Introduction to Fluid Mechanics and Fluid Engineering
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Lecture - 16
Dynamics of Inviscid Flows

Today, we are going to start with our discussion on dynamics of inviscid flows.

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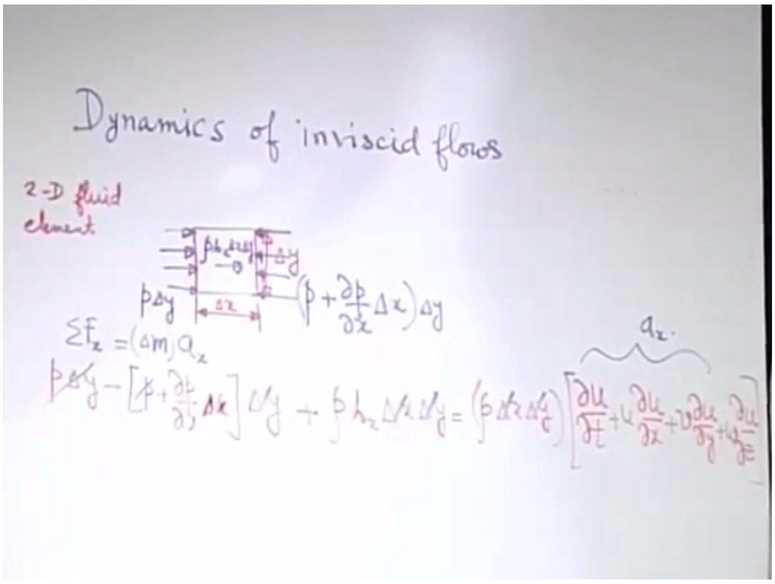

In the last chapter, we were discussing about the kinematics. So we were not discussing about the forcing parameters, which are involved to influence the flow. So we have discussed about the motion. Now we are going to see that what are the forcing parameters, which influence the motion and how they are related to the motion. When we talk about inviscid flows what we essentially mean is initially we will discuss about cases where viscous forces are not present.

So it is a simplified situation of the reality but at the same time, it will provide us with a lot of important insight, which we will use later on when we will be discussing about the dynamics of viscous flows. So when we will be considering or focusing our attention in this particular chapter, we will be considering cases when viscous forces are not there or negligible. To start with a discussion on this what do will try to do?

We will try to write the equation of motion for a fluid element where viscous forces are not present. So when viscous forces are not present, the kinds of forces which are there are the surface forces in terms of the normal components, which are manifested through pressure and some body forces which maybe like the gravity forces. Keeping that in mind let us say that we want to write the equation of motion for a fluid element.

Let us say that it is a 2-dimensional fluid element, it need not always be 2-dimenisonal but if we are writing the equation of motion along a particular direction, then like for simplicity we can take it as a 2-dimenisonal one for illustration. So let us say that we take a 2-dimenisonal fluid element as an example, fundamental it is always 3-dimensional so the third dimension you may consider as 1 or some uniform third dimension.

Let us say that these dimensions are delta x and delta y. We will quickly identify what are the forces, which are acting on the fluid element only along x. So we will identify forces along the x because we are interested to write the equation of motion along x. So other forces we will not show, so it is not a complete free body diagram, only the x component of forces will be shown. So here you have force due to pressure.

So if p is the pressure here then p times delta y maybe times 1 where 1 is the width is the force that acts on the left phase due to pressure. Force that acts on the right phase due to pressure is what we have encountered such situation earlier, so p+this*dx times delta y*1. Along x these are the only surface forces because other phases will have surface forces along y, body force maybe there.

Let us say that bx is the body force per unit mass, so rho*bx*delta x*delta y is the body force component along x because rho*delta x*delta y is the mass of the fluid element. So we can write the Newton second law of motion for the fluid element. That is we can write resultant force along x=mass of the fluid element times acceleration along x, maybe you can write delta m, it is a small mass to acknowledge that.

So we will try to simplify this expression. Resultant force along x is p*delta y-p+this one with respect to*delta y. This thing then+rho bx delta x delta y=the mass of the fluid element is what? rho delta x delta y times acceleration along x. What is acceleration along x? This we have discussed in the kinematics. So what is that? This is the acceleration along x. This we have derived in a kinematics.

See when we were discussing about the rigid body type of motion of fluid elements, then we did not use this expression, we were using an expression as if the entire fluid is having a particular acceleration, this regarding the deformation within it. So now the different gradients of velocity will become important, which was not there or which we kept ourselves abstracted off when we just wrote some acceleration when it was moving like a rigid body.

Now we are more detailing it, so we are looking into the detailed expression that reflects that acceleration. So this is acceleration along x. Now you can cancel various terms. So first this term will go away and then like you will have okay let us just correct it a little bit it was delta x right, we did not consider it delta x, so just change this dx to delta x because we took our element as delta x.

And then we cancel delta x*delta y from all the terms because these are small tending to 0 but not actually=0.

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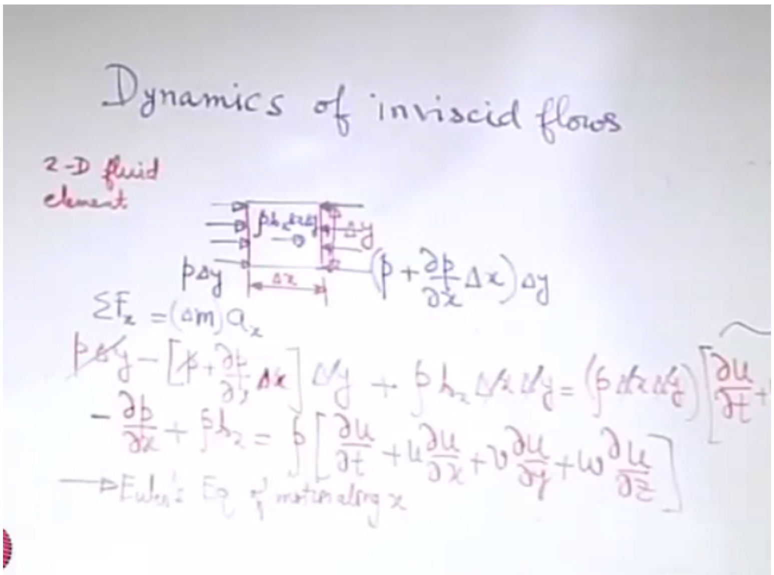

So what we are left with we are left with the simplified expression+rho bx=rho. This very simple expression is also known as Euler's equation of motion along x. Similar expressions we can write for the motion along y and z. We are not repeating it because it is very trivial. Now what does this equation of motion contain, if you look into it, it is fundamentally like Newton second law of motion where viscous forces are not considered.

So this right hand side is something like the mass*acceleration. Left hand side is the effect of the force which is acting. So one force is because of the pressure gradient and another force is because of the body force. So these 2 forces are considered, so it is just a different way of writing Newton second law of motion for a fluid where viscous effects are not present and any other force other than this body force of this particular form we are not considering.

Let us take an example to illustrate that how we can make use of this.

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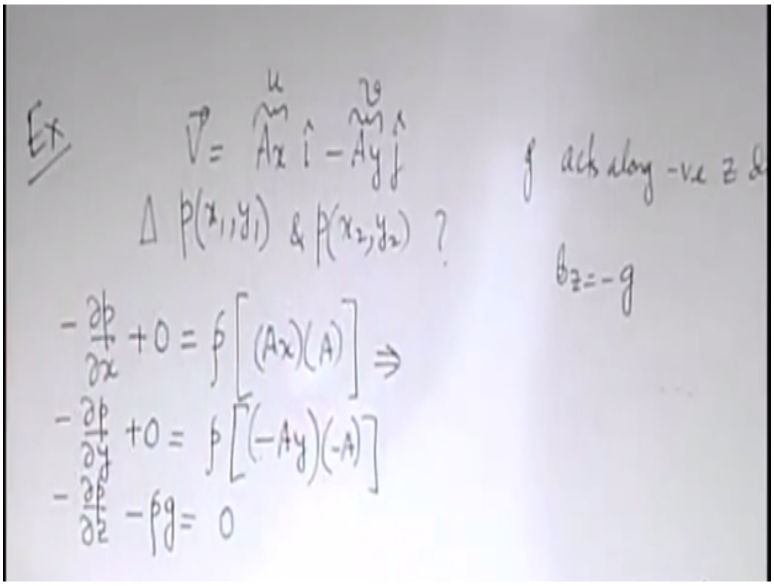

The example is like this let us say that you have a velocity field v given by say Ax i-Ay j where A is a constant number and the dimension number to adjust the dimensions in the 2 sides. We are interested to find out what is the difference between pressure at 2 points given by x1, y1 and x2, y2. It is given that g that is the acceleration due to gravity acts along negative z direction.

So the question is what is the difference in pressure between these 2 points okay. The problem is very simple but it will at least give us some idea of how to make use of this expression. A and B are not functions of time, so it is a steady flow field. Let us write this equation say along x for this one. So if you want to write this Euler's equation along x, so we have – of partial derivative of pressure with respect to x+what is bx?

There is no x component of body force. Body force only acts along negative z, so this+0=rho. Velocity field is not a function of time here; A is a time independent constant that is given. So the time derivative will be 0. Then so what is u and v here? This is u and this is v okay with a –sign of course includes the –sign. So rho so u is Ax*A that is this term, the other terms are not there because u does not have any dependence on y and z.

So the other terms are not there, so this is the equation of motion along x. What will be the equation of motion along y? Just it will be similar to this. There is no body force along y and rho, right hand side what is going to happen, this u is only going to be replaced with v. So the term that will remain relevant is only v*partial derivative of v with respect to y okay. So it is –Ay*-A.

Let us consider the z component. Now you have a body force along z. So what is that? So Bz=-g so –rho g=the right hand side u will be replaced by w and there is no w component of velocity, it is a 2-dimesional flow field so it is 0. So it is possible to integrate these expressions to find out how p varies with x, y and z. So let us integrate that let us say we integrate this one with respect to x.

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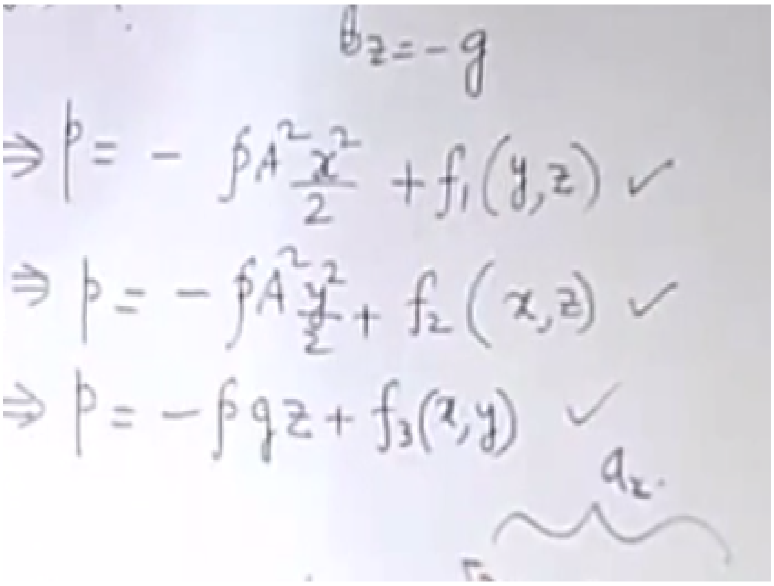

So we get p here as what –rho A square x dx will become x square/2+function of y and z right. For this it will become p=very similar –rho A square now y square/2+a function of x and z and what will this give? p=-rho g z+a function of x and y. All these 3 expressions are representing the same pressure field. So we can compare these to get these 3 functions. So let us compare this and get the 3 functions.

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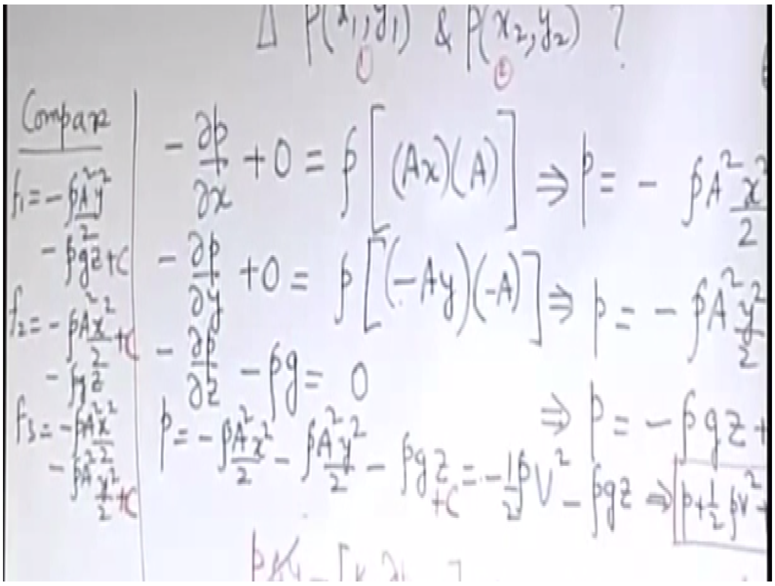

If you compare what functions you get, what is f1? F1 is a function of y and z. So –rho A square y square/2 then –rho gz, this is f1. F2 function of x and z, so –rho A square x square/2-rho gz and f3 –rho A square x square/2-rho A square y square/2 okay. So the expression for pressure becomes –rho A square x square/2 rho A square y square/2-rho gz right. So this you can write -1/2 rho A square x square+A square y square is u square+v square.

So that is the square of the resultant velocity. So let us say capital V square-rho gz that means p+1/2 rho v square+rho gz=0. Now when we write this, there is a lack of generality here. What is the lack of generality? When we considered this f1, f2, f3 we did not consider a constant. So fundamentally we should also have a constant there where that constant maybe eliminated depending on a choice of a reference frame but that is not done priori.

That is after you get the general expression then only that is possible. So a very important thing here is that each of these should be augmented with a constant. So what it means is that this+c will be there. So this will become in place of 0 it will become some constant. This looks very familiar to you; it is like a Bernoulli's equation. Now do not get a wrong impression here that the Bernoulli's equation is always valid and that is why you can write it in this form.

There is a specialty of this problem because of which the Bernoulli's equation gets valid between any 2 points 1 and 2. So this is point 1 and this is point 2, solving the problem is trivial you can find out p1-p2 while substituting the velocities respectively at x1, y1 and x2, y2 that is a straightforward exercise. **“Professor - student conversation starts.”** Yes, the direct comparison of the equation is possible because they represent the same pressure.

This is by observation, see I mean when you write, when you say that these two are equal or these three are equal, it should be such that it does not contain any function of x, y, or z which falls beyond the functions written here right. I mean there are certain things which you can do just by common sense and this is one of the big things in mathematics which you can do by a little common sense and that is what is expected when you solve such problems. **“Professor - student conversation ends.”**

Now when you come to this conclusion that it is like a Bernoulli’s equation in fact it is of the same form and we therefore can apply it between any 2 points 1 and 2. It is not a general conclusion that we must remember and we will do it vigorously to show that when it is valid and when it is not valid. This is very, very important because all of you are very, very habituated in using Bernoulli’s equation anywhere and everywhere you like.

So we will try to see we will try to restrict you so that you do not apply it anywhere and everywhere and we will see that when it is applicable and when it is not, but before that this problem at least tells us that this is a very easy problem and it demonstrates that in this case it is possible to apply it between 2 points 1 and 2. So what is the specialty of this problem? Let us look into it.

See for every problem there is one aspect that is how to solve a problem that is fine but there is a greater aspect how to develop a more detailed insight on what the problem is about. So we are now trying to do that, problem is solved, but it is not enough. Let us see that what insight it gives us. Try to find out what is the rate of deformation and angular velocity of this flow.

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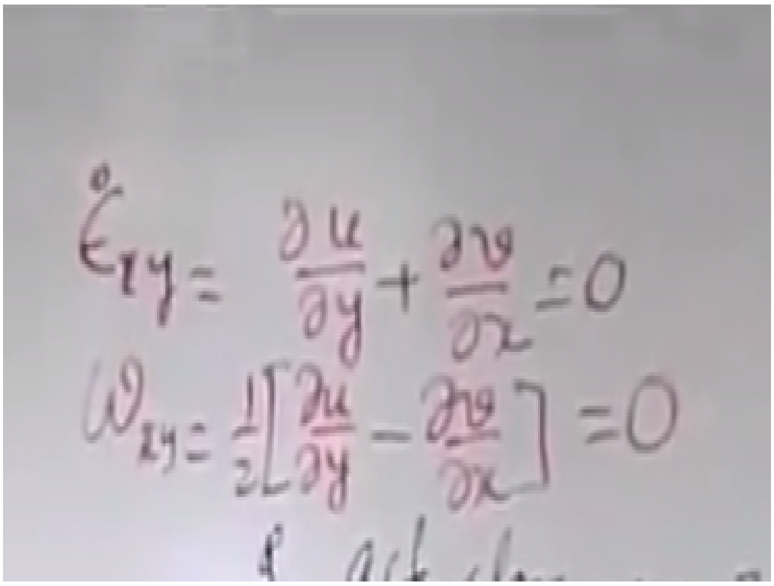

So if you recall that if you want to find out the say rate of deformation epsilon dot xy, what is that? So in this case what is the value of this? Identical=0, angular velocity in xy plane, 1/2 of this one because each of this terms are 0, you have this also identically=0. So what does it show? It shows that if there is a fluid element located in this flow field, it does not have any shear deformation, it does not have any rotation.

That means it its edges were originally parallel to x and y, those will remain always parallel to x and y. If it is incompressible what will happen? It might stretch along say x, so it should reduce its length along y, so that the volume is preserved.

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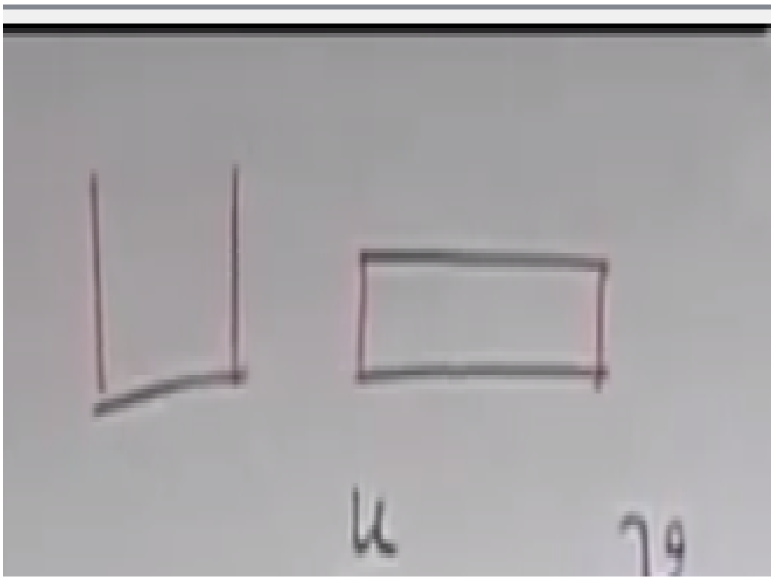

So it is like if the fluid element was originally like this, maybe it will become once like that and it will change its configuration in such a way that angularly there is no change. Only there are changes in linear dimension but ensuring incompressibility because it also represents an incompressible flow field. That you can check by checking the divergence of the velocity vector is 0, so it is an incompressible flow.

So that is one important observation. So the important observation is it does not have any shear effect. Next is it does not have any angular velocity, so it is like an irrotational flow because irrotational flow has no angular velocity or no vorticity so to say. Now let us try to see that what will be the equations of the stream line in this case. So we are interested to find out the equations of the stream line.

It will give us even a deeper insight and we will relate it to one of the movies that we saw in our previous lecture.

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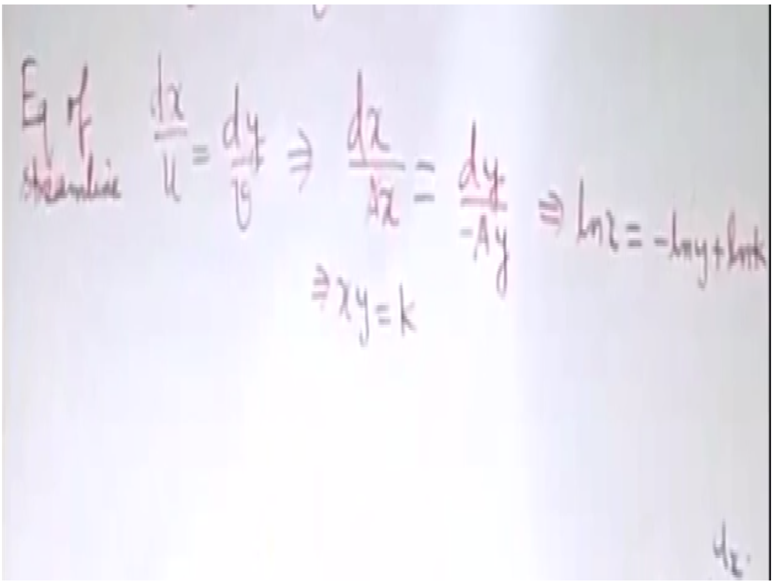

So if you write the equation of the stream line it is dx/u=dy/v right. This is the equation of the stream line. So you have dx/Ax=dy/-Ay A is !=0. You can cancel that and if you integrate it you will get ln x =-ln y+a constant let us say ln k. So this gives an equation of the stream line of the form xy=k, which is like a rectangular hyperbola.

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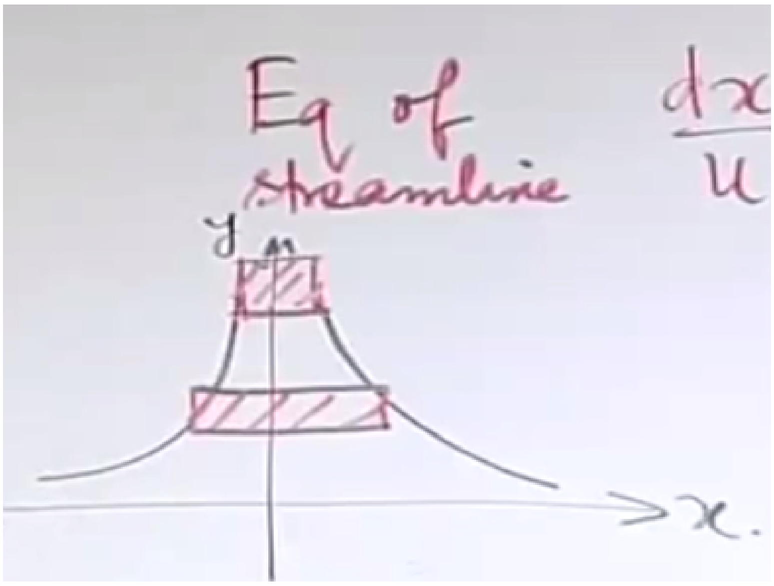

That means if you have say if you consider this as x axis and may be this as y axis, it is possible that you have your stream lines in this way. So if you have a fluid element originally like this maybe the fluid element is coming down along the stream line. I am just trying to make you recall one of the movies that we showed to you that the fluid element is coming down like this with no angular change, no rotation, no shear deformation but only the lengths of the respective edges are getting altered.

So it is a case of pure linear deformation, no angular deformation okay and then in such a case we have 2 things satisfied, one is there is no effective viscous effect because the viscous effect comes through what? Viscosity* the rate of shear deformation. So if the rate of shear deformation is 0, it does not matter whether viscosity is 0 or not. So inviscid effect is not always through viscosity=0.

It maybe the rate of shear deformation=0 because eventually we are interested about whether the shear stress is 0 or not. So if the shear stress is 0, it does not matter whether it is 0 because of mu=0 or because of rate of shear deformation=0. Here it is 0 because the rate of shear deformation is 0. So it does not have any effect of viscous shear, it does not have any effect of rotationality.

So for it is effectively like an inviscid and irrotational flow and for such a flow, we will show later on that you can apply Bernoulli's equation between any 2 points in the flow field. This regarding where they are and we will now go into the more vigorous way of establishing this very important consideration. So to do that what we will do we will leave this example and go back to the Euler's equations of motion along the different directions.

So we have written the Euler's equation of motion along x, which is there in the board. Similar equations are there along y and z. Now what we are interested to? We are interested to write or to find what is the difference in pressure between 2 points.

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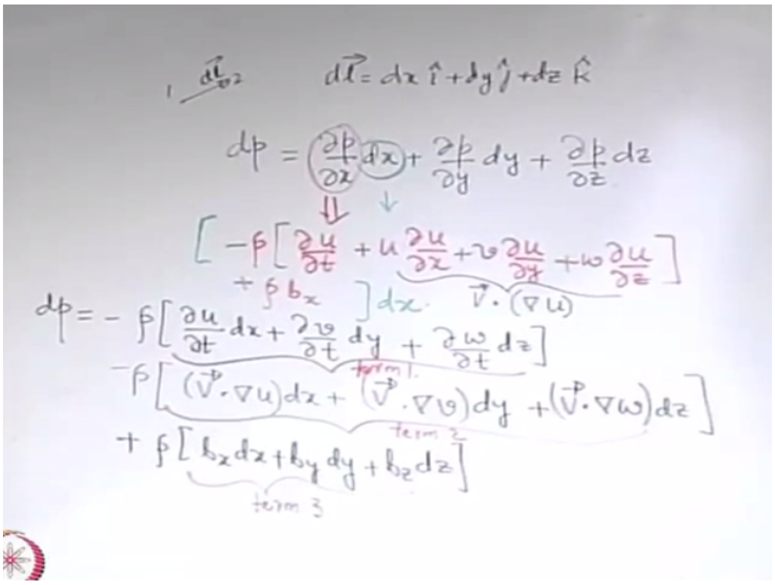

So let us say that we have 2 points, 1 and 2 which are quite close so that they are connected by a position vector dl, dl is given by dx i+dy j+dz k okay. So this is the position vector that we are looking for. What is our interest? To find out what is the difference in pressure between points 1 to 2. So whatever we did in the previous problem a bit more informally we will now generalize it for very general case that is what happens in that case.

To do that we will note that if you want to find out the difference in pressure between the 2 points, here pressure is a function of what x, y and z. So you can write this as the sum of these 3 partial derivative terms. Each of these terms we can substitute from each component of the equation of motion. So the first term you can substitute from the x component of the equation of motion, which is written below. So let us write that, so this will be – of.

So this you are writing now the + of this one, so that means this term will become – the right hand side. So that will become – of rho. Then + a body force, so +rho bx dx, with dx multiplication we will come separately. We are just isolating the dp dx term, so we are not writing the dx together with this. So if you also consider the dx term together with this then it will be the entire thing multiplied by dx.

Now we will try to write it in a compact form because like it is possible to utilize some of the very well-known identities of vector calculus to simplify it. So what we will do is we will write this particular term in a vector calculus notation. So we can write this as v dot with gradient of u right. So then you will get these terms. Keeping that in mind that other terms will also give similar expressions like what we will change for the second term in place of this u it will be v, in place of this u it will be v, in place of bx it will be by like that.

So it is very, very analogous and we can write the general expression for dp as –rho now we will collect all the terms, we will keep all the terms of similar type together. This is one term, then next we will write that acceleration term, that is the convective component of the acceleration term is the temporal component. Then –rho and the body force term okay. So these 3 types of terms are there.

Just for the writing convenience, we will call it term 1 and there is a logic behind that these terms are containing expressions of similar nature. So we can simplify them in groups. Let us write or let us try to simplify terms 1, 2 and 3 separately. So we will do that keeping in mind that the term 1 is the transient term and when we are simplifying we will be keeping in mind that we will be utilizing the vector dl which connects the 2 points, which are closed to each other.

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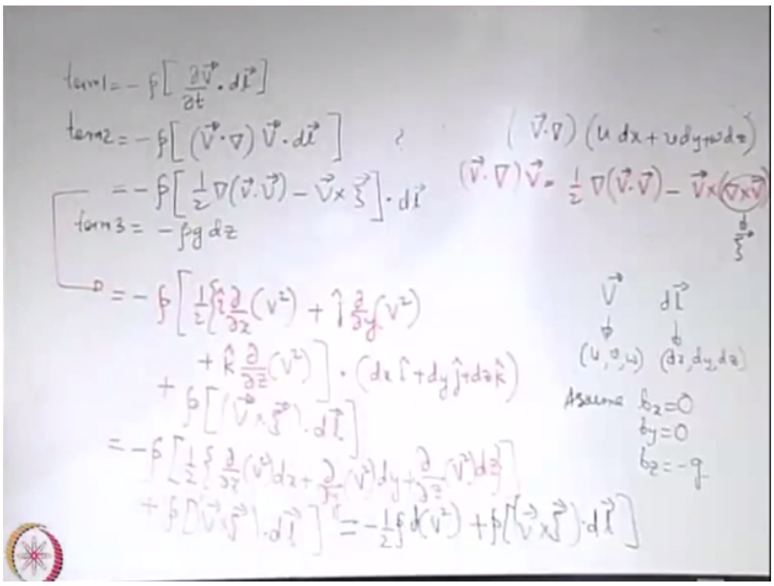

So the term 1 is –rho. Now can you write it in terms of the 2 vectors v and dl. Remember v has components u, v, w; dl has components dx, dy, dz. So if you write for example like this dot with dl then that expression and this is the dot product of this with this of course you have a partial derivative of that one. So it is just writing the same expression in a vector form. Let us write the term 2.

How do you write the term 2? –rho again let us try to write using the vectors v dot. Let us check whether this is alright or not. See and then we have to keep in mind that this is a scalar term right. So first of all whatever is a vector operator it should give back a scalar so you have one dot product and a dot product and the product of that is expected to give back a scalar.

You can just check, let us check that, so you can write this as v dot del then u dx+v dy+w dz. So it becomes of a same form as that given by the term 2. Now it is possible to simplify the v dot del v using a vector identity what is that? So v dot del v=one important thing we will see that whether the bracket is to be put here or after the v and still this is not complete. So let us tentatively write it.

This is a very well-known vector identity. Now you see that what is this? v dot v is a scalar. The gradient operator operating on it makes it a vector and this is very clear, this is a vector. So this should be a vector. So when you have v dot del, this is a scalar but this being a vector it keeps it a vector. So whenever you write an identity, these are certain common sense things that you should check.

Because depending on what you operate the same thing may look may become a scalar and vector very easily depending on how you put your cross products and the dot products. Now why we are putting in this particular form is because here you get the vorticity vector and we were finding out that the condition of rotationality or irrotationality has some influence on the pressure difference between the points.

And this vector solely is responsible for whether it is rotational or irrotational okay. So we will put that simplification here. We will put this as –rho 1/2 in place of the curl of the velocity vector will write the vorticity then dot dl. For the term 3, what we will assume? It is again a very general term, but we will assume that the gravity is the only body force which acts along the negative z direction as we considered in the problem that we discussed just before this.

So what we will assume that bx is 0, by is 0 and bz is –g because that is the common thing that we encountered in many problems but if there are other components of body force you know that how to simplify like you can just put the corresponding components here. So then that will become term 3 will just become –rho g dz. Since it has just only one scalar component, it is not useful to write it in a general vector form.

It will not give us back many things, so dp is sum of term 1, term 2 and term 3. We can simplify the term 2 and term 3 further let us try to simplify the term 2 one more step. So –rho let us now consider the dot product of this with dl so 1/2 what is v dot v? v dot v is v square where this capital V is the resultant velocity. So that we are writing this dot with this one sorry that is the first term.

And you also have a term +rho v cross vorticity vector dot dl. You can recognize that it is like a scalar triple product of 3 vectors like A dot B cross C okay. So we will keep that simplification for a moment and just consider the first term. What does the first term look like? 1/2 that is the first term of the term 2 and then rho. You can clearly see that the first term of the term 2 will become what? It is like it will become d dx of v square.

It is a sum of the 3 partial derivatives will give the total one. So this will become at the end a simplified form-1/2 rho d of v square. So this is like not d dx just the total d, so this is partial derivative with respect to x*dx, this is partial derivative of y*dy and that with respect to z*dz. So that is given the total d+the whatever term that is remaining. Now let us put back all the terms together in the equation.

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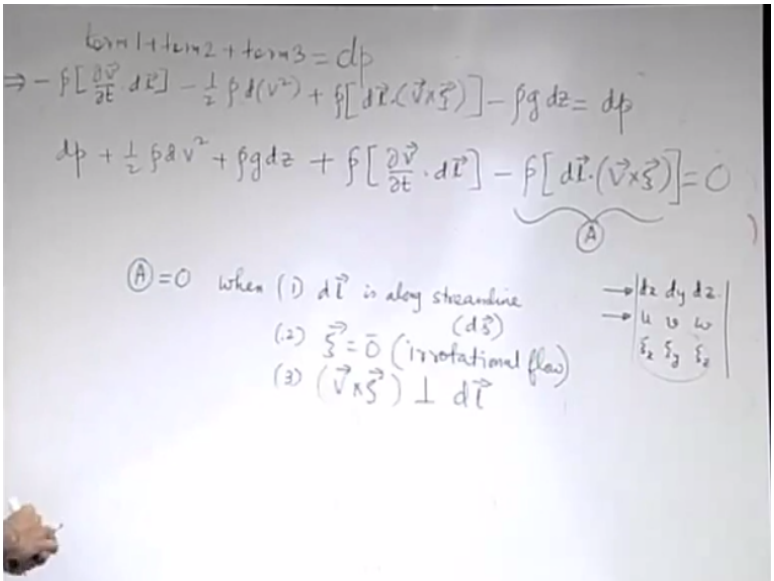

So what is our equation? Our equation is term 1+term 2+term 3=dp that means –rho that is the term 1. So let us sorry dp not 0, it was dp. Then term 2, in place of term 2 we will write -1/2 rho dv square+rho may be let us write dl dot v cross zeta that is the term 2 and term 3 is –rho g dz=dp. This is the compact form and it is possible to simplify it even for the based on certain special cases. So what special cases we will be interested in let us see.

So what special cases maybe let us take all the terms in the same side. So you have dp+1/2 rho dv square+rho g dz, up to this you can find some similarity with the Bernoulli's type of equation that you have encountered earlier but you are getting also some 2 extra terms. Let us write those extra terms. So + then –rho this = 0 because these 2 terms are like beyond what you have encountered many times, we will try to put more attention to the last 2 terms.

We will put the first important attention on the last term because this particular term in a case when it is a steady flow, this trivially goes away. So there is no big controversy or there is no big uncertainty in that that is quite understandable but the last term there are many possibilities when the last term can become 0. What are the cases?

So if you just write it in a determinant form, when you are having such a scalar triple product, you can write it in terms of determinants where each row of the determinant will represent the components of the vectors taken in the particular order. So you have like dx, for dl you have dx, dy, dz; for v you have u, v, w; for the vorticity you have okay. Now let us consider a case when this say rho 1 just look it into mathematically say rho 1 is a scalar multiple of the rho 2.

When it is possible? When the direction of dl and the direction of v are the same. Then one will just be the scalar multiple of the other because direction wise they are representing vectors oriented identically. So when that is possible? What is the special dl for which that is true if it is located along a stream line? So if you consider this as like term A, so we will identify certain cases when A becomes 0.

So A=0 when certain cases one is dl is along stream line. Let us call the stream line direction as ds, s for stream wise coordinate. When that is the case, we do not care whether it is an irrotational flow or not, it does not matter whether it has nonzero components of the vorticity vector. **"Professor - student conversation starts."** Yes. There is nothing called as stream line flow, first you have to understand.

There is a stream line in a flow, there is nothing called stream line flow okay. Next, this is what, this is the line element that you are considering. These are the components of the velocity vector. What is the definition of a stream line? Such that tangent to the stream line at any point represent the direction of the velocity vector. So tangent is this direction dl, small elemental direction and this is the velocity vector direction.

So if they are located in a same direction that means they are parallel vectors. That is the definition of the stream line, it is nothing extra. **"Professor - student conversation ends."** If dl is located along a stream line, then we do not care whether it is a rotational or irrotational flow but if it is not then if the vorticity vector is identical = 0 then A will become 0, no matter whatever is like no matter dl is located along the stream line or not.

So vorticity vector is a null vector, this is irrotational flow. So you can clearly see that if it is a steady and irrotational flow, these 2 terms go away and then sum of these 3 is 0 that means if you integrate that the integration will give a constant of integration and that is what we actually saw in the example the problem that we discussed before going through this derivation.

There is a third case, there could be many such cases but a third case say you have the v cross this vorticity vector is perpendicular to dl. These 2 cases are more common cases that we have encountered. This is not a very common case you have encountered but this mathematically you cannot rule out.

You have a vorticity vector, you have a velocity vector, you can find the cross product and take an element in a direction which is oriented along that cross product and then if you take such an element then for such an element also for steady flow it will appear that the Bernoulli type of equation is valid. So this is not a Bernoulli type of equation. This is in fact one step before that where we do not make any explicit assumption on how that rho or the density varies.

So this is still the Euler equation of motion. So this is more general way of writing the Euler equation of motion where you are considering all the individual components and trying to write that in a vector form but at least we can understand that this term become 0 under what cases.

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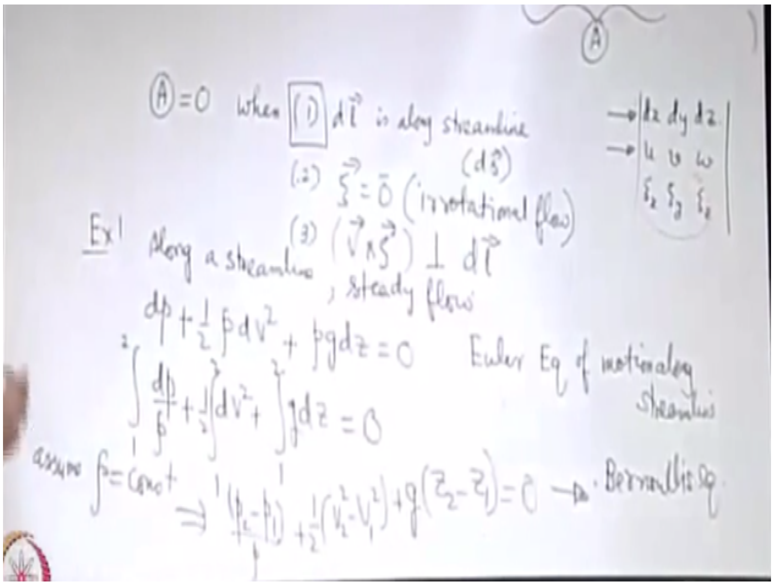

So let us say that we are considering one such case let us say that we take an example we are considering along a stream line. That is what do you mean by along a stream line that means we are interested to find out these changes. See this relates what? This relates change in pressure, change in velocity, change in elevation with respect to a change in position vector from point 1 to point 2.

So when we are considering along a stream line that means we are interested to evaluate that change by moving along a stream line. So never consider something like a stream line flow. Again I am repeating there is nothing called as stream line flow. In flow, there are stream lines but it is not a stream line flow. So when you have a stream line we are looking for the difference in like these variables along a stream line.

So when you have along a stream line and let us say steady flow as the first example. This is example 1. So then what you will have, then A term will become 0, this term because of steadiness will become 0. So you have dp+1/2 rho dv square+rho g dz=0. This is known as Euler equation of motion along a stream line. Now this is valid both for compressible as well as incompressible flows. You are not yet committed of how the density changes.

So now we are interested to see that how the density changes. To do that we will let us say we will write it in this form dp/rho+1/2 dv square+rho g dz and try to integrate it. So we will try to integrate it, the rho will not be there because rho we have already divided by rho. So when we try to integrate it what are the points over which we are integrating? We are integrating with respect to 2 points 1 and 2 which are located on the same stream line.

Because we have considered along a stream line that is we are considering this particular case which has made the term A=0. So when we do that this = 0 that is still valid for any type of flow compressible or incompressible. Now we would make an assumption that rho is a constant, assume, that is the special case of an incompressible flow. So then what you can write, you can take the rho out of the derivatives.

So you can write p2-p1/rho+1/2 v2 square-v1 square+g*z2-z1=0 okay. This is nothing but the Bernoulli's equation. That is p1/rho+v1 square/2+gz1=p2/rho+v2 square/2+gz2. So it is in fact the Bernoulli's equation. Now you tell that what are the assumptions that we followed in deriving this? So this is the Bernoulli's equation. We will come into the physical significance of this Bernoulli's equation in a next lecture.

But let us at least try to identify that what are the assumptions that we utilize to derive these. So what are the assumptions?

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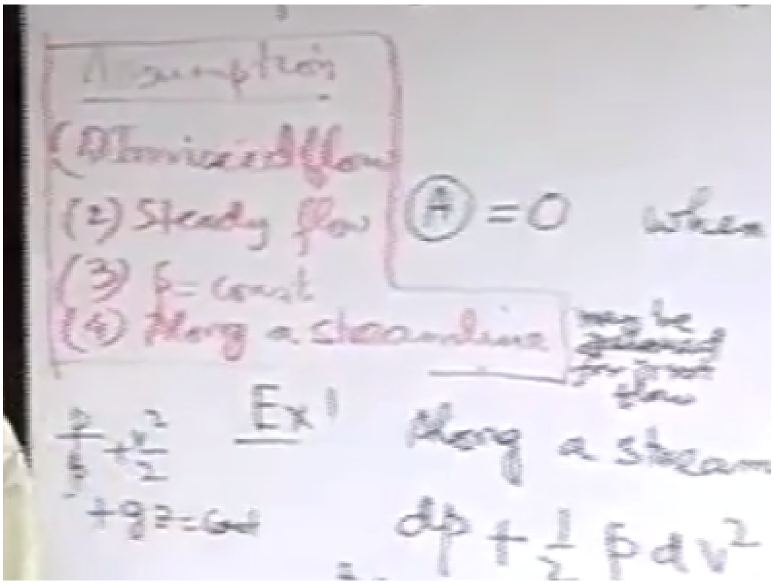

So first start with the most basic assumption, when we wrote the equation of motion what we assumed? Okay, only gravity is the only body force, it is okay but it is like it is very, very explicit what is not so explicit is inviscid flow. So inviscid flow is very important. Then steady flow, density is constant, it is a special case of incompressible flow, not irrotational flow. We have not taken this condition 3.

When we take irrotational flow, we get a more freedom then we need not be restricted along a stream line, but when we are considering along a stream line then it need not be irrotational. If it is irrotational fine, if it is not irrotational still okay. So along a stream line that is what we considered in this example. So these are the 4 assumptions that we have considered in deriving this.

Now these are the assumptions that we commonly use because commonly we utilize the Bernoulli's equation along a stream line. At the same time, we must understand that these are not always the cases, inviscid flow is the most important thing. Now can you tell that if you are thinking about Euler's equation along a stream line out of these which assumption is not necessary? Say the Euler's equation of motion along a stream line.

Density=constant is not necessary, so density=constant is the additional assumption beyond the Euler's equation. So after you make that assumption, you have to also keep in mind that I would say the most important assumption is inviscid flow because many times we tend to apply the Bernoulli equation in case when viscous effects are very much present, maybe many times you have solved such problems in your earlier high school exercise problems to solve like to get the velocity pressure and so on.

We will see that that is not fundamentally correct, in some cases you can get rid of that and still get some qualitative picture. We will see that when and when not but fundamentally it has to be inviscid flow. Steady flow is for this version of the Bernoulli's equation but you can also have an unsteady version of the Bernoulli's equation that we will see later on maybe in the next class that where we retain this term.

And we can write Bernoulli's equation by considering even the unsteady flow along a stream line. So only for this version, it is steady flow and that is the standard Bernoulli's equation but we also have unsteady Bernoulli's equation. So for unsteady Bernoulli's equation, the steady flow assumption is not required. Rho=constant is always required because you are taking rho=constant and taking out of the integral.

And along a stream line is required for this special case when you are not bothered about whether it is irrotational or not. If it is irrotational, then this need not be the case. So may be relaxed for irrotational flow. So what is the summary? The summary is if it is an irrotational flow and other conditions are satisfied that it is inviscid, steady and rho=constant, you can write p/rho+v square/2+gz is constant need not always be along a stream line.

So this is constant no matter whether you are considering the points 1 and 2 anywhere in the flow field that is very important. So points 1 and 2 may be located anywhere in the flow field still this equation is satisfied if it is an irrotational flow. If it is not an irrotational flow, then 1 and 2 have to be located along the same stream line. So these are very, very important fundamental assumptions that go behind the Bernoulli's equation. We will stop here today. We will continue again in the next class. Thank you.