Introduction to Fluid Mechanics and Fluids Engineering
Prof. Suman Chakraborty
Department of Mechanical Engineering
Indian Institute of Technology – Kharagpur

Lecture - 17
Dynamics of Inviscid Flows (Contd.)

We will continue with the Bernoulli's equation about which we were discussing in the previous class.

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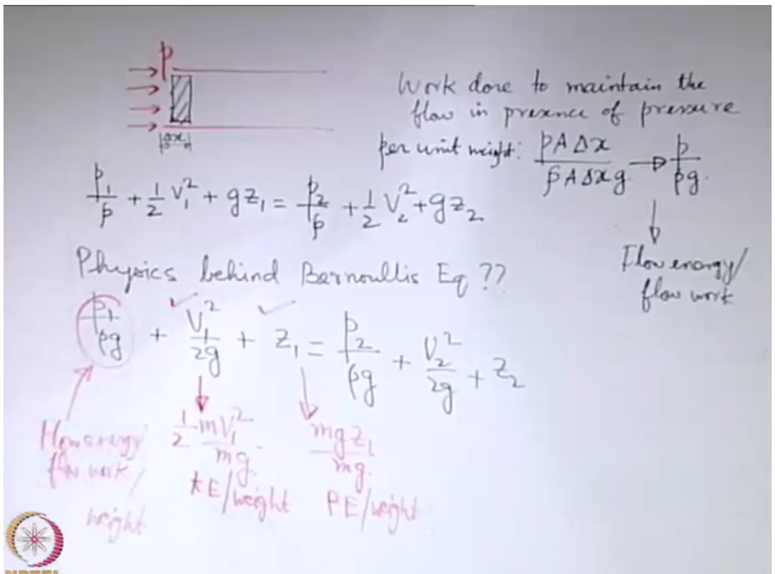

So the Bernoulli's equation as we have seen is taking the form. What are the assumptions under which this was derived? Inviscid flow is the most important one I should say then this version is for steady flow we will see, what is the version for unsteady flow subsequently. Density is constant and we derived it along a streamline where it requires no other restrictions.

But if you want to apply between any 2 points who are necessarily located on the same streamline then it has to be irrotational flow or maybe a special case when the cross product of the velocity with the velocity with the verticity that is perpendicular to the line element that is been chosen. So these are some of the discussion that we made in the precious lecture. Now when we come to this form, so let us say that we are considering it along a streamline there are points 1 and 2 and this equation is valid.

So what does this equation say that is very, very important after all we are not mathematician, we are not just bothered about that here there is an equation where we can plug-in values to get a numerical answer. What is even more important for us to appreciate that, what is the physics behind the Bernoulli's equation, so that is what we will try to learn. Because once we appreciate the physics properly we will be perhaps able to utilize this equation.

In certain cases, where this equation may not be exactly valid but in a somewhat approximate sense. To understand the physics, it may be better to appreciate the physical consequence of each and every term in the equation. These terms maybe written in different ways, this is one standard way of writing it but in engineering sometimes what we do we divide all the terms by g and write it in this form so p/Rho g is the first term.

Then we have V square/2g the second term and z as the third term, so this also is one of the forms and one of the common forms in engineering. This particular form has terms which have dimensions of length because the third term is dimension of length and all terms same equation must have the same dimension, so all other terms have units of length. Now what we will try to see is that expressed in term of units of length, what do this length is physically represent.

We will start with the more obvious terms that is we will start with these 2 terms, these are more obvious and easy to interpret and in some way very trivial. So you can write, say V1 square/2g, how you can write it, you can also write it at like half m V1 square/mg, right. That means this in a way represents kinetic energy per unit weight. Similarly, this can be written as mg z1/mg, so this is potential energy per unit weight.

So you can see that these terms are therefore representatives of energy per unit weight. Energy per unit weight in fluid mechanics is known as a Head, so that is the term given as Head is energy per unit weight. So loosely this may be called as kinetic energy head or maybe velocity head in a more simple term. So Head is a length-wise representation of energy. I mean giving a dimension of length to the energy by normalizing it with respect to the weight.

Let us look into the p/Rho g term. So this is like kinetic energy per unit weight, this is potential energy per unit weight. I guess you have learned this term as pressure energy or something like that but I do not understand anything called as pressure energy, it is something very absurd the terminology which is enforced maybe on whoever and you are just habituate accepting that. So if you are in general asked that, what is p/Rho, you will say pressure energy.

But what is pressure energy right we have learned many types of energies but I cannot remember. Well I was a very bad student but still I cannot remember that I have understood something fundamental as pressure energy, so let us try to see that what is that terminology. Maybe terminology is not so important we may use some other term also for that but what does it physically represent.

We can understand the kinetic energy and potential energy what do this physically represent. But what does this physically represent is something which is not very straightforward from this and if we want to make it very straightforward by calling it as pressure energy perhaps we are trying to suppress our lack of understanding of it. So let us not do that. Let us expose our lack of understanding and see that what it should be.

So let us say that you have a pipe and fluid is trying to enter the pipe. When the fluid is trying to enter the pipe let us say that the pressure at this inlet section is p. The fluid is having a particular velocity it is entering the pipe. Now see this is a flowing system. So there is a pressure in the fluid, this pipe is not that, this pipe is like in vacuum. So you have a continuous flow going on like this. So this is filled with water and water is continuously entering and leaving like that.

Now, if the water which is entering the pipe has to (()) (07:42) over a distance it has to do that in presence of the pressure. And therefore it has to do some work. So what is that work, let us say that it undergoes a displacement of delta x. Why we are considering just a small displacement because we will consider that this pressure is remaining constant what that displacement, it maybe variable pressure.

So we will consider only a small displacement over which the pressure is supposed to be a constant. The whole idea is based on that we are interested to calculate the work done in presence of pressure. So if A is the area of cross-section of the pipe then what is the work done to maintain the flow in presence of pressure? Let us say we want to find it out, so to do that we will first consider a small displacement.

So if A is the area of cross-section then because of the presence of this pressure p what is the work done? p*A*delta x. Now just all the other terms we will also try to express this work per unit weight for the fluid. So if we want to express it work done to maintain the flow in presence of pressure we amplify this as per unit weight. So then this has to be divided by the weight of the fluid element that covers up to this, weight of the fluid rather that covers up to this delta x.

So what is the weight of that fluid? Rho*A*delta x that is the mass that * g is the weight. So this becomes p/Rho g. So you can see that, that is same as the first term that you can see in this Bernoulli's equation. So this, what it represents? It represents the work done to maintain the flow in presence of pressure that is fundamentally what it represents. So if there is no flow then this term would have been absent.

So the fluid must possess this additional or the fluid must be capable of transferring or transmitting this additional energy through its motion so that it can overcome whatever pressure is there and still it can maintain the flow. So this extra energy the fluid must possess to maintain the flow in presence of pressure. This is known as flow energy or flow work. Now if you give it a name Pressure Energy none of us have any problem, it is the name is up to you.

So if you are happy with giving the name you give that. **"Professor-student conversation starts"** Yes? (()) (10:50) That is the matter of terminology. Usually consider it per unit weight and call it flow energy or flow work, but if you have to be more precise you may call it flow energy per unit weight. It is the sense that is more important. So if you just write it here this is flow energy or flow work, this is expressed per unit weight that one has to remember.

This also you can call as pressure energy, there is nothing wrong in it but one has to understand. **“Professor-student conversation ends”** So this equation is saying that sum total of these 3 forms of energy is getting conserved along a streamline under the assumptions that we made. Now the question is, is this energy been possessed by the fluid or what? To understand that let us consider one example. Say, you are there in an Airport and there is a conveyer belt.

Now the suitcase is put on the top of conveyor belt the suitcase move from one place to another place because of the motion of the conveyor belt. And the conveyor belt just as an analogy, consider it like a fluid flow, so it is like moving the suitcase from one place to another place, is it holding the suitcase anywhere? No. So the suitcase is like something which is put on the fluids it is just like some energy. So you have sum total of these 3 energies that is somehow being there in the flow and it getting transmitted from one place to another. So it never possessed.

Therefore, the Bernoulli’s equation essentially says physically that the sum total of the flow energy, kinetic energy and the potential energy per unit weight remains conserved as it is transmitted from one point to another in the flow field along the streamline maybe that we are considering in this example. It is not possessed it is just transferred. So the flow is just acting like the medium which is not holding the energy.

But which is transmitting or transferring the energy from one place to another place. This is like a sort of statement of conversation of mechanical energy. But we will see that under restricted cases it may also take different forms not that this is the only form that is there for the conservation of mechanical energy. So we have now a sort of clear picture on the significances of different terms in the Bernoulli’s equation.

The next point is that whenever you are talking about energies in different terms you must have a reference. Like when you say potential energy you have a datum with respect to which you are calculating the height z. So this z is not in an absolute scene it does not make any big significance because eventually in this equation g1-z2 that is what is important, and that is independent of the choice of the reference.

So if there are 2 points 1 and 2 say this is 1 and this is 2 so this difference in the height between 1 and 2 vertical heights is that what is important. So if you have this as g1 and say this as z2 with respect to some datum say this is the datum then it independent of the choice of the datum g2-z1 remains the same. But still when you want to prescribe it as in an absolute scene you require a datum or a reference.

For the kinetic energy term there is a velocity and you require a reference for that. So there should be a reference frame with respect to which you are prescribing this velocity. Typically, this reference frame is a reference frame at a risk. So you are writing here the absolute velocity. And this reference frame for pressure is also very important. We have already discussed when we were discussing the statics of fluids.

That you can prescribe pressure also with reference to something, if we prescribe with reference to the atmospheric pressure we call it a gauge pressure. So you can as well prescribe the pressures in the different terms in terms of reference pressure term and then you can substitute it as gauge pressure. Important thing is whatever reference you use for postulating the different term it should be the same in the left hand side and right hand side that is very common and obvious conclusion.

Now we will see that what could be the other variants of this Bernoulli's equation and more fundamentally the Euler's equation. So we will next see the unsteady case. So till now we have discussed about the steady case but let us take the example 2. In the example 1 we considered a steady flow.

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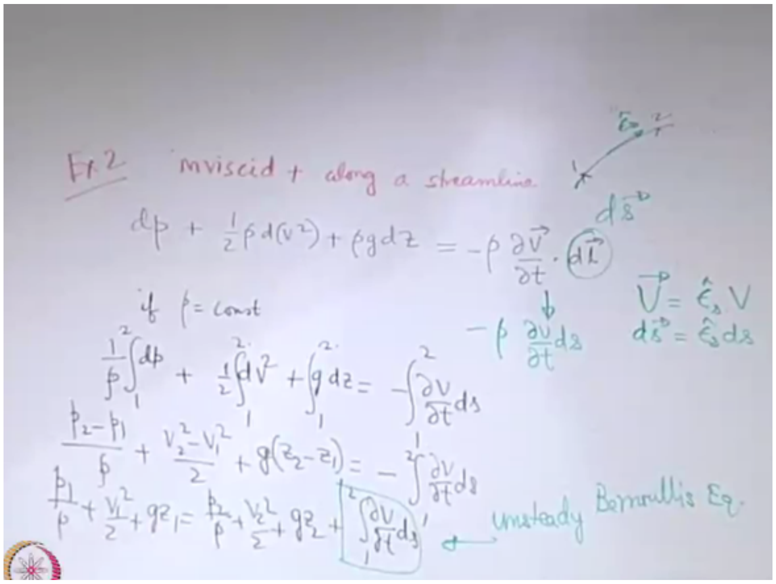

So in the example 2, we will keep all our previous assumptions as valid that is inviscid flow and flow along a streamline but we will not consider it to be steady a priory. So we will consider inviscid + along a streamline. **"Professor-student conversation starts"** (()) (16:38) Rho constant we will take in a later stage. We will start with the Euler's equation form. **"Professor-student conversation ends"**

So Euler's equation form let us try to write that so dp + half Rho d V square + Rho g dz this were the first 3 terms. This was =, now 2 extra terms were there which we dropped off by consideration of steady flow and along a streamline. So now the term which is their along a streamline that is dropped but the term which is there because of unsteadiness that now will not be dropped because now we are considering the unsteady flow.

So what will be that term? –Rho dot dl. Now we have considered along a streamline so our dl is ds where s is the streamline coordinate. We will subsequently write special forms of the Bernoulli's equation using the streamline coordinates, but for the time being let us say that we are considering this as the streamline direction so we call it ds. Now when we are considering the streamline V is already oriented along the streamline.

Because that is the definition of streamline. Tangent to the streamline represents the direction of a velocity vector at each and every point. So if you write in terms of the streamline coordinates so V will be let us say that epsilon s is a unit vector in a stream-wise direction. So V, you can write the magnitude of V times the unit vector in a stream-wise direction. So if this is the streamline maybe this is the stream-wise direction.

And ds again you can write epsilon s * magnitude of the length of the element. So when you take dot products of these 2 you can just write it in a simple scalar form for along a streamline. So you can write this one where these are just magnitudes, keeping in mind that we have written this along a streamline. Now next what we can do, we can take Rho=constant. So this is Euler's equation of motion for a general case where the density maybe a variable.

Density need not be a constant. But if you take the density as a constant then you have dp/Rho+dV square + gdz = -. Let us integrate it from point 1 to 2 along a streamline and if Rho is constant we will take Rho out of the integral when we are evaluating the integrals. So 1/Rho will come out, take the integral from 1 to 2. So we can write p2-p1/Rho p2 square – V1 square/2+g*z2-z1= this one.

You can of course rearrange the terms and write p1/Rho+V1 square/2+g z1 just like the standard one form p2/Rho+ V2 square/2+g z2+1 extra term. So the extra term has now appeared because of relaxing the requirement of steadiness. So now it also can be an unsteady flow, this is known as unsteady version of Bernoulli's equation. We will work out some problem subsequently to illustrate the use of this one.

So what is clear is that whatever term we have dropped because of steadiness now that term has appeared and it is just creating an extra effect and the meaning of this term is quite clear because it gives a variation of effect of the variation of the velocity with respect to time. Now we will take a third example when we consider irrotational flow. So let us take the third example.

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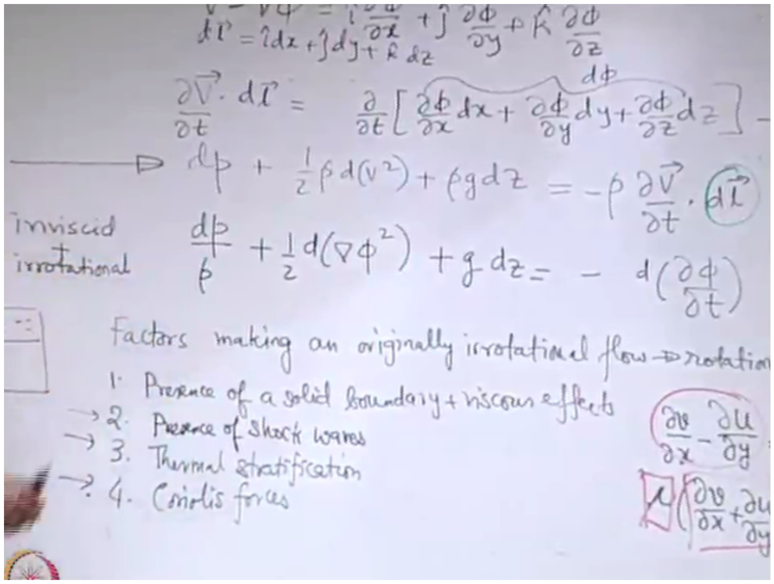

Example 3, Irrotational flow. So when you take the example of irrotational flow let us see that what happens to the equation. So when you have the irrotational flow the thing is that dl we are not writing as ds because when we are writing irrotational flow we are keeping in mind that it points 1 and 2 are taken such that they need not be along the same streamline. So if they need not be along the same streamline what is the consequence?

The consequence is that the equation is just like this equation what is there in this example 2, but we will not substitute dl with ds, we will just keep dl as it is. But when it is an irrotational flow we know that the other term where there were vertices vector that term will become 0. Not only that, you can write V as the gradient of scalar potential which is the velocity potential that we discussed. So we can write, so this is like what this is, okay.

Now, what is dl? So let us try to write what is this expression for partial of V with respect to dot with dl. Now we have to keep in mind that dl need not be along ds. But that is okay because we have considered an arbitrary dl with if x, y and z component, it is not necessary it has to be along a streamline. So V. with a partial derivative with respect to d dot dl, what will be that? So we can see that it is a sum of the 3 partial derivative terms for variations along x, y and z.

So what is this? This is the total change in the velocity potential. You can also write it as sorry, d of, the partial derivative of with respect to t because you have to remember that d and this del operators they are interchangeable mathematically. So it is just possible to consider this in place of this and the underway so you can write this as the exact differential of the partial derivative of the velocity potential with respect to time.

That is one observation. The other observation is that you can also express g directly as a gradient of a scalar potential and therefore you can express this Bernoulli's equation solely in terms of the potential. So you can write as, this is not the Bernoulli's equation; this is actually the Euler's equation form. The state prior to the Euler's equation.

To keep in mind that all Euler's equation is the more general form when substituted Rho=constant and integrated that gives a Bernoulli's equation. So this is the Euler's equation form. Even the Euler equation form can be simplified with that. This is the Euler's equation in terms of the velocity potential, valid under what assumptions? Inviscid flow and irrotational flow. So this is valid for inviscid + irrotational.

There are no other assumptions because it does not require to be along the same streamline and in this stage it is not necessary to make Rho=constant. If you make Rho=Constant and then try to integrate it then that Rho=Constant work as **"Professor-student conversation starts"** Oh sorry. RHS should be divided by Rho, correct. So it does not require Rho to be a constant in this stage. But if you integrate it by taking Rho=Constant it will give Bernoulli's type of form. This is Euler equation type of form. **"Professor-student conversation ends"**

Now you can see that there is some special requirement inviscid and irrotational. Now there is very important and interesting relationship between these 2. Fundamentally we could try to answer this questions. If there is an irrotational flow, is it true that it has to be inviscid number one. Number 2 if it is an inviscid flow is it true that it has to be rotational. Remember these are not very simple questions to answer.

And we will try to look into very basics of looking into these issues. Let us say that you have an irrotational flow. Say there is a free stream which is having an irrotational flow that means it has null verticity vector. Now the question is; is there any agent that can make the flow from irrotational to rotational. So will the rotationality be preserved.

So there are certain factors which can create situations such that an irrotational flow the flow which was originally rotational now becomes rotational. So what are those factors? So the factors, making an irrotational flow and originally irrotational flow to a rotational one. One of the important factors is presence of a solid boundary and viscous effects. Presence of a solid boundary is there in many wall bounded flows.

And viscous effects are common for fluid with some substantial viscosity. If that is there that means even if the flow was originally rotational physically it will not be able to retain its irrotational state, that means although you may start with a irrotational assumption the viscous flow assumption will not hold that irrotational state physically. We will see that mathematically it will not be able to we will not be able to reflect this directly in such an elementary level.

It is possible to look into that mathematically but not in such an elementary level, but physically we have to at least appreciate, that if it was irrotational there is no guarantee that internally it will rotational and the factors which disturb that irrotationality one of the factors is the viscous effect in the flow presence of wall boundedness. There are other factors I am just listing those down not necessary that we will discuss in detail.

One of the other important factors is presence of Shock waves. What are Shock waves? Shock waves are created by situations in highly compressible flows when there is an abrupt discontinuity in the fluid properties. So there is like a way front across which there is a jump in all the properties of flow and that takes place with a condition that across that there is a change in state from a supersonic to a subsonic flow. So a Mac number >1 to a Mac number m <1.

Now I mean the detailing of how Shock waves take place and all we are not going to discuss here because we are entire specialized discussion on compressible flows. But at least we, we will try to appreciate that these are, these are situations where there can be abrupt jump discontinuity fluid properties and those are the those are the situations where originally irrotational flow may become rotational, even if viscous effects are not otherwise important.

Then the third one is a Thermal stratification. Thermal stratification is a like if you have 2 fluids of different densities and maybe that is simulated by a case when you have the same fluid one single fluid but you are heating it up. So once you are heating it up the fluid will become lighter and the lighter fluid will occupy the positions which are higher and higher just because of the density gradient.

So the Thermal stratification means there is a thermally stratified layer that is been created because the density gradient is been created by the temperature. So hottest one are there at the top and cooler and cooler ones are at the further bottom, so you create a density gradient. But the density gradient is not created by changing pressure but created by the change in created by the temperature gradient prevailing in the system.

And that also in a direction oriented against the gravity that is known as Thermal stratification. So if you have such stratified layer then it is possible that makes the flow rotational from irrotational. Then other forces like there maybe Coriolis forces or Coriolis effects can create a rotationality in the flow if it was originally rotational. So you see like the earth when it is rotating it has the Coriolis effect and if you consider the ocean currents,

…so there are rotationality in the ocean currents which are we dominantly created by the Coriolis effects, so from even if it was if the earth was stationary that is hypothetical case to think. It might be possible that, that was irrotational but because of the Coriolis effects being present that is converted to a rotationality effect. So there are many factors, these things just show that these are very natural factors these are not any artificially imposed factor on the system.

And these natural factors have a tendency to create a rotationality in the flow. So we cannot ensure that if we have a irrotational flow as a reference case or as a undisturbed flow that will remain as irrotational. But if it is inviscid and then if we consider that the effects 2, 3 and 4 are not there in assistance then if it is inviscid and irrotational originally it will remain irrotational forever.

Because let us say that effects 2, 3, 4 are not present only effect 1 is present. The presence of solid boundary still will not be able to create, create a rotationality if the rotationality was not originally there. Because the message that the solid boundary is there cannot be propagated through the fluid, viscosity is that messenger which propagates the presence of a solid wall into the fluid.

So if the viscous effects are not there the fluid will be dump in responding to the presence of the wall and then flow which is originally rotational will retain its irrotationally, that is one of the very important for understanding. If you look into this mathematically you see that you may lead to know where because if you have a irrotationality. Let us consider a 2-dimensional flow, so if you want irrotationality then you must have the angular velocity in the plane that should be =0.

That is irrotationality. 2-dimensional irrotationality. And inviscid flow, what is the requirement the requirement is that effectively requirement boils down to the shear stress is not there, that is the net effect that is there because of the viscous effect so for a Newtonian fluid it is Mu* this one, this is 0 shear stress. Now you can clearly see that there is no relationship between these 2 if you ensure that this is 0 this is not ensured to be 0.

Until and unless these 2 terms are individually 0. We have seen such an example where you have a fluid element which was originally of a particular orientation it does not change anything angularly it just gets stressed along one direction and reduced in length along other direction that example we saw in the previous class. But that is a very special case. In general, if you have a irrotational flow so you have terms a-b=0.

That does not ensure that a+b=0, until and unless a and b are individually 0 that is a very special case. So you are relying on what? You are relying on having Mu identically =0 to have inviscid and irrotational flow. Physically sometimes it is not a very absurd way of looking into things.

Though mathematically you cannot ensure that. See Mu is a fluid property, so if you have an irrotational flow still physically it is possible to have a viscous effect.

Because the flow is likely to have a viscosity, this term is not =0, so you can have a viscous effect but irrotational flow, irrotational flow is also called as a potential flow because velocity potentiality exists in irrotational flow. So that type of case when these Mu is not 0 this term is not 0 but this term is 0 that is that can be called as a viscous potential flow. It is mathematically very much possible, nothing is denied that.

But if you just look into it in a bit more physical terms, what is an -- what is the origin of the thought of a irrotational flow. We found that it is a conservative velocity field. Because the velocity because the field vector field is conservative we could write it as a gradient of a scalar potential. Now when you have a conservative field physically it means that there are negligible dissipations in the system.

Just like if you have a conservative field as gravity so if you think of a conservative force, force field in the particle mechanics you neglect the effects of friction because friction will no more keep the force field as a conservative one. Now if you think of the velocity field, the velocity field is not exactly like a force field but you may think it analogically because it is also a vector field.

So in a velocity field what could create a disturbance in a conservativeness is the presence of a dissipation and that dissipation is to the mechanism of viscosity. So if viscous effects are strong then physically it may not help in retaining the flow field as a conservative field. So physically it might be very common that if it is in irrotational if it remains if it is wants to remain irrotational it has to be inviscid.

Because viscous effects will create sort of dissipations in the flow just like what friction does in a force field, so that is one important conceptual thing that we need to keep in mind. So as we were discussing that is not very straightforward to give the answer to this question that is irrotational and inviscid are there relationship between these 2 but I hope you have now some kind of physical picture on this understanding.

We will now workout maybe one problem which will be based on the concept of say the Bernoulli's equation that we have discussed. There are many applications of the Bernoulli's equation and we look into some of the important applications in today's class and then maybe in the next couple of lectures. But before we work out problem which is not based on a very common application but it is still not a bad example.

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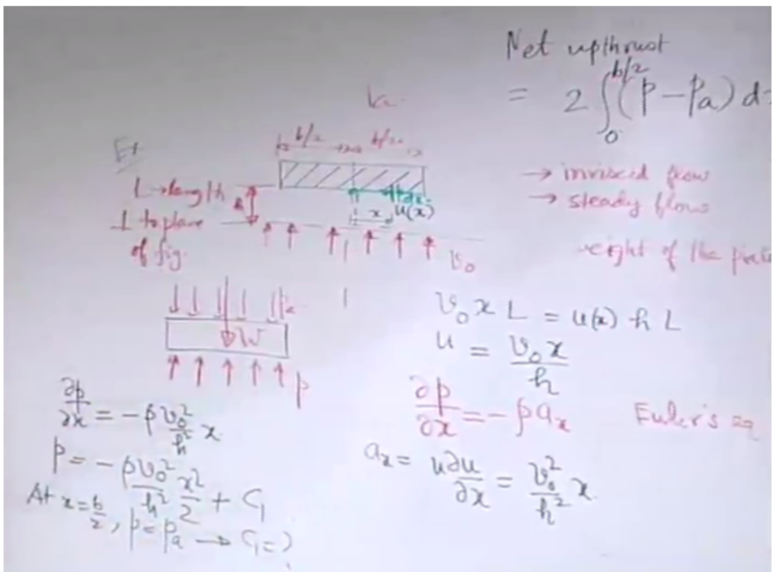

We refer to this example as such to work out a problem in the context of fluid kinematics. There is something like plate a rectangular plate and there is a bottom plate there are holes in the bottom plate through which fluid air is blown like this. If you recall, you walked out such a problem. And the velocity-- we will continue with that problem and try to work out a different problem on this.

So we have a uniform velocity say we not with respect to which air is entering through this force. And let us have a coordinate system like this were this is symmetrically located with respect to the plate so let this be b/2 and b/2 which are the half of the dimensions. Let us say that L is the length of this plate perpendicular to the plane of the board. So L is length perpendicular to the plane of the figure. The gap between these 2 let us say the gap is h and our assumption is that it is inviscid flow. And let us say steady flow.

We are interested to find out what should be the weight of this plate to keep it in such a position. Okay. Weight of the plate pot. The density of the fluid is given. Classical design example we have discussed about this but just to iterate say this is an electronic chip you want to cool it by blowing air because it has become hot with heat generation because of the electrical effects. Now you are blowing air there is a because of this because of the air velocity it has come to a flotation effect and it will come to a equilibrium height where it will remain stable based on each stage.

So this is the height then what is the weight of the plate here you consider as a chip. So what is the weight of that. It is not a very absurd question and it might appear to be a very absurd question in that flow field these are given now what is the weight of the plate or weight of the chip, but once we look into it carefully we will find it is not very absurd it should follow from the basic considerations. So when we do that the first thing is we need to find out how velocity varies.

Because for anyone any type of calculation that we have seen involving the kinematics or even the dynamics of flow the velocity field is very important. So from this given consideration we have to find out what is the velocity field. So if you recall that we earlier considered like at a distance say x from one end and we found that what is the rate of flow entering and rate of flow leaving this control volume is marked by the dotted lines.

So the rate of flow that was entering is V naught * x now the length perpendicular to the plane of the board is L. This a volume flow rate and what it leaves here let us say u is the function of x because of the assumption of the inviscid flow u does not vary with y. So you can take just u as a function of x*h*L. So it is as good as writing A1 V1 = A2 V2. If you write A1 V1= A2 V2 what are the assumptions under which that is valid?

I am going to hammer this on you again and again and again, because many times you have use this without keeping in mind as assumptions. So let us write A1 V1 = A2 V2. What are the assumptions in which these are valid? So 1 and 2 are the 2 sections that we are looking for over which we are having equivalent constant velocities V1 and V2, so if they are not constant these have to be replace by the average velocities over the sections.

But there are even more important assumptions which are inbuilt here. What are those? If we put say Rho A1, V1 = Rho A2, V2 where Rho 1 and Rho 2 are the densities at the sections 1 and 2. Let us say these are average densities over the sections then what is the requirement under which this will be valid. See you have to keep in mind why do derivations in the class you have to keep in mind how this was derived.

This was derived by dropping the unsteady term in the continuity equation and integrating the remaining terms in the continuity equation that means the only assumptions was it was unsteady flow, a steady flow so unsteady term goes away. So when it is steady flow it need not be constant density so you can write it still in this form. If Rho 1and Rho 2 are the same then it becomes A1 V1 = A2 V2.

So it has 2 assumptions one is the steady flow and other is Rho is a constant because you have canceled or maybe whatever function of Rho is there in the left hand side same function is there in a right hand side. Say Rho is a function of something else say time. So left hand side and right hand side so same functions so they are canceled out but Rho cannot be a function of time because you already considered a steady flow, so it cannot be a function of time.

So it is just like a constant which is in the left hand and the right hand side. That is how these 2 got canceled. And when we say Rho=constant the other thing again I am going to hammer on you, keep in mind Rho=constant is a special case of incompressible flow. But incompressible flow does not require Rho to be a constant. This is often like even in some of the best of the textbooks this confusion is retained.

So we will see that when assumptions are taking for the problem it is written that incompressible flow. Well, incompressible flow can be handled without requiring Rho to be a constant. So whenever we consider Rho to be a constant we specially, specifically we say that Rho is a constant that is what our assumption. Incompressibility is not good enough to ensure that Rho is a constant but if Rho is a constant it has to be incompressible. Okay.

Now, so this is something that we have already derived. And let us write the velocity u as a function of x so that is h, sorry V0 x/h. When you have this V0 x/h then it is possible to find out the acceleration which we earlier found out but we have to see what is that, that we want to find out, so we have to make a strategy for solving this problem. We know what I u and you also know that is V.

We have found out V by using the continuity equation, that also we did in the previous example of similar type that we walked out. But we will not concentrate on finding out V we will concentrate on finding out a strategy for solving the problem. See what are the forces which are acting on this? So there is a, so if you consider a sort of free body diagram for the plate or if you want to think it as a chip.

So there is a pressure distribution from the bottom, there is a pressure distribution from the top which is because of the atmospheric pressure. Let us say that it entirely surrounded in a uniform atmospheric pressure which says p atmosphere which his along all the sides except the inside part. Now because of this difference in pressure, let us say this is p so if p is > p atmospheric pressure there will be a up thrust on it.

And that should be balanced by the weight to keep it equilibrium. That means if we find out what is the resultant force due to pressure on this chip that will give us an insight on what is the weight because then we can use the conditions for equilibrium. To do that, what we will do, we will find out how pressure varies with x. So how pressure varies with x? We have the Euler's equation of motion along x.

So what is the Euler's equation of motion? That was the Euler's equation of motion along x Euler's equation along x. And acceleration along x, acceleration along x is only one term will be there the other terms will be 0 because u is a function of x only. So you can write, so let us say that we want to integrate this expression from x=0 to let us say x=b/2 and it will be similar in both the sides so 2* that will be the total integral to get the force.

So let us say that at a distance x we take a small strip of dx and we are interested to find out what is the pressure on this strip. So if we integrate along this so we are integrating with keeping y=h which is the constant. So the other component of velocity has no affect because of no penetration V0 here. So we can integrate along this surface so we will have p=-Rho V0 square h square * x square/2+ let us say some constant.

The constant in general could be function of y because it is a partial integration with respect but we have already fixed y as y=h. So let us say that it is a constant C1. If we, how can we find out this? We know that at x=b/2; p=p atmosphere. So at x=b/2 p is p atmosphere. So from here you can find out what is C1 by putting this boundary condition. So then you know p as a function of x. So what is the net up thrust that is p upward from the bottom p atmosphere from the top.

So p-p atmosphere dx that is the total force acting on the element of thickness ds and length perpendicular to the plane of the figure L so that is area on which it is acting. 0 to b/2 is half of that 2* that is the total force, right. So this must be = weight for equilibrium. Okay. So remaining exercise is very straightforward. See eventually when you find out C1 you will get it in terms of p atmosphere.

So will get an explicit expression of p-p atmosphere from this equation, which you substitute here and integrate is the very simple polynomial integration, so that will give you the weight of the chip or the plate. So this is a simple illustration of the use of the Euler's equation, we will in the next class we will use the Bernoulli's equation for solving some other problems. But before that in the next class what we will do we will just create a small introduction for that.

We will write the Euler's and the Bernoulli's equation in terms of a different coordinate system, that is a streamline coordinate system. So why we are interested to write it in a streamline coordinate system because we know that along a streamline under certain conditions these equations are valid. So if we write it for 2 points along a streamline it may be very convenient if we use the streamline coordinates.

So we will just briefly see that what are the streamline coordinates and what how they are related with the other coordinate systems. So let us just consider the streamline coordinates. So the streamline coordinates are like this.

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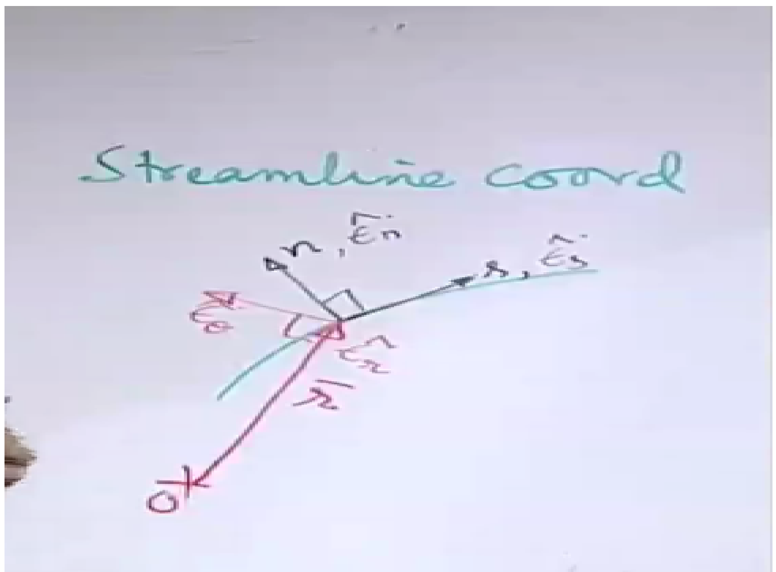

So if you have a streamline we consider tangential to the streamline as s and normal to the streamline as n. Okay. Many times there is confusion between these coordinate systems and the coordinate system that is used in a cylindrical polar coordinate system. So we will try to avoid that confusion from the very beginning. Say, if you are using a polar coordinate system. So if you have this as the origin of the pole then what how the coordinate is represented?

It is represented by one radius r, this is the radial direction and perpendicular to that theta direction. So we have unit vectors along this epsilon r and epsilon theta. And unit vectors along this as epsilon s and epsilon n, both of these are orthogonal system. But you have to keep in mind that they are not the same. Many times there is a confusion many times I have seen students calling this as the radial and this as a tangential direction, no.

You can clearly see that this is not a tangential direction, only for a circular geometry normal to the radius is a tangent, but not for all types of curves. So this is fundamentally called as radial and cross-radial direction. So epsilon r is a unit vector along the radial direction, epsilon theta is the unit vector in a cross-radial direction which is perpendicular to that or orthogonal to that whereas these are sort of tangential and normal directions.

So you should not confuse between these 2 coordinate systems, and we will write the Euler's equation of motion in this streamline coordinate system basically, that we will do in the next class, let us stop here today. Thank you.