Introduction to Fluid Mechanics and Fluids Engineering
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Lecture - 18
Dynamics of inviscid Flows (Contd.)

We are now going to discuss the Euler's equation of motion in streamline coordinate system.

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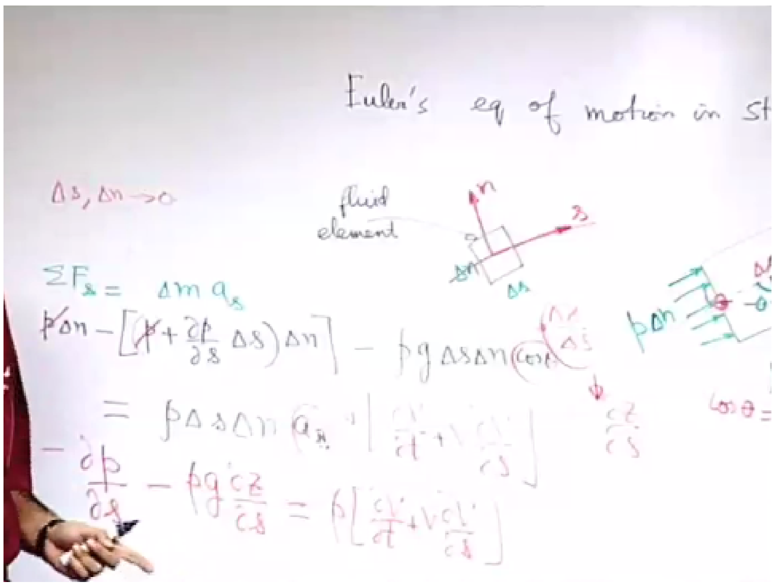

So let us consider a streamline coordinate system as we were discussing in the previous class. Euler's equation of motion in stream-wise coordinate or streamline coordinates. So we consider that there is a streamline and we consider a coordinate system such that you have a coordinate system s, n where s is given by the stream-wise coordinate, fundamentally it is not exactly the tangential coordinate but it is oriented just along with the streamline.

But affectively it is just like tangential coordinate. The tangential coordinate is coordinate along direction which is given by a slope and this is along a direction given by the curve so it moves, it is a coordinate system that is align with the streamline itself. But locally it is as good as like a tangential coordinate and n is the normal coordinate. So in this coordinate system, let us say that we have a small element of fluid like this, this is a fluid element.

But this fluid element has a specialty; we have now considered this fluid element to be sort of coaxial with the streamline at a given location. Let us try to identify all the forces which are acting on the fluid element, we separately draw it for clarity. So let us say that we have a fluid element like this and let us consider that the central line of the fluid element is such that it is representing the streamline locally.

Now we are going to identify all the forces which are acting on this fluid element. So what are the forces which are acting on this fluid element? Again, there are forces which may be resolved in the coordinate directions s and n. We will first write the equation of motion along s, so we will only identify the forces which have component along s. So we consider the left face where you have a pressure distribution.

And what is the resultant force on this if p is the pressure. Let us identify the element by its dimensions along n and s say delta s and delta n, so p*delta n*1 is the width perpendicular to the plane of the figure. Then, there will be a pressure distribution here p+ this*delta n. Remember whenever we are talking about the Euler's equation the most fundamental assumption that we are making (()) (04:00) inviscid flow, so now question of any shear component of force.

Then, on the other faces there are forces which are acting along that n direction, so those will have no components along s. But something else will have a component along s, what is that? The weight of the fluid element should have a component along s. So let us identify that, let us say this is the weight of the fluid element and let us say that its makes an angle theta with direction of s. So what is the weight of the fluid element?

Let us say Rho is the density of the fluid element, so Rho*delta s*delta n*1 is like the volume of the delta s*delta n*1 is the volume on the fluid element, Rho* that is the mass and that *g is the weight. One important thing is although this is a curvilinear coordinate system but remember this is not a rectangular coordinate system this is a curvilinear coordinate system but when you take small elements this almost behaves like a rectangle.

Although it is a curvilinear coordinate system that is the one of the advantage of taking a small element. Now we can write this component of this force as, you can see from this figure the component of the force along the direction of s as related to cosine of the angle theta. And that maybe described in terms of the difference in vertical elevation between the points 1 and 2 which are located say at the centers of the 2 faces, the left face and the right face.

Let us say that these points are located at a vertical elevation of delta g. So delta G is what, delta G is the difference in height between the points 1 and 2 which are the centers of the faces. So you can clearly see that this angle will also be theta and you can therefore write that cosine of the angle theta is given by delta g/delta s because the other length that is the actual length of the element.

Now we can write the Newton's second law of motion for the fluid element because we have identified all the forces which are having some components along the s direction. So resultant force along s should be the mass of the fluid element times acceleration along s. Let us simplify this so you can write p*delta n- the other term then the weight component should have –low g delta s, delta n into Cos of the angle theta=Rho delta s, delta n*acceleration along n, sorry acceleration along s.

Now you can simplify this equation by considering the previous discussion that we have regarding the description of the Cos theta. First you can cancel certain terms, next delta s*delta n both these products get canceled out from both the sides, we have to remember we are considering delta s, delta n small tending to 0 but definitely not =0. We can express cos theta as delta g/delta s and we can express acceleration by, what should be the acceleration along s?

See the express should be similar as we had acceleration along x, y or z. So it should be this one. See what is the advantage of using a streamline coordinate? The advantage of using a streamline coordinate is, that since the fluid always tangential to the streamline with the respect to the streamline it is locally like a 1-dimensional case because you are not having cross-stream-wise velocity components.

So this is like a one dimensional case, of course the velocity maybe the function of time but otherwise in terms of spatial variation it is only a function of s because flow is always tangential to the streamline that is how the streamline is defined. And for further simplification we have to keep in mind that in the limit as delta s tending to 0, this is as good as, so it becomes the derivative in place of the differences because in the limit as delta is tends to 0 this will become the derivative just from the basic definition of the derivative.

So what we can say here that –partial derivative of p with respect to s – this one =. You can clearly see that this if you integrate with respect to s this will become the Euler's equation of motion in the streamline coordinates. We have derived that from a different point of view by considering the equation of motion along x, y and z and then using that to find out the net changing pressure.

So if you want to find out the changing pressure across between 2 points located along the same streamline you can integrate this, integration is obviously along a streamline because you are using a streamline coordinate systems during so the integration is along that line only and it will follow exactly the same form as that of the other form that we have developed by this time. You can clearly see that this V dv ds type of term you can write V ds square/2.

So you have all the terms like d ds of something and integrate with respect to s this will become like change in pressure, this will become like Rho gdz, this will become Rho dv dt multiplied by ds and this will become d of d square/2. So then the partial derivative and d will become similar because V is a function of s only provided V is not a function of time, but otherwise you have to keep in mind that if V is a function of time also this partial derivative nature has to be maintained. So for unsteady case one has to written it like a partial derivative.

Now you can write similar expressions for the n direction. So for the n direction if you write, let us see how this looks like.

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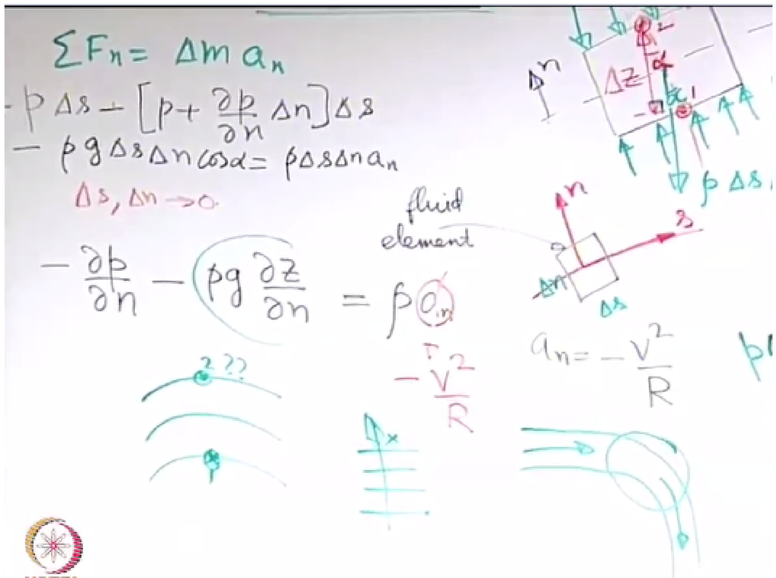

So equation of motion so along n direction. Usually we are happy to write it along the s direction because we feel that, that gives us enough information because flow is along that s direction. But along the n direction something interesting also is possible and let us try to look into such possibilities. So the same element let us draw it maybe again and we are interested now to capture what happens along the n direction.

So we will now write all the force components with an understanding that we are interested for the forces along the n direction. So let us represent those forces in the n direction. Now you have from this side you have what is the force, p*delta s and from this side p+ partial derivative of p with respect to n*delta n that*delta s. Again this, have its own weight, so one has to consider the weight of the fluid element, and when you consider the weight of the fluid element.

Now we are not bothered about the left and the right face we are bothered about the bottom and the top face. Let us say again that the centers of these faces are located at a vertical elevation difference so this is say 1, this 2 and there is a difference in vertical elevation or the height between the points 1 and 2. So that is given by delta z. So delta z is what delta g a symbolically either case represents the vertical elevation between the 2 faces which are contributing to that forces along that direction.

And again let us say that this makes an angle theta with the, the vertical direction is making an angle theta with the n-axis of the element. Now if we write, if we describe the weight of the fluid element in the same diagram that is oriented along g, -z of g s V. And it is again Rho delta s, delta ng. So it has a component of again its component along the direction of n is this * cos theta, remember this theta and this theta are not the same.

This is the different figure differently defined theta. So whatever was defined theta here, it is actually 90 degree – that, so if you want to avoid a confusion maybe let us give it a different name so that later on when you look into it you are not in confusion. So let us say this alpha which is like 90 degree-theta. So you can see that the Cos alpha component of this is what contributes to the force along the n direction and Cos alpha is given by, what is Cos alpha, delta z/delta n, right.

Just like what we got from a right angle triangle in this figure, similarly here also we can. Okay. So let us write the equation of motion for this case, resultant force along n=the mass of the fluid element times acceleration along n. Left hand side will be very similar to what was along s except the s's are been replaced by n's. Okay. So let us write the left hand side.

So p* first –p*delta s*delta-- sorry –p*delta s then + p+ partial derivative of p with respect to n*delta s then again your positive n direction is outwards and the weight component is in the negative of the direction so –Rho g delta s, delta n*Cos of the angle alpha = Rho delta s delta n*an. Okay. **"Professor-student conversation starts"** (()) (18:32) Which one? Okay, so this is the first one is + and this one is -, okay. **"Professor-student conversation ends"**

Now we will also consider the limits as delta n, delta s all are tending to 0. And then what follows, you have -, so delta s*delta n will cancel from both the sides – what will be the Rho g, this one=Rho an. Important thing now will be what is an, okay. So you have to keep in mind that you are considering a fluid element in the limit as this size is tend to 0 it becomes like a fluid particle. So you can think of a particle moving along a curve.

The basic particle mechanics when the particle is moving along the curve. So what is the normal component of acceleration if the particle has a tangential component of the velocity as V. What is an? V square/r but not in the positive n direction but in a negative n direction, so –V square/r what is r, r is the local radius of curvature of the streamline at that point where we are considering the fluid element.

So the very important distinction between the tangential and the normal components come through this description of the acceleration. So this will now become –V square/r. So let us consider a very simple case to illustrate the use of this. So what does it say? It says that if there are curves streamlines there will be a pressure gradient across the streamlines. So let us say that you have curve streamlines like this.

When such curves streamlines are possible? Let us say that you have a pipe bend, so there is a pipe which was carrying fluid like this. Now the fluid is changing its direction it is moving in a different direction, it is very much possible and very, very common, pipes are bend to change the direction of flow. So in the pipe bend if you consider the streamline, streamlines are like oriented along the direction of the flow, so you have curve streamlines.

Let us say that this entire bend is in the horizontal planes, so that there is no change in the vertical elevation. Say there is a pipe which is bend, in a horizontal plane. So then just for simplicity this term is not there. So when that term is not there you can clearly see that the pressure gradient along the normal direction should be given by V square/r. So if the streamlines are radius of some radius of curvature and if you know the pressure at say this point 1 you should be able to predict what should be the pressure at this point 2, if you know the radius of curvature of these streamlines.

If the streamlines are such that they are parallel to each other then the radius of curvature tends to infinity that means, there is no cross-wise pressure gradient because of the curvature. So this is only the curvature affect. So because of the curvature of the streamline there is no cross-wise pressure gradient. There might be cross-wise pressure gradient because of a change vertical elevation.

But here we are considering it in a horizontal plane so that is not coming into the picture. So this n component of the equation of motion is very important because it gives a pressure gradient because of the curvature of the streamlines in the cross-stream direction which you cannot get from the equation of motion from the s direction. Okay. Let us try to work out maybe 1 or 2 simple problems to illustrate the equations that we have developed.

We will deliberately try to solve the examples where we can use the stream-wise and cross-stream-wise components or maybe the polar coordinate systems like that.

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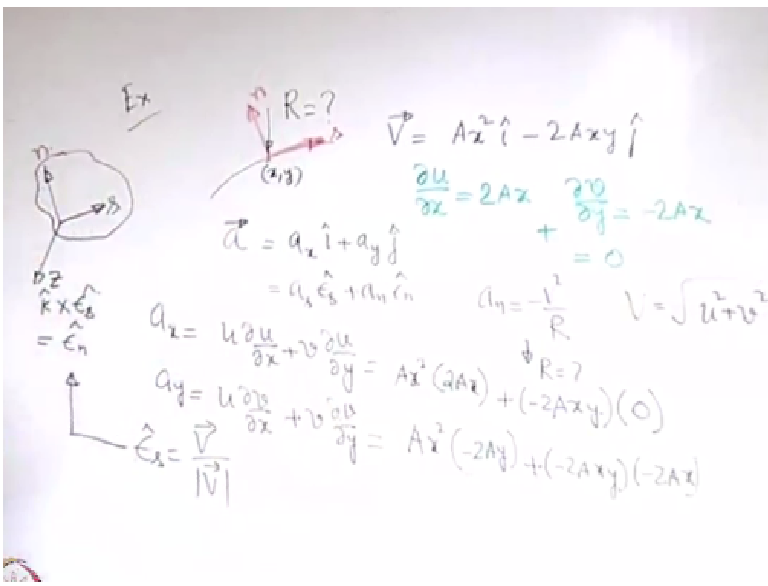

So let us take an example where let us say that you have a streamline and the velocity vector is given not in the streamline coordinate system but in the x, y coordinate system. So at a given point x, y located on the streamline the velocity vector is given like this say it is Ax square i-2Axy j where again A is the dimensional constant and x and y are the coordinates. We are – what we are interested to find out.

So we are interested to find out what is the radius of the curvature of streamline at this point. Let us say we are interested to find out that given this information. To do that, first let us see that what type of flow it is. So that may not always be necessary for solving every problem but it is not a bad idea to see that whether it is an incompressible flow or not. So we need to check whether the diversions of the velocity vector are 0 or not.

You can clearly see that the sum of these 2 is 0. In fact, that is how I try to define the velocity fields so that it becomes an incompressible flow. Now when you have this incompressibility in mind you can also try to write this expression in terms of stream functions and so on. But here objective will be just to find out the radius of curvature. To do that, let us say that we are interested to find out the acceleration. See what can be our strategy?

Let us make a strategy and then we will find out relevant quantities. Say we find out the acceleration at the point A. So this acceleration you can find out in terms of the x and y components so acceleration say it becomes ax i + ay j, okay. So it is straightforward from the formula of acceleration it is possible to find out. Now this acceleration maybe resolved in 2 components.

This is also resolved in 2 components but the components that we are very much looking for in streamline coordinate system are s and n. So this may be resolved say along s, s is virtually like the tangent but again I fundamentally told that it is not a tangent it is basically located oriented along the streamline. So s and n coordinates are there, so this may also be resolved in such a way that you can write it as as* unit vector along s + an* unit vector along n, right.

And once you get that you know that you know that an= -V square/r, where V is the square root of the square of the components of the velocity, so that should give us what is R, okay. So before calculating anything by a (()) (27:51), it is not a bad idea to make a plan of why we are going for such calculations. So now we will not of course will not go into the detailed calculation because it is just a very trivial differentiation algebra.

But at least we should try to figure out that how do you find out the unit vectors along s and n because that maybe the only botheration for solving this problem. So when you write the acceleration components say let us quickly write the acceleration components, so what is acceleration along x, here it will be, okay this is acceleration along x, acceleration along y, so you can write acceleration components along x and y.

So you can see the numerical values of the coordinates x and y are given you may substitute to get some values. Now how do you get the epsilon s? How do you get the direction of the unit vector in the stream-wise coordinates? yes. So V is oriented along s so you can write epsilon is as V/ the mode V that is the unit vector in the direction of s. How do you get epsilon n? Yes, they are perpendicular but it is well-known that how exploit that.

There are many possibilities in which their dot products maybe 0. So if I have a vector like this they are perpendicular and dot product is 0, but like it may give this instead of this, right. Well, if you cannot do it with a dot product you have to think for a cross product. So let us see why and for what purpose we are going to do that. So if you have, see think about the coordinate system. Let us say we have a coordinate system like s, n and g.

So s, n are the things which are happening in the x, y plane basically and g is perpendicular to that. So this again forms like system of orthogonal vectors, may be the coordinate is the curvilinear one but still orthogonal one. So when you have such orthogonal basis so just thing s like x, n like y not that they are x, y we know that they are different from x, y but just to draw an analogy the usual unit vector components and vector components along this.

So can you relate the unit vector along n with the unit vector along s and unit vector along s, sorry unit vector along g and unit vector along s. So unit vector along g is k, so k cross with unit vector along s should give the unit vector along n, right. So that means once you get unit vector along s you can easily find out because unit vector s is gives me in terms of i and j the unit vectors along x and y and then it is possible to utilize the cross products.

k cross i and k cross j to find out unit vector along n in terms of the unit vectors along x and y. And once you find that out the remaining work is very trivial, so you can write this in terms of the acceleration along s and acceleration along n. So how do you write the acceleration along s and n? So acceleration along n is what is our interest so what is acceleration along n? Say you know what is acceleration along x and y.

So x, y are the coordinate systems like say this is x and this is y. And one importantly you know the resultant acceleration, say this is the resultant acceleration. So what is the acceleration along n, this is nothing but the resultant acceleration dot with the unit vector along n that will give its component along the direction of n. Very straightforward vector resolution. And then you can equate it with V square/r and get the value of r. Okay. Let us look into another example.

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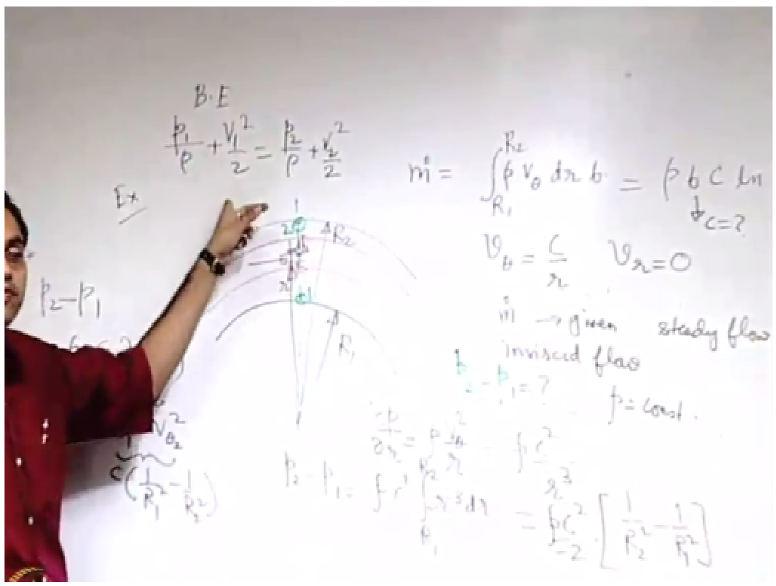

Let us say that you have curve or a bend that we were discussing. Say there is a bend in a pipe line so that the streamlines which were originally parallel to each other they may remain parallel to each other but, they are now curved instead of being straight, so let us say that the streamlines are becoming like this. So this is like a bend in a dark, bend in a horizontal dark so that we are not bothered about the change in height between various points.

Everything is taking place in a horizontal plane. And let us say that these are parts of circles so if these are parts of circles you have let us say the inner radius of the bends as R1 and outers radius of the bends as say R2. Circular curves are special cases of general curve where the radius of curve is a constant. So when you have radius of curve as a constant the good thing is then if you use the polar coordinate system R theta that is equivalent of using the s in coordinate system.

Because then your r is always oriented along n, because the R is remaining constant and it is always the normal direction to the cloud that is the specialty of the circle which makes the manipulation much easier. What is our objective in this problem? Our objective in this problem let us say that the velocity field is given. Typically, when the fluid flows in a bend its velocity field maybe approximated in this form.

So we will write the velocity components in the polar coordinate system V theta = c/r and Vr=0. What is the name of this type of flow? We have discussed earlier, Free Vortex. So this, not that it is really a Free Vortex. See, important one important aspect science and engineering is to approximate or model the reality with something which is very close to that. It does not mean that actually the flow through a bend is given by this velocity field but it has a lot of closeness with this situation, that is the only reason why we have chosen this velocity field.

Now you have seen, so the 2 types vortexes you have seen. So this is likes a Free Vortex and you also have seen like a something like a Force Vortex. You have to keep in mind that like no matter like whatever is the vortex, if the Free Vortex or the force vortexes you have a V theta component for a Free Vortex V theta is c*r and Vr=0. So in general what can we call as a Vortex type of flow?

A Vortex flow is a flow which has only V theta component of the velocity and Vr=0. So different description of V theta may give different types of vortexes okay, so fundamentally vortex is something which we will have only the cross radial component of velocity not the radial component of velocity, and that cross-radial component of velocity is a function of the radial coordinate, that is also a very important consideration.

Now this velocity field is given, c is not given but say somebody experimentally measures what is the mass flow rate through this dark and say that is given M. M. is the rate of mass flow per unit time, say kg per second that is given. And assume that it is a steady flow so this M. does not change with time and also assume that it is an inviscid flow.

If this is an inviscid flow, then many interesting things are possible. One of the interesting things is that if the flow is originally irrotational it is likely to remain irrotational. So you can clearly see we have discussed this earlier that the Free Vortex flow is an irrotational flow. So this flow is an irrotational flow. Now what is our objective, our objective is to find out the difference in pressure between 2 points 1 and 2.

One is in the just close to the, or adjacent to the inner radius within the fluid and 2 is just at the outer radius, so we are interested to find out what is p2-p1. There are certain ways in which we can do it. Let us look into that what are the ways. So one of the ways is you have dp or like the expression that we just derived now. The partial derivative of p with n=Rho V square say local radius of curvature, right. Just now we derived this. It is not a very old derivative.

Just now we have derived this. So here n direction is like the r direction. And when you are writing here V, this V here has only V theta components it is a vortex flows, so this as good as V theta square/r. **"Professor-student conversation starts"** Yes. What about the Rho (()) (40:30)? It is in the bend in the horizontal dark, so that means there is no change in vertical elevation. **"Professor-student conversation ends"**

See every keyword given for the problem definition has some implication and it is important that you put enough attentions to all the keywords which are given to describe the problem. So this is the form and if you look into books you will see that this start with this form this might give you a false impression that this equation is fundamentally derived for a R theta polar coordinate system, no.

This is fundamentally derived for the s, n coordinate system but special case of a circular geometry will make n as good as r and special case of situation where your V theta is only V component will give this as V theta square, so that you have to keep in mind. So whenever you have again I am repeating whenever you have a formula type of thing just try to be aware of the origin of the formula that will help you in solving a problem which is exactly not this a bit change from this one. Now you can-- the remaining work is easy.

So you can just substitute V theta square that is C square/r square so this is become Rho C square/r cube. So you can integrate this with respect to r, so you will get p2-p1= Rho c square, r to the power -3dr from r=r1 to r=2. Okay. So you will get this as Rho C square/-2 1/r square – 1/r square. So the final expression p2-p1= Rho C square/2 * R2 square – R1 square / R1 square – R2 square that is the final expression.

Now the question is you do not know what is C. C is not given, it is just given that V theta is inversely proportional to R. So to find out C you require the description of a mass flow rate. So what is the mass flow rate? Let us say that we consider a small element at a radius r we consider a small element of, so this is local R. So at a local R we consider a small element of width dr. There is a flow across this element.

So what is the flow across at this element? Let us-- so we have to know what is the shape of the section of the dark, this is just like one of the it is an edge view in the plane but it has its perpendicular direction. Let us say that in the other direction it has a uniform width. So let us say that with perpendicular to the plane of the figure = say b. Okay. So width perpendicular to the plane of the figure is b that means.

That you know that, that area of the strip is d*dr and what is the volume flow rate that is flowing through that? See V theta is perpendicular to that. So V theta* V*dr that * Rho is the mass flow rate. And integrated with respect to that r=r1 to r=r2 will give you the total mass flow rate. So you can write M. integral of Rho V theta dr * b from r=r1 to r=r2. Okay. So no V theta you can write C/r. So from here you can find out what is the value of c, okay.

And when you say value of c you can substitute here to get if you know given mass flow rate it is why important it is by some measurement device you can measure the mass flow rate and by that you can directly what is the pressure difference between these 2. Problem is solved but as I have told quite a few times our job may not be completed done let us try to look into the problem in a bit different way.

Let us say we are very crazy and we just simply interested to find out the pressure difference between the point 1 and 2 by the use of the Bernoulli's equation. Why, because we are always tempted to use such an equation. And anytime whenever a problem fluid it is given I have seen that it's like a every student feel so happy that when the Bernoulli's equation maybe is used. So let us say that there is such a happy student who is interested to use the Bernoulli's equation.

And when that is being used so if you apply the Bernoulli's equation between points 1 and 2 okay. So a better student will say that have you checked that located along the same streamline? And we can clearly see they are not located along the same streamline but the other student will say that still let us apply that so if you apply that p1/Rho+V1 square/2=p2/Rho+V2 square/2. Other assumptions are they are like inviscid flow it is already given steady flow this is given.

And we are assuming that Rho is a constant that is there additional assumptions that we are making. In fact that assumptions we made earlier while taking out Rho form the integral in t he mass flow rate calculation Rho is like a constant. So what is the logic of the other student? Logic of the other student is somehow if I get the get my answer right I will not care that how I have solved the problem, which is true for most of the students.

So then, you can find out what is p1/p2 and this is a very simple exercise. So you can write p2-p1=Rho* Rho/2*V2 square sorry V1 square – V2 square. And V1 and V2 are like V theta 1 and V theta 2 because they have no other components, so this is as good as V theta 1 square – V theta 2 square. So this is as good as C*1/R1 square - 1/R2 square. This exactly the same what we go through this and the within straightforward because it is an irrotational flow you can use the Bernoulli's equation between any 2 points no matter whether they are located on the same streamline or not.

So this is not a magical or a coincidence that is like this but you have to keep in mind that once you are applying the Bernoulli's equation between 2 points, see whenever you are solving problems what we see every year is just I am narrating my experience, people always write, writing the Bernoulli's equation between points 1 and 2 you have to be very, very careful that whether you can write Bernoulli's equation between those points1 and 2.

It is irrotational flow you do not care really where they are located but if not you have to make sure they are located along the same streamline. So if they are not located along the same streamline even if your final answer comes correct it is fundamentally wrong and it is not acceptable. Here you get rid of that complication simply because it is an irrotational flow. So you can apply Bernoulli's equation between any 2 points in the flow field.

No matter whether they are located along the same streamline or not provided other assumptions of the Bernoulli's equation they are valid. Now we will work out some additional examples to illustrate the unsteady Bernoulli's equation. So till now we have discussed about the steady version of this equations, but we have made a remark that it is possible that you have an unsteady version. So look into such an example.

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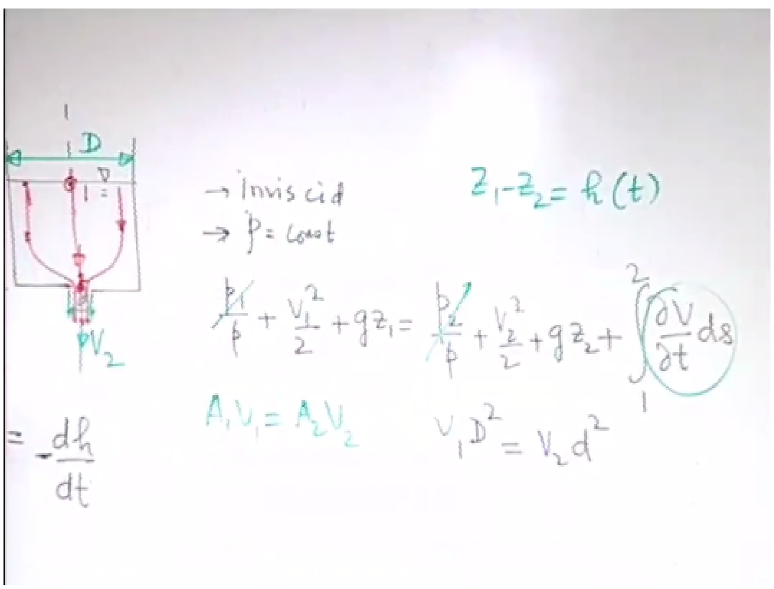

We will again consider a case which you have encountered many times in your earlier experiences with solving fundamental very simple flow problem. So you have a tank which has water to some depth and there is a small open in at the bottom of the tank through which water is being drained out. So you are interested to find out how the height of the tank or the depth of the tank is changing with time because of the water that is being drained out.

Let us say that the original height, so let us call this as H and let us say that at t=0 you have H=H0 which is the original depth of the water in the tank and then you have open say some exit line or escape route for the water so that water is coming out. Let us say that the, this is a cylindrical tank so let us say that you have this as the diameter capital D and the diameter of the escape as small d. So our objective is to find out the time which is may be required to empty the thanks as a special integration.

But here the general interest will be to write the equation of motion for this case. So let us say that we are interested to write, now here you can clearly see it is an unsteady flow okay. Since it is an unsteady flow you when you are using a Bernoulli's equation type of equation you have to be careful. It is if you are not sure whether the steady or the unsteady version has to use you retain the unsteady term and see that whether it is appropriate.

So let us say that you have, let us consider some streamlines. So how in the streamlines looks like originally they will be parallel all the streamlines but when they come close to the constriction they will try to come close to each other because the streamlines then have to be confined through this small hole. So they are coming closer to each other so you can see that their curvatures of the streamlines they are created close to the exit point.

Similar thing is possible if you have like a hole in the side not in the bottom, fundamentally there is no big difference. Now if you take say 2 points 1 and 2 located on streamline say you have one streamline and you have 2 points 1 and 2 and let us say that other assumptions of the Bernoulli's equation are valid that is it is inviscid flow Rho=constant these 2 are the other assumptions that we are considering, we are considering the equation along a streamline which on which there are points 1 and 2. And let us write the unsteady Bernoulli's equation.

So can write p1/Rho+V1 square/2+gz1=p2/Rho+V2 square/2+gz2+, okay. So this extra term is there because of the unsteady version of the equation. Now let us see where you are considering the points 1 and 2, so 1 is expose to atmosphere and 2 is also expose to atmosphere, so at both location the pressure is the atmosphere equation. So this 2 are the same, so these get canceled out.

You also can clearly see that the difference between the heights are 1 and 2 g1 and z2 this is h which itself is a function of time, because as water is coming down h is decreasing. Now V1 and V2 you may relate V1 and V2 with the capital D and small d by writing A1 V1= A2 V2. Again, so this is a form of the continuity equation for constant density steady flows. What are these V1 and V2? These are fundamentally the average velocity at the sections where 1 and 2 are located.

But because we are considering inviscid flow we are implicitly assuming that this velocity profiles are uniformed that means V1 and V2 are as good as the velocities locally at the points 1 and 2, only valid if it is a uniform velocity profile otherwise it has to be replaced with an average velocity. So A1 is proportional to capital D square again A2 is proportional to small d square so we can write this as V1, D square capital D2 = V2*small d square. So you can relate V1 and V2.

Now you can also write one important thing that is what is V1, V1 is, it depends first of all you can write that V1 is dt. Because what is the velocity have this section. Locally it is the velocity representative of the rate at which there is a changing level of this. So H being a fixed type the rate with respect to h changes is given by dh dt. Now you have to see that you must have an appropriate fine adjustment in this equation, think of A1 V1= A2 V2.

If you take V2 as this the velocity the velocity sense with respect to it comes say V2 is positive then, but if you write V1, and V1 is dh dt, dh dt itself is negative. So it should be adjusted for this equation with a – sign, so - sign or + sign is based on your sign conventions. If somebody writes this with a positive sign actually there is fundamentally no wrong in that because it is your sign convention.

So the sign convention is like where you may say V1 is positive in the downward direction or in the upward direction so that it is up to you. But you have to keep in mind that V1 and V2 sign convention should be consistent. So V1is like positive here and dh dt itself is negative so-- sorry V2 is positive downwards dh dt is itself negative so it has to be adjusted with a – sign, so you can relate V1 and V2.

The third thing is how do you approximate this term? How do you approximate this term is very, very important because you explicitly do not have an information on how V varies with time. So you have to make an approximation. See in engineering we always do approximations to get feel of what is the magnitude of this term and is it important with relation to the other terms. If as compare to the other terms this term is not important then we can drop it and we can utilize the steady version of the equation.

But if not, we may not be in the position of drop it and we should retain it even in an approximate form. So in the next lecture we will see that how to treat this special cases of this unsteady terms. So we stop here for this lecture, thank you.