Introduction to Fluid Mechanics and Fluid Engineering
Prof. Suman Chakraborty
Department of Mechanical Engineering
Indian Institute of Technology - Kharagpur

Lecture - 15
Fluid Kinematics (Contd.)

We continue with our discussion on stream function.

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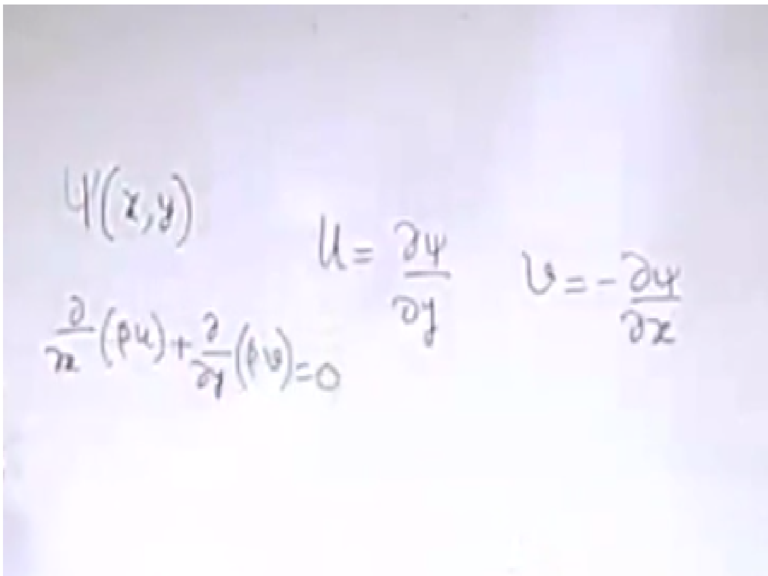

So the stream function we defined as a function of x and y with an understanding that it should satisfy the requirement of mass conservation constrained by the continuity equation and special case is giving that parametric form 2-dimensional incompressible flow. So the important thing to keep in mind that is defined for 2-dimensional incompressible flow in the way in which we are defining it.

For other types of flow, special case is again it may be defined with a slightly adjusted manner. Now say we are interested to define it for a 2-dimensional compressible flow but or a 2-dimensional steady flow may be compressible may not be compressible but 2-dimensional steady flow. When you say 2-dimensional steady flow, your continuity equation becomes whether rho is a constant or not based on that it will come out of the derivative or not.

So this is a bit more general case than the case that we had considered and for this also it may be defined just replace u with rho u and v with rho v. Then the similar parametric form maybe defined. So it is not that this is hard and first rigid definition but this is like a definition for the most common case that we are talking about. Now the stream function also has some relationship with the rate of flow.

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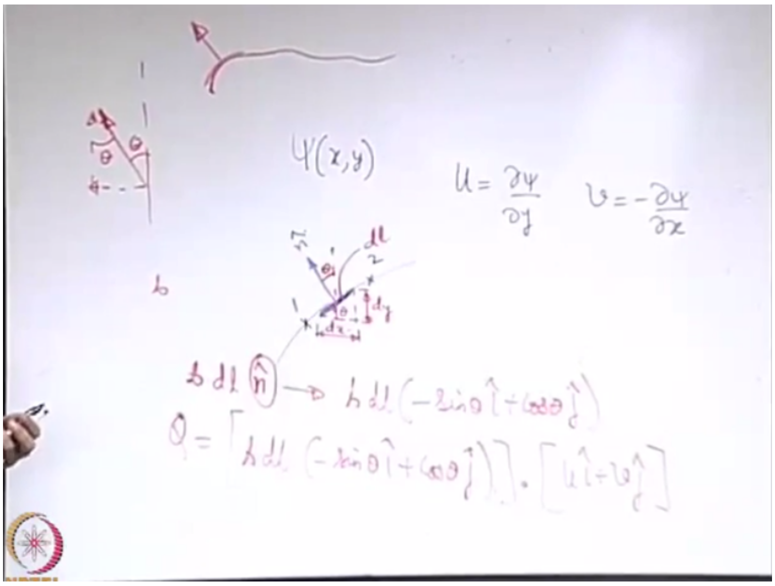

To understand that let us say that we have some line element and we are marking 2 points 1 and 2. We are interested to find out what is the total rate of flow between these points 1 and 2. How we can do that? To do that let us say that points 1 and 2 are located so closely that they are or maybe let us consider a small element within that domain bounded by points 1 and 2 that small element is given by a length of dl which is almost like a cord of this curve.

Now let us say that the length of this is dl. So if you consider it like a vector sense, it has outward normal vector like this say n cap and it may be expressed as a function of the important angles. So if you mark this angle as theta then you can write dl or decompose dl as dx dy. Length of this is dl, let us say that b is the width perpendicular to the plane of the figure.

So it is possible to write this dl and subsequently dA which is dl*b as a function of like dx dy and the orientation given by the angle theta. So dl in a vector form is like dl n cap. It is just like the area in a vector form is the magnitude of the area times the unit vector normal to the area and away outward to the area. So here the area is represented by dl, b is the width so it is basically b*dl that is the dA times the unit vector.

So the unit vector in terms of theta you can write. How you can write? Say the tangent to the curve makes an angle theta with x. So the normal to the curve makes an angle theta with y. So how can you write n cap in terms of theta. So you have to resolve the magnitude this vector with deduction n cap and magnitude 1 because it is a unit vector in x and y components.

So the n cap is like this. It will have x component like this and y component like this. So x component will be what? So remember that this angle is theta that means this angle is theta. So x component is –sin theta and y component is cos theta. So b dl-sin theta i+cos theta j. So this is basically the area now if you want to find out the volume flow rate, volume flow rate is nothing but what?

The dot product of the velocity with the area because if you have an arbitrary area what will give rise to a volume fluid only that component of velocity which is perpendicular to the area. That gives a net flow rate right. So if this is an area and the velocity vector maybe arbitrarily oriented but its component normal to the direction of the area is what is only important. That means dot product will give the component along that direction.

So the dot product of the velocity with the area vector will give the component of velocity along the area vector. Area vector means area normal, so the product of that will give the volume flow rate. So the volume flow rate Q will be this one, b dl-sin theta i+cos theta j, this dot product with the velocity vector. What is the velocity vector? ui+vj. We are assuming it is a 2-dimensional flow because we are discussing it in the context of stream function.

So everything is a 2-dimensional concept that we are discussing. So what it will become? So let us call it maybe dQ because we are talking about a small length and the flow rate across this small length is expected to be small.

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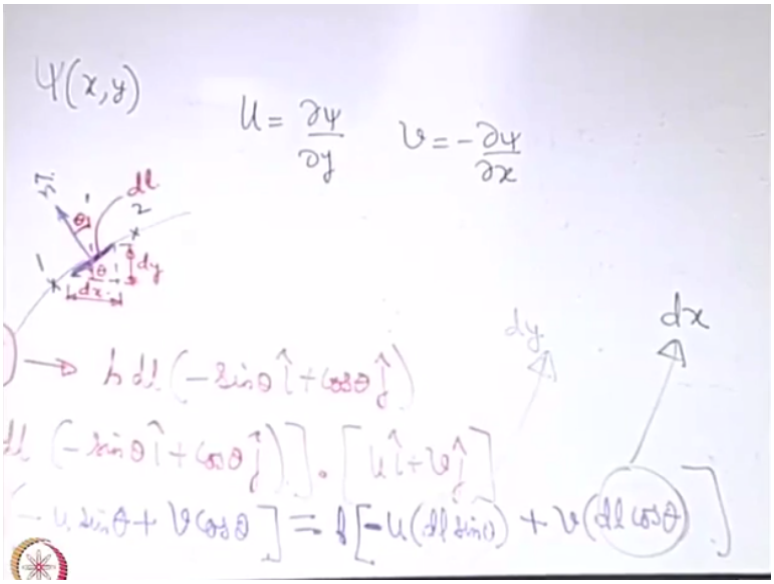

So the next step of simplification it will be b dl times-u sin theta+v cos theta. So you can also write it as b*-u*dl sin theta+v*dl cos theta. From the figure you can clearly see that dl sin theta is dy and dl cos theta is dx. So you can substitute dl sin theta with what you can substitute this with dy and you can substitute dl cos theta with dx.

So dQ is b*-u dy+vdx okay. Try to find out its relationship with the expression for d psi that we derived just in the previous lecture. What was d psi? **"Professor - student conversation starts."** So it was vdx-udy. So this is nothing but v*d psi. **"Professor - student conversation ends."** That means if you want to find out the net flow rate between the difference in flow rate between 1 to 2.

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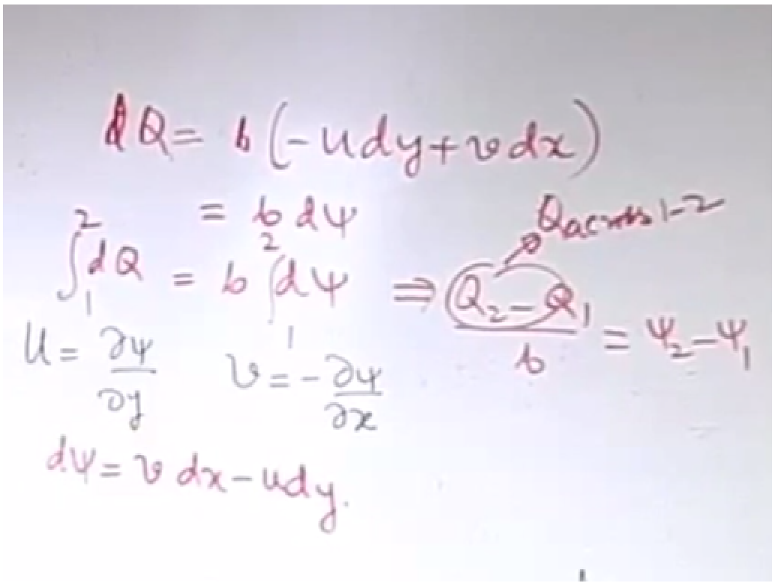

So dQ from 1 to 2 that is b d psi from 1 to 2. So the difference in flow rate between the points 1 and 2 Q2-Q1 per unit width is given by the difference in stream function. That means what is this Q2-Q1? Q2-Q1 is the net flow rate that is taking place through the element 1 to 2 because it is basically sum of the flow rates so far such differentially small elements like dl. So Q2-Q1 is the net Q across 1 and 2.

You can clearly see that if psi 2=psi 1 that is if the values of the stream function are same, then there is no flow and it is quite logical because we have defined streamlines in such a way that stream lines are such that the velocity vectors are tangential to the stream line. So a very important corollary of that is there cannot be any flow across a stream line. So if you have a stream line like this you cannot have any flow across this.

Because all the velocity vectors are tangential to it. There is no normal component of velocity and normal component of velocity can only give a cross flow. Because it is so you can clearly see that from the definition of stream function also it is reflected in that way because stream lines are characterized by constant steam function. So you do not have a change in stream function along a stream line and therefore no flow across a stream line.

So the difference in stream function between 2 points gives a quantitative indication of what is the flow rate across the line element that joins those 2 points per unit width. Per unit width is important because it is just a 2-dimensional concept. So with this background on stream function let us work out maybe one or two problems from your textbook related to stream functions.

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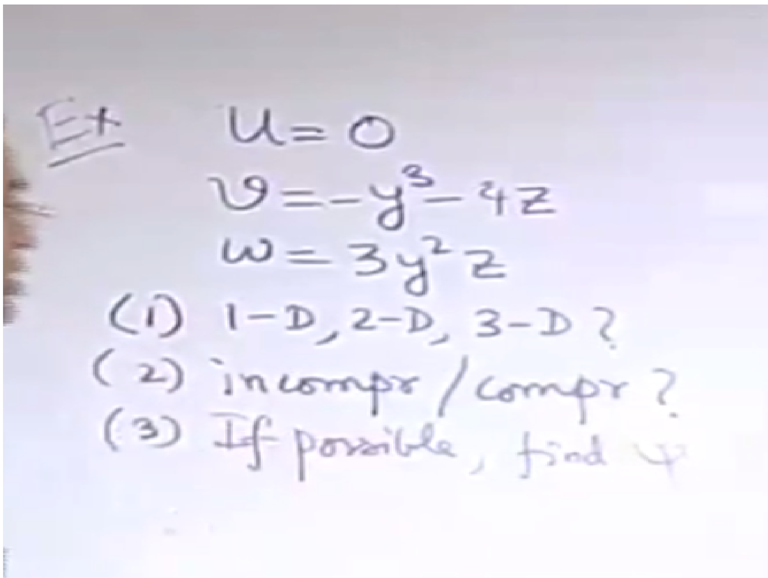

So let us work out maybe one simple problem to begin with say you have flow field given by the following components u=0, v=-y cube-4z and w=3y square z. These are the velocity components which are given. Now the questions are as follows number 1 is the flow 1-D, 2-D or 3-D perhaps you cannot have a more simple question than this. The next one whether it is incompressible or compressible flow.

And the third part is that if possible define a stream function for the flow. See if possible is very important because the stream function is not defined for all types of flow. So if that stream function is defined then only will find out the stream function that is the whole idea of the problem. So let us try to work out this problem. So the first part it is a 2-dimensional flow because it has 2 velocity components fine.

Incompressible or compressible flow so how will you test? The divergence of the velocity vector you should find out because that gives a rate of volumetric strain. So if the rate of volumetric strain is 0 that means it is incompressible flow.

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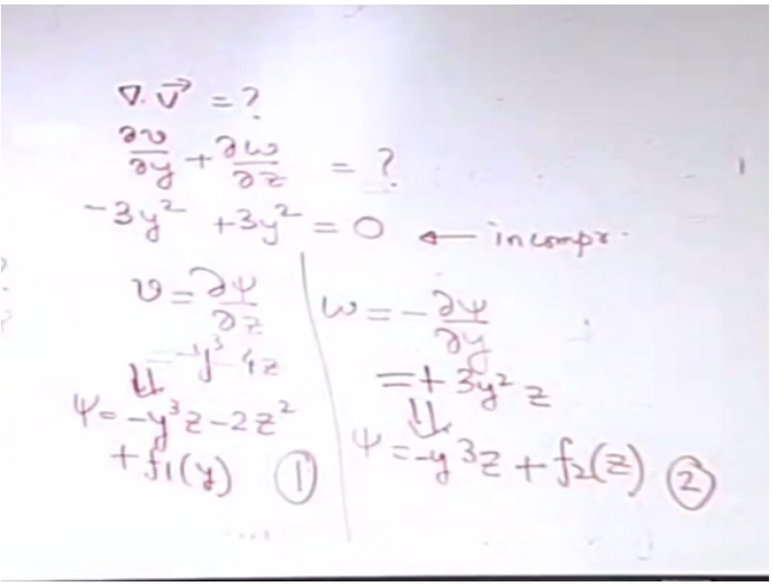

So we have to check what is the divergence of the velocity vector. So here it will boil down to the checking of what is this. So the first one is -3y square and the second one is +3y square so this identically becomes=0. That means it is incompressible flow. So now you have a 2-D flow from the first part you are when incompressible flow from the second part seems it is 2-D and incompressible, the stream function itself is defined.

So we can now attempt to find out a stream function. How do you find out a stream function? So now it is like it is in a yz plane that we are talking about the flow. So you have to define in some way v and w in terms of the stream function. So how do you define v? Maybe again +- sign is not that important, what is most important is it should be of opposite sign to make this equation trivially satisfied=0.

So let us write the corresponding components **"Professor - student conversation starts."** Yes. This is a v component; this is a w component, so these are written in such a way that this equation is satisfied. That is what I am always saying that do not write take it as a formula. Now see that with a slight change the components of velocity given as y and z components instead of the previous ones you see that you are facing a dilemma that should not be there.

Again the objective will be that given a form of the continuity equation just write a parametric form which satisfies v and that will automatically give you the definition of the stream function for that case. So do not confuse it with the case which we were discussing for developing the theory flow in the xy plane, now the flow is in yz plane. So that definition does not work here.

So this is given by –y cube-4z. So if you integrate this with respect to z what will be psi? Yes –y cube z-2z square then+some function of y because again repeating this is partial integration with respect to z. So when you are integrating with respect to z as a variable even y is a constant with respect to that. It cannot be function of x here because there is no dependence of x, I mean x is just like it is a flow taking place in a yz plane.

So where does the dependence of x comes? So you have to understand it physically. Then, next here that is -3y square z so if you partially integrate with respect to y. So psi will be y cube. Yes. Sorry this is + right, psi=-y cube z+function of z. **“Professor - student conversation ends.”** So you have say this as equation number 1 and the equation number 2. Both are representing the same psi.

So you have to compare 1 and 2 to get f1 and f2. So 1 and 2 are the same.

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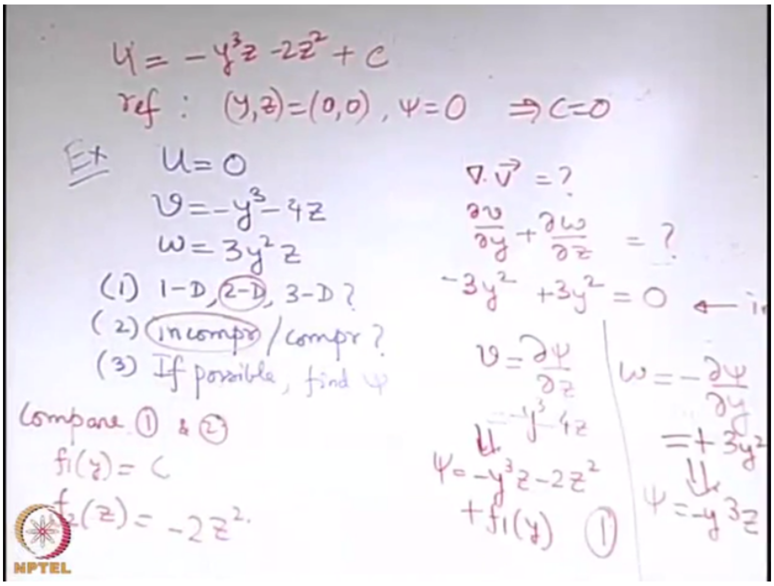

So if you compare 1 and 2 what follows? What is f1 y? It maybe some constant at the most or you may choose your reference in such a way that the constant itself is 0. Let us just keep it as a constant and see that how you can choose it to make it 0 and what is f2? F2 z is -2z square.

So the expression for the stream function becomes –y cube z-2z square+some constant. Now this constant you may arbitrarily choose because there is no absolute value of stream function so to say. You have a reference with respect to that you find out a change because see the velocity component is defined as the partial derivative of the stream function. So it is not defined on an absolute sense with respect to the value of the stream function.

So if you have the value of the stream function defined arbitrarily say you define that your reference or origin is such that at yz=0 psi=0 remember it is not a must. It is just your choice and always such choices are convenient because then you come up with that expression which will have c=0. It does not matter whether c is any arbitrary value or not because at the end difference in stream function is what is important that c will get canceled if you find out the difference in stream function.

But for working convenience, you may set your references in that way.

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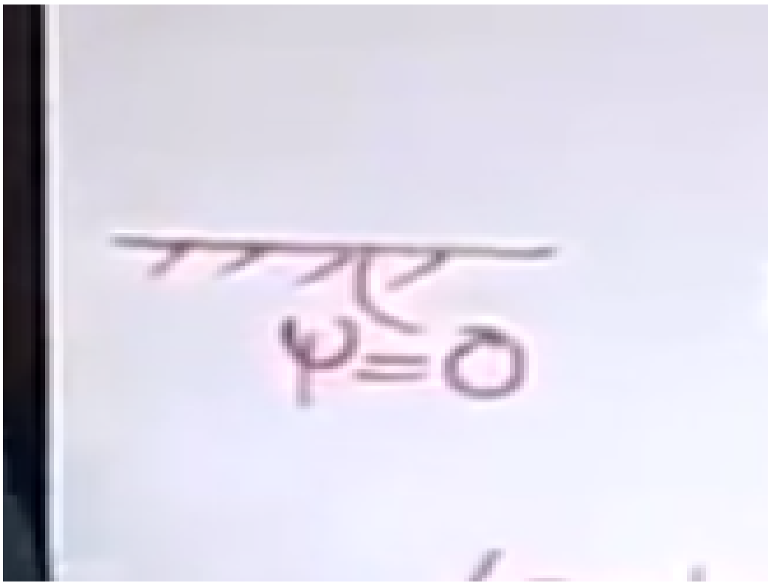

In most practical purpose is you have flows on solid boundaries. Usually, the solid boundaries are considered to be so solid boundary itself is a stream line right because there is no flow across it. So by its physical sense any shape solid boundary is itself a stream line. So you can give it a value of a particular stream function if it is a 2-dimensional incompressible flow.

So classically just as a matter of convention we give it psi=0 as a reference. Again there is no sanctity with it, you might give 1, 2, 4, 5 whatever but it is just a reference, a data with respect to which you want to calculate other stream function. So solid boundaries are classically referenced as 0 stream functions. So let us maybe move on to our next problem which may be similar to this but let us just workout another problem with respect to the concept of the stream function.

So to do that again we will be solving a problem from your textbook.

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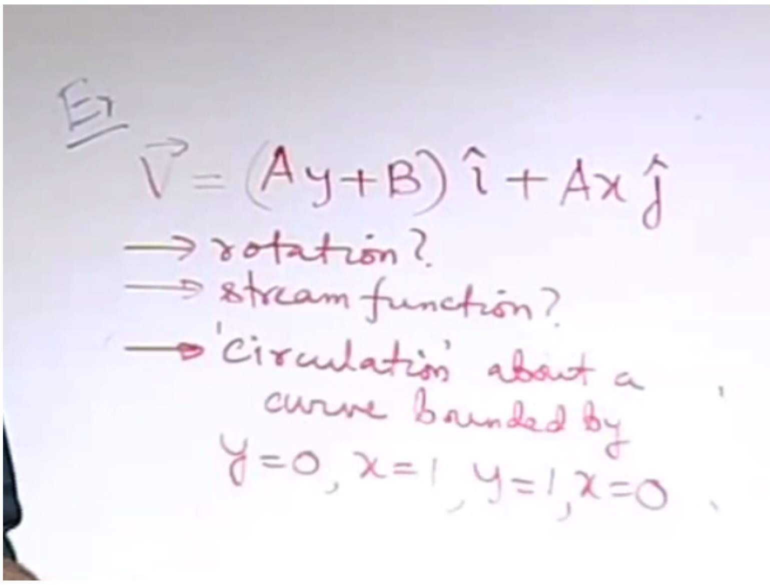

So we have the flow represented by velocity vector in this very much similar to the previous problem Ay+B i cap+Ax j cap where A and B are dimensional constants. These are given in terms of some numbers. We are not going into that. You have to find out the fluid rotation and the stream function.

Again, this is very simple case to talk about but whenever we are trying to understand the fluid rotation, we will now try to understand a very important concept which is asked in this particular problem. That is what is the circulation about the curve; we will now learn what is the meaning of circulation about a curve bounded by the following lines. What are the lines y=0, x=1 then y=1 and x=0.

So we have come across a new terminology in this problem called as circulation. So we will keep this problem a bit aside tries to learn what is the meaning of the terminology circulation and then we will try to apply it. The terminology circulation is not a very new terminology, it is very much related to what we have already learnt and we will see that it is very much related to the concept of vorticity.

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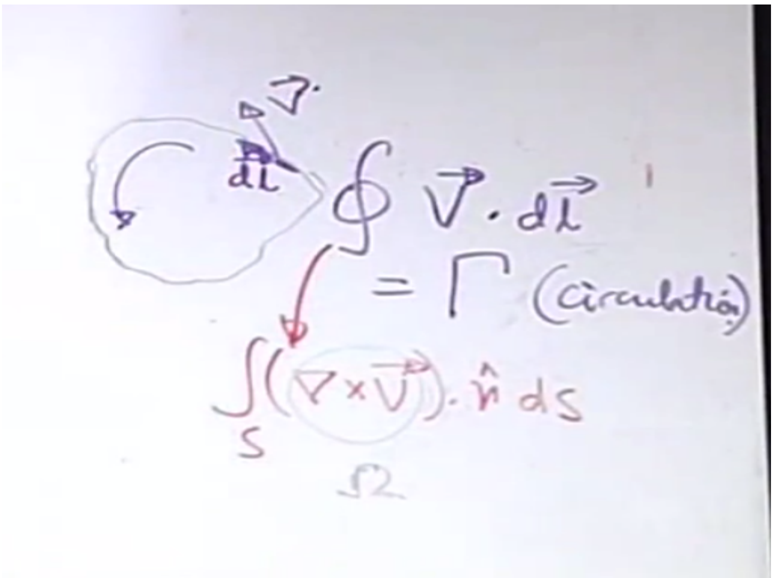

How it is related to that? Let us say that we have a line a closed contour. A contour which is closed by some curve. Now if you take a small line element located on the contour. Let us say that we take a line element like this. So you may have a velocity vector say v and you may assign a vector to this line element by giving it directionality. Say you are traversing along a clockwise direction or an anticlockwise direction.

Here we are traversing along an anticlockwise direction. So if you find out the v dot dl, the dot product of the velocity vector with this one and find the contour integral that is the integral of this over this closed path by traversing in a particular direction that is known as circulation along the curve, which is given by a symbol capital gamma, this is circulation. Now it is possible to express this in terms of an area integral by using the Stokes theorem in vector calculus.

So you can express this in terms of curl f or here the curl v where the vector function f is here v where ds is an element of the surface and s is the total surface that is bounded by this closed curve that is very, very important. So you can clearly see that if you talk about the value in a plane, it is possible to make out from this that first observation is this is nothing but the vorticity vector.

So this vorticity vector when you utilize this vorticity vector here, so the circulation is nothing but how we will express it in terms of vorticity? It is just like roughly vorticity per unit area or it is in the other way. It is like vorticity*the area is the circulation so it is just the other way not the vorticity per unit area but vorticity*area is the circulation.

So let us take an example before going into this particular mathematical form to figure out that how you can calculate what is the circulation. Now to do that let us take an example which is not the same example but we will utilize what we learn from that example to solve this problem.

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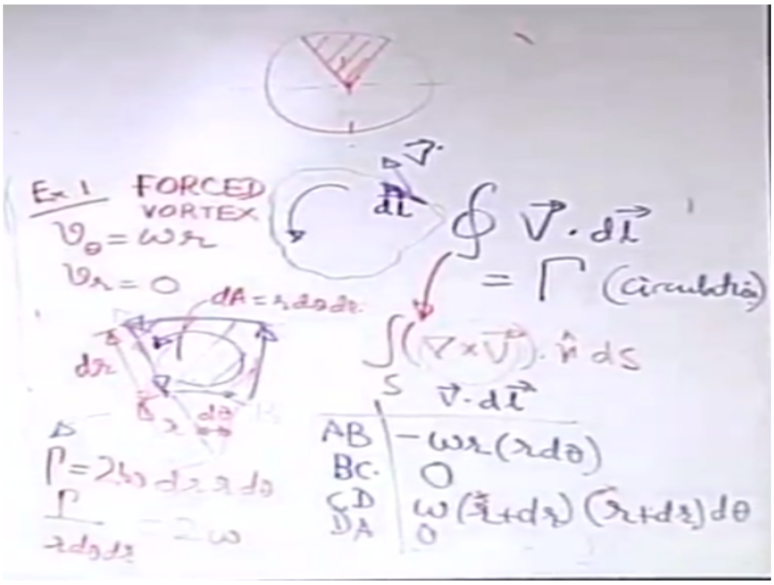

Say you have another example where you have the velocity components given by polar coordinates say v theta=omega r and vr=0. There is no vz, a 2-dimensional flow okay. Let us say that we want to find out what is the circulation around the closed contour. So a closed contour in an element in a polar coordinate form, you can generate in this way. So you consider that at a distance r I am magnifying dr to show that as if dr is>r.

But it is just for clearly drawing the figure of dr and this angle say is d theta. So what is the contour that we have chosen? We have chosen the contour which is formed by this element in the polar coordinate. So these like A, B, C, D. Now we are interested to find out the circulation. So first we start the traversal from the point A, so what is the v theta defined? v theta is defined as omega r.

So we are going from A to B in this direction. What is v dot dl? Here dl is also along AB because it is a small arc, it is almost like tangent to the curve, v theta is also tangent to the curve but the thing is your positive theta direction is anticlockwise. So v theta and dl so v theta is in the opposite direction as that of your dl if you are trying to traverse the path in this way.

So when you go from A to B, v theta is opposite to that of your dl that is drawn. So v theta dot dl will give a –sign. What is the magnitude of v theta dot dl? So for AB v theta dot dl will be omega r*rd theta with a –sign. So what we are writing here is v theta v dot dl okay. Then for BC, v theta is along the tangential direction, BC is along the normal direction. So their dot product is 0.

So this does not come into the picture, it is 0. For CD what is v theta? So for CD what is dl? The dl is from C to D, you are traversing it in a particular direction. So you are traversing it like this for CD what is v theta? **"Professor - student conversation starts."** Omega*r+dr and what is dl that is r+dr*d theta okay. **"Professor - student conversation ends."** And for dA it will be 0 just like before.

So when you sum it up this contour integral will be the sum of this 4 line integrals. So sum of this 4 line integrals will give what is the total circulation? That is sum of this so what will be that. You see the first term-omega r*r d theta will be canceled with 1+omega r*rd theta. Then there will be terms omega dr r d theta. There will be term omega*dr square d theta. Then any more term?

So one of the terms this has got canceled with omega r*rd theta, then omega r*dr d theta we have written, then omega r*dr d theta, so another omega r*dr d theta right. So 2 omega r dr d theta. So if you find out what is the circulation per unit area, what is the circulation per unit area? This divided by rd theta dr that is dA. The dA of this shaded one is r d theta dr. So that is nothing but 2 omega.

Actually this omega is the angular velocity and 2*that is the vorticity. So this is like the vorticity. So what we get from here this is an example of illustration of the same concept, this is just from a vector calculus theorem, this is just detailed illustration of that. So you can conclude from this that in such a case you can find out the circulation per unit area=vorticity. So from that you can relate vorticity with circulation.

So if you find out one you can automatically find out the other. This type of example where you have v theta is proportional to r and vr=0, this is known as a forced vortex. So it is something like you are creating rigid body type of rotation in a flow by having an angular velocity to the system. So an example is like we have discussed in the class earlier that you have a cylindrical tank.

And you start rotating the cylindrical tank, in the limit as you are neglecting the viscosity it takes the shape of a parabola of revolution type that is an example of a forced vortex. So you can have a forced vortex with this type of velocity. So velocity is 0 at r=0. You have to keep in mind one very important thing. So the circulation per unit area is given by the vorticity.

Let us say that somebody has taken an area like this, so this is a circle somebody has taken it area like this. This is the center of the circle and is trying to find out what is the circulation. We will see that in certain cases this definition will be restricted if we include the origin. So let us take another example that will illustrate that what is the corresponding problem.

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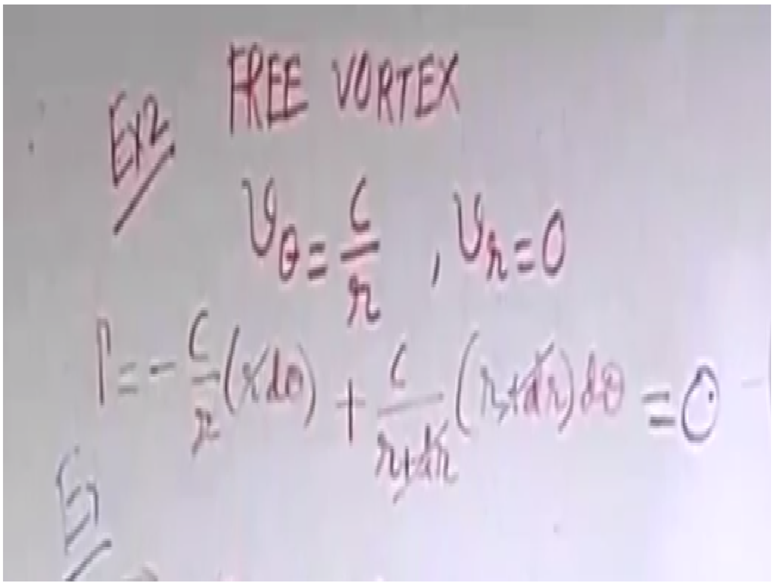

So take an example 2. With understanding of example 1 and example 2, this example problem that we are going to solve it will be easy for us to appreciate. Example 2 is something which we call as free vortex. What is a free vortex? So free vortex is defined as the v theta component is inversely proportional to the radius. Here it is directly proportional to the radius; it is inversely proportional to the radius.

And vr is=0, so this kind of a situation you get in a kitchen sink. So you open the tap and what you will see that as it comes so the sink is first say you fill up the sink first with water close the valve, so that the water cannot go out. Now suddenly open the valve, you will see that as the water comes to the outlet and goes to the pipe, when it comes to the center of the sink it is coming out with a very high velocity.

So there and it has a sense of rotationality in the flow, so the v theta is inversely proportional to the radius. It is singular at r=0 mathematically because at r=0, it is as if v theta tends to infinity and you cannot have an infinite velocity. So you have to keep in mind that this is something where this r is defined in the limit as r tends to 0 plus not r=0, r=0 is a point of singularity for such a case.

So when you are writing, now you try to figure out that what should be the circulation. So the circulation here just we will have similar element AB, BC, CD, DA. We will straightaway write because by now you know how to write it. So for AB what will be the circulation, so – c/r*r d theta. For BC it is 0, for CD +c/r+dr because new r is now r+dr*r+dr d theta and for dA it is 0.

So very simple observation is this fx will get canceled out so it is 0 okay and clearly it is so because you can also find out the vorticity, you will find out the vorticity is also here 0. This type of cases where the vorticity is 0 is known as irrotational flow.

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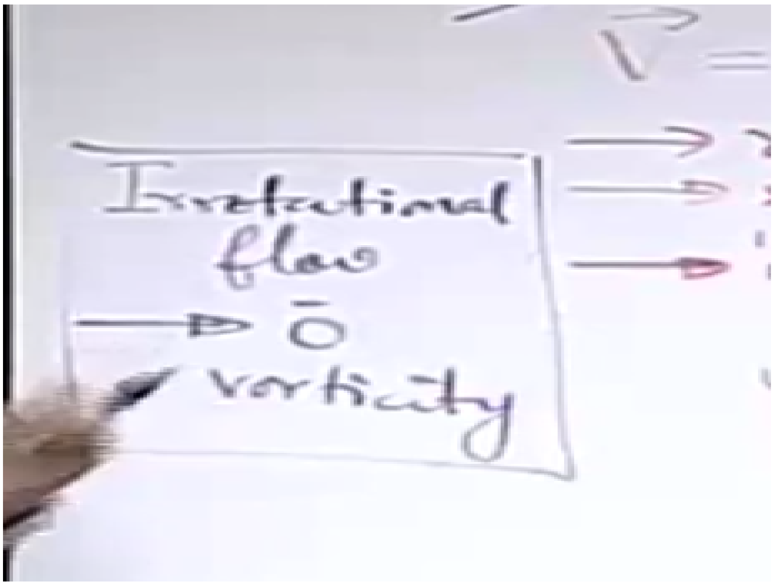

So irrotational flow is a flow with 0 vorticity. We will come into the concept of irrotational flow soon. Irrotational flow means 0 or null vorticity vector that is the very simple definition and from the name itself it is quite clear that there is no element of rotationality in the flow that is why it is called as irrotational flow. Now if in this example you take a close contour like this, this by definition is a close contour.

You try to find out what is the circulation, so for these 2 edges the circulation will not be there, for this edge the circulation will be there and clearly when you submit up over the contour, it is not 0 if you take such an element. What is wrong with that? A very important conceptual mistake is there by choosing this element. You have taken the element by including the point of singularity.

So you cannot take an element which contains the point of singularity to establish the relationship between the circulation and the vorticity. So obviously this is not a valid element. **“Professor - student conversation starts.”** Yes. No, no, no that is I mean that is like it is just coming, it is that is not a circulation, it is like when you will always see that when it comes we will see now that practical example.

When r=0, it is not defined so it is always a forced vortex close to r=0. So what is the practical case? The practical case is never purely forced vortex or purely free vortex.

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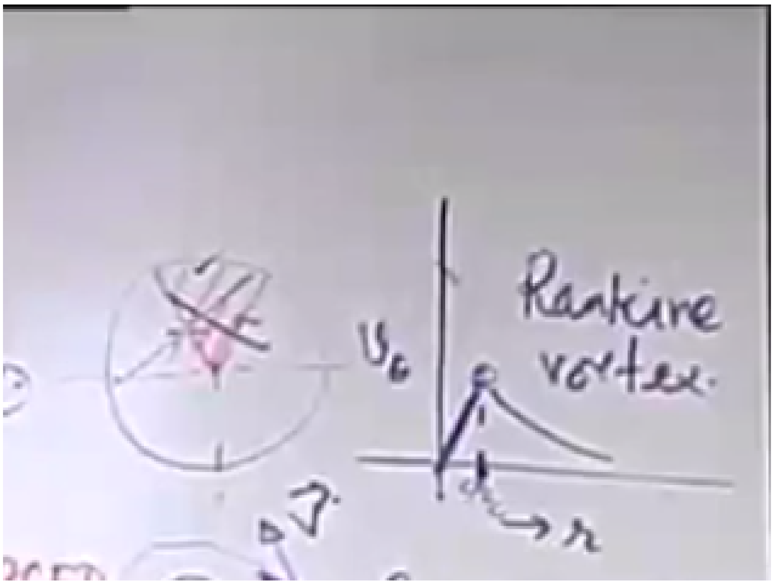

So the practical case if you draw the velocity profile, v theta versus r, you know that at r=0 if you use a completely free vortex understanding then v theta will be undefined so actually always very close to r=0 it is a forced vortex. So close to r=0 v theta will be=omega r so it will be something like this and away from that it may become a free vortex. So it is like a rectangular hyperbola type.

There is a rotation close to this one, so this is known as this is the practical example, free vortex or forced vortex neither of these are like very practical examples but their combinations are quite practical. This is known as Rankine vortex. **"Professor - student conversation ends."** So it is a combination of free and forced vortex. So up to a particular radius say critical radius, it is like a forced vortex, beyond that it is like a free vortex.

A very classical example that occurs in nature is a tornado. So tornado very close to the eye of the tornado, it is up to that it is a forced vortex. So it has a strong element of rotationality, beyond that it is like a free vortex. So tornado maybe very well approximated by a Rankine vortex, which is a combination of free and forced vortex. You have to keep in mind that if you have such a combination you have to satisfy that at this point this critical radius.

I mean where you have like transition of behavior from free to forced vortex that v thetas should be same as given by the considerations of free and forced vortex because you cannot have a discontinuous velocity field. The velocity field in the physical sense is continuous. Now if you come to this example, we will try to utilize this example to find out the circulation.

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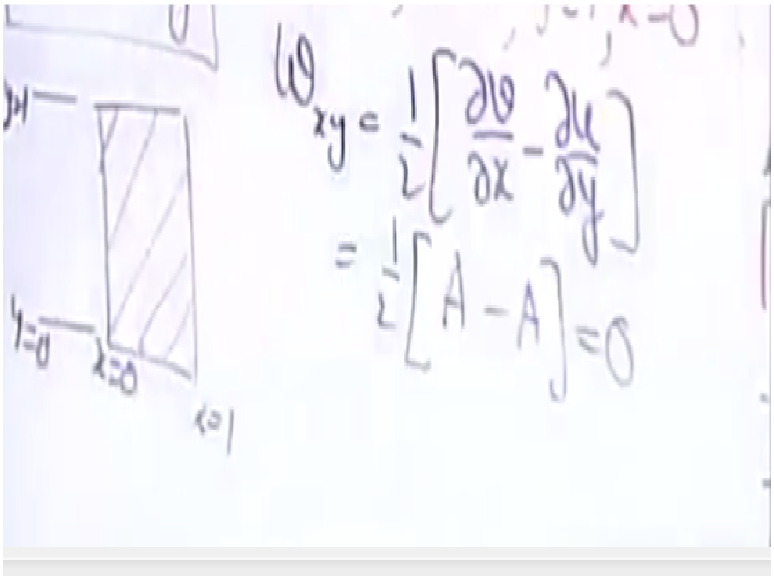

So first what is the rotation? So the rotation is given by the first you find out the curl of the velocity vector so it is a 2-dimensional field so the curl of the velocity vector eventually will boil down to that you want to find out the angular velocity in the xy plane. So what is that? Half of this one so this is basically if you find out half of the curl of the velocity vector and take its component in this plane wherever it is important it will also boil down to the same expression.

So that will be A-A that is 0. The rotation is 0 that means its vorticity is 0 and by the relationship between circulation and vorticity, now you have a curve bounded by what y=0 and 1 and x=0 and 1, so it is like a rectangular contour. This is x=0, this is x=1, this is y=0, this is y=1. So this is the area bounding the contour. So this does not include any point of singularity.

So this is a valid area and therefore the circulation about this curve is expressed in terms of the vorticity, since the vorticity is 0, this is also 0, you need not work it out but I would suggest that you work it out yourself and check just by following this that you are getting 0 that will a good exercise for you and stream function we already know how to do it, I am not going into the details of it.

The whole purpose of this problem was to demonstrate the rotation and the circulation part. Next, we will go to the concept that is very much related to the rotationality of the flow again and that concept is given by the name of velocity potential. So we will see that what is the velocity potential and what are the important considerations that go behind this.

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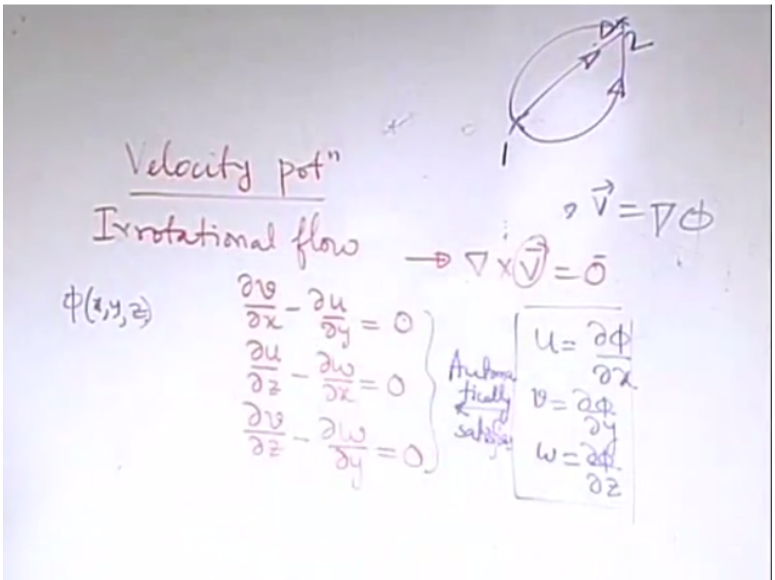

So velocity potential, let us consider the case of irrotational flow. So this is defined only for irrotational flow so for nothing else. Therefore, we must find out the condition for irrotationality. So what is the condition for irrotationality? The curl of the velocity vector is a null vector. So what it will boil down to? So if you write the 3 scalar components of this by expanding in a determinant form and equate the scalar components to 0, you will get these.

So from this you can see that if you now parameterize say u=del phi/del x w=del phi/del z where phi is a function it is in general a function of x, y, z. Then, this definition automatically satisfies these requirements. This is not a magic; this is very much obvious because from the theory of vector fields you know that if the curl of a vector is a null vector then that vector maybe represented as the gradient of a scalar function.

So if the curl is 0 or a null vector, it is possible to express v as gradient of a scalar function phi. So that is what which is the mathematical basis of such a coincidence. It is not a coincidence but directly it follows from a important vector identity. Now such a field where the curl of the field is null vector, the field what we talk about is a general vector field, here it is a velocity vector field.

We call it as a conservative velocity field, so in general in field theory if the curl of particular vector field is a null vector that field is a conservative field. It may be expressed in the form of gradient of a scalar potential maybe with + or – sign but it is all the same because again with – or + this will be 0. So when you express the vector field as a gradient of a scalar potential what is the significance?

Think about the case of a gravitational field. So when you say or having a displacement of a particle from 1 to 2 in a gravitational field, it does not matter whether you are going by path this, this, or this provided only gravity is the important force. Then the work done for going from 1 to 2 is independent of the path and just is dependent on the difference in the potential that is the potential energy in this case.

So that path dependence comes from the conservative nature of the field. So in a conservative field, so similarly this is talking about a conservative velocity field not a conservative force field but the concept is very much analogous so whenever you have such a conservative field the field is expressively in form of gradient of a potential in such particle mechanics in a gravitational field that potential is the potential energy that we know.

So here we are talking about the velocity potential. So now we are interested say about a relationship between the velocity potential and the stream function. Now when we are interested about the relationship they must be comparable. So when, under what case, under what circumstances they are comparable? So what is our objective now? Objective is to find out a relationship between stream function and the velocity potentials.

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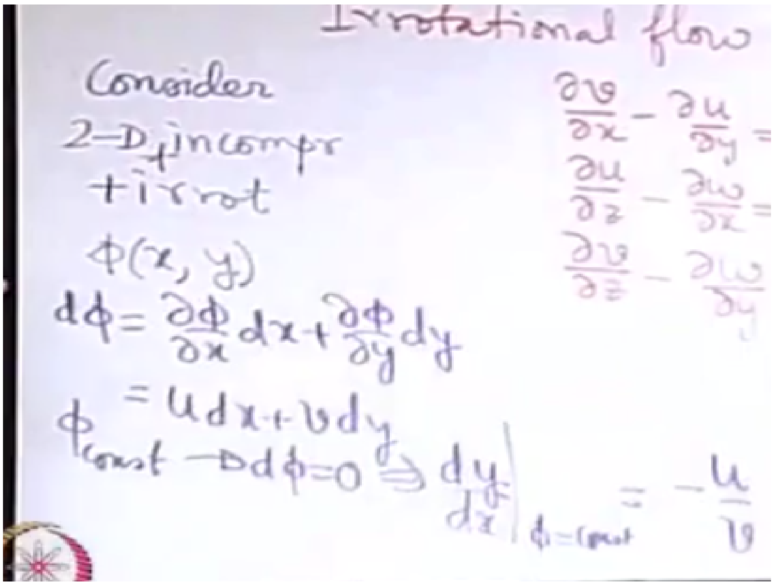

What type of flow should we consider? Consider what? So we should consider a case when both are defined. When is stream function defined? It is defined for 2-dimensional incompressible flow need not be steady, 2-dimensional incompressible flow when the velocity potential is defined for irrotational flow. So it has to be 2-diemnsional+incompressible+irrortational.

So that both phi and psi functions are defined. On this basis, we want to compare the two. So when you are considering a 2-dimensional irrotational flow, you have only the relevant components as this u and v. So now let us find out what is say how we can express d phi exactly in the same way in which we expressed d psi. So what is d phi? d phi is this one. So it is udx+vdy.

Because u is defined as the partial derivative of phi with respect to x and so on. Now if you have a line of phi=constant just like we have psi=constant then that will mean d phi=0. That means for that line which is having phi=constant, you can have dy/dx for phi=constant=-u/v. Let us now recall what was the case with the line of psi=constant, so psi=constant so what was the expression for d psi?

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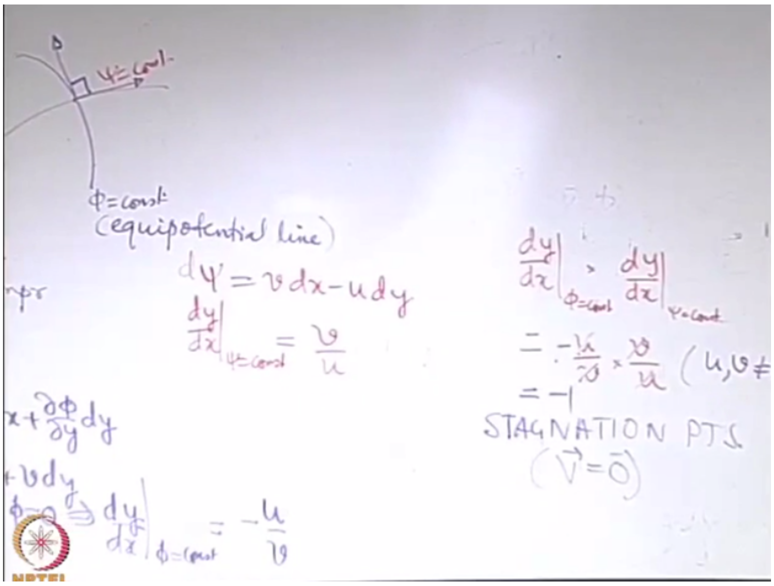

So what was d psi? vdx-udy, so what is dy/dx along a line with psi=constant that is d psi=0 that is=v/u. Now if at a common x, y if you want to find out the product of this dy/dx for phi=constant and dy/dx for psi=constant what is that? That is -u/v*v/u okay. It is not trivially -1 that is the very important thing because who has guaranteed that u and v are nonzero. If at least one of these components you have 0 then you get a division by 0.

So you have to make sure that these are nonzero. So if these are nonzero then only you may cancel out this and when you are canceling out both, you are ensuring that u, v both are nonzero because both appear in the denominator. So then in that case this is -1 and what does this -1 signify that if you have a line of constant psi and if you have a line of constant phi, they are orthogonal to each other.

So you have phi=constant and psi=constant. So we do not say that they are perpendicular because it is not that the curves are perpendicular but their common tangents act at the common point. So here you have a tangent like this and here you have a tangent like this. So these are perpendicular to each other and therefore the important conclusion is that phi=constant lines, which are called as equipotential line.

So this is equipotential line, so this is very much analogous to the equipotential line that you get in say electromagnetic field theory. So the phi=constant and psi=constant, these lines are orthogonal to each other everywhere in the flow field except for certain points which are the points the points are where the velocities are 0. Those points are known as stagnation points. So in a flow field the points where the velocity is at 0, those are called as stagnation points.

So stagnation point is a point where v is 0 and at the stagnation point you do not have such a relationship because at a stagnation point you cannot really work out this. So it is not true that the stream lines and equipotential lines are orthogonal everywhere in the flow field. They are orthogonal at each and every point except for the stagnation point where it is not defined in that way.

Now finally what we will see, we will see a relationship or a governing equation for the expressions for phi and psi. So if you say that you are interested about this case when you have both 2-dimensional incompressible and irrotational flow.

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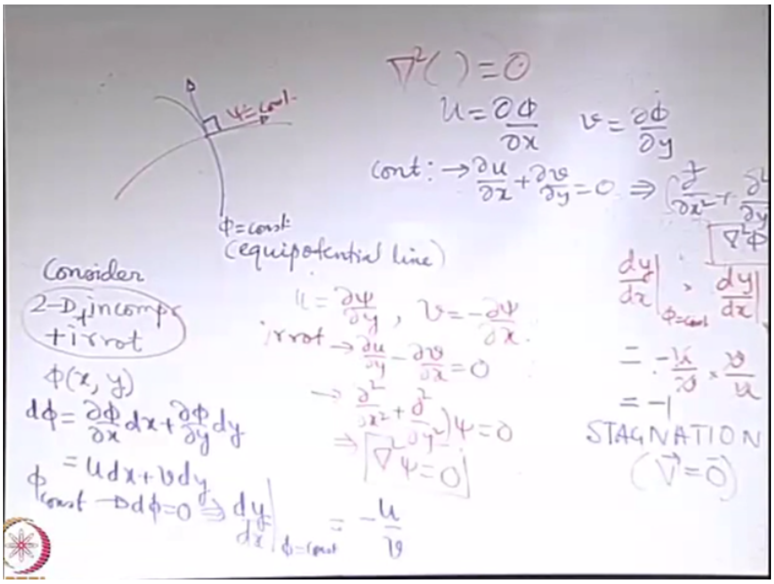

So when you have a 2-dimensional irrotational flow you have u=this and v=this. Now if you want to satisfy this with a continuity equation, continuity also has to be satisfied. So you have this=0 that means you can write or Laplacian operator phi=0. Similarly, if you start with a definition of the stream function, you have u=and v=- this one. Now this automatically satisfies continuity but for the case when you are considering both phi and psi at define it also has to satisfy the irrotationality constraint.

So for irrotationality you must have this=0 okay. So for irrotational flow, now if you substitute this you will get the same thing just replace phi with psi. So you get Laplacian of psi=0. So you can see that for 2-dimensional incompressible and irrotational flow, both phi and psi satisfy the Laplace equation. This prototype equation Laplacian of phi that is del square of phi=0 or del square of psi=0, the general prototype is del square of something=0.

That is known as Laplace equation, so both phi and psi satisfy the Laplace equation. It does not mean that their solutions are same because boundary conditions are different, although governing equations for phi and psi are the same but their boundary conditions are different and therefore solutions are different. So in summary what we can see that we have defined what is the stream function, we have defined what is the velocity potential.

We have seen that there is a relationship between these two when we have both defined that is 2-dimensional incompressible irrotational flow and both in terms of their governing equations and also in terms of their orthogonality. Now what we will do will finally we will look into some visual demonstration of certain types of flows and end up the discussion today.

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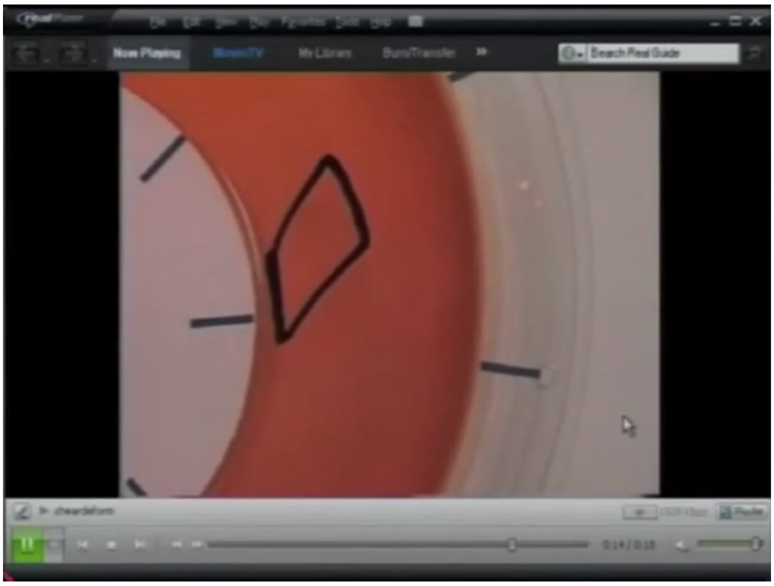

So in these visual demonstrations, we will see some example. So first you see this is a shear deformation. See there is a line element which is marked and this is a flow between 2 concentric cylinders. We have seen such a case when we were discussing about viscosity. So you can clearly see that the fluid element is deforming. So from a rectangular shape the fluid element is coming to the deform shape.

So this is the angular deformation that we are talking about, we just play it again so that you can see it again. So originally it was rectangular but because of the shear you see that how it is getting deformed okay.

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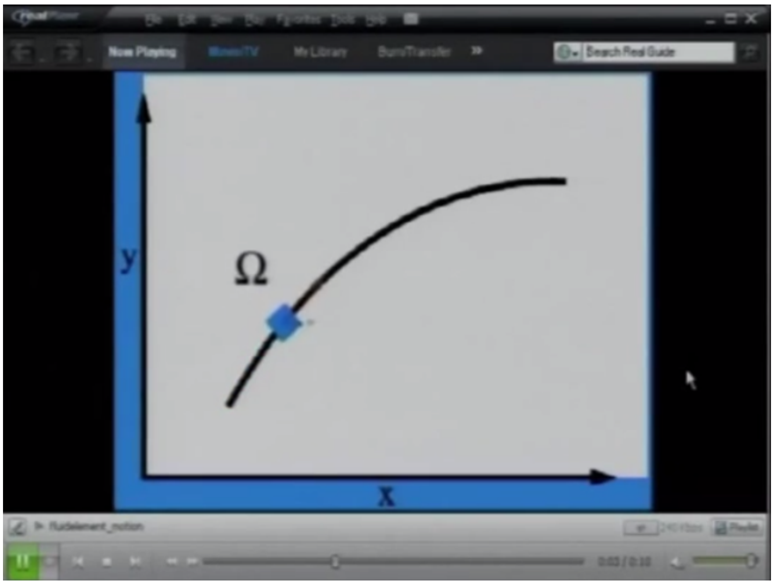

Now we will look into a second example after this where we will see that like if you have a general case when a fluid element is moving along a path then like it can have a general type of behavior. So it can have deformation so here it is just rotating. You see that it is not deforming. So it is just a pure rotation.

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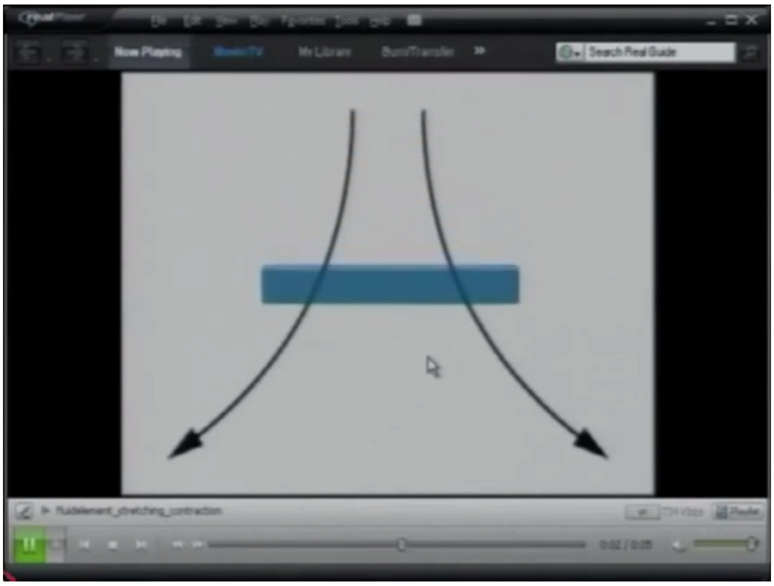

Now let us look into a third example where we see a flow. This is a very interesting case, this is a case when it does not have any type of angular deformation neither shear deformation nor rotation. So it is as if just it is getting stretched. So I will leave it to you as an exercise. You find out what are the velocity components u and v that will lead to this condition. What are the important restrictions you must have both the partial derivative of u with respect to y and partial derivative of v with respect to x=0.

Then both the angular velocity as well as the angular deformation will be=0 and then such a case is visible. That is known as the stagnation point flow.

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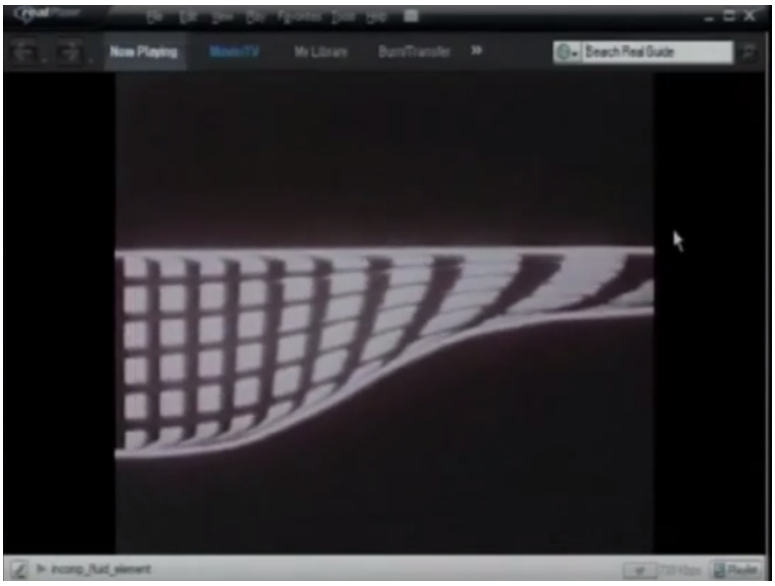

Finally, let us look into a general case of an incompressible fluid element, how this fluid element flow? So this is a visual demonstration of the continuity equation. So you see that whatever flow is entering the same flow is leaving. So if you have a constraint passage, what is happening? What is happening is that the velocities are increasing to compensate for the decreasing in the cross section area.

So this is what like a1, u1=a2 u2 that type of expression that we have seen. So we stop here today. Thank you very much.