Introduction to Fluid Mechanics and Fluid Engineering
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Lecture - 20
Dynamics of Inviscid Flows (Contd)

In our previous lecture, we were discussing about the venturimeter and we will continue with that as an example.

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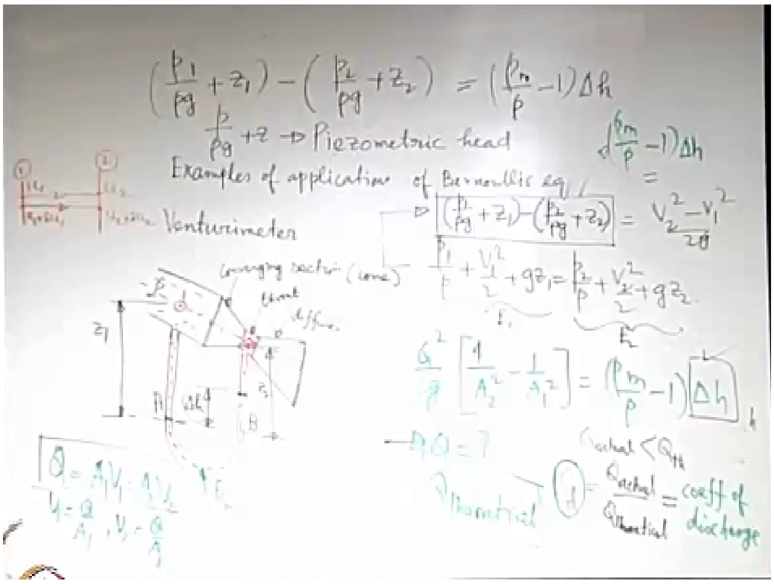

So if you recall what was the purpose of the venturimeter. The purpose was to measure the volume flow rate through a pipe. Now for that we utilize this type of an arrangement where you have a converging section which is sometimes also called as a converging cone. Then you have a throat where the area of cross section of the entire arrangement is a minimum and then a diverging section and the purpose is obvious that this part has to fit with the pipe.

So if you have reduced the cross section area somehow you have to increase it so that it again matches with the pipe. So this black color portion is like a fitting which is fitted with the pipe to measure the flow which is occurring through it. The question is why you have such a converging section we have seen that it gives rise to an accelerated flow so it gives rise to a high velocity and therefore a change in pressure.

And we have seen that it is not just the pressure that is important, but the sum of the pressure and the elevation head together that is p/rho g +z that term together is something which is changing because of the change in the kinetic energy head and we gave it a name called as piezometric head. So the difference in piezometric head is reflected by the reading in a difference in the heights of the liquid columns in the 2 limbs and that delta h therefore will be an indicator of the rate of flow which we found out by utilizing the Bernoulli equation.

The question is that is it a very reliable way of finding out the flow rate. The answer is straight forward in some sense that it is not that reliable. The reason is that we have utilized an ideal type of equation. All the assumptions which are there inbuilt with the Bernoulli equation in a steady form are inbuilt with these and therefore all the idealities which are also inbuilt with these form of equations those are assumed to hold/

At the same time, we understand that in practice such idealities do not hold true. What is the biggest deviation in practice? it is never a frictionless flow. So you have viscous effects and because of the viscous effects something happens or quite a few things happen. Now one of the important things is because of the viscous effects if you have this as the total head or expressed in terms of different unit's total energy it is a representative of the total energy at section 1.

And this is the representative of the total energy when we say energy here we only mean the mechanical form of energy at section 2. We assume these 2 are equal because there is no loss because it is a frictionless flow. In practice, there is a loss so you expect that if you call this as E1 and if you call this as E2 which one is more E1 or E2 is more. **"Professor - student conversation starts"** E1 is more. **"Professor - student conversation ends**"

E1 is more because you expect that there may be a loss of energy and the loss of energy will be because of the travel of the fluid from the initial point to the final point. So when it is travelling from 1 to 2 it will have a loss. So that means E1 will be more so that when you come to the section 2 it is E2+ some losses the losses which have taken place between 1 and 2.

In one of our later chapters we will try to characterize these losses in a more formal way, but we will just keep in mind that there are certain losses because of viscous effects and these losses will give you a guideline of like what is the direction of flow. Say you know nothing you are given some E1 you are given some E2. If E2 is > E1 you must be assured that flow is taking place from 2 to 1 not from 1 to 2.

So it is basically taking place from a high head to a low head. It cannot be the other way because where will the head come here. Now here because of this loss what will happen? See this eventually will bowl down to a larger drop in the piezometric head. So if it is flowing from 1 to 2 the piezometric head which is coming into the picture the piezometric head drop that is the difference between the 2 terms present there in a bracket this you expect to be more.

Because you expect a more severe drop in pressure because of overcoming that frictional resistance effects. So that means whatever delta h you read here is not ideal delta h. See in this formula what delta h you put to get the ideal Q. This delta h is what you experimentally observe. We have to keep one thing in mind what is the basic principle of measurements that we use in experiments that we have an expression in which we have certain measurable quantity, certain easily measureable quantities.

And we express some more difficult to measure quantities in terms of the easily measureable quantities. So here delta h is something which you measure easily flow rate you do not measure directly, but use this formula to write flow rate expressed flow rate in terms of the measured delta h. So here maybe to get Q ideal it would have been better if you could put delta h ideal, but that you cannot do because delta h is what you are reading from the practical thing.

So it is giving the delta h which has got manifested because of by considering all practicalities. So this delta h also considers the practicality that there is some loss of energy to overcome the fluid friction effects that means this delta h is higher than what delta h would have been if it were a frictionless flow, but you cannot help this is what you read experimentally and it is what you put here.

That means even in terms of theoretical flow rate it is not giving the correct theoretical flow rate it is over estimating that because you are putting an over estimated delta h the reason is straight forward because experimentally you cannot reveal an idealistic picture experimentally you reveal the real picture where the delta h is much more severe than delta h you could perhaps get in a friction less condition.

So one important thing we realize that if you consider no other effect this particular effect alone should give you that Q actual should be < Q theoretical which you calculate by using this formula, but there are other important reasons also. What is the other important reason? See when you have written v 1 square/2 actually we were bothered about the velocity at the point 1, but the velocity of the point 1 how did we evaluate?

We evaluate by using this Q= A1 V 1=A2 V2. So by that we implicitly presume that V1 and V2 are like same as the average velocities over the section that is possible only when the velocity profiles are uniform over the sections 1 and 2, but because of the viscous effects we have seen that those are not uniform. So there is a non ideality because of viscous effect not only in terms of the frictional resistance, but also in terms of putting V square/2 term.

So there is also some error in that. So these 2 errors are very, very significant one error is a frictional resistance another error is the miscalculating the velocity expression or misinterpreting the velocity expression. So when you put the velocity here ideally you should have put a velocity here in such a way that these would have represented the kinetic energy across the section 1.

Again in our later chapter we will see that how to exactly put that, but here we will just appreciate that we have not put it correctly. So whenever we make a mistake in writing something the first and foremost thing is to appreciate that we have made a mistake. So let us appreciate that this is not correct. There is some error in it. Now incidentally engineers are such classes of people who are happy to get the final result disregarding may be some mistakes which have already been done.

And then to adjust that mistake let us say that some adjustment factor is put let us say we call a new coefficient Cd as Q actual/Q theoretical. So if somehow this coefficient is known to us whatever by magic. We will see what magic will tell us this number, but if somehow you get this value of Cd then you straight away multiply that with the Q theoretical that you get from here to get what is the Q actual.

If it is very close to the final result that you want in many practical engineering applications people are happy. So we have to see that what is there in a Cd which will try to make us more and more happy and this coefficient is known as coefficient of discharge. So this takes into account that we have realized that if it is an ideal case altogether this would have been=1. So deviation of this from 1 represents the extent of non ideality in a flow and not only non ideality in the flow in general.

But more specifically how that non ideality has got manifested in the prediction of flow rate. So that means what is the total influence of this friction in terms of delta h and what is the influence of the inaccuracy in a velocity distribution that has already got inbuilt in the corresponding expression for energy. To look into that, we will not go into all sorts of details, but we will just consider one thing that see at the section 1 say we have sort of uniform velocity profile somehow we have maintained.

We will later on see that it is not easy to maintain that, but fortunately we will be easily maintaining such a situation when the flow is turbulent and when we will be discussing about turbulent flows we will see that turbulence is a kind of situation which will create almost a uniform velocity profile over the section so that will in some way take care of some of our acts of ignorance in writing or describing the correct velocity profile.

But even then let us try to see that at which section the error will be more severe at section 1 or section 2. To do that let us say that we consider 2 streamlines which are very close to each other. Say you have a stream line like this and another stream line which is very close to each other both streamlines are connecting the sections contained by 1 and 2. So here 1 and 2 are points, but let us say that these are sections which contain the points 1 and 2.

So we have 1 streamline and we have another streamline. Now these 2 streamlines are very close so close that let us say that here the velocity is u 1 here the velocity is u2, here the velocity u1+delta u 1 and here it is u 2+ delta u 2 where delta is a small change in comparison to the other value. Now if these streamlines are very close to each other what will happen there will be negligible difference in pressure between these 2 streamlines.

Always remember that there is a difference in pressure between the streamlines because of the curvature effects of the streamline, if they are very close that effect is negligible. So if these 2 are very close there is negligible difference in the pressure head between these 2 streamlines at 1 and at 2 and if they are very close there is negligible difference in the height also that is z coordinate.

So what we can say is that the difference in kinetic energy heads between the 2 points it remains same for the streamline above and the streamline below because the other terms they do not change. So how we may reflect that in our analysis let us try to do that.

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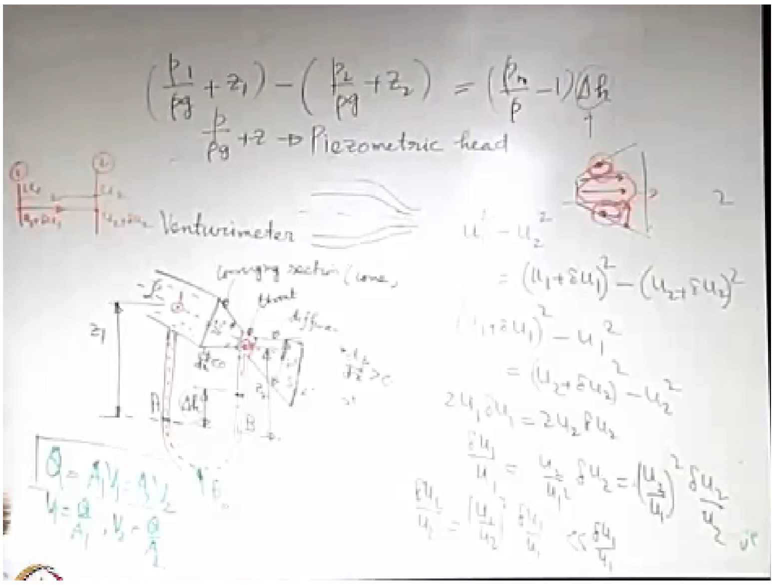

So what we intend to write is u 1 square-u 2 square= U1+ delta u1 square- u12+ delta u 2 square. So let us try to simplify it keeping in mind that delta u 1 and delta u 2 are much smaller as compared to u 1 and u 2 respectively. So you are clear that why such an equation has come because other terms like p/ rho g and z those are the same so they have cancelled out when you consider the Bernoulli equation for the streamline above and streamline below.

And when you have subtracted that effect has gone so these terms remain. Basically u square/ 2 and all those things, but that division by 2 gets cancelled out that is how these terms come up. So if you now write it like this. So what you have here. So you can write this as u1+ delta u 1 square that you can bring in one side u 1+ delta u 1 square- u1 square= u2+delta u 2 square-u 2 square.

So you can write it as 2 u1* delta u1. So when you write this is like A square- B2 square formula when you write A+B it is 2 u1+delta u 1. Delta u 1 is much, much smaller as compared to 2 u1 so only 2 u 1 and the difference only delta u 1 is there so that is= 2 u 2 delta u 2.

So we are interested to express delta u 1/u 1 in terms of delta u 2/ u 2 to see that what is the relative error in relative change between the velocities into adjacent streamlines. So you will have delta u 1/ u 1 as if we are dividing both sides by u 1 square. So you write as u 2/ u 1 square delta u 2. So here you can write this as u2/u1 whole square * delta u 2/ u2. That means delta u 2/u 2= u 1/u2 whole square * delta u 1/ u1.

Now which velocity you expect to be more u 1 or u 2. Look at the sections 1 and 2 here the area is large so the velocity will be less. So u 1/u2 more is this reduction in area u1/u2 will be lesser and lesser square of that will be small. So from this our conclusion is that this delta u2/u2 is expected to be much, much < delta u1/u1 provided there is a great reduction in section.

That means if it is approximately uniform at section 1 it will be even better at section 2 because the non-uniformity is much less. What this represents a non-uniformity when you go to a different streamline along the same section you expect the velocity to be different and that difference give rise to a non-uniformity. So when you have this non-uniformity, but again see this is an estimation because for estimating the non-uniformity we again utilize the ideal equation which is like the Bernoulli equation.

But we have considered that even for a non ideal case this is not very, very invalid because whatever is the frictional effect that also has got cancelled out when you have subtracted the 2 equations. Assuming the frictional effects are also same as the fluid flows from 1 to 2 along the 2 streamlines above and below. So even if frictional effects are considered and these Bernoulli equations for the 2 or the modified Bernoulli equations considering the frictional effects they are cancelled or they are subtracted one from the other that effect will be cancelled.

So this is not a bad estimation. So this estimation shows that if the velocity is such that you are going towards cross section of reduced size if at the bigger cross section of velocity was more or less uniform the smaller cross section it is expected to be even more uniform. The reason is quite clear that if there were streamlines like this streamlines will more converge to each other because they are now confined to be there within a very small space as compared to how they were earlier.

So if the streamlines were quite a large distance apart so if the streamlines were like this now when their streamlines are confined so what will happen all the streamline will try to converge. So when the streamlines try to converge you see that distance between the streamlines corresponding streamlines become smaller and smaller and eventually different streamline represent the sort of like different states of flow.

So if you have them quite close to each other and almost parallel to each other that non-uniformity in the velocity is almost like it is not totally nullified, but it becomes a better situation. So by having a section 2 like this which is like a convergence section it is not bad. It sort of eliminates one non ideality. The other non ideality because of negligible friction that may be reduced to some extent by minimizing the length travelled between 1 and 2.

Because the frictional resistance will be related to how much length the fluid has travelled against the viscous effect. So how do you reduce that? One of the ways is like you have this angle this cone angle quite large so that it converges quickly to a small section. In practice this angle is like typically kept as like 20 degree or so. These are like design consideration of this device.

It is not that 20 degrees is a magic number and it is always kept like that I am just giving you a rough idea of what is the range in which it is kept in practice. Now there are different issues like you cannot make it as large angle as you like there are issues of manufacturing the device and so on. So it is not that whatever angle you want and you propose one has to also fabricate it and put it in practice.

One particular aspect on which one may not make a comprise is if this is like by putting by locating this section 1 where you are having this manometer limb. It should be preferably somewhat away from the place where the deduction has started so that this disturbance is not influencing the velocity at this point significantly and that is why it is kept little bit away from this one.

And roughly it maybe if the diameter of the pipe it is roughly like distance d away I mean it is again a rough estimate there are more accurate estimate for each device. So the connection of the pressure tapings are also very important that is where they to be put. So one is here then this is roughly like 20 degrees and this creates a good accelerated flow if you achieve it in a very small or a short distance it is good you have less frictional resistance.

And smaller the cross section you expect that more will be your resolution in terms of these delta h. So the experimental objective is the delta h is more it is better because that is your reading if it is very small your error in resolution will be affecting your result significantly, but if the readability of this is good then the error corresponding error is less and that is why you are trying desperately to reduce the cross section area so that there is a change in the kinetic energy head very severely which is manifested in terms of these delta h.

So after this section has come and then what you have to do then you have to revert back to the pipe diameter again. So you have a diffuser which is like a diverging section. So you have a converging cone you have the throat where you have the minimum area and then you again have a diffuser. The question is what should be the angle of this diffuser I mean do not get confused with the sketch that I have drawn in the board it is just because of lack of space that I have drawn it not to scale.

So this angle does not represent what is there in reality. It just represents the shape, but not really the sense of the angle. So what should be this angle? Now again there are 2 conflicting requirements. In engineering is such an area where when you want to design something there are 2 aspects that you have to keep in mind. One is it should satisfy the fundamental scientific requirements so that the device is based on a thorough scientific principle.

The other important thing is that it must optimally satisfy the performance requirements. So what are the corresponding influencing parameters always you will see that there will be opposing parameters. So opposing parameter means if you increase this angle then something good will happen and something bad will also happen. So let us see that if we increase this angle of that diverging section.

First let us see that what good thing will happen that is very obvious. So if you increase that angle of the diverging section what good thing will happen. If you increase the angle of this what is the good effect of that? That portion will decrease in length. So in a relatively short length this device will merge with the pipe. So the loss due to frictional resistances will be less.

So just like what would have been a good effect of making this angle large the same logic holds there also, but one of the logics that does not hold is that there is a great difference between an accelerating and decelerating section. This is an accelerating section, but this is a decelerating section. Why this is a decelerating section? So if you see the area of cross section is increasing.

So you expect the kinetic energy head to reduce that means if you expect the kinetic energy head to reduce that means p/ rho g+z the piezometric head will increase to compensate for that. So if you say let us say that you are having a horizontal venturimeter so if you have a horizontal venturimeter z1 and z2 are the same or maybe 2 and 3 here you consider another point z 2 and z 3 are the same.

So then if you go from a point 2 say to a point 3 you expect what? If these 2 are located at the same height, then what do you expect that pressure will increase or decrease pressure will increase. So when pressure increases that means so if you consider the direction of the flow as x. So you can write the dp dx as the rate of change of pressure with respect to x in the converging section dp dx was what < 0.

But here in this particular section dp dx will be > 0 because pressure is increasing with x what is the consequence? The consequence is see you expect that if the pressure is decreasing with x that is fine because then a higher pressure is creating a drive for you that is if pressure is decreasing with x that means $p_1 > p_2$ and in some way it is trying to create a driving force for you.

On the other hand, here from 2 to 3 the pressure is opposing you because as you are moving from 2 to 3 you are experiencing higher and higher pressure. So that means it is a sort of effect that tries to inhibit the motion of the flow. So that is why this type of pressure gradient is called as adverse pressure gradient. So this type of pressure gradient is called as a favorable pressure gradient.

So favorable and adverse the English names are quite clear. Favorable means which favors the flow adverse means which is not good for the flow. So when you have an adverse pressure gradient which is like this dp $dx > 0$ what happens the flow has a tendency to be decelerated because of that kind of a pressure gradient. So if you try to sketch that what happens to the streamlines in such a case.

So the flow tries to move like this, but because of the deceleration effects and the deceleration effects are more severe close to the wall. Why because viscous effects propagate from the wall. So at those locations what will happen is the flow may not be capable enough of being racked main or the core flow because it is slowed down so severely that it just creates a locale rotation, but it does not contribute to the main flow.

So that type of thing is called as a flow separation. That means you have a main flow like this. Now the flow the fluid particles this poor guy is close to the wall they are so severely disturbed because of the adverse pressure gradient which are acting on them that they really cannot maintain the flow and they might even reverse their direction of flow. So local vertices are created close to the wall. How do these vertices contribute?

They contribute in a sort of negative way. See these vertices by virtue of the rotation have some energy, but that energy is not contributed to the main flow the main flow is like this which is moving. Now here this energy which is there because of the rotation of this vertices because of flow separation that does not contribute. So effectively as if some energy is taken away from the main flow to sustain the rotation of these vertices.

So effectively there is a kind of loss of energy of the main flow and that loss has been created because of this flow separation and this flow separation effect is stronger more is this angle of diffuser. The reason is more is this more severe will be adverse pressure gradient because more severe will be the pressure increase over a given length the length become smaller. So this is a conflict with the requirement of the frictional resistance. So we have seen that if we increase the length of this one or may be reduce this angle.

Then this effect will be less so the adverse pressure gradient effect will be less if you make this angle quite small so that this length is large, but if this length is large the direct frictional resistance will be more. So these 2 are conflicting parameters in the design that is where you have to come to an optimal design where you cannot keep this angle maybe as large as this.

And the common optimization is that this is typically like 5 degrees, 6 degrees like that much less than the angle of the converging section. So this is something we have to understand very carefully that why in the practical design the diffuser angle is much, much less than the converging section angle. When you have the converging section you do not have such a case of flow separation so only the frictional resistance is because of the length is the only important resistance because flow separation will be there when the flow is decelerated.

But in the converging section the flow is accelerated. So it does not suffer from a resistance because of adverse pressure gradient in fact the pressure gradient here is favorable which makes it move in a much more convenient manner. So the design aspects are quite clear that why you should have different angles for converging and diverging sections and what are the parameter we should decide the range of these angles.

And keeping these things in mind if one design these device quite well by minimizing the losses then the coefficient of discharge which is the ratio between the actual flow rate and the ideal flow rate it is actually very close to 1 0.98, 0.97 like that. So somehow if the device is very cleverly designed some of the non idealities are taken care of in some way not that it becomes ideal, but our ignorance about non ideality does not get manifested so much.

The reason is that one is we are using a continuously converging section in this way and the diffusing section is also say properly well designed. Now this venturimeter is therefore a very common device which may be used in a pipeline to measure the flow rate at the same time this is not a very inexpensive device. It is not very highly expensive, but at the same time for very routine applications one might look for some cheaper devices which are broadly following the similar principles.

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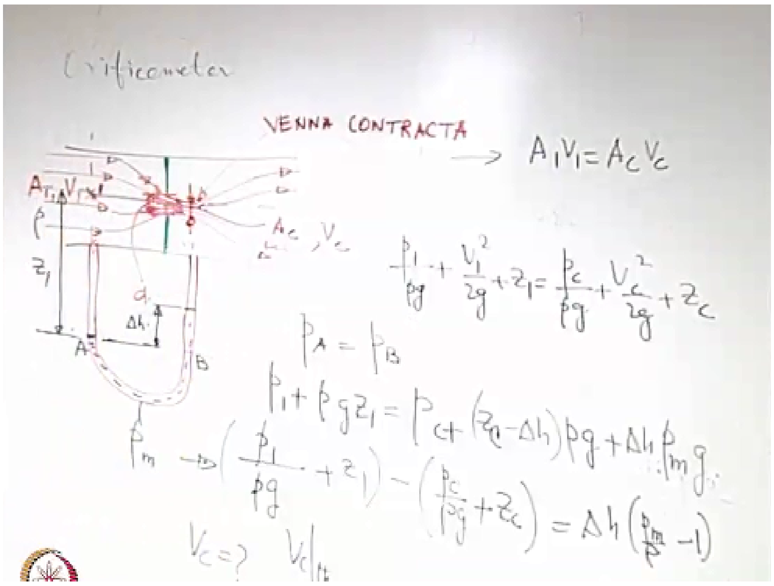

And let us see one such device. So that device we call it as orifice meter. So orifice meter is another application of the Bernoulli equation. So orifice meter is something like and the purpose is the same that you have a pipeline you want to measure the flow rate through the pipeline. So what you are doing here you are putting an obstruction in the form of a plate. So this is like a circular plate if it is a circulate pipe it maybe a circular plate with a hole at the center which is called as orifice.

So here also what happens here if you consider the streamlines the streamlines were originally say parallel to each other not that they always have to be, but just as an example. Now you know that because of this constriction in the streamlines have to be forced to flow through this small section. So streamlines will converge like this and then when the stream lines pass through this.

So there is also streamline at the center so when the streamlines pass through this constriction after that what happens that is very interesting. So after this streamline pass not that they become parallel because of the inertial effects the streamlines go on tending to converge. So there it is not that after coming out of this they become parallel. So they go on converging till the streamlines come to a condition where the distance between the extreme streamlines is a minimum

And then the streamlines tend to diverge again from that and the divergence is again to match with the pipe contour so that type of streamline behavior is there. Qualitatively, it is important to first appreciate this because from this you will get an apparent similarity with the venturmieter. What is that? In the venturimeter you try to have a deduction in the area available for flow.

Here also you are having the same thing, but what is a difference. Difference is in the venturimeter you had a gradual transition from a bigger area to a smaller area and here you are trying to have a more abrupt transition. An abrupt transition is something which is not so good because the flow does not get enough opportunity to be adjusted to that abrupt change and that may create additional losses not only that there are more uncertainty in the measurement.

Why there are more uncertainly in the measurement let us try to see. Again our policy will be that we will try to measure the pressure difference or to be more fundamental the piezometric head difference between 2 points. What are the points that we should choose see? when we are choosing a particular point, we are making a tapping in the wall of the pipe. So as if we are making a hole and fitting a manometer that is the arrangement.

The arrangement does not change here. The philosophy also does not change, but implementation becomes more difficult why. See here you have taken it at a distance substantial enough from here so that this effect in the curvature of the streamline is not important. You are interested about the pressure at this point actually not actually you are interested about the pressure at a point which is at the central line, but at the same section.

There may be a difference in this pressure if the streamlines are curved, but if the streamlines are parallel that will be not. So the pressure here and the pressure here will obviously mean almost the same, effect of streamline curvature will not change anything. Here also if you want to utilize the same principle you should come to a location where there is negligible streamline curvature.

And that is there only at the place when this has come to a minimum. So if you consider a curve which has come to a minimum that tangent is parallel to the axis. So at this location where the distance between the consecutive streamlines or the extreme streamline they are minimum here almost streamlines are parallel to each other. So there is negligible error because of neglecting the curvature of the streamlines at that location.

And this location where the distance between extreme streamlines is a minimum is known as a Vena contracta so that is the name Vena contracta and that is located somewhat away from the orifice. It is not exactly located at the orifice. So if you connect this limb of the manometer at the position of the Vena contracta then your analysis is quite good. Question is how will you know where the Vena contracta is located.

One has to do a lot of experiment to figure it out and it depends on the flow condition. So it is not like a universal location where it will always be located. So it is not that trivial to put the manometer location correctly that is one of the big errors because we are assuming that the manometer limb has been put at the section of the Vena contracta and we are writing our equation accordingly, but actually it may not be.

But let us say that this is put in the section of the Vena contracta. Let us say that area of cross section of this is Ac and the velocity of flow through this section entire section is uniform and is=Vc. Again we are assuming uniform velocity profiles which is a deviation from the reality and with such a kind of abrupt change the deviation from the reality is more severe.

Now here also let us say we consider this as section 1 or maybe a point 1 on the section 1, but it is a uniform velocity profile we consider v1 to be same throughout the section. Let us consider A1 as the area of cross section which is basically if capital D is the diameter of the pipe then A1 is pi capital D square/4. Let us say that small d is the diameter of the orifice and let us utilize the subscript O to indicate the orifice.

So let us say Ao is the area of cross section of the orifice which is pi small d square/4 where small d is the diameter of this orifice. And let us say that Vo is the velocity through the orifice again we consider it is a uniform otherwise there is no meaning of the term velocity through the orifice it will vary across the section. So if you write the Bernoulli equation between say 2 points.

Let us say we have a point 1 and a point c. Point c is located at the same streamline as that of 1, but in the Vena contracta section. So we are writing the Bernoulli equation between points 1 and c along the streamline which is identified by these black color. What is the equation p1/ rho g + v1 square/2 g+ z1= pc/ rho g+ Vc square/ 2 g+ z2. So again the question comes that how will you find out the difference between p1/rho g+z1 and pc/ rho g+ zc that is by using the manometer principle.

So let us say that you have the depth of the limb as marked in the figure and let us say that delta h is the difference in the reading of the 2 limbs of the manometer. So using the principle of manometry you can write that if you have A and B as these 2 points. You have pressure at A= pressure at B. So pressure at A is nothing, but pressure at 1+ if rho is the density of the fluid that is flowing through the pipe+ rho g and let us say that this is the (()) (40:31) also with respect to which we measure the height.

So rho g z1= pressure at B= pressure at C+ z1-delta h rho g+ delta h * rho of the manometric fluid *g where rho m is the density of the manometric fluid. This is like the same equation what we had for the venturmieter there is no difference. So from here you will be getting a difference between p1/ rho g+z1-pc/rho g+ this is zc+ Zc. What is that? That is= delta h * rho m/rho-1 * g.

So g is already there so only this. So that you can substitute in this expression and you can write A1 V1=AC VC. So you can eliminate V1 by expressing it in terms of Vc. So from this expression what you will eventually get you will get Vc by combining this manometric equation and A1 V1= AC VC, but when you get this Vc let us call it say VC theoretical because again we have used the theoretical equation.

This assumes that you know the area of cross section of the Vena contracta which you actually do not know.

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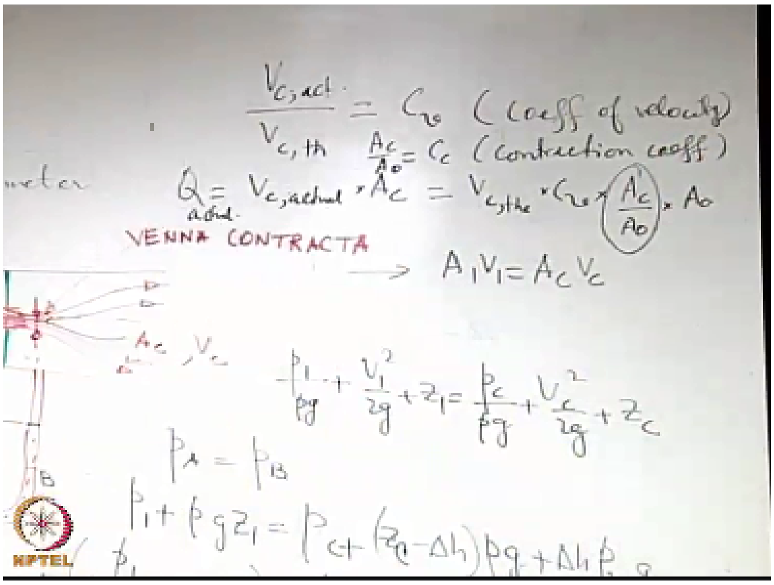

Now the actual velocity Vc actual/Vc theoretical this is not= to 1 because of certain non idealities which have not been considered in this equation just like the volume flow rates are not also same the velocity calculated are the actual and theoretical consideration they are not going to be the same. So this is again considered to be a coefficient Cv this is called as coefficient of velocity.

Coefficient of velocity you have to keep in mind it is also a coefficient of ignorance used by the engineers because actually we do not know what is the velocity we can only estimate from our reading there are some kind of theoretical velocity, but there is a difference between these 2 and because of losses the actual will be less than theoretical so this will be < 1. Now if you want to find out the flow rate Q.

Q is the actual Q it is Vc actual * the area of cross section of the Vena contracta this is Q actual. Vc actual you can express it in terms of Vc theoretical. So Vc theoretical * Cv. Now see area of cross section of the Vena contracta you cannot really measure when you are doing experiments. What area of cross section you know with more confidence you know area of orifice because that is like it is usually given the geometrical construction and everything the manufactures knows exactly what it is.

So you can change the basis from Ac by writing this as Ac/ area of cross section of the orifice * area of cross section of the orifice. By changing the basis from Ac to Ao. So this is again another coefficient which is a coefficient of ignorance. We do not know, but we expect that the manufacturer has done a lot of experiment to figure it out and this is again not a constant it depends on many things that what is the ratio of the big diameter to the small diameter.

What is the velocity of flow? So it depends on many things, but if the manufacturer has done lot of experiments and has calibrated the device again something more standard then the manufacturer can give a data on that. So this we call A_c/A_0 as another coefficient. So A_c/A_0 this we call as CC which is called as contraction coefficient.

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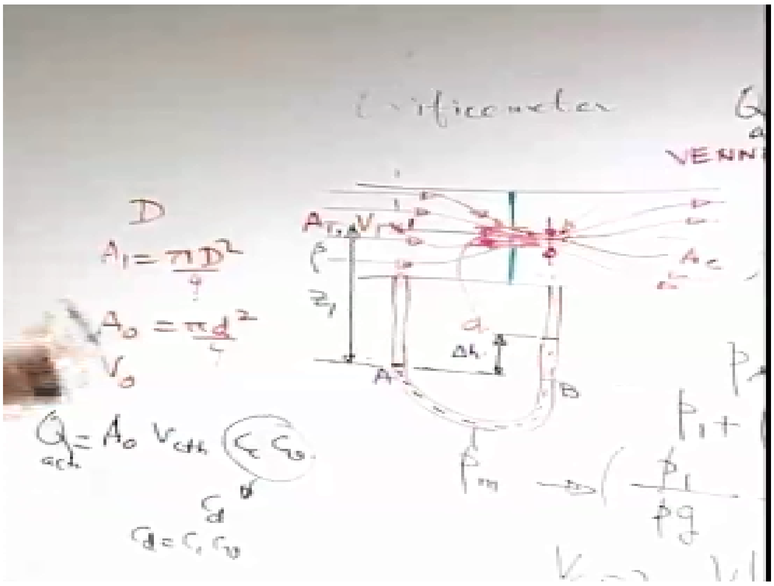

So we can say that the final expression is you have $Q = A_0 * V_c$ theoretical both of which you have determined A_0 you know area of the orifice. You know the V_c theoretical from this simple analysis multiplied by CC * C_v. And this we call as C_d here coefficient of discharge. So this is the sort of ideal flow rate, but the thing is this be different from the previous case because in this case the areas and velocities are referred to 2 different sections.

Area is referred to orifice but velocity is referred to Vena contracta so that is a basic different, but otherwise notionally it is like a sort of ideal velocity and this is an actual velocity. So by the definition of the coefficient of discharge it is like Q actual/Q ideal. So you can say that C_d= CC* C_v because these 2 combined non idealities are there in the calculation the C_d is much less than what you get in a venturimeter.

So here the C_d in such a device maybe say.0.7 .6 .5, 0.7 like that it is not as close to 1 as that for venturimeter that makes it a more inaccurate device than the venturimeter, but the advantage is that is much cheaper than the venturimeter. You just have to put a plate with a hole in a pipe and put the manometer tapping properly. Classically if capital D is the diameter of the pipe this manometer tapping is kept at the distance of roughly say capital D from the plate and this is roughly like capital D/2

This is one of the standard engineering practices of putting these tapping. It is not necessary always that one has to put that, but with a lot of experiment that has been found at than these 2 represent the proper sections with the kind of considerations that we are looking for. So these type of device is known as orifice meter and this plate with the hole is called as orifice plate.

The whole objective is to reduce the cross section area so that the velocity is increased and the piezometric head is reduced and the reduction in piezometric head is measured through a manometer. So same principle as that of a venturimeter, but much less accurate one. In reality there is some device which is in between that is called as a flow nozzle.

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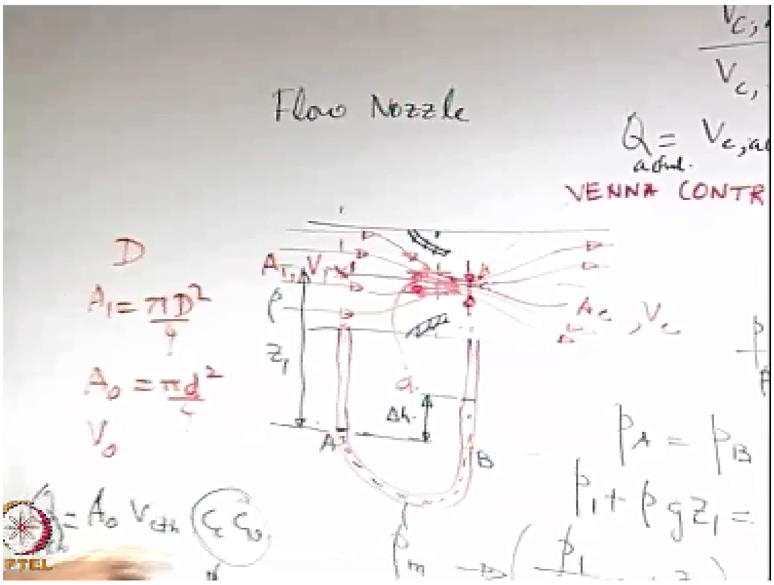

So what is a flow nozzle. We will not go into the detail construction of a flow nozzle. We will just try to go through the philosophy because it is something in between the venturimeter and orifice meter. It is the cost is in between the accuracy is also in between. So what it does is instead of putting a sharp orifice with an abrupt change puts a kind of nozzle at the wall. To have a more gradual change of the cross section of the area.

It does not make it as good as the venturimeter, but sort of compromise between the venturimeter and the orifice meter. So that is the flow nozzle its performance is also a compromise. So with this kind of flow through the orifice let us consider a very simple example to illustrate it that in what other conditions these types of concepts of Vena contracta also come into the picture.

And one such example is something which you have encountered many times that if you have a tank and if you have a hole through the tank there is a water jet that goes out.

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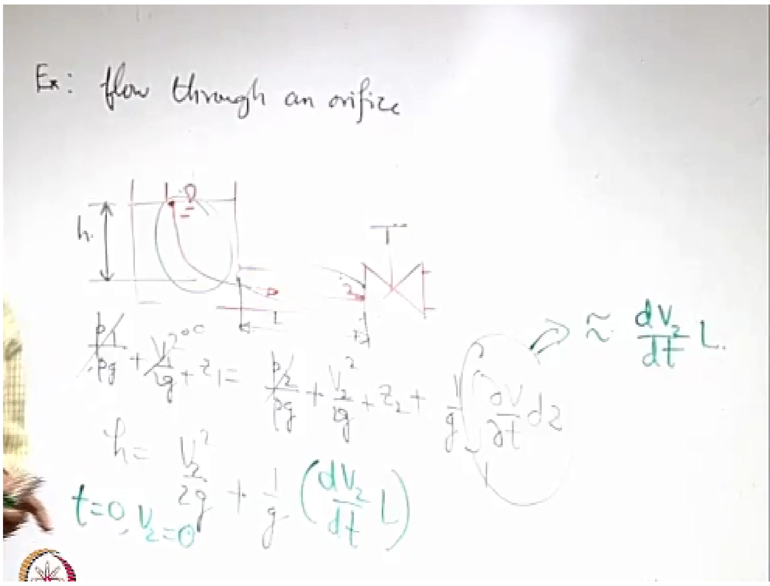

So example of flow through an orifice. Let us say you have a tank like this and there is a hole through this hole some water jet comes out. Let us try to draw a streamline. Say let us say you are coming from a free surface. The stream line gets bent or curve to accommodate this one. All the stream lines which are there they are getting bent or curved and just like the previous case the stream lines or convert to a location of minimum distance of separation between these 2 before they divert and then maybe the water is falling like this.

So the location where the extreme streamlines come to a minimum distance of separation is somewhere is a Vena contracta here, but not at the orifice. So this is the place what we are looking for and let us say we identify a stream line from going like this and we want to apply a Bernoulli equation from in between the points 1 and 2. Along the streamline assuming it to be ideal and which let us make certain approximations.

So that it matches with the high school thing that you have learned. So what are the approximations we will make we will make that we will assume that is a steady flow that is number one. Number 2 we will assume that it frictionless flow and then we will also assume that there is area of the thickness of the orifice is such that it is much less than the area of cross section of the main tank.

So if you have that then you neglect v 1 as compared to v2. So if you write say p1/rho g + v1 square/2 g+z1= p2/ rho g+v2 square/2 g+ z2. So once you have that what happens see 1 and 2 we are assuming both are at atmospheric pressure. So you cancel the 2 pressures v1 you neglect as compare to v2 and z1-z2 let us say that is=h which is a function of time maybe, but at a particular instant therefore you can write v2= root 2 gh a very famous formula known as Torricelli's formula because Torricelli' has derived this.

This you know from high school physics. Question is other than the approximations that we make one very important thing deviation that we have made from the high school physics. We have not considered the area 2 to be at the tank orifice why because we have considered the pressure at 2 to be p atmospheric. If there is a serious stream line curvature, then there is no guarantee that throughout 2 pressure is p atmospheric.

Because of the stream line curvature there will be a difference in pressure as you go across it. Only where it is a Vena contracta that is true because stream lines are parallel. So there is no normal gradient of pressure across the streamline. So whenever you have called it that same as p atmosphere you have to take this section 2 at Vena contracta. So that means this is not the velocity at the exit.

So if you want to find out the flow rate if you write the flow rate it should be A0 into the V0 where O is 0 or O is the orifice, but this is actually the velocity at the Vena contracta. So you must compensate for this. You can write this also in terms of the coefficient CC, Cv like that. So your v0 is not same as root 2 gh and you can therefore write this q in terms of the coefficient sum coefficient c times root 2 gh where this coefficient c takes into account this deviation that it is not actually at this section that you are considering.

But at a section which is located at the Vena contracta. So that you have to keep in mind so that is how. So this is like a coefficient of velocity times the area of cross section times root 2 gh. Where this is like a coefficient that takes care of that non ideality. Now finally we will come into one example where we show the use of unsteady Bernoulli equation for a practical device.

So let us consider that we have similar arrangement like a tank with a pipeline. In this pipeline there is a valve so you have a valve this valve when it is fully closed it does not allow the water be discharged through this pipeline Now suddenly this valve is made open and water is allowed to flow. So you have to find out that how the velocity changes with time assuming the flow velocities to be uniform over each section.

So then if we consider a stream line between the say point 1 and 1. And if you write the Bernoulli equation here the velocity is clearly a function of time. So you have to write p1/rho g+ v1 square/2 g+z1= p2/rho g+v2 square/ 2 g+z2+ integral of 1/g that is the extra turn that you get because of the unsteadiness. P1 is like p atmosphere and when the valve is open it is also (55:28) to atmosphere.

So this is when the valve is suddenly made open that is what we are trying to analyze. So then these 2 pressures are the same because this is atmosphere when this value is totally open that is exposed to atmosphere. Let us say L is this length of this pipe and you may neglect v1 as compared to v2 if the area of cross section of the tank is much larger than that of the pipe so let us say that you neglect that z1-z2 is like h let us say.

So you have h= v2 square/2 g+1/g now you have to approximate these terms. So these clearly has 2 parts. One part is like you may consider the part within the pipeline and another part within the tank. So what is this at each and every point you are locally finding this time derivative of velocity and integrating this over this entire length. So how you are doing it. You are doing it by considering maybe this part and this part.

Clearly for the part within the tank the velocity is much less than the part within the pipe. So this may be approximated to be as good as the part within this length L and because the area of cross section is not changing here like v is not changing with the length. So this is approximately same as like dv2 dt* L. So this is like 1/g * d v2 dt dL. So from this consideration you can integrate this by considering at time=0 v2=0 because time=0 is the time at which the valve is suddenly kept open.

And then like you can separate variables and integrals to find out how v2 rise with time it is a very simple integration. So the whole idea is that how you utilize this unsteady term properly to find out an estimate.

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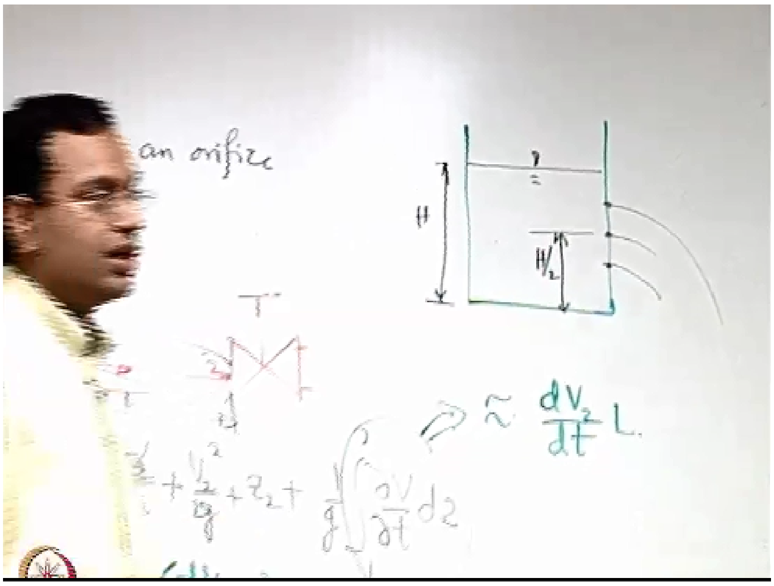

So I will end up our discussion of this class by giving you a very simple exercise. Again like a junior class level problem. So you have a tank like this you have 3 holes in the tank. If this height is h the central hole is at h/2 and the others are symmetrically located one at the top and one at the bottom. So when water jets are ejected like this which one will travel the greatest distance.

This is like a entrance examination problem you have solved it many times. Now you try to figure it out. Can you tell from your maybe memorial whatever which one should be the most? Not the upper one not the lower one, but the middle one and now with all these background that we have developed your objective will be to find out yes the middle one will be like that.

Second is what are the approximations or assumptions under which that analysis will hold true. So I hope that you will complete that exercise. So with that we stop our discussion today. In the next class, we will start with the new chapter the conservation equation for control volumes. Thank you.