Introduction to Fluid Mechanics and Fluid Engineering
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Lecture - 14
Fluid Kinematics (Contd.)

We were discussing about the continuity equation last time and let us workout a problem from a textbook to go ahead.

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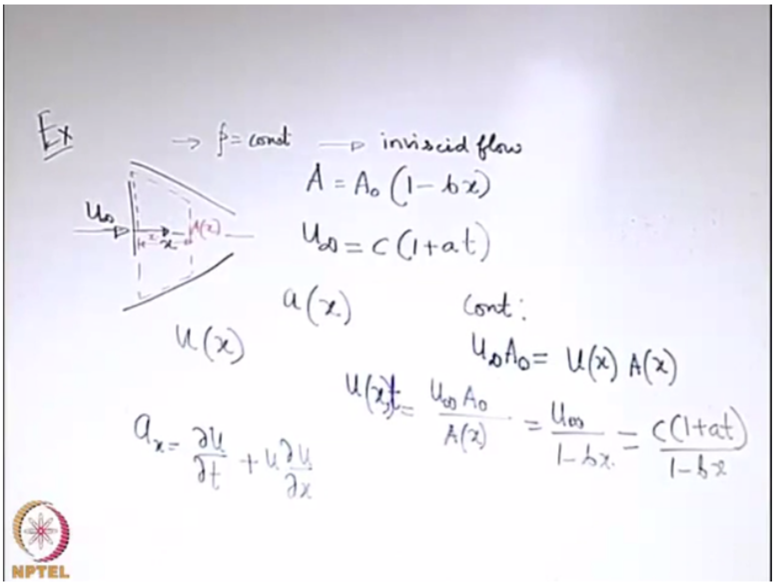

So we have a nozzle like this and it is carrying a fluid with density=constant and some other input data are given for the problem. The area of the cross section of the nozzle, it varies with area of the cross section of the inlet in the following manner where x is the coordinate measured from the entrance and other information that is given that the velocity at which the fluid enters the nozzle let us say that is u infinity that varies with time.

So it is of the form of c*1+at where these A0, b, c, a these are some given numbers. These are dimensional parameters to adjust the dimensions of the expressions. We are asked to find out that what is the acceleration as a function of x? One important assumption is density=constant. The other assumption that may not be stated explicitly in the problem but we will assume to go ahead is that it is an inviscid flow.

So when we assume it as an inviscid flow that means we are not bothering about the variation of velocity along the transverse direction. We are assuming that at each section the velocity is uniform. Now to find out the acceleration we have to go step by step. We have to first find out what is the velocity. How do you find out the velocity here? If you are given this information how do you find out the velocity?

Say u at a given x how do you find that out? You know the u at x=0, you know the A at x=0, you know the a at given x, so you can relate that with u at given x through the integral form of the continuity equation. So if you take a control volume like this one maybe where say this distance is x, so here the area a is the function of x and how do you express u infinity in terms of u at x?

See in this control surface, the surfaces which are the lateral surfaces across which there is no flow. So only flow is through the inlet and through the surface at x. So you can write from the continuity equations, it is density=constant so it is as good as u infinity*A0=u at x*A at x. As we discussed earlier, this u should have ideally been the average velocity at that cross section but because it is uniform it is same as at the center line.

There is no difference. So you can find out what is u at x that is u infinity A0/A at x. So that is u infinity/1-bx because A0/A is 1/1-bx. You have to also keep in mind that u infinity is not a constant but it is a function of time. So it is c*1+at/1-bx. Once you know that what is u as a function of x, the next thing that you can do you can utilize the expression for acceleration. So it is not only a function of x but also function of time.

So the remaining work is very straightforward that acceleration along x. It is just a 1-dimensional type of flow but here u here we would write it is not just u as a function of x but it is also a function of time. So here we write it u xt to be more precise because at a given x it will also vary with time and then you can differentiate this expression that is the straightforward exercise to find out the acceleration component along x.

There is no acceleration along any other deduction because it is just a 1-dimensional flow. So here you can see an example where you have both the temporal component as well as the spatially varying component of the acceleration. So given this acceleration, it is also possible to find out that how much time a fluid particle will take to traverse say from one end of the nozzle to the other or given in fact the velocity components.

Or just here one component of velocity you may work out that what should be the time necessary for a fluid particle which is injected here to move along the center line from one end to the other if you know the velocity component. So it is just like tracing the path line and finding it out when along that path it moves and comes to the end of the channel. So that is the straightforward extension of this one okay.

So we will move on to our next concept and that concept is related to the angular deformation of the fluid elements. So till now we have discussed about the linear deformation of the fluid elements and from the linear deformation of the fluid elements, we found out that we got as a consequence a very important equation known as the continuity equation.

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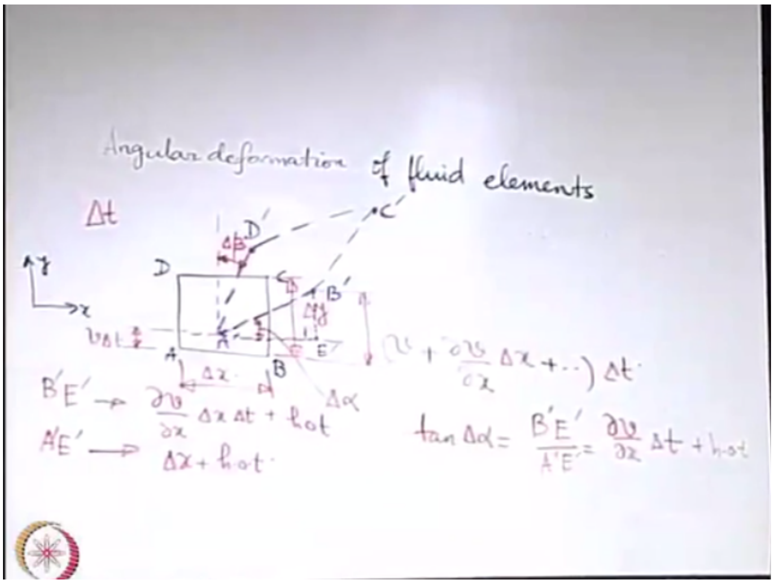

For the angular deformation, we will try to first sketch that how a fluid element when deformed angularly will look and then try to quantify it in terms of the velocity components, so angular deformation of fluid elements. Let us say that we start with rectangular fluid element like this. When we are thinking about angular deformation, the deformation is in terms of change of an angle and that angle change may take place in a plane.

So although the general fluid element is a 3-dimensional element but you can always take a 2-dimensional element and consider the deformation in the plane because no matter how complicated the deformation is it may be resolved in different planes. So let us say that this is one such plane in which the deformation is taking place. So what is the special plane? This is a xy plane.

So for example if there is a rotation with respect to the z axis then that occurs in this plane. So it does not mean that if we are focusing our attention on a plane we are actually restricted to 2-D. We are actually restricted to one component of the deformation and other components will be very, very similar. Let us say that name of this fluid element is A, B, C, D and it has its dimensions say delta x and delta y along x and y.

Now let us stretch our imagination and assume that this fluid element has got deformed with time. When it has got deformed, it is possible that it has got deformed in 2 ways, one is its volume might have got changed which is like extension of the linear deformation but its shape might have also got distorted which is more common if it is under shear. So if its shape is distorted let us say that maybe it has come to this shape.

This may not be a very regular shape. This is just a schematic. So now if you see that what are the important parameters that are characterizing this deformation? If you draw 2 lines through this new location of A, say one along x and another along y, the first important parameter that will come very much apparent is this angle say delta alpha. Here we are considering a small interval of time delta t within which this deformation has occurred.

Because if we allow large time, we will not be able to keep track of the deformation fluid is under continuous deformation. So we take a small time interval, in that small time interval the element ABE which was originally oriented along x now is oriented at an angle delta alpha with x. Similarly, let us say that this angle is delta beta. Our objective will be to quantify the time rate of change of these angles say alpha or beta in terms of the velocity components.

Here u and v because it is in xy plane. To do that we may make some simple geometrical constructions, it is not actually a construction but just to figure out what is happening. So if you think of right angle triangle like this maybe A prime, B prime, E prime. This right angle triangle is important.

Because in that right angle triangle if you know that what is B prime, E prime then you may possibly be able to express delta alpha or tan of delta alpha in terms of that and A prime E prime. So let us try to figure out what is B prime, E prime, that is the next objective. To do that we first understand or we first try to figure out that what is the vertical displacement of the point A.

See to know B prime, E prime our consideration is the vertical component of the displacement. So first we find out what is the vertical component of the displacement at A, also we find out what is the vertical component of the displacement at B, the net difference between these 2 is this length B prime, E prime that we are talking about. So what is this displacement?

Let us say that the velocity at the point A is given by the 2 components u and v, u, v are the components of the velocity vector. So at A, if you allow a time of delta t what should be the vertical displacement? v*delta t. Next, we try to find out that what is the corresponding vertical displacement for the point B. If this is v, we have to find out what is v at B? So v at A is v, what is v at B? That v we have to write here.

So what is that v? v+higher order terms. So here the change is because of change in x so that is why partial derivative with respect to x. So from A to B when you go, it is not change in y but change in x that is why this term is appearing then that*delta t. So we can say that B prime, E prime that is nothing but the difference between these 2. So that is like this. Similarly, what is A prime, E prime?

A prime, E prime is like delta x+some higher order term, it is not exactly delta x but it is why it is not exactly delta x because when the fluid element has got deformed. Now this component along x gives the linear deformation along x. So linear deformation along x is something, it is not 0 in general. Therefore, it is not same as delta x but it is actually delta x+the change in delta x.

So delta x+the strain along x times delta x and that is much smaller than delta x itself and we have figured out earlier that what is that. So if you go back to that you will see there is delta x+some higher order term. That is the new length along x. So if you find out what is tan of delta alpha then that is=B prime E prime/A prime E prime, so that will be like this and may be some higher order terms.

So the delta x from the numerator and denominator it has got canceled out. It is like this is a negligible term.

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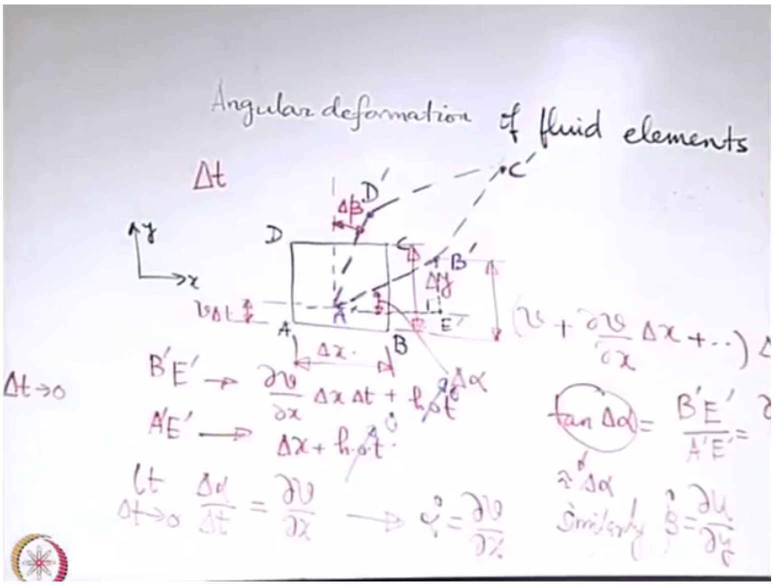

Remember we are talking the limit as delta t tends to 0 so when you are taking the limit as delta t tends to 0, this higher order term is tending to 0, this higher order term is tending to 0, this higher order term is also tending to 0. So effectively in the denominator you are left with only delta x in the numerator you are left with only this term. Other terms are vanishingly small in comparison to this dominant term.

So in that limits now as we know that if we are allowing a very small time interval delta t, this angle delta alpha also will be very small. So in that case this tan delta alpha is as good as delta alpha. The conclusion from this expression is that we can write this with limit as delta t tends to 0, we have already taken that limit but now we are just writing it more formally delta alpha/delta t=the partial derivative of v with respect to x.

And this is nothing but the rate of change of the angle of this line element with the horizontal so let us call it alpha dot that is partial derivative of v with respect to x. Similarly, we can say that what is beta dot. So similarly just by same consideration what will be beta dot? It will be the partial derivative of u with respect to y. So whatever geometrical consideration you had on this side if you have it on other side, it will exactly lead you to the same thing.

So we have got a quantification of the rate of change of these angles and these are like rates of deformation so to say. Now this rate of deformation we have to quantify in terms of certain parameters. So when we quantify the rate of deformation we have to keep certain thing in mind that we have to give a definition to what is rate of deformation. These are just changes in angles.

Now how we translate that into a more formal definition?

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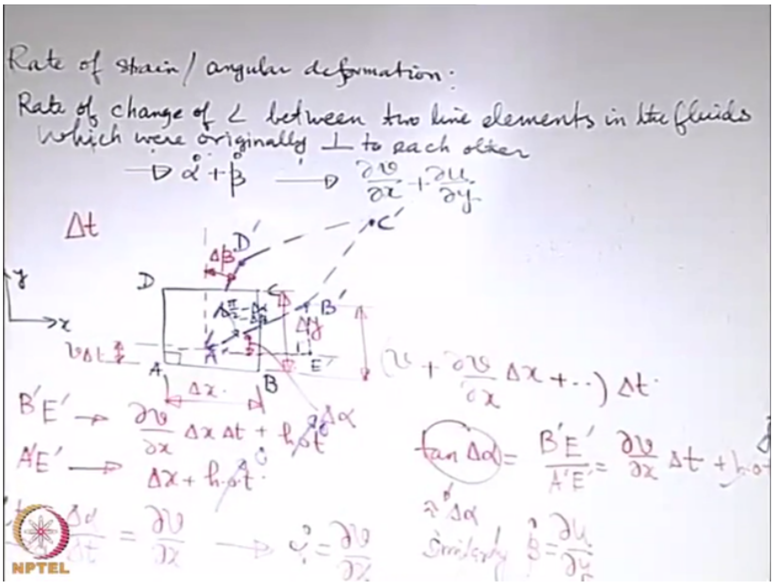

So to have a more formal definition we may say that we are defining the rate of strain or the rate of deformation in the following way. Rate of strain or angular deformation in what way? See there is a very important quantification of the distortion in the angle. What is that? If you consider 2 line elements AB and AD, they were originally at an angle 90 degree with each other.

Now these line elements are no more at an angle 90 degree with each other. They are now at an angle 90 degree-delta alpha-delta beta. So this difference between 90 degree and this one gives an indication of the angular deformation because if there was no angular deformation this angle would have remained as 90 degree. So what is the change in this angle? So what is the specialty of this angle?

We are trying to identify what is the change in angle between 2 line elements in the fluid, which were originally perpendicular to each other. So these are representatives of 2 line elements, which were originally perpendicular to each other, but with deformation they are no more perpendicular to each other. So what we are interested to find out that what is the rate of change of angle between 2 line elements in the fluid which were originally perpendicular to each other.

So if we want to quantify that we now know that we can quantify that in terms of delta alpha and delta beta and the rate in terms of alpha dot and beta dot. So what is the change? The change is pi/2-delta alpha-delta beta. So it becomes delta alpha+delpa beta that is the total change. So the total rate of change is alpha dot+beta dot. It is like delta alpha+delta beta is the change.

The rate of change is you divide those by delta t and take the limit as delta t tends to 0. So this is nothing but okay.

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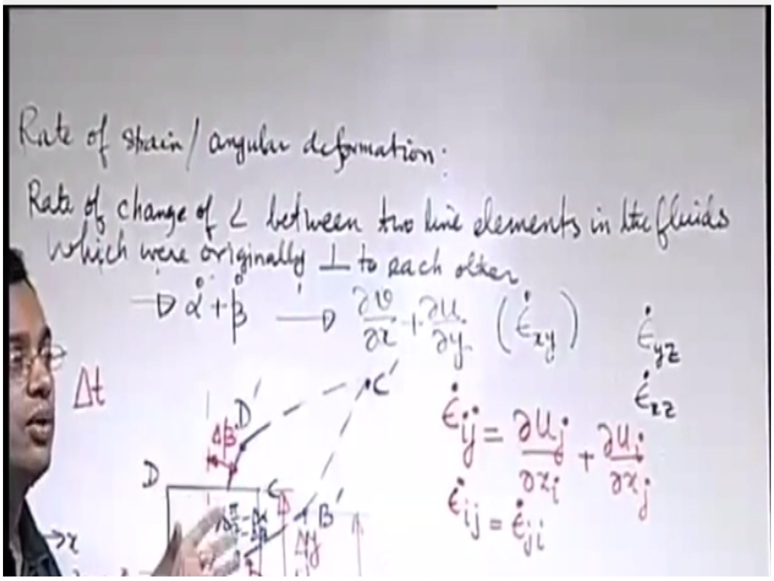

So this we may call as say epsilon dot xy as a symbol, epsilon for strain, epsilon dot for rate of strain, xy to indicate that it is actually an angular strain in the xy plane that is just a nomenclature. One may use of course different nomenclatures but I mean these also works. In this way, it is possible to write epsilon dot yz and epsilon dot xz. You see that specification of this epsilon dot requires 2 indices.

So what are the indices? The indices are the coordinate directions along which your original line elements were oriented and now because of deformation those line elements are not oriented anymore along those directions. So if you want to write it in an index notation say if you want to write now it as epsilon dot ij how will you write it? Just think in place of x and y i and j. So what will be this one?

So this is the component of velocity along j, so partial derivative of uj with respect to xi+partial derivative of ui with respect to xj. The advantage with this notation is now once you write it, you do not have to bother whether you are writing in xy plane, yz plane or xz plane. It is like you just replace i and j with the particular indices 1, 2, 1, 3, 2, 3 like that. One important observation that you can see is epsilon dot ij is same as epsilon dot ji.

Just replace i with j and j with i. So it is symmetric, so if you write it in the form of a matrix where you can see there are 2 indices each index varies from 1 to 3 that means you can write a 3 by 3 matrix using these components. Again out of that, you will have 6 independent components because of the symmetry. So this may be written in a matrix form that is the components of this, not only that what is the other thing that you get from this.

See this is also a second order tensor because it may be shown that it maps a vector on to a vector and not only that, it requires 2 indices for its specification in the Cartesian notation. So we have seen the rate of angular deformation but one important thing is we have seen examples earlier that there are cases when the fluid element is not having angular deformation as such like this but it maybe rotating like a rigid body.

So if the fluid element is rotating like a rigid body without any angular deformation then there also will be change in some angle. That will not be an angular deformation but the change in angle because of rigid body rotation. So that is also an aspect of angular deformation and in place of deformation we can just call it rotation because it is not actually a deformation.

It is a change in some angle because of a rigid body type of motion. So we will now see that what is the way in which we may parameterize or we may describe the angular velocity of a fluid element and that angular velocity definition should be such that whenever there is a rigid body motion, the angular velocity will only be there but the rate of deformation will be 0. So keeping that generality in mind, we have to define the angular velocity.

These are all definitions. We know that what we want to qualitatively represent and we are now trying to put into some mathematical definitions to quantify those physical features that is what is the exercise we are undergoing at the moment. So the next objective therefore is finding the angular velocity of the fluid element.

Now when you want to define the angular velocity of the fluid element it is not such a straightforward picture as that of rate of strain or rate of angular deformation.

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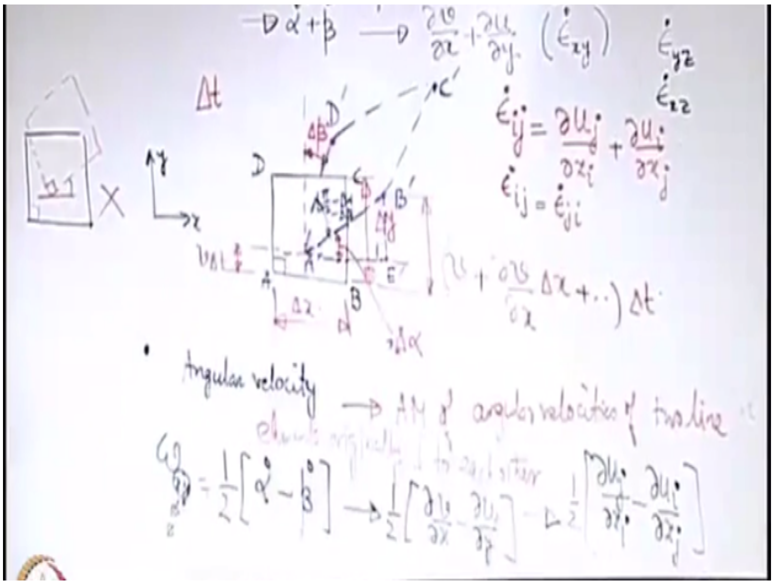

Why it is not so straightforward let us try to see, angular velocity. See whenever you were thinking of angular velocity, we were discussing about the rigid body motion but that is the special case. Usually, fluid is not under a rigid body motion just like this case. So when it is not in a rigid body motion you cannot really have a unique angular velocity of all the line elements in the flow.

So the line elements say AB it has some angular velocity, the line element AD it has a different angular velocity. That is why these angles delta alpha and delta beta they are different. If it was rotating like a rigid body, then what would have been the case? Then the case would have been like this. So if you have the original fluid element, now you have the fluid element maybe like this, so that this angle is preserved.

That is a rigid body rotation but that is not happening here at least in the example that we have drawn in the figure. So if that was the case we could clearly say that what is the rate of change of this angle and the time rate of change of that angle would have given the angular velocity, very straightforward. Here those angles are different and therefore we have to come to our angular velocity definition with a compromise not exactly same as we do for rigid body mechanics.

But keeping in mind that in the special case that it becomes a rigid body, it should follow the rigid body mechanics definition of angular velocity. So when we say that we may define the angular velocity in this way that it may be thought of as the arithmetic average or arithmetic mean of the angular velocities of 2 line elements, which were originally perpendicular to each other okay.

Now if you see what is the consequence, the consequence is as follows. So if you have a case where you have the fluid element having an angular deformation so that it is not a rigid body rotation. So arithmetic mean of the angular velocities of the 2-line element that means 1/2 of the angular velocities of AB and AD, which were originally perpendicular to each other. So what is the angular velocity of AB? That is alpha dot.

What is the angular velocity of AD? According to this figure, it is –beta dot because alpha dot in this figure is in the anticlockwise direction, if you take that as positive beta dot is in the clockwise direction we should take it as negative because we are already putting the magnitudes of alpha dot and beta dot in terms of the u and v. So this will become half of. So let us call that this is angular velocity in the xy plane.

Or sometimes people give an index instead of this z because this is also angular velocity with respect to z axis. Consider the special case when alpha dot=-beta dot. When alpha dot=-beta dot that is the case which is represented in this figure because when alpha dot=-beta dot that is even in terms of the total deformation delta alpha=-delta beta and then delta alpha+delta beta that becomes 0.

So the angle which was originally pi/2 remains same as pi/2 and that is the rigid body deformation case that we are considering. So in that case when alpha dot=-beta dot it becomes as good as either alpha dot or beta dot. So in that limit it corresponds to the definition of angular velocity in rigid body mechanics. So whenever we are defining something we have to keep in mind that in a special limit which is already known it should be consistent with that special limit.

The definition should not violate that special case. Now what we can see is this also we may write in terms of an index notation and how is it possible? You can write this as half of this one. You can write similar components of rotation in the other planes. So this is rotation with respect to z axis, you can have rotation with respect to x axis, y axis and similar such terms will be there.

This half factor is put as the matter of definition to ensure that in the limit when it is rotating like a rigid body its angular velocity is same as the definition of the angular velocity of a rigid body. That is why that adjustment factor comes. Importantly, if you forget about that adjustment factor, it is the term that is there in the square bracket.

That is actually dictating what should be the angular velocity in terms of the velocity components half you can take just like a scale factor or an adjustment factor for merging the definitions in the limiting cases. Now if we see clearly we have come up with 2 different types of angular motion representations. One is through this epsilon dot ij; another is the omega the angular velocity.

And it is possible to see that when they are combined with each other what do they actually represent? If you look into this expression like for the omega you can clearly see that if you write it in a matrix form now.

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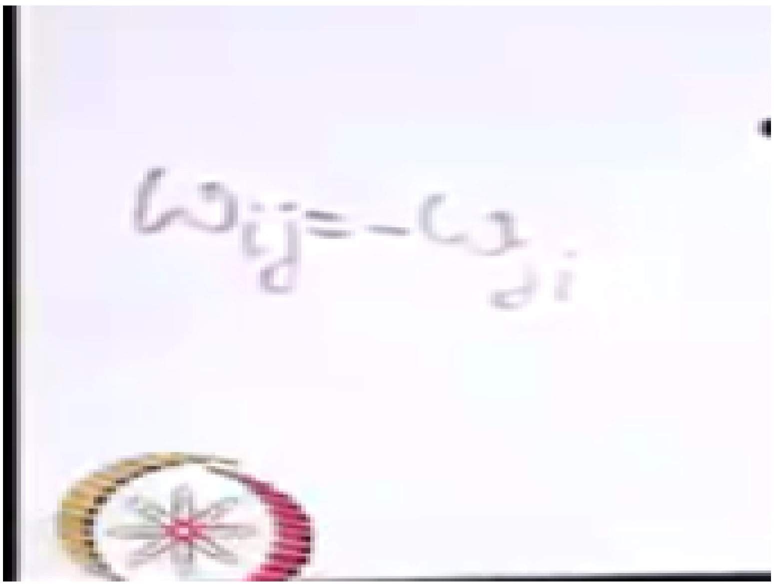

What is the relationship between say omega ij and omega ji? So you will be replacing j with i and i with j. So it will be – of so if you write it in a matrix form it is a skew symmetric matrix. Now it is possible to combine these 2 deformations and write it in some way which is again a very straightforward thing.

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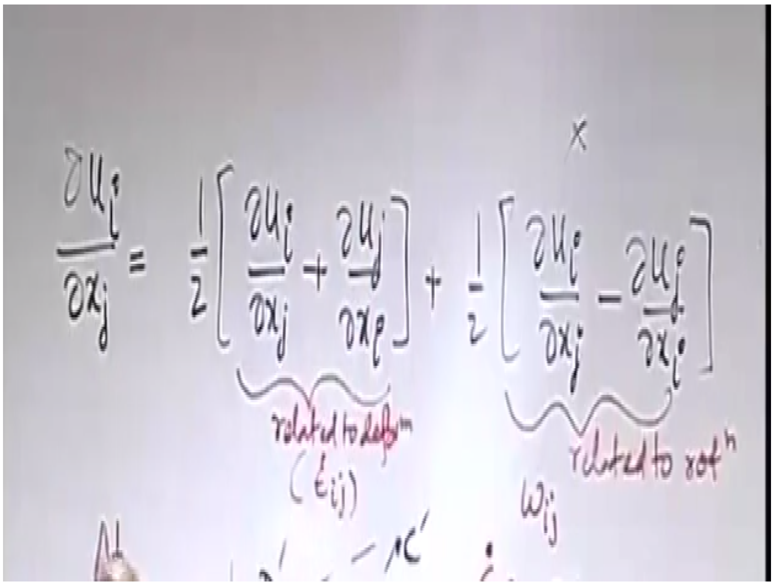

That is if you write the partial derivative of ui with respect to say xj, you can write it as this one plus of. Why it is important because this is a general velocity gradient and a general velocity gradient is what is related to what is expected to be related to deformation. Now you see that when you have a general velocity gradient out of that only one part is related to deformation and another part is not related to deformation.

It is related to just angular motion like a rigid body, so this is related to deformation and this is related to rotation. You have to clearly understand the demarcation between rotation and deformation and this is like epsilon dot ij sometimes when 1/2 is considered with epsilon dot ij, it is give just a different name say gamma dot ij. Just a matter of writing the symbols, there is not great sanctity in that.

And this one of course we know that with the 1/2 it is omega ij. So what we can see from here is a very important thing. So when we write the general velocity gradient which should be in general a function or parameterization of the deformation out of that clearly we distinguish that one part is not related to deformation. So only this part is related to deformation.

And whenever you relate shear stress with the velocity gradient, this part is what is important because this part is giving rise to angular deformation. The other part is the rigid body motion, so it should not be relating the constitution of shear stress in a material in a fluid. So whenever we have discussed about the Newton's law of viscosity you clearly see that this is the term that we had actually taken.

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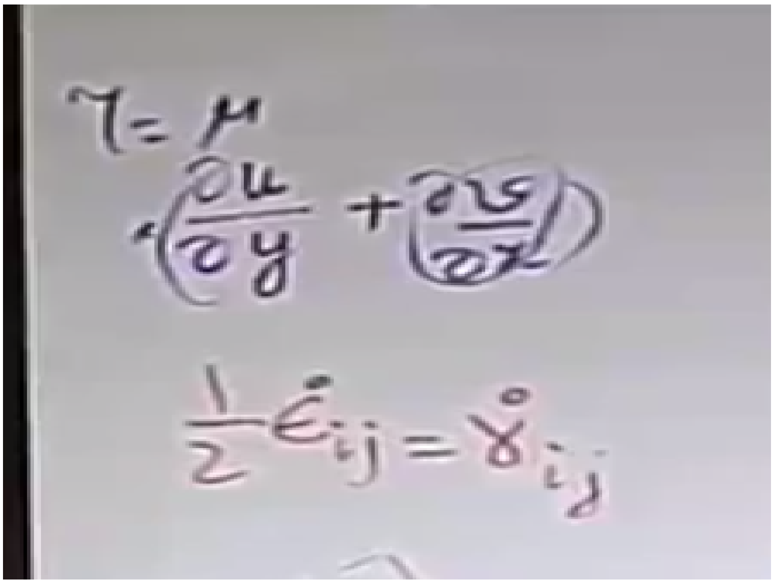

So if you write if you look it in terms of say u and v, it was what was the quantification of this deformation we wrote partial derivative what we wrote, we wrote tau=mu times this one. Actually, we should have written it this+this one because we were considering a unidirectional flow, v was 0 and that is why this term was not considered.

We will look into it more that is so whatever form of Newton's law of viscosity we have discussed in the fundamental way it is correct that is the shear stress is linearly proportional to the rate of angular deformation but the quantification of angular deformation that we made earlier was based on a very simple case and assumption. So as we advance and proceed more and more we will come into more and more vigorous ways of writing the Newton's law of viscosity.

And we will again take it up later on in one of our chapters. We will not take it up in this chapter because this just bothers about the kinematics. So this does not bother about the force, so just because it has come in this context. I am just reminding you that it is not that the Newton's law of viscosity is like mu*du/dy it is like the rate of deformation maybe expressed by other terms also.

In that special example, which we took up in our earlier lectures we consider there is no other velocity component therefore those other terms were not appearing. The other important observation is that although it looks something which is nontrivial but actually there cannot be a more trivial expression than this because it is just like you are writing a form every matrix can be written as a sum of a symmetric and as skew symmetric matrix.

And that is what we are trying to do. So eventually that symmetric matrix is being represented by the components of the angular deformation and the skew symmetric matrix is represented by the components of the rotation. So in a more formal way since we are dealing with tensors, so matrices are some of the ways by which we may represent say a second order tensor in a notational form.

So in general, these rules are for tensors and we can say that any tensor maybe decomposed as a sum of a symmetric and as skew symmetric tensor. So matrix is a very special example illustration way of writing it. Next, what will do, we will see that their interesting quantities or parameters which are related to the angular velocity of the fluid and based on that and we will define a term called as vorticity.

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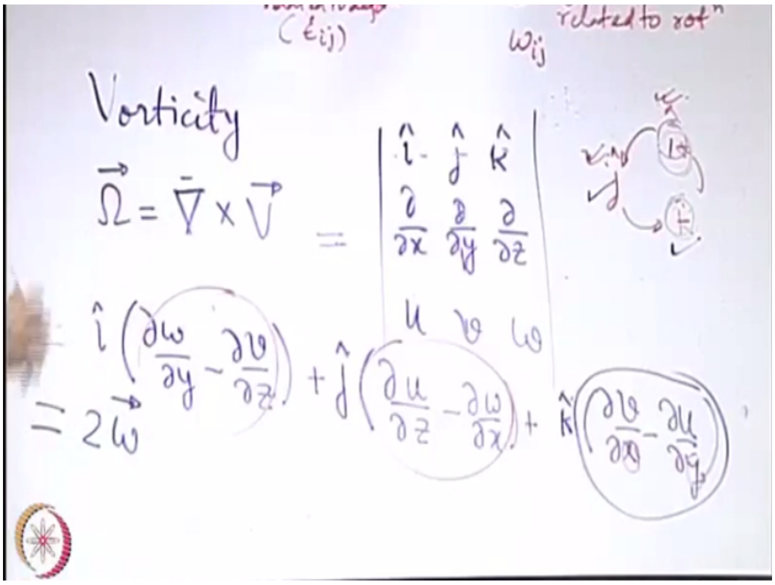

So what is the vorticity? See the entire subject of fluid mechanics in its fundamentals was developed by mathematicians. So we are trying to give a physical basis or physical insight to whatever concepts that we are discussing because we have to keep in mind that after all we are trying to learn it in an engineering context and we have to understand that what physics goes behind these mathematical derivations.

At the same time, when the subject was first developed, it was developed in a sort of true mathematical way and in certain cases quite a bit abstracted from any physical reality and in that perspective the term vorticity which was defined as a vector say it was defined as the curl of the velocity vector. Now when it was defined as the curl of the velocity, it was just defined as the mathematical quantity.

But important thing is that such quantities are always defined in a general mathematical theory known as field theory. See field theory is something which is so general that it is applicable equally in electromagnetics than in fluid mechanics and so field theory talks about a vector field or maybe a scalar field but in general a vector field and what are the different rules that govern the behavior of the change of vector in a vector field.

So when you are talking about electromagnetics, you are having certain parameters. When you are talking about fluid mechanics, you are having different sets of parameters. These are physically different sets of parameters, but when you look into the mathematical field theory, there is hardly any distinction between fluid mechanics and electromagnetics in terms of the basic mathematical theory that goes behind.

We will see later on a couple of examples where you will find it very much analogous to as if there is an electrical field and something is happening with the electrical field. Now this vorticity when it is defined in this way. So let us try to expand it in terms of its Cartesian components. Let us see that will give us a fair idea of what physically it tries to represent. So we will write it in a determinant form.

That is the vorticity vector, so let us write its components. What are the components? So we are now expanding with respect to this alright. Now think of the special case of rotation in the xy plane that we were talking about. So if you have a rotation in the xy plane, you can clearly find out that this is one term that came into the picture when we defined an angular velocity. In fact, this is just double of that.

If you are interested about rotations in the other planes, you will easily find out that those are given by half of these components. So this is a vector, which physically is nothing but 2 times the angular velocity vector. So vorticity is a mathematical definition but what it gives physically a sense of rotationality in a flow. So if the rotationality in the flow is very strong, we say a vortex is created in the flow and the strength of that is given by the vorticity vector.

So that is the physical meaning of a vorticity vector. So if you have say a line element say this type of a symbolic line element, you put it in a fluid. If the fluid has an element of rotationality, it will change its orientation. Otherwise, it will move parallel to itself in the flow. So that will give a visual understanding of whether the flow has an element of rotationality or not and that mathematically is quantified by the vorticity vector.

If you see if you know one of the components of the vorticity vector, it is possible to generate the other one intuitively without going into all the cross products. So many times the common mistake is like in the sign. So which term will come as positive and which term will come as negative, for that you have to just keep in mind that you maintain a different cyclic order for the x, y and z components.

So like if you follow the cyclic order, see when you are writing the z component, first comes the x and then comes the y. So first comes the x derivative and then the y derivative. Similarly, when you are writing the x component first comes the y derivative. So when you are writing this component first comes the y derivative and then the z derivative. When you are writing the j that is the y component first you are coming with this one and then with this one.

So if you maintain this cyclic order, it is possible to reproduce that without doing the cross product each and every time. In summary, we have discussed the quantification of the linear and angular motion of the fluid elements. Now based on this, we will come up with a few important conceptual understandings. So those conceptual understands are based on 2 important terminologies in fluid mechanics.

One is stream function; another is velocity potential. Let us see what is stream function.

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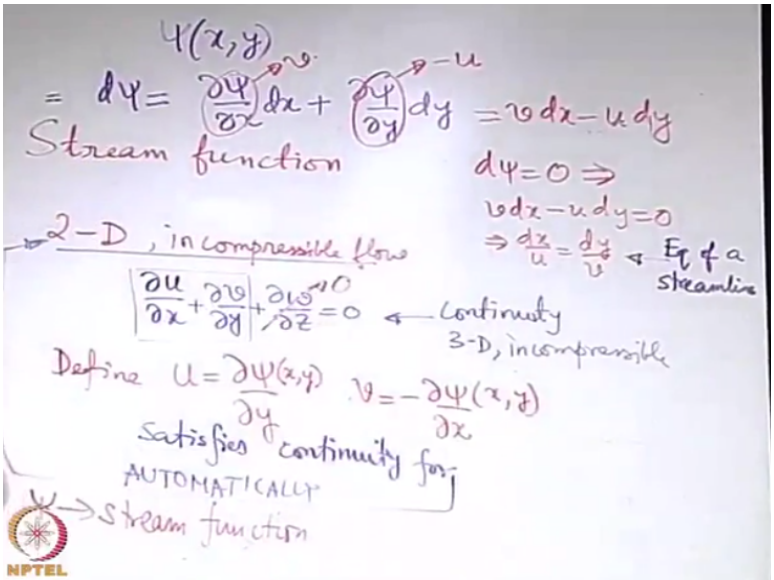

So stream function is defined as follows. To look into the definition let us first figure out that what is the motivation behind defining such a function. Let us take an example of 2-dimensional incompressible flow and write the form of the continuity equation for that. What is the form of the continuity equation for 2-dimensional incompressible flow?

If it was a general 3-dimensional flow, then it would have been this equal to 0 for a general 3-D but incompressible flow because this is the condition for incompressibility zero volumetric strain. So this is for continuity for 3-D and incompressible flow. Now when it is 2-dimensional that means the third velocity component is not important, so you are left with these 2 terms.

Now let us say that you are interested to find out the velocity components u and v. We will later on see that this is not the only equation that governs the change in u and v, there are other differential equations which need to be coupled but just in a notional or a mathematical form if instead of the 2 variables u and v, you could transform into a single parameter.

That is express u and v in terms of a single parameter that satisfies automatically this form of the continuity equation. Then u and v maybe parameterized with respect to that new function. So what type of parameter that we may choose, let us say we define u=partial derivative of psi with respect to y and v=-partial derivative of psi with respect to x where psi is a function of x and y.

If you do that then what is its effect on this continuity equation. You see that this definition satisfies the continuity equation automatically provided it is continuously differentiable up to the second order. So this definition satisfies the continuity equation for 2-D incompressible flow. So continuity for this special case which special this special case automatically.

So the objective is that we are trying to define the velocity components in terms of a mathematical function, which ensures the satisfaction of continuity equation automatically because no matter how complex or how simple the flow is it should satisfy the continuity equation. So this definition cannot violate that. See why we are restricted to such a case because for a more general case, it is not easy to find such parameter.

So if you had a 3-dimensional case, then it would have not been possible to find out such a parameter automatically that satisfies this general equation. So this definition is restricted for 2-dimensional incompressible flow. This function psi is known as stream function. One could give many names to this function psi but why the name stream function? What is the specialty about a stream function?

The specialty is that it has some relationship with the concept of streamline that we have learnt. What is that relationship? To figure out that let us write psi mathematically as a function of x and y. So when we want to write d psi how we can write d psi? psi is a function of x and y. So what should be d psi? Plus, right, now let us substitute the u and v. Remember one thing, one could also write u as – of this and v as +of this.

So there is no sanctity with this +or –. Only thing they have to be of opposite sign to satisfy the continuity equation. So different books will take either this as + or this as – or maybe this as – this as + nothing is wrong because your objective is to satisfy the continuity equation nothing more than that. So if we take this particular definition then in place of this like the first term you can write v and this as –u.

So this will become vdx-udy, so what is represented by d psi=0, d psi=0 is represented by vdx-udy=0 that means dx/u=dy/v. What is this? This is nothing but equation of a streamline. So what is the very important conclusion? Important conclusion is along a stream line there is no variation in stream function that means one stream line represents a particular constant stream function. That is the stream function=constant along a given stream line.

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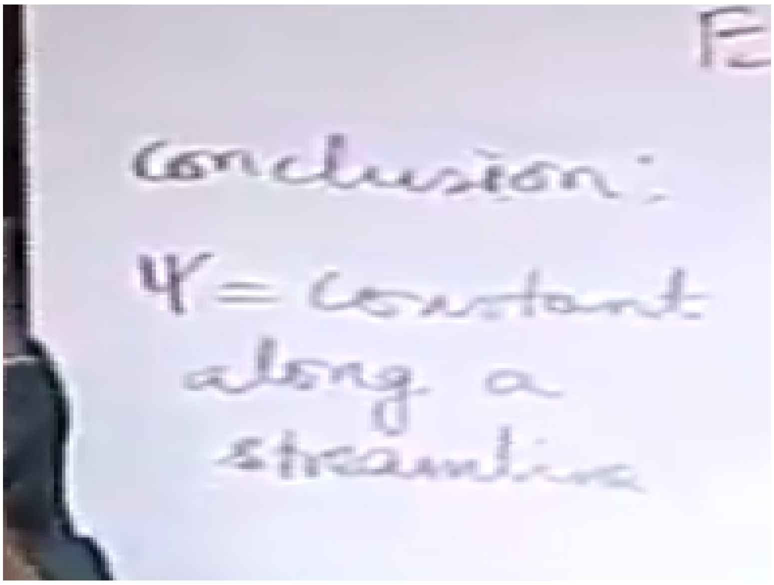

So the conclusion is psi=constant along a stream line and that is why the name stream function. Another important thing you have to keep in mind, what is that important thing? When you say psi=constant along a stream line, you should not be confused that always psi is defined. When psi is defined? It is only defined for 2-dimensional incompressible flow. Stream line is defined for all types of flow.

So this relationship is not for all types of flow, is only for that case when both are there. Stream line is always there but always psi=constant along a stream line is not relevant because always psi is not defined, only for 2-dimensional incompressible flow this definition works and only for that case we may say that it is constant along a stream line. It does not mean that the stream line is not there if it is not a 2-dimensional incompressible flow.

It is very much there but the stream function is not defined in this way. So you cannot have analogy or relationship between those 2 okay. We stop here and we will continue again in the next lecture.