rate process. So when the system tends to control volume this volume effect almost tends to a surface effect. So at the surface of the control volume, at the time t it represents that there is some rate of transport or rate of influx of the quantity that you are looking for. So it is like the rate of inflow of the quantity across the surface of the control volume that is called as the control surface, surface of the control volume.

Similarly, the term which is there for the region III in the limit as if delta t tends to 0 this represents a sort of the fluid with a property which is ready to leave the control volume at that instant. So when it is divided by delta t and in the limit as delta t tends to 0, then N III term that sort of represents the rate of out flow of the quantity that you are looking for. So this, when you have the difference between these 2 it is nothing but rate of outflow - inflow of the quantity across the control surface.

So that is the physical meaning of the last 2 terms. So this first thing you have to appreciate that these are not volumetric phenomenon, these are surface phenomenon and these surface phenomena are occurring as a consequence there is some property which is entering the control volume there is some property leaving the control volume and the living one is like with a positive sign and entering one is with the negative sign so it is like the net rate of out - in just in a very qualitative way.

And of course we will try to write it in a more mathematical way in the subsequent steps. Let us try to do that.

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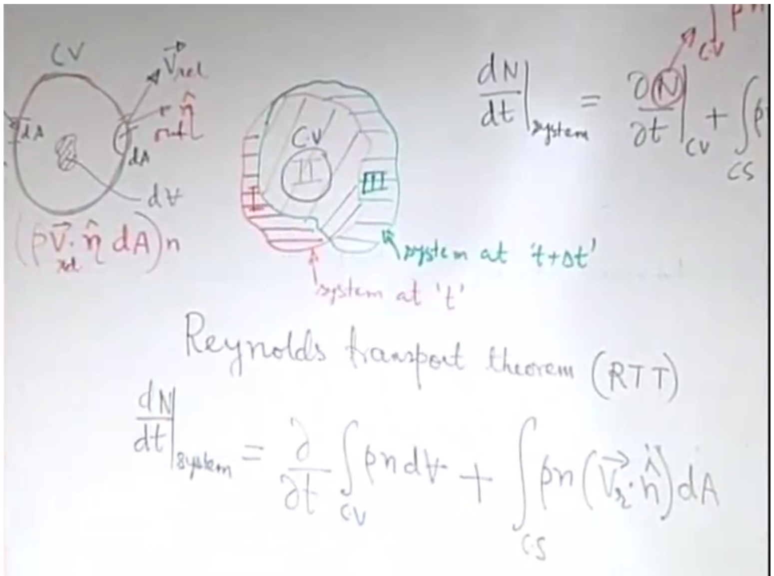

What we will keep in mind is that we have already defined that n is the N per unit mass. Now let us say that there is a boundary located on the control volume. Let us draw the control volume separately. Let us say that this is a control volume and we are interested to develop a mathematical expression for the rate of outflow of the quantity.

So how we do that? So rate of outflow depends on what here we are talking about fluid flow. So if there is a property the property is being transported by the fluid. So if the fluid enters it takes some property with itself. If the fluid leaves it also takes some property with itself. So let us say that you identify a small area on the surface over which the fluid has a velocity V. So the fluid let us say is going out of the control volume over this surface may be it is a part of that surface over which it is leaving.

So it is a part of a outflow boundary. So out of this total boundary of the control volume there may be some part over which there is no inflow outflow may be it is like a wall. There may be some part across which fluid is entering or leaving may be it is like a hole. So this is such a place across which the fluid is leaving as an example. Though the fluid is leaving with a particular velocity and it is arbitrary, some arbitrary velocity vectors the area that is being identified we have seen it many times that area has an important directionality.

So this area has to be identified with a unit vector n so do not confuse these with the other end that we put as a symbol for the property per unit mass. So this is the unit vector or maybe you may call it eta just to avoid the same symbol. Now what is the total rate of property that is coming out of this one. We have to keep in mind that n is the property per unit mass. So if we find out what is the rate of flow of mass over this multiply n with that that will be the rate of transport of n through this flow of mass.

So first of all our objective is therefore to find out what is the rate of flow of mass over this. What is that? So for the flow of mass what is important is, what is the component of the velocity normal to the area so that is given by the dot product of v with n. So v dot n is the velocity component normal to the area that multiplied with dA is the volume flow rate through the area that multiplied by the density rho is the mass flow rate through the area.

And that multiplied by n which is the property per unit mass gives the rate of flow of the property through this area and if you integrate it over the area over which the fluid is flowing out then such an integral this is an area integral that should give the rate of outflow. Now what about inflow? Let us see that is it different from these or is it same as this. Let us a take a different case may be just based on the same control volume where you identify again a small now fluid is entering the control volume.

And let us say that we identify the normal eta in this way and other things hold similarly so what is the rate of inflow of the quantity through this again it goes by the same principal and if you evaluate this algebraically you see a difference between the outflow and the inflow. What is the difference? Difference is here v dot n will be positive. Here v dot n will be negative. That means when I have an outflow - inflow algebraically we may treat it n the same way.

We will just use these term automatically the positive dot product will tell that it is outflow and - will tell as inflow. So we do to have to separately treat the outflow - inflow. It is just like a (())

(35:10) and therefore this total term integral of rho n v dot n dA integral over the control surface. This is the total integral control surface is the surface of the control volume. So wherever there is some velocity, we will put that velocity. The velocity will locally vary along the surface that is why this integral is there.

Whenever v dot n is positive it is outflow whenever v dot n is negative or v.eta rather. Whenever v.eta is positive it is outflow. Whenever it is negative, that is inflow. So we need not have a separate consideration for outflow - inflow. This algebraically takes care of everything. The other point and which I believe is a very important point is this V is not the absolute velocity of flow because the flow depends on what is the velocity of the fluid relative to the control volume.

Let us say the control volume constitutes a tank and what are these coming out of the tank through a hole and let us say the tank is moving with a velocity say 1 meter per second in a certain direction and the fluid velocity is also 1-meter per second in the same direction. So there is no net flow of that what are relative to the tank so there is no mass that comes out. So this V that what we have put here it should not be the velocity of the fluid in an absolute sense but velocity of the fluid relative to the control volume.

So this must be amplified with v relative. So all these v is that we have talked about these are be relative and involves this v relative means velocity of the fluid relative to the control surface. So you have now seen that we may write the expression of this system derivative in terms of the control volume derivative in a bit more compact way than the previous step that is we may write dN/dt for the system is equal to this for the control volume + integral of rho n vr.

We just write in short vr as v relative. vr with eta dA over the control surface. We may simplify this a bit more if we want by considering that what is N? N is like a let us try to simply the first term in the right hand side. So if you have a control volume the control volume has different small chance of volume element. Let us say dV is a small chunk of volume element in the control volume. We are using this symbol for volume to distinguish it from velocity.

So a small volume element, so what is the total property of this volume element? n is the property per unit mass. so if you multiply that with the mass what is there in this volume that will be the total property within this volume and that integrated over the control volume we will give the total N of the control volume so the N of the control. This is the partial derivative with respect to time of N of the control volume. So what is that N of the control volume?

This is integral of so first what is the elemental property that is rho n dv. This is the volume integral you have to remember. So these are symbolic. So like although you are expressing it with just a single integral sign, it depends on what kind of coordinate system you are using and so on. So if in a more mathematical way this should be a double integral, this should be a triple integral like that, but it is a symbolic representation that to keep in mind.

So this is like the total property N within the control volume. So the final expression we may write in this way that is dN/dt of the system we may express very neatly in terms of the rate of change within the control volume. So we can say that dN/dt of the system = the partial derivative with respect to time of integral low n dv of the control volume plus. So what objective we have achieved by this exercise.

We have now been able to express the rate of change of some quantity in a system in terms of the rate of change within the control volume and that 2 are adjusted with each other by a term which physically represents the net rate of flow across the control surface. So this sort of represents a conservation and this is known as Reynolds transport theorem. The good thing about this theorem is its generality.

Till now we have never committed what this is n and because we did not have to commit what is this n we may try to apply it for different cases with n parameterized in different ways physically representing the different principals of conservation. As an example, we will start with the conservation of mass. In the fluid mechanics we will be discussing about 3 conservation principals in this chapter, conservation of mass, conservation of linear momentum, and conservation of angular momentum. So we will first start with the conservation of mass.

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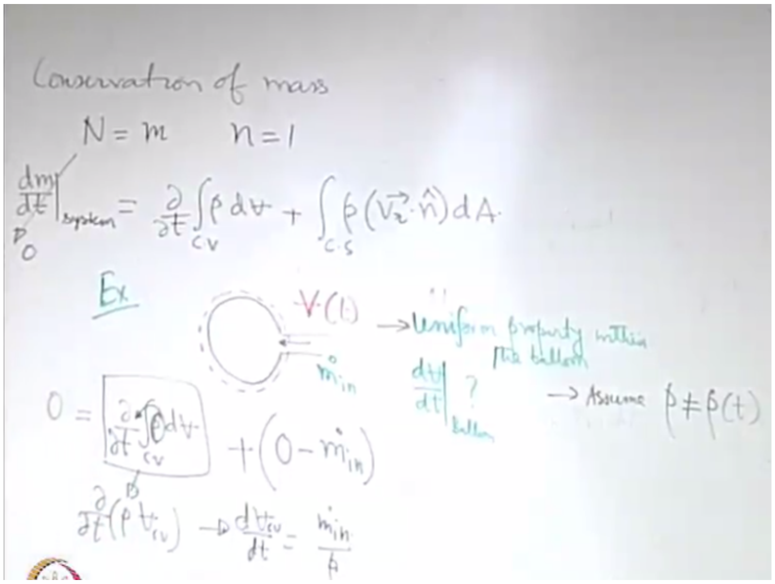

Conservation of mass. When we talk about the conservation of mass what should be N. N is what? What is the property that we are conserving here total mass of the system. So N = m which is the mass of the system. So what is n is N per unit mass that is so let us substitute that in a Reynolds transport theorem and write dm/dt for the system =, now look into the left hand side. Left hand side represents what?

Left hand side represents the time rate of change of mass of a system. What is the definition of the system? A system by definition is of fixed mass and identity. So the mass of the system does not change. mass within the control volume changes because something across the control volume is entering and leaving, but when you are considering the system, it is as if it is a conceptual paradigm that you are considering all the mass which comes into your analysis and that does not change.

That may occur different positions at different time. Sometime that mass is out of the control volume, sometimes it is inside the control volume may be again it is leaving the control volume going somewhere else. You are not keeping track of the mass, but if the conceptual mass which is the mass that you are considering as conserved so that does not change with time. So this is always 0 by the definition of what is a system.

So the integral form of the conservation of mass fundamentally is this one. This is very, very general and it all depends that the special cases depend on what further considerations that we make. See regarding the choice of the control volume we have again remain very, very general. We have never committed that the control volume is stationary or fixed that is why in fact this v relative we have introduced so that even if the control volume is moving, it has no consequence, it can still be applied.

The other thing is we have not committed that the control volume is non-deforming. That means what is the example of a deforming control volume, let us take an example. Let us say that you have a balloon. Initially the balloon is very small, but you are pumping air into the balloon so it is getting inflated. So if you consider the control volume like this, which is encompassing whatever air is there within the volume, then that volume is changing with time.

So this is if you consider air within the balloon as the constituent of the control volume and the balloon is being inflated, then the volume of the control volume is a function of time. So here we have this v as a function of time. This is an example of a deformable control volume. So if you have a deformable control volume that is also not ruled out here, because we have not committed here that v is non deformable.

So in the most general case, v may be moving and v may be deformable. Moving and deformable are 2 different things. When we say it is moving, it need not be deformable. It is like it may be moving like a digit body and what is deformable it might be locally stationary but deforming, and when this moving and deformable it is the most general case that is by move as well as continuously deform.

So all those possibilities are there and let us try to say that in such a simple case how we make an analysis. Let us say that we are interested to find out let us say that we have given, let us consider may be a simple problem related to this. Let us take an example that you have a balloon like this. There is rate of mass flow of air into the balloon which is being supplied by the pressurizing mechanism so some air at a given rate of m dot n is entering the balloon.

The state within the balloon is such that you have a density it is considered that the property within the balloon is uniform. So uniform property within the balloon is an assumption our objective is to find out what is the rate of change of volume of the balloon with respect to time of the balloon. So how we do that? We use this integral form of the conservation. Left hand side becomes = 0. What happens to the right hand side? Look into this term integral of rho dv.

When we are calling that or when we are stating that its uniform property within the balloon that means what? The density is uniform, but it may change with time. We have earlier seen that uniformity does not ensure steadiness. So rho in general could be a function of time, but at a given time rho is uniform everywhere. So if at a given time, rho is uniform everywhere, it may be possible to take the rho out of the volume integral. So the integral is testing the rate of change within the volume.

So if you take rho out of the integral then this term, this is the only the first term this became rho * the volume of the control volume. Because if you take rho out of the integral, the integral of dv becomes the total volume of the control volume and what is the remaining term + so what does it represent? It is the net rate of flow of what. Here the quantity is mass. So it is net weight of outflow - inflow of mass. So outflow is 0.

There is nothing which is coming out and what is coming in so outflow is 0 and inflow is m dot n. So you have the differential equation relating the volume density and m dot n in this way. Now you may simplify further if you assume that the density is not changing with time. Otherwise you have to know that how the density is changing with time and it is not a trivial situation. Density is definitely expected to change with time.

So if you consider that the density is invariant with time, it is not a very practice assumption but at the same time we have to keep in mind that it is not a rigid tank. If it was a rigid tank and if you are supplying the air what will happen, the density will increase with time always, but because it is a flexible balloon, then whatever mass is coming in this balloon is getting adjusted to that. So it might be possible that the density is changing, but changing only slightly.

If it was rigid that assumption would have been a very bad assumption, but it is flexible, may be it has capability enough to adjust to that. Whether it has capability enough to adjust to that it depends on mini parameters. It is actually one of every toughest problem in mechanics because it needs the understanding of what is the elasticity of the balloon material. So based on that how it adjusts to that change in terms of the new density within inside is something that they need to be looked into more carefully.

But if you make an approximation that as if it is not only uniform, but another assumption that rho does not change with time, rho is not a function of time. Then it is possible to find out what is the rate of change of the volume of the control volume with respect to time that is m dot n/rho. So at least we have got a fair idea that it could be a deformable control volume and if it is a deformable control volume it is not a trivial thing to deal with.

The first assumption may not be bad, because if it is pumped well enough it will assume a homogenous distribution of density quite quickly, but this is perhaps not very correct assumption. It may be approximate, but it is not so good. So we will leave apart the deformable control volume and what we will try to do is, we will try to now consider a special case of a non-deformable control volume. So we will make 2 important assumptions, first assumption is.

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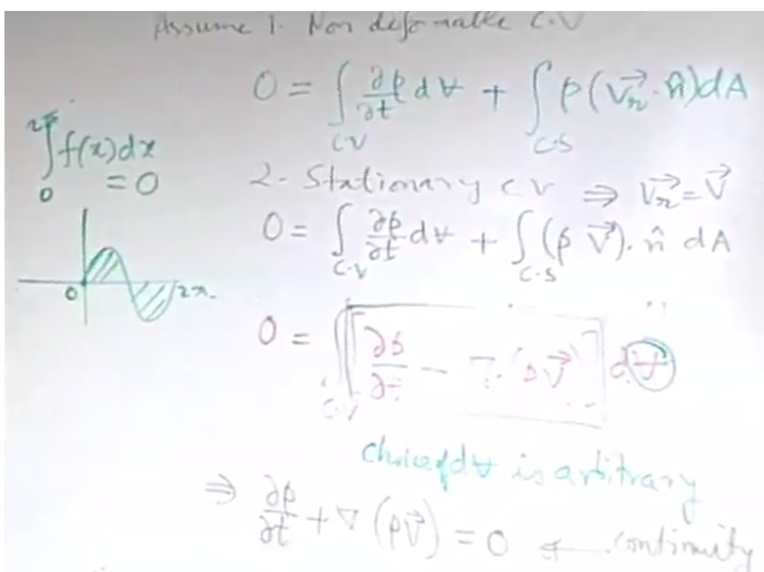

Non-deformable control volume to further simplify that equation. So if you have a non-deformable control volume, then what happens? See now we are looking into a very important aspect of mathematics. You have the derivative outside the integral. Question is can you take easily the derivative inside the integral that is, is it permissible that you take this partial derivative any partial or ordinary whatever it is this derivative within the integral sign, yes or no?

In general no because if this v is a function of this time then in general no, but if this v is not a function of time, you can take this derivative within the integral. If this v is a function of time then you can take that, but with an adjustment of certain terms and that is given by the Leibniz's rule for differentiation under integral sign, but when you are considering a non-deformable control volume, we are easily able to take this the time derivative within the integral.

So then our form becomes 0 =. Next we will make another assumption. The assumption is that the control volume is stationary is fixed stationary control volume. So if you have a stationary control volume, then what is the consequence? Consequence is v relative = v. The relative and absolute velocities are the same. If the control volume is not moving so then the equation becomes + integral of rho v dot n dA.

We will simplify this further by what understanding? by a very important, but straight forward understanding that it is possible to express this area integral in terms of a volume integral by using the divergence theorem. So the divergence theorem what does it state? F dot n with n dA over the control surface = the divergence of f dv. So this volume, the area has to be an area which is completely bounding the control volume.

That is the only important assumption. F is any general vector field. So here what is the f here? Rho * v is the f here. So in this particular example f is rho * v. So the next step is 0 = integral of now you write it both are now volume integrals. So this one + so the next term has also become a volume integral. So you are now left with what term. Integral of something dv = 0. The big question is does it mean that the term in the square bracket which is the integral it has to be 0.

Yes, or no? In a special case may be yes. so let us say that let us think of say a function this volume integral is a bit more general, let us consider a one dimensional case you have integral of fx dx. If integral of fx dx = 0 can you say that fx = 0. In general, obviously it may be in a special case like if you have sin x type of thing. So let us say you have integral from 0 to 2pi. So this area and this area are same and opposite.

So the integral will vanish, but the function fx is not vanishing at all points within the interval, but you have to keep in mind that is not for any arbitrary domain of the integration that is only for a particular limit. Here the choice of the elemental volume is not based on a particular domain. It is absolutely an arbitrary choice of an element within the total volume chunk. So you have to keep in mind that the difference between this case and this case is one very important thing.

Here these dx is really restricted with the limits may be 0 to 2pi, but here these choice of dv is arbitrary. That is a very important thing. Since the choice of the dv is arbitrary, then where does it land up? If that is arbitrary then if this has to be 0 then the integral should definitely be 0 because it is for each and every arbitrary choice of the elemental control volume and that means that you must have the integral = 0. So it is not for a general like interpretation for any case, but it is an interpretation based on the arbitrariness in choosing the control volume.

So with that understanding, now we get this form of the equation. You can recognize that this is the continuity equation that we derived in a different way earlier. so this gives you not only the continuity equation in a differential form as a special exam, it gives you a kind of mathematical skin or understanding of how to convert an integral form into a differential form. So we started with the Reynolds transport theorem integral form and we could show that we can come up with the differential form of the same conservation.

So this is again representing the mass conservation principal, but in a differential form. So it is possible to convert one form to the other keeping the physical meaning intact. So that is one of the important strings of the Reynolds transport theorem that it is possible to derive almost all conservation equations in whatever form you like by starting with the most general integral form of the Reynolds transport theorem. So we will stop here today and we will continue with more examples on this conservation in the next class.