Introduction to Fluid Mechanics and Fluid Engineering
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Lecture - 19
Dynamics of Inviscid Flows (Contd)

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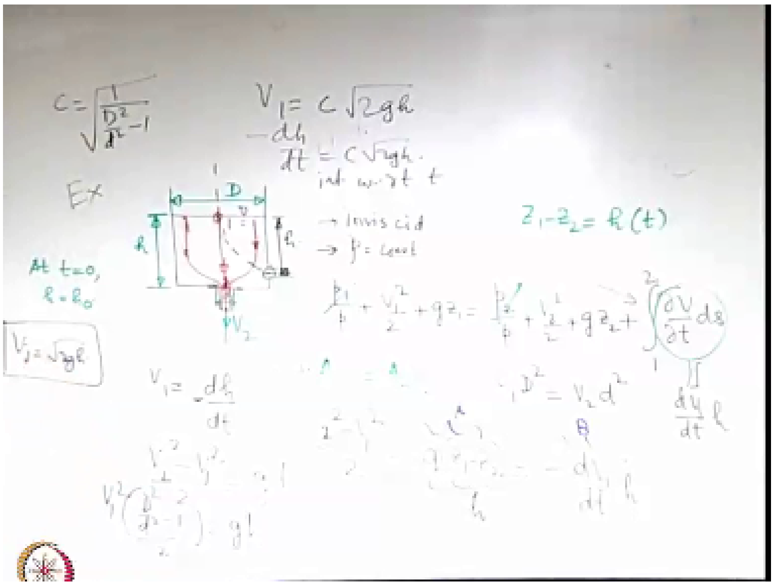

We continue with the example that we were discussing in the last lecture that that there is a tank and through the bottom of the tank the water is being drained out and the height of water in the tank is therefore changing with time. Our objective is to find out how the height changes with time. Now we were discussing about what is the significance or impact of the unsteady term that is retained or that should not be retained or should be retained is our doubt in the Bernoulli equation.

Now if you try to approximate it in some way. See in engineering we try to get a feel of the order of magnitude. So we may try to approximate it by a certain term which should be like our derivative of velocity with respect to time. Time some height. So let us say that if this dvdt was a constant if it was a constant not that it is a constant. If it was a constant, it could have come out of the integral and then it would have been some equivalent constant dvdt times s2-s1.

So s2-s1 maybe roughly like the height if you take the streamline which is just along the axis then it is exactly=h, but you cannot just write it as sum equivalent dvdt*h because V is changing with time in an unknown way so you do not have really an equivalent constant dvdt, but you may make a kind of approximation. You can say that I approximate this dvdt with dv1dt.

Why if you see that except for very close to the outlet the streamlines are almost parallel to each other and when the streamlines are almost parallel to each other it represents a case when that v is not varying very much. See why v is varying see there is a flow rate confined between these. So when the streamline the distance between these two streamline remains the same you have to say A1 V1=A2 V2.

So A1 is like this A2 is like this both are like as the cross sectional area with the streamline as envelope. In fact, you can have a large number of streamlines their envelope will look like an imaginary pipe or a tube that is known as a stream tube. So it is a collection of streamlines making an imaginary tube within with the fluid is flowing. So if you consider such a tube you can always see that the extent of that tube that remains almost the same till you come to the exit where it is really accelerating.

Because now the area available to it is so small that it has to get adjusted to itself. So when the area is very small and it has to get adjusted to itself that is only a small portion in comparison to the tank extent. So if you approximate this dvdt with dv1dt it is wrong, but it will give us some picture some idea what is the effect of what is the impact of this term? So if you make an approximation that this is=dv1 dt times h.

You have to remember that both are functions of time h is a function of time v1 is also a function of time. So if you write this equation in a bit different way you can write say v2 square-V1 square/2+ or – g V1-V2= - of and z1-z2=h which is itself a function of time. So these are valid locally at each and every interval of time at that time you have dvdt and you have an h.

Now if you try to compare different term say we want to compare these term with these term. So if these 2 terms are compared then let us say this is term A and this is term B. So when can you neglect term B in comparison to term A when you have this mode of this/ mode of gh when this is much, much <1 then B is much < A. So if the condition well h is something which you do not consider locally because this is like h is always a local constant.

That means whatever h is a function of time here in term a same h is there in term B. So only that means you are comparing dvdt with g. So the rate at which the change of velocity of the free surface it is there that is it is sort of acceleration if it is comparable with the acceleration due to gravity then you cannot drop this term and then you should retain this term at least frame a differential equation it cannot be solved analytically.

But if this is the case which is true for most of the practical cases then it is possible to drop this term. The second important point is irrespective of whether you drop this term or not A1 V1= A2 V2 is what you are always using. The reason is straight forward the origin of these does not come from steady flow. Although this is valid for steady flow it does not mean that it cannot be used for cases when the flow is unsteady.

Because the fundamental way in which it was derived from what from a continuity equation. First by dropping the partial derivative of rho with respect to time=0. So if rho is a constant partial derivative of rho with respect to time is 0. It may still be unsteady flow because the velocity may be function of time, but rho not being a function of time was the first thing to drop the first term in the continuity equation the derivative with respect to time.

For the other terms then how we came up with this we integrated this that differential form of the continuity equation and then if there say rho at the inlet and the exit sections are equal again if rho= constant that is valid then you have A1 V1=A2 V2. So a very important thing is for A1 V1 for A2 V2 to be satisfied it is not necessary that it has to be a steady flow only thing rho should not change that is a very important thing that we have to keep in mind.

So even when it is varying with time you can use that. Now let us say that this is the case so that we can drop the term B. So if we can drop the term B then you can write V2 square- V1 square/2=g h. Now what is V2 or you can express V2 in terms of V1 so V2 is V1* capital D square/ small d square. So it is V 1 square capital D square/small d square -1/ 2=gh. And the remaining work is very straight forward.

You can find out so V1 is of the form sum constant *root 2 gh where that constant is basically D square/d square-1 by that okay square root of that. See this gives a contradiction what is the contradiction? When small d is very small you consider the limit as small d/ capital D tends to 0 that it is a very big tank of a large cross section area and there is a very small hole through which the water is coming down.

Then how does this work? Yes, how does this work? C is almost if c is almost 0 then V1 is almost 0 I mean practically it is true that if it is a tank of very large area and if there is a small hole the velocity at which the free surface is coming down is not perceptible it is very small so that is okay. Let us not bother about that too much. Let us just try to complete this one by writing this as –dhdt= c root 2 gh.

Now if you integrate with respect to time you can find out how h varies with t this is a very simple work. Now try to relate this with a kind of again formula that you have used earlier in your studies. So let us think that this hole is not located here, but located at the side. This is a different example just I am drawing in the same figure to save the effort. So let us say that now this height is h which is changing with time.

So there is no hole here, but there is some hole here. There is a nozzle that is fitted and water is coming out. So when you are doing that the way in which most of you have done is like you have assumed the velocity that which the jet is coming out is root 2 gh. This is known as Torricelli's formula. So how you have arrived at that equation. You have used Bernoulli equation between 1 and 2 at that time you are not very careful about whether they are along same stream line or not just out of pleasure you are applied between 2 points.

And then when you applied between 2 points you put V1=0 you put p1=p2 the different between the 2 height h and so V2 will come root 2 gh. So what are the assumptions under which that is validated that is not a very bad formula. Torricelli's derived it long back I mean in a historical perspective it is a great development because nowadays we can speak these big words but the subject when it was fundamentally developed this itself was not a very trivial matter to resolve.

So then when Torricelli's found out this expression what are the assumptions in which these expressions you expect to work still. So one of the things was taken as V1=0 that means V1=0 when capital D is much, much > small d. So V1 is approximately tending to 0 the other approximations are that you are having a streamline like this with respect to which you have the points 1 and 2.

And the unsteady term does not appear in that analysis and it is assumed to be an inviscid flow. The greatest deviation from realities is because of the assumption of the inviscid flow. So that is one of the very important features that we have to keep in mind. So with that assumption this formula is not illogical, but a very important thing is we must keep in mind that some of those assumptions are to be questioned.

One of the important assumption is like capital d is much, much > small d which is true if it is a very large tank and from that there is a small hole through which water is coming out and dropping of the unsteady term and we have discussed that how this unsteady term this particular term in what condition it may be dropped or not. So this is a very simple problem.

But if you try to look into this problem very carefully it will give you a lot of insight on the use of Bernoulli equation under different condition. And I would encourage you to think about it more deeply under what conditions different terms are important in different ways not just satisfied with finding h of the function of time, but to write the differential equation of maybe say V1 as a function of time in a very simple case and in the most general case.

And try to compare them that what are the terms that are making them to be different. We will consider another example in the unsteady Bernoulli equation in the use of the unsteady Bernoulli equation that is given by the next problem.

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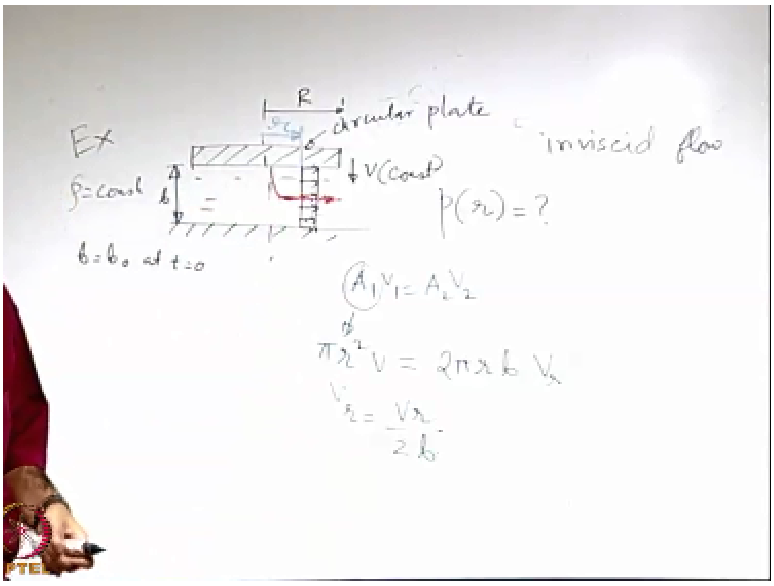

Let us say that you have 2 plates these are circular plates. We have solved problems with rectangular plates just for a change let us consider that it is a circular plate. So this is like this plate is coming down with a uniform velocity V. And this is a circular plate. The radius of the plate is r and say we are considering a coordinate system the local coordinate as small r. So small r is the local coordinate at a radius r.

Now with this we are interested to see so the bottom plate is stationary. There is some water with rho=constant and when this plate is coming down what is happening water is squeezed out of the plates because whatever water was there say originally this was B0. So B= B0 at time=0, but as this is coming down this B is changing B is decreasing. So where will that water go that water will be squeezed out radial to make sure that the continuity is maintained.

So we are interested to find out how the pressure varies with R. Assume inviscid flow and flow is constant that we have already defined or we have already assumed. So as we have seen that in all these cases it is important to get a feel of the velocity profile. So if it is an inviscid flow the velocity variation over the section is not there so that velocity is uniform over each section, but these uniform velocities is changing with radius.

So how you can find out it you have to think that what is the rate at which this is pulling water downwards with the same rate at which it is being squeezed out. So if you consider a local radius R what is the rate at which this is coming down. So when you write A1 V1= A2 V2 question is how do you write V1 V2 A 1 and A2. What is V1? V1 is the rate at which so it is like an artificial flow imposed by the movement of the top plate.

So that flow velocity is given by V1. So what is that A1 *. So what is A1? So if you consider only up to a local radius of small R. So A1 is pi* small R square. So A1 is pi* smaller square what is V1? V1 is V because this is a uniform rate. This is uniform this is not a function of time this is constant= what is A2 2 pi r*B. B is a function of time * V2 or V as a function of R. Let us write Vr just to emphasize that it is v at a radius R.

So you can write V at a radius R= V divided by Vr/2v. Now so this is a velocity at a radius R. Next we are interested to find out the pressure. So if we are satisfied with inviscid flow and flow= constant we can consider a streamline that connects two points. Any 2 points say 1 and 2. So the streamlines how the streamlines will look. So the streamlines will virtually look like this.

So the flow is being squeezed out in this way so that is how a streamline will look. So let us take any 2 points located on the streamline and write the Bernoulli equation between those 2 points located on these identified streamlines.

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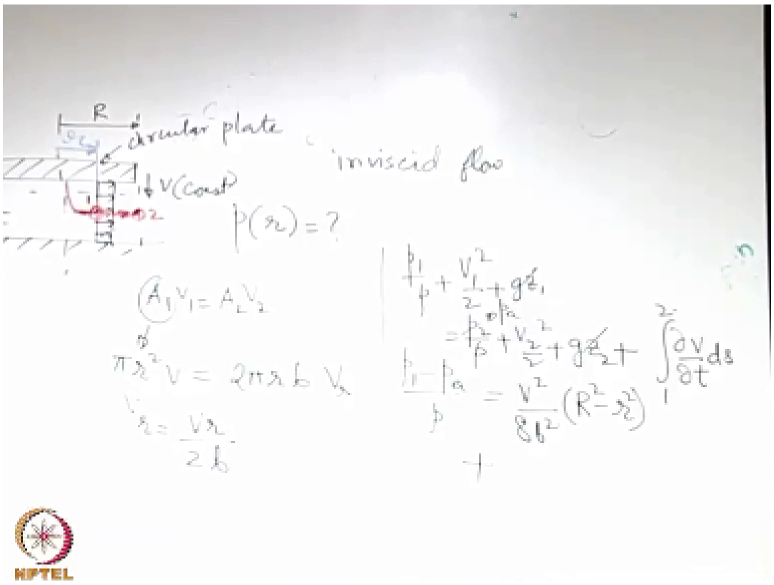

But because it is an unsteady flow we need to retain the unsteady term in the Bernoulli equation. So p1/rho + V1 square/2+gz 1= p2/rho+V2 square/2+g z2+ let us say that we apply that between 2 points. One point is located at R= small r and another point 2 is located at R= capital R. So when you have such a case you are getting rid of many things. One is between the points 1 and 2 there is no difference in height.

So of course if this gap B itself is narrow then even if there was a change in height because of taking the points 1 and 2 not exactly along the same line that term itself is not that large, but like if you take them along the same horizontal lines they are identically the same. Then you are interested to find out p1 and p2 you know p2 is the atmospheric pressure. So because it is at the exit plane.

So you are interested to write p1-say p2 is p atmospheric p1-p atmospheric/ rho= now v2 squares- v1 square/2 so V2 square- v1 square/2 is V2/ 4 b square * capital r square-small r square because V is having only this component. Then + this term so by 2 will be there. 8 b square then + let us calculate the third term.

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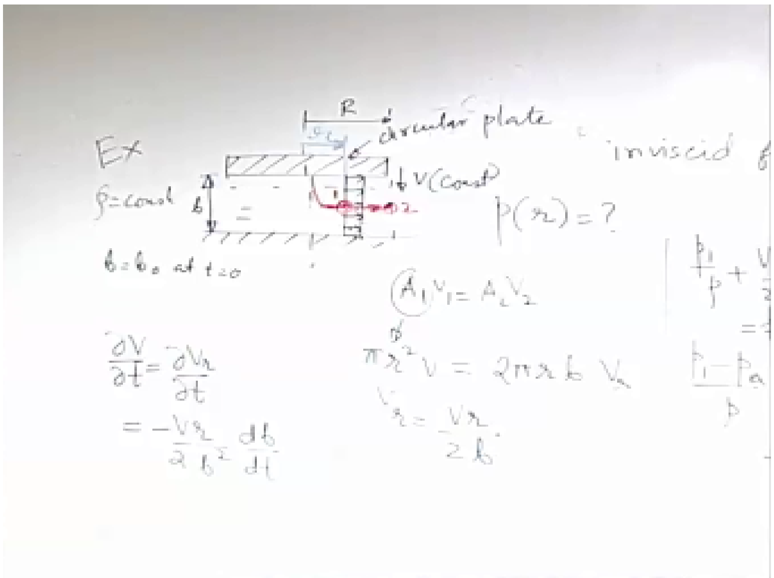

So what is the partial derivative of V with respect to t that is the partial derivative of Vr with respect to t that is the only v component that is there which is a function of t here. B is a function of t here. So this will be- Vr/2 b square * db dt and –dbdt=V. So –dbdt=v. Just like the previous tank problem that we were considering. So this term becomes v square r/ 2 b square.

So that you can substitute here and ds will be dr because you have chosen your streamline in such a way that the change in s is like change in r.

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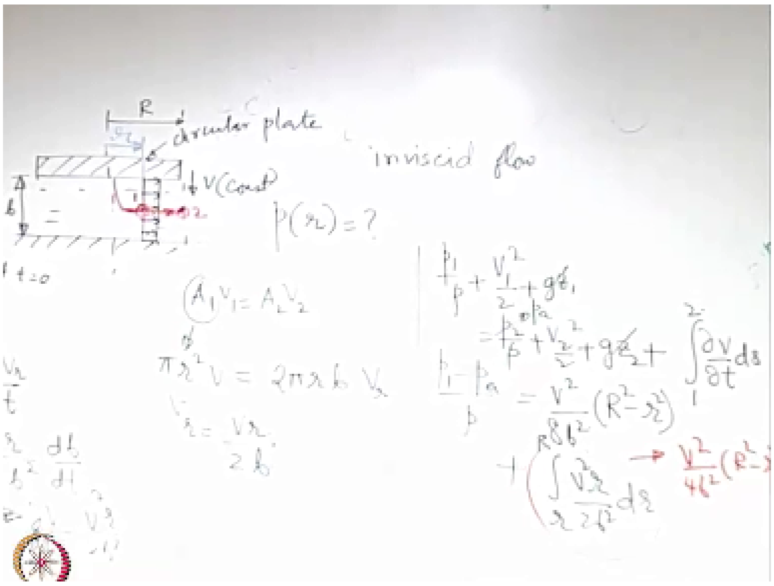

So this is from integration from small r to capital R v square r/2 b square dr very straight forward to complete it. It becomes v square/ 4 b square* capital R square- small r square. So at a given instant you can see the pressure at the radius small r is varying with time because b is a function of time. So this only a given instant you can say. So at different instance you have different values of b.

And you can find out what is the value of b at a given time how because you know dbdt is –v. So b= b0-vt. So if you are given a particular time so this will give you b=b0-vt. So if you are given a particular time you can find out what is the value of b at that time then you may substitute the value of b at that particular time to get the pressure at a radius. So you can clearly see that the unsteady Bernoulli equation how it can be utilized.

Now the next topic that we are going to discuss in the context of these Bernoulli equation is the use of such equations. See the Bernoulli equation has been one of the very popular equations in fluid mechanics not just because of its simplicity, but because of its applicability in an approximate sense in terms of quantifying the nature or the principle of working of many engineering devices.

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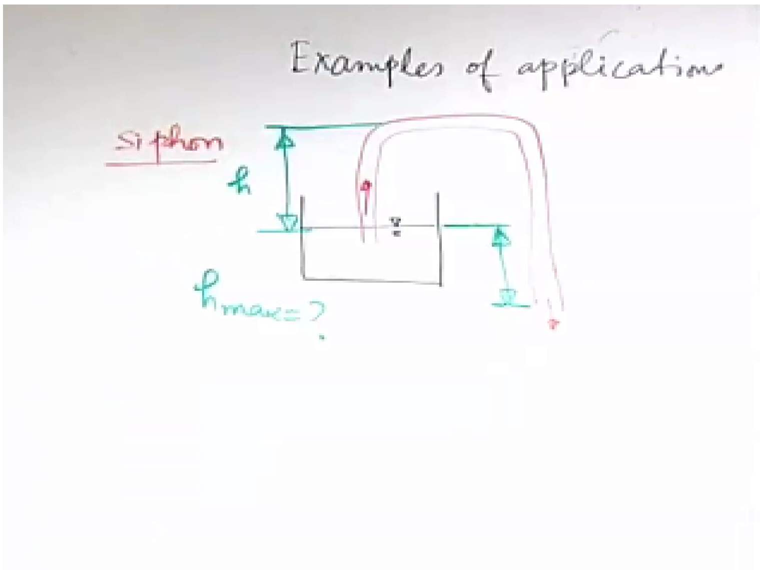

And we will look into such examples of applications of Bernoulli equations. So some of the examples we will not detail very much, but we will only get the essence the details of most of these examples are uploaded in a course website note on the application of the Bernoulli equation. So if you go through that in details you will get all the detail picture because we are going to discuss subsequently about certain devices.

These devices have certain intricacies and we will only highlight the major or important features, but for the other detail feature you should refer to those notes. Now before coming to any device of very great engineering application we may come up with a sort of a very primitive device which you have already heard of something called as a siphon. So if you have say water in a tank like this.

And you are having a bent tube which is sort of sucking water and ejecting water to a different place from the tank so this is called as a siphon. The apparent amazing features of the siphon is out of nothing it is pulling the water in a upward direction that is the apparent amazing feature, but if you look into it a bit carefully it is not at all any amazing feature because eventually when it is discharged it is discharged at a level below.

So the actual head difference which is working on it is this one which is a favorable one because effectively it is coming from this elevation to this elevation and this net elevation difference is actually giving it a velocity. So with that velocity the water is being sucked. So the fact that is going up is nothing very special because eventually it comes down and it gets ejected from height which is less than or below the level of the tank.

But the good thing is that while doing it can traverse a vertically upward distance. Question is how much distance it can vertically traverse? So what should be this say if you call this as h then what is this h max? This is given by a practical consideration. Let us try to identify a streamline which connects the points say streamlines will be bent like this, but let us just consider a streamline which is confined between that points 1 and 2 which are almost like located on a vertical line.

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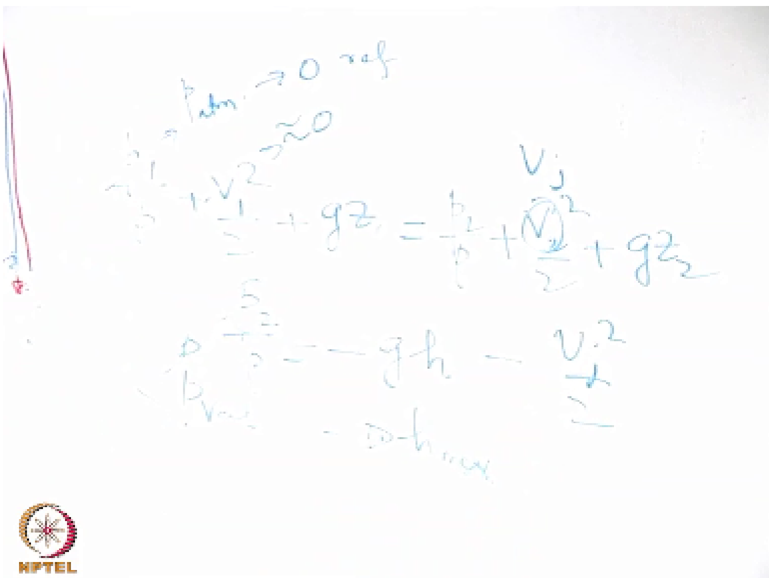

So if we are interested to write the Bernoulli Equation we can write p1/ rho+v1 square/2. So every time whenever we are writing the Bernoulli Equation we are not repeating the assumptions, but you should keep in mind that what are the assumptions on the basis of which we are writing it. So p1/ rho+ v1 square/2+ gz1= p2/rho +v2 square/2+ (()) (29:14). Now you can clearly see that at the level 1 you have pressure as the atmospheric pressure.

So this is p atmospheric. V1 is approximately=0 just like the Torricelli's equation because the area here is so large that the velocity with respect to which this level is changing is very small as compared to the velocity here v2 is same as the velocity at which the jet is ejected here if the area of cross section remains the same. So V1 is small because A1 is large as compared to the area available at 2.

Then V2= Vj that is the velocity at which the jet is coming out if the cross section is same and you can find it out that Vj is nothing, but approximately root 2 g * capital H by writing the Bernoulli equation between 2 points on the same streamline whether if you continue with that streamline it goes like that and comes out. So the net elevation difference will remain this one.

If you write the Bernoulli equation of a streamline between say point 1 and say point j which is located here. Now when you write that one what you will get? You will get p2/rho= g* z1-z2. So g* z1-z2 is –gh- vj square/2. So you can clearly see that if you take the atmospheric pressure as 0 reference. So this is written by taking atmospheric pressure as 0 reference. So this is a like a (()) (31:22) expression.

So when you take the atmospheric pressure as 0 reference then p2 is negative because h is positive vj square is positive. That means the pressure at this point is below atmospheric. So if it is below atmospheric it may come to a state when it comes to the local vapor pressure. So when the pressure falls below the local vapor pressure then what happens then vapor bubbles are formed.

So when the vapor bubbles are formed. So when the vapor bubbles are formed it is nothing very special that vapor bubbles are formed, but what is special is that when this vapor bubbles are transported or moved to a different place where the pressure is again higher they will collapse again to formal liquid and once they collapse what happens basically then they were occupying a large volume, but when they collapse again to be converted to liquid again there is a volume change.

So it creates an unsteadiness in the flow and it can create a lot of vibration and noise and that is not so good for the flow and that type of phenomenon is known as gravitation. We will see in details what is cavitation when we will be discussing about the fluid machinery which will be our last chapter in this particular course. So we will not go into the details of like what is cavitation at this stage.

But we have to keep in mind that it is better if we keep the pressure at 2 below the local vapor pressure that is below the vapor pressure which should be there at that corresponding temperature so that vapor is not formed. So that means we are keeping a restriction that p2 must be less than the vapor pressure at that local temperature of the fluid. So then you can see that you get I h max from that.

And that is the maximum h with respect to which you should design your system. So that you do not have a problem with formation of vapor. So the siphon in principle maybe designed to be very like tall in height in terms of this vent tube, but in practice one should not make it too tall because if you make it too tall it is possible that the pressure is so low that vapors are formed and that can create other disadvantages in terms of operation of the device.

The next application when we consider we will keep in mind that now whatever applications we are going to study our objective will be to have a Bernoulli equation utilized in devices through which we are interested to measure the velocity or flow rate in a pipeline.

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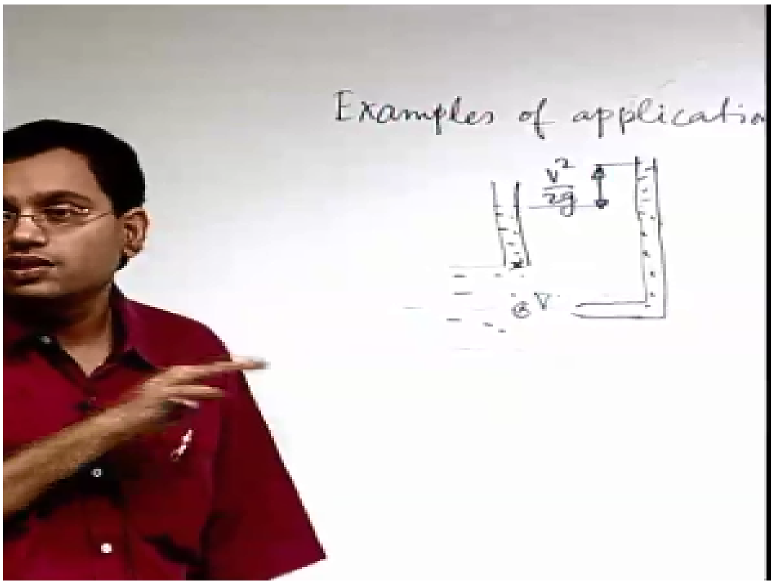

So let us take an example let us say that you have a pipe like this a horizontal pipe. Now water is flowing and you make certain holes in the pipeline. What holes you make? So first you make a hole like this so when you make such a hole what will happen the water will rise and it will come to a height. The height with respect to which the water rises will be an indicator of the local pressure at that location.

Pressure at (()) (34:52) we are interested about the central line. So if we are interested about a point in the central line what we are doing we are sacrificing one thing we are not able to exactly probe at the central line at the same actual location we are proving at a point which is different from the central line and we know that it is very much possible that the pressure at the central line should be different in general from pressure at this.

When they are different when you have a curvature of the streamline. We have just in the previous lecture seen that if you have the gradient of pressure in the direction of n you will only when the streamlines have a radius of curvature which is non infinity, but here if we consider that the streamlines are parallel to each other then you do not have that effect of the stream line curvature in terms of the pressure gradient.

So whatever is the pressure here should be the same as the pressure here. So then this is an indicator of the local pressure. Now say we are interested to have an indication of the velocity so for that what we can do we can have another tube where we make a penetration in the wall, but before that we have the tube directly confronting with the flow. So this tube and this tube is different this is not directly interfering with the flow.

But this is directly interfering with the flow. When it is directly interfering with the flow it is bringing the flow to a standstill or a dead stop. So it is creating like a stagnation point where the flow comes to a dead stop it cannot go further. So whatever water was coming here it comes to a dead stop what it will do it will it (()) (36:39) tube and the question is will the rise will be > this one or < this one. See this rise was the function of the pressure.

Now the entire energy which was there in the flow if we assume that assumption on the Bernoulli equation those are valid. Now we have made the kinetic energy to 0 so the entire energy now contribution of pressure term plus the kinetic energy term will be successful to make it go further up because where will that energy go you have made the fluid to a dead top you are assuming that is a frictionless flow.

Then where will that energy go. It will obviously make the fluid rise to a greater height and the difference between these 2 heights is if this points are very close to each other the pressure almost the same the difference between these 2 heights is just v square/ 2g. So from this principle V is the velocity of flow at this point. So from this principle it is possible to make an estimation of the velocity and if you know the estimation of the velocity.

And if you assume it to be uniform then you can also have an estimate of the flow rate. Now if it is not uniform you can keep it at different radial locations and you can even find out how velocity varies radially because these tube you can put at different radial locations. So this is put at r=0 at the central line, but you can also keep it away from the central line. So at different radius if you put it will give you a picture of velocity at different radius.

So it is possible even to get a velocity profile if this is quite accurate of course there are many doubts about the accuracy of such a simple arrangement, but it gives as a conceptual understanding.

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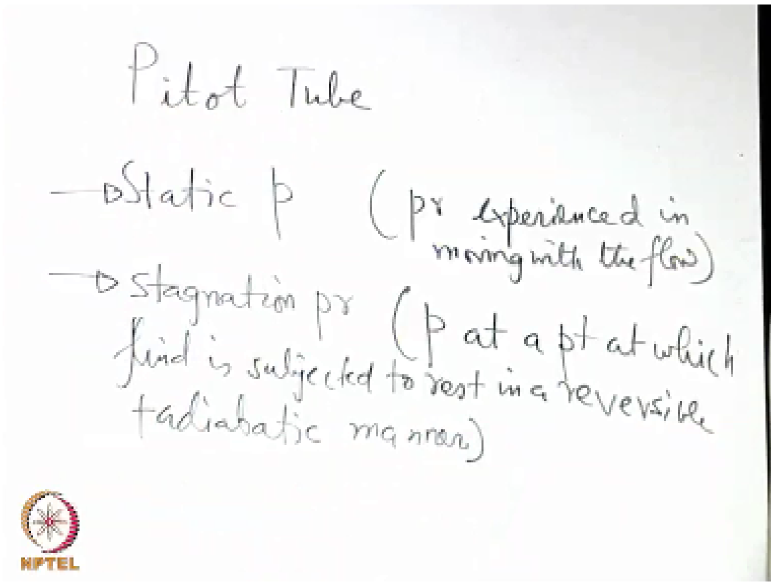

So the device is based on this conceptual understanding is known as a Pitot Tube. So the last t is silent so it is pronounced as Pitot Tube. So this of course is to honor the name of inventor of this device and it is a very simple device and the working principle of this device is based on 2 important definitions which we will tell now. One is known as static pressure. So what is a static pressure?

Static pressure is the pressure which is there because of the intermolecular collisions so that means if one is moving with the flow then what is the pressure felt because of just moving with the flow is the static pressure. So this is a pressure experienced in moving with the flow. So this is the result of the intermolecular collision and this is the pressure that we fundamentally define.

Now we are also going to define something called as stagnation pressure. So what is the stagnation pressure? Stagnation pressure is the pressure that is there at a point if the fluid is subjected to 0 velocity at that point in a reversible and adiabatic manner. So pressure at a point at which fluid is subjected to rest in a reversible and adiabatic. We will not go into the details of the reversible and adiabatic processes.

Because these you will learn more in details in the thermodynamic course that we will have subsequently, but important understanding in our context is that one of the important requirements of this is it is a frictionless flow. So that means when the fluid is subjected to rest at a point you have to make sure that it is subjected to rest in a frictionless manner. So whatever is the pressure that these tube is getting is the stagnation pressure.

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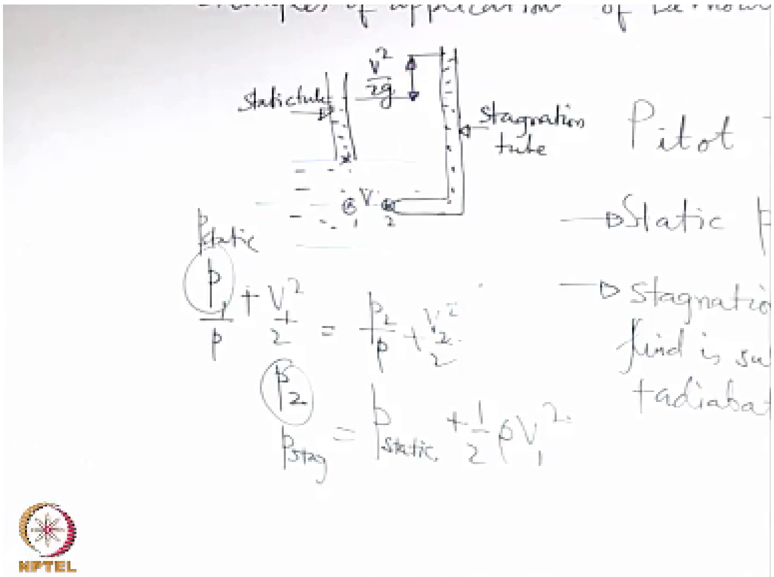

So this is also known as stagnation tube because its reading gives an indication of the stagnation pressure and this is known as the static tube. So if you want to write the Bernoulli equations between 2 points 1 and 2 which are located in such a closed manner that point 1 if you have pressure as Ps or say p1/rho+v1 square/2. We are not writing g z1 and g z2 they are so close that the difference in height is negligible= p2/rho+ v2 square/2+ g z2 we are not writing again.

So what is V2 V2 is 0 because it is a stagnation point. So the definition of the stagnation point is velocity is 0. So you can see that you can write p2 which is the stagnation pressure as p1 which is the static pressure. This is same as p1 this is p static+ 1/2 rho v1 square that means stagnation pressure is a sort of property of the flow if you know the velocity of the flow.

But you have to keep in mind that this equation is derived by considering a frictionless condition. And frictionless condition is valid when you are subjecting the flow to rest in a reversible and adiabatic process. So the definition of the stagnation pressure is to be kept in mind. Stagnation pressure is not just pressure at a stagnation point. What is a stagnation point? The stagnation point is a point where you have 0 velocity, but it does not mean that pressure at that point is a stagnation point.

Pressure at that point will be a stagnation pressure only if the flow is subjected to rest in a frictionless manner because the stagnation pressure is defined in that way. It is not just sufficient it is necessary that you must have the velocity to be 0 at that point so that the pressure (()) (43:36) is a stagnation pressure, but at the same time it is not velocity subjected to 0 in any way, but it is subjected to 0 in a frictionless way.

The second important thing is since these points are very close to each other and you can just say stagnation point stagnation pressure at a point just as a property which is dependent on the local velocity. So stagnation pressure did not always be measured through a stagnation point. So if you want to say find out stagnation pressure at a point you can simply say that it is the static pressure which is the regular of the normal pressure + 1/2 rho v2 that is a definition.

So the stagnation pressure does not mean that you have to bring the fluid to rest at that point to get a pressure. It is like how you physically conceive that pressure not that. So it should not give you the false idea that whenever the velocity is non 0 stagnation pressure is not defined. It is definitely defined. It is just a physical way of looking into it interpretation. Now the next we will discuss 1 or 2 important flow measuring devices.

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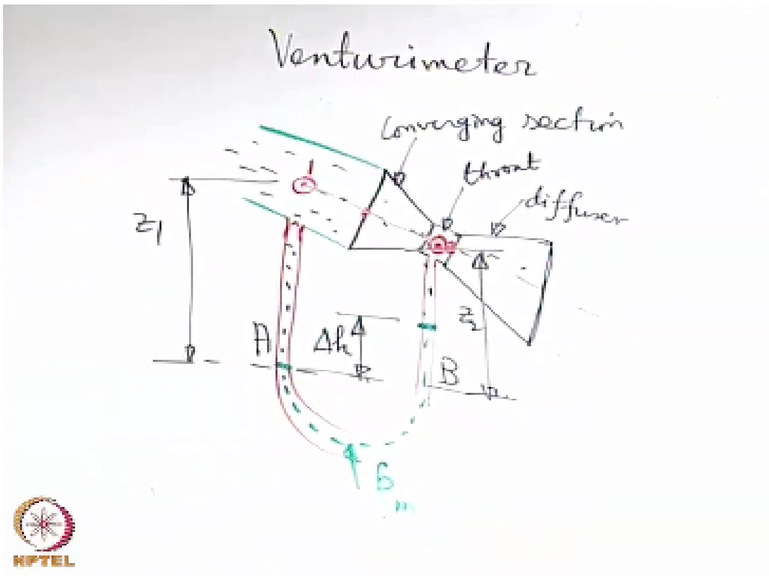

And the first device that we will discuss is known as a venturimeter. So what is a venturimeter? Say you have a pipeline and you are interested to measure flow through a pipeline. So what you are trying to do say you have a pipeline like this you want to measure what is the rate of flow through the pipe? So how will you do it there are many ways in which it can be done is one of the ways is by utilizing a device called as venturimeter.

So what is done a part of the pipe is like replaced with a device. What is that device? The device is like this so you have an accelerating section by having a converging cone and then you have a zone of uniform cross section and then you will again come back to the pipe dimension. So this is known as diffuser. This is a known as a throat and this is a converging section.

So what is the objective? The objective is by this way you are reducing the cross sectional area. So to maintain the continuity in a steady state what you are doing. So if you consider now the points let us say that you consider points 1 and 2. The point 1 was having the velocity as same as that of the velocity of flow in a pipe. Now at the point 2 the velocity will be more or less. It will be more because the area of cross section has reduced.

So since the velocity is more. Now if you write the Bernoulli equation assume that it is a friction less flow. Then p/rho+ g z that term will be what that term will be changing and if we can find out a measure of that change then it is possible to find out the velocity through the Bernoulli equation how we do that? Now let us say that you make a tapping of a manometer that means let us say that you consider a hole in the pipeline and a hole here.

And connecting that with a manometer. So when you are connecting that with a manometer see we have not taken the point 1 at the inlet of the converging section, but at some location which is sufficiently away from that because here the streamline curvature effect will tend to become more and more dominant. So you want to take it away from such a place where the streamlines are almost parallel to each other.

So pressure at these point and maybe pressure at these points should not be very different because of the streamline curvature effect. So we are having a manometer in which we have a fluid. Now in which limb the fluid height will be more or in which limb it will be less. Let us write the equation the Bernoulli equation along the streamline between the points connecting the points 1 and 2.

So let us say you have a streamline that connects 1 and 2. So you can write p1/rho+v1 square/2+g z1= p2/rho+v2 square/2+ (()) (49:23). At the point 1 if you have this as the height of the limb and at the point 2 if you have this as the height of the limb. Now I have drawn it in this way. Do you accept that it should be like this? let us a fluid a marker is there as a manometric fluid here.

We call it rho m the density of the mercury. Now is it an acceptable sketch in this case the remaining is filled up with water. So if water is flowing through this tube let us say this is filled with water. The same fluid which it is flowing here. So is this acceptable? By this you are expecting that pressure at 1 is > pressure at 2. We will see that may not be correct also let us see.

But this figure is correct? How that is possible let us see. So let us say that this is the difference in height that we measure so that is = delta h. So when you measure this height delta h then from that delta h it is possible to write the equation of the manometric principle that is you can write that if you have 2 points A and B at the same horizontal level you have PA= PB.

So when you write PA= PB. Let us say that you are writing say this is your reference for measuring z1 and z2 in the Bernoulli equation. So this is your z1 and this is your z2. You can use any datum, but this is a convenient datum.

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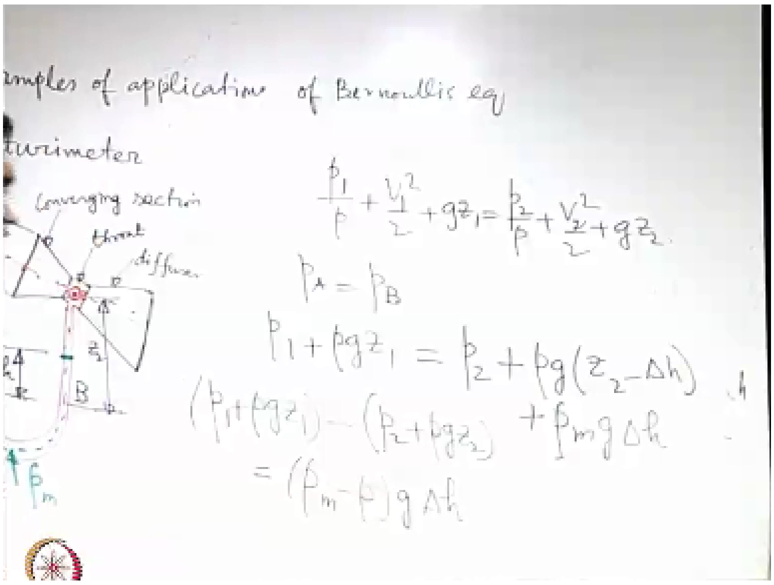

So you can write p1+rho g z1 that is= pressure at 8 where rho is the density of the water that is flowing through the pipe=P2+ rho g z2- delta h+ rho mg delta h. So when you are finding out the difference in p1 and P2 p1-p2 you see that you can clean up the expression by noting that it is not just p1-p2 that is important. You have p1+rho g z1- p2+ rho g z2 that is what is going to be important.

So if you write p1+ rho g z1- p2+ rho g z2 then that is rho M-rho * g * delta h. So in this figure you are expecting that delta h is positive. Isn't it? This is just the dimension. Rho m say this is mercury so we know that it is much heavier than water. So rho m-rho is positive that means we are expecting this to be positive. So what this reading gives us. This reading gives us not the difference in p1 and p2, but the difference in sum of p1+rho g z1 and p2+ rho g z2. So it is not giving us the pressure difference. So what it is giving us.

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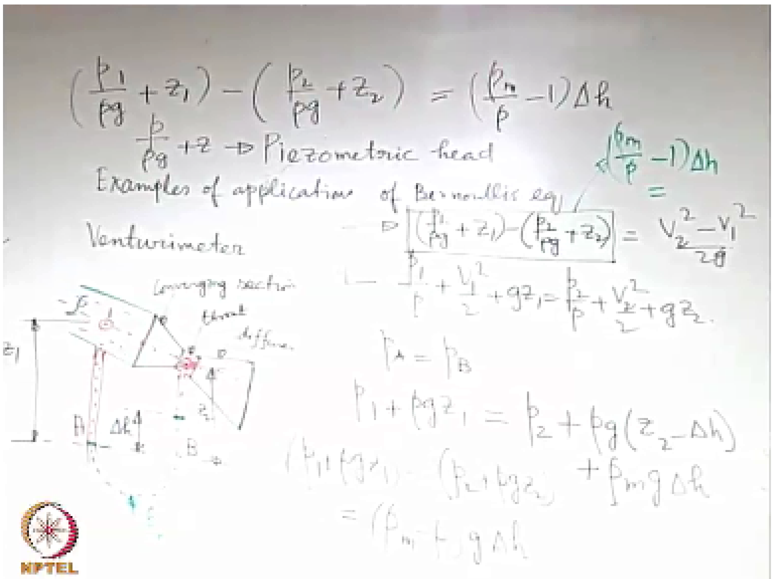

So let us write this in a bit more explicit way. So let us write it as p1/rho g+z1. So we are dividing it by rho g. So p2/rho g because we know that in this process we will get something called as head which we use as a terminology for this calculation. So this is rho m/rho-1* delta h. This delta h is very important because this is what experimentally you can read. So when you read experimentally delta h you see that it is an indicator of not just the difference in pressure, but the difference in pressure head+ the elevation.

So when it is flowing from 1 to 2 it is possible that p1 < p2, but p1/rho g +z 1 is> p2/rho g+z2. So this flow is taking place from a higher value of this collected term to a lower value of this collected term. This collected term which is given by p/rho g +z is known as a piezometric head. So p/rho g+z this is called as a piezometric head. Why it is called as a piezometric head?

The reason is that if you say have a pipe and if you puncture the pipe or if you penetrate the pipe say and if you have a tube through which the water goes up. It is just like that static tube that we considered in the previous example then the elevation that it assumes here is the elevation because of its vertical location + because of the pressure at static pressure at that point and this tube is commonly known as a piezometer tube.

So that is why the name piezometric head. So in the manometer in this kind of an example we do not measure the pressure difference, but we measure piezometric pressure or piezometric head difference. In terms of head it is called as piezometric head. If you express in terms of pressure units, it is called as piezometric pressure. So always keep in mind. In this case manometer is not measuring pressure difference.

It is measuring piezometric pressure difference. These are very, very fundamental mistake that people make. See as I told in a very introductory class that we are bound with certain intuitions that it will flow from high pressure to low pressure and you can clearly see that with a very simple example where it is not actually a practical example because we have considered a frictionless flow, but even that it gives a very important insight that it did not be from a higher pressure to low pressure.

It is basically from a high piezometric pressure to a low piezometric pressure. Now fortunately what is important for this equation is only the piezometric pressure because if you see like if you write it in this form you will get p1/rho g+z1-p2/rho g+z2 that is= v2 square-v1 square/2 g. So this is something which is a very simple term for us now because from the manometer we have got an explicit expression for that is rho m/rho-1 * delta h.

So this we can write as rho m/rho-1* delta h and this is = now you can express V2 and V1 in terms of the volume flow rate. So if Q is the volume flow rate.

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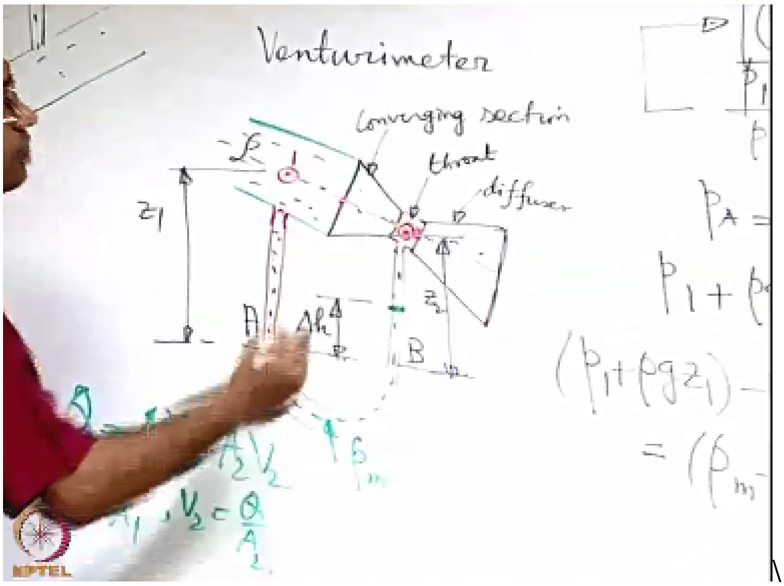

Then you can write as Q=A1 V1=A2 V2. Again what are the assumptions? Rho is constant and it is a uniform velocity profile over the section that is inviscid flow. Viscous effects are not there. So you can write V1 as Q/ A1 and V2 as Q/ A2. So if you substitute that in this expression.

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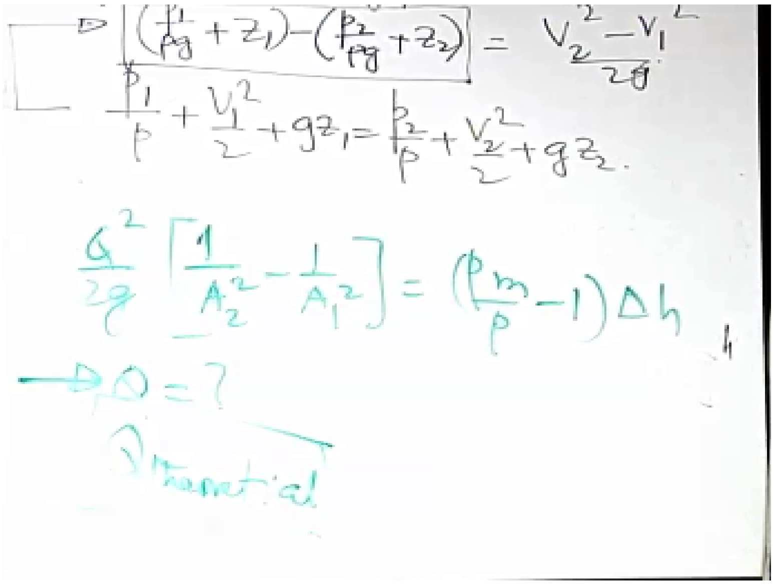

It is possible to express V2 square-v1 square as Q square/2 g*, no this g is there so because of division by (()) (58:24). So Q2/2g* 1/A2 square- 1/A1 square= rho m/rho-1* delta h. So from here, you can solve for what is Q? Remember it is very theoretical. Why it is theoretical because it has considered many idealizations which do not actually occur in practice. So we will keep this in mind and in the last class we will try to identify that what are the idealization which were here which need to be rectified.

And what are the important design considerations that should go with this device matching with the non idealization. That we will discuss in the next class, but if it was ideal just by getting the delta h reading you could get what is the flow rate through the pipe because A1 and A2 you know are the areas of cross sections of 1 and 2 which are given geometrical parameters. So we stop here today we will continue with that in the next class.