Introduction to Fluid Mechanics and Fluid Engineering
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Lecture – 13
Fluid Kinematics (Contd.)

We were discussing about the continuity equation last time and we will see now that there may be a different way of deriving the continuity equation not only one different way but there could be many different ways of looking into that. We will look into one such alternative way of deriving the continuity equation here and in our subsequent chapters, we will look into other possibilities.

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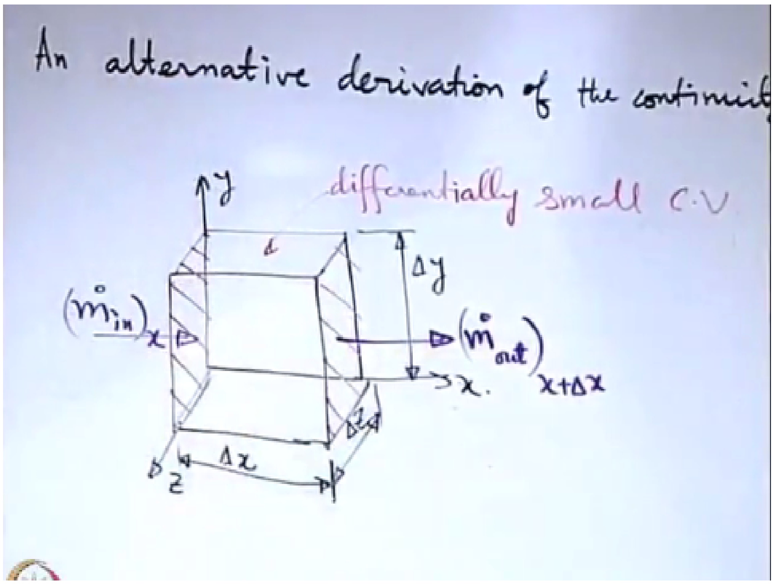

So, more number of different ways we look into it gives us a better and better insight of what is there actually in the continuity equation. So, we look into an alternative derivation of the continuity equation. In this alternative derivation, our objective will be to look into the entire thing from an Eulerian viewpoint that means, we will identify a specified region in space across which fluid is flowing and that we call as a control volume.

So, in a control volume of a particular extent, let us for simplicity in deriving the equations, assume that the control volume is of a rectangular parallelepiped shape, so it has its dimensions along x y and z as say delta x, delta y and delta z, which are small and in the limit, we will take all these as tending to 0. So, this is a differentially small control volume that is the entity; differentially small control volume.

Now, what is happening across this control volume? Some fluid is coming in, some fluid is going out and that is occurring over 6 different phases and each phase has a direction normal and basically the mass flow rate across that phase is taking place normal to the direction of that respective phase. So, if we consider the flow rate along x, then we should be bothered about which phases; we should be bothered about these 2 phases.

Because these 2 phases have direction normal along x, let us see, what is the mass flow rate that gets transported across these phases? So, there is some mass flow rate that enters the control volume along x. Let us symbolize that in this way, across the opposite phase say there is some mass flow rate that goes out and that occurs at x + delta x, if this is x, this must be x + delta x.

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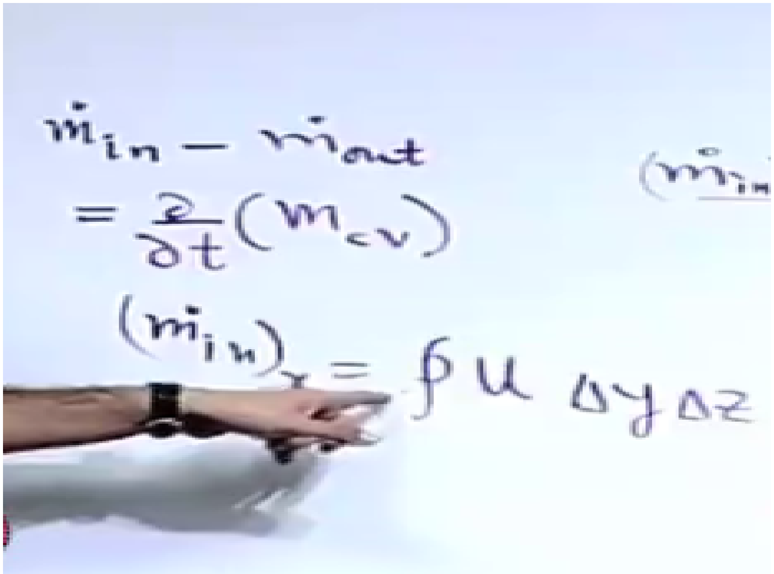

So, how do we characterize the difference between these 2 because what we have to remember, we have to remember the mass balance? What is the mass balance, what is the rate at which the mass is entering, say m dot in, it may be along x, y or z - m dot out that also may be along x, y, z. So, what is the difference between these 2? Say, there is some mass flow rate coming at the rate of 10 kg per second.

And say, there is a mass flow rate that leaves the control volume at the rate of 8 kg per second, so what the remaining 2 kg per second will do? That will increase the mass of the; mass within the control volume; see control volume has a particular volume; it does not have a fixed mass. So, if it is a compressible flow say, it is highly possible that the mass inside these changes.

So, that remaining 2 kg per second may contribute to the rate of change of mass within the control volume, so we can say that mass flow rate in - out is nothing but the rate of the; time rate of change of mass within the control volume. Again, why do we use a partial derivative here? Because by specifying the control volume here by some fixed coordinates, we are assuming that we are freezing its locations with respect to position.

And trying to see what happens in that frozen position with respect to time, so, that is why a partial derivative with respect to time. So, when you say, m dot in – m dot out, you have to remember that it has like contributions for flow along x, y and z, so we can try to write what happens along x, similar expressions will be along y and z, so m dot in x. So, what is; how do you calculate a mass flow rate given a density?

So, to calculate the mass flow rate, you require first to obtain the volume flow rate, so what is the volume flow rate? Volume flow rate is the normal component of velocity perpendicular that is velocity component perpendicular to the area times the area, so the component of velocity along x is u. What is the area perpendicular to that; * delta y * delta z that is the volume flow rate that multiplied by the density is nothing but the mass flow rate, very straightforward.

So, when you write m dot out at x + delta x, basically you are looking for the value of this function at x + delta x, you know the value of the function say at x, again you can use the Taylor series expansion. In the Taylor series expansion, when you expand it you keep in mind that delta y * delta z is fixed. So, what you can do; you can always take it out of that derivative and think about the expansion of rho * u.

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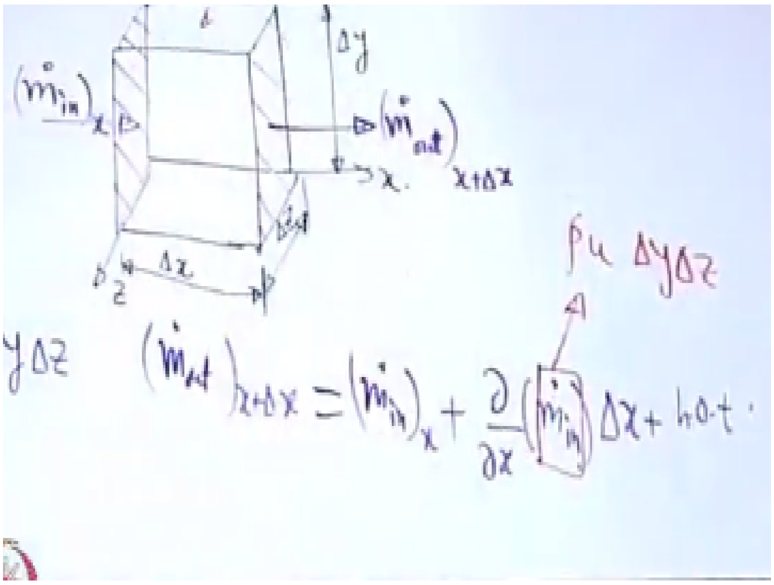

So, this will be m dot in x + higher order terms, let us substitute what we can write in place of m dot in x, so that is rho u delta y delta z.

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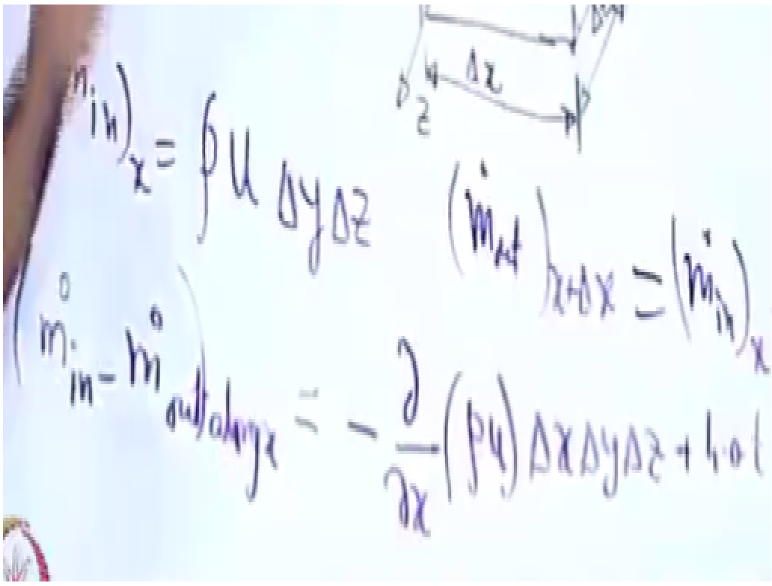

So, clearly if we write what is m dot in – m dot out along x, what is that? So, this term will come in the other side, it will be minus of that into delta x delta y delta x + maybe - higher order terms, right just by simple transformation or taking one side terms in the one side to the other, so the minus sign appears.

Similarly, you get what happens along y and z and you can write therefore the expression of m dot in - m dot out, we will do that but before that let us see what happens to the right hand side. So, what is the mass within the control volume? So, we have to write basically the time derivative; partial time derivative of that, so what is the mass within the control volume? Let us try to write what is the mass within the control volume, what is that?

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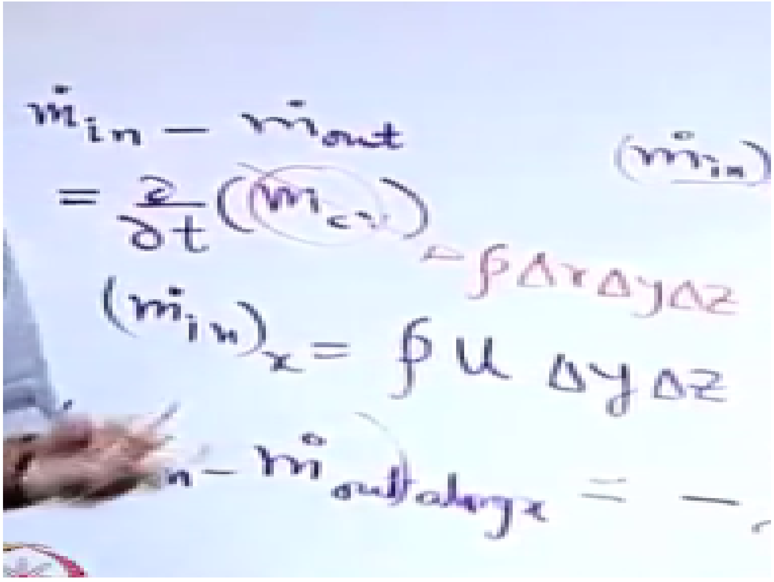

Yes, so it is the density times the volume, so rho; if it is the density that times delta x * delta y * delta z that is the mass within the control volume. So, what we can do now; we can write a mathematical expression of this physical balance because we now know how to write all the terms and we take the limit as delta x, delta y, delta z tends to 0, so that delta x, delta y, delta z that gets cancelled from both sides.

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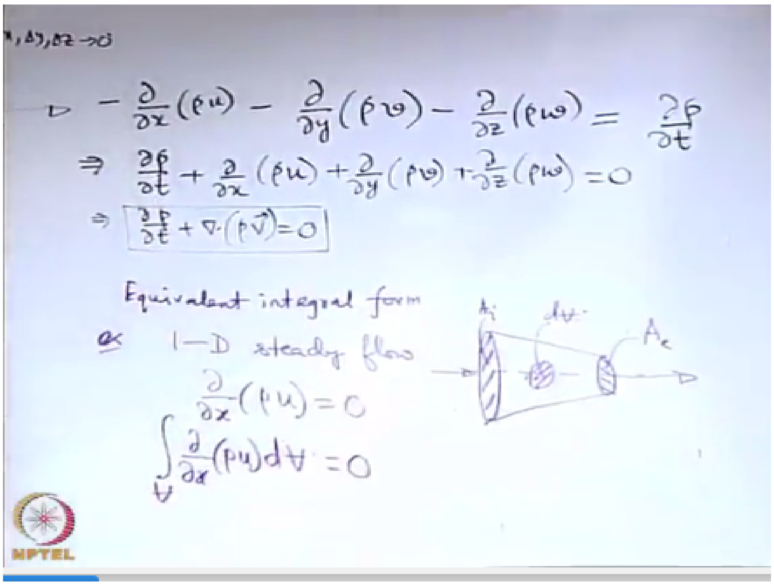

And in the higher order terms some small terms remain which in the limit as that tends to 0 that term will be 0, so with that limit that is delta x, delta y, delta z all tending to 0, so you will have for flow along x that will be the case, flow along y, what should be that term and flow along z. See, the individual velocity components are responsible for flows along certain directions and that is what we have to keep in mind that is equal to, what?

Because delta x, delta y, delta z gets cancelled, so you can write this in the well-known form that we saw in the previous class or equivalent vector notation, which is the continuity equation. So, we have seen at least 2 different ways of deriving the continuity equation and keep in mind these are not the only 2 ways but at least these have given us some insight of what this law or what this equation is talking about.

We will have a couple of important observations related to this before we go on to a problem where we illustrate how to make use of such equations. So, the first point is that you see this is a differential form now, let us say that you want to express in an integral form, so how will you do it? We will later on formally see one methodology by which you can convert easily from differential to an integral form.

But without going into that formality, let us look into a very simple example by which we see that how to do it. So, we are now interested about equivalent integral form. What is that equivalent integral form? We will not go for the most general case that we will study later but we will consider a very simple case as an example, one dimensional steady flow say the flow is taking place along x.

Maybe just for your visual understanding, let us say that the flow is taking place through a nozzle like this and nozzle is something where you have; when the flow is entering, it is entering with a higher velocity and as it is moving along it the area of cross section, gets reduced so the velocity of the flow gets increased, so it is. You may assume that it is something like a conical shape maybe, frustum of a cone or something like that.

The shape is not important for us, we will just keep in mind that there is an inlet section with area Ai I and there is an outlet section with or exit section with area Ae and the flow is taking place along x, so one dimensional steady flow, so how do you simplify this differential form when you have a steady flow, the time derivative is not there? When you have one dimensional flow, it will just boil down to only the x component of velocity.

But it may be compressible or I mean otherwise, rho may be a function of position or whatever, so we are not committing ourselves to a constant rho and we are just putting the rho inside; keeping the rho inside. Now, what we will do is; we will try to integrate this over the entire volume of the nozzle, so that means what we are trying to do; we are trying to integrate it.

So, we have a small volume element say, a small volume element dv. Why we require a small volume element to consider? Because u is specially varying, so we are taking u at a location, where u at that particular x and then we are integrating that over the entire volume by considering such elemental volumes, so that is; that integrated over the entire volume that should be equal to 0.

It is very straightforward, if the function is 0, its integral should be 0. On the other hand, if the integral is 0, function need not always be 0, right but we will see later on that there are certain cases when if the integral is 0, we may say that the function itself is 0 under certain important considerations but not for all considerations but here it is the other way, which is more straightforward.

Now, when you have it, see our objective is to convert this volume integral to an area integral because we are interested about the areas across with the fluid is flowing. So, if you recall in vector calculus, there is a theorem called divergence theorem; Gauss divergence theorem which converts the volume integral into an area integral or vice versa. So, what is that theorem?

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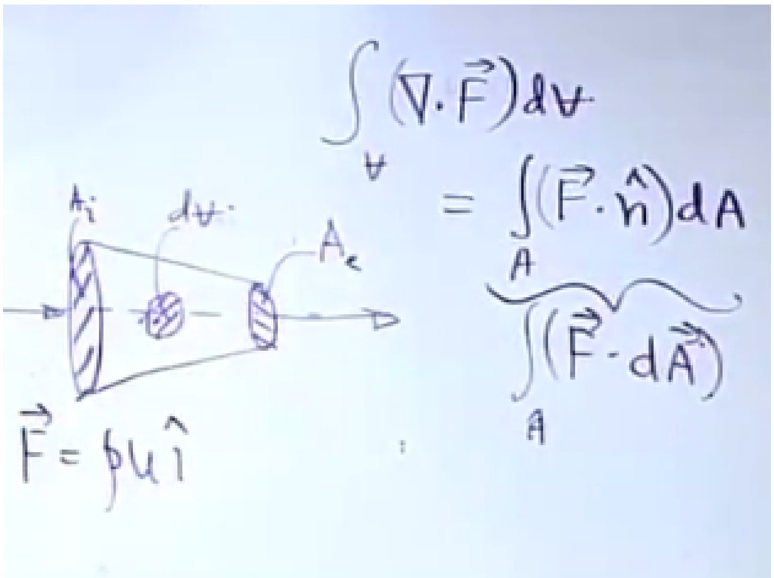

So, if you have; if you have say, F as a vector function; general vector function, so if you have a divergence of F over a volume that is given by area integral, sometimes this is also written in an equivalent notation as F dot dA by giving the area directly a vector sense, so giving an area vector sense is like magnitude of the area times the unit vector normal to the area, so that anyway takes care of this.

So, when you write dA as a vector, it is n cap dA scalar, you have to keep in mind that it is always the outward normal that is considered to be the positive direction of any area. So, if you see this theorem, if you want to convert it to an area integral, you have to cast its form in a divergence, so say we want to cast it in a form of a divergence. So, what should be the vector function here, F.

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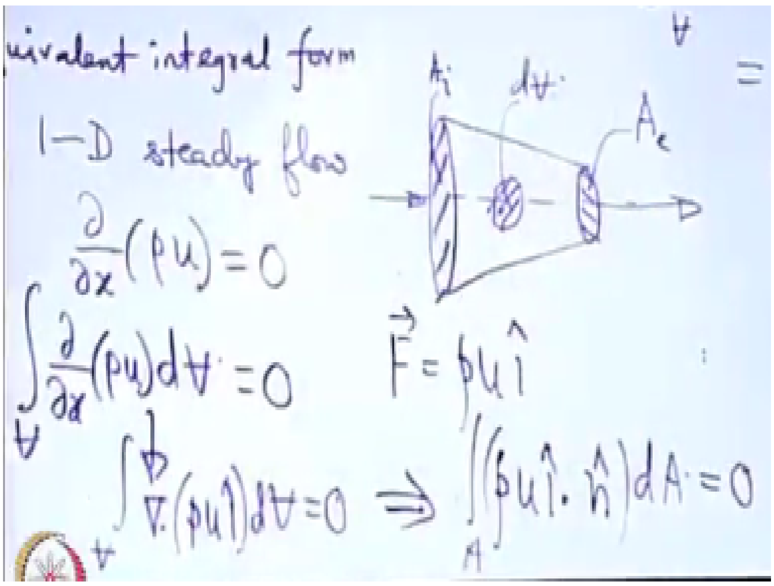

So, what F should you choose such that it is in the form of a divergence, yes rho ui, right, so the divergence of that will give you this partial derivative, so we can write this also as divergence of rho ui dv. Now, if you look at this theorem, this is the mathematical statement of the theorem but it has a very important understanding. What is this A and what is this V? It is not any arbitrary A, an arbitrary V, this A is the area of the surface that bounds the volume V closely.

So, when you have the volume V here, it is bounded by say, lateral surfaces and these cross sections. So, when we are writing this in terms of an area integral that area integral should consider Ai, Ae and the lateral surfaces also; lateral surfaces at the end will not be important because there is no flow across those surfaces, so those are like irrelevant from flow computation considerations.

But fundamentally, it is the entire surface that is bounding the volume, so you can write this by using the divergence theorem as how do you write this? Rho ui dot n dA over the area A that is = 0. Now, let us look into this form, so when you consider this dA that area element now you have as we mentioned 3 types of like; one is inlet, another is exit and another we may call as wall, across which there is no flow.

So, when you have the wall, it is not necessary to calculate to bother about this integral for the wall because there is no flow there, so we will therefore break it up into 2 integrals; one for the area Ai, another for the area Ae, other areas are not relevant. So, when you consider the area Ai, what is the n cap for the area Ai? –I.

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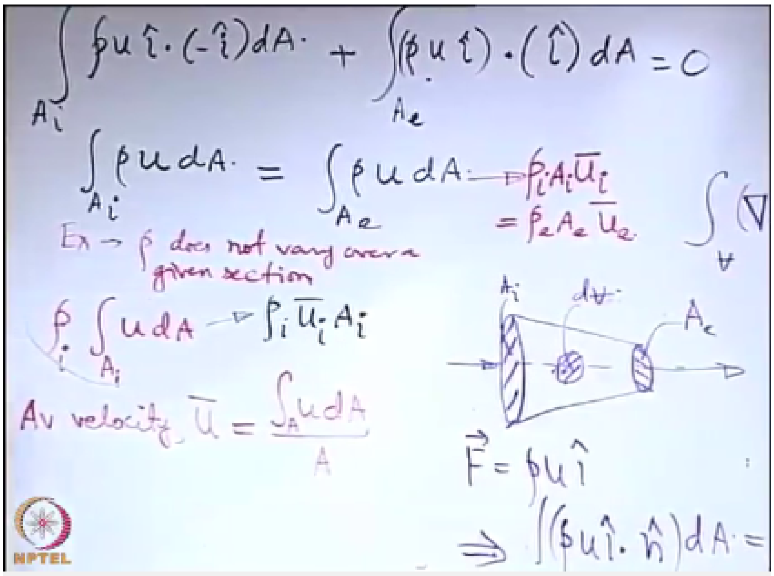

So, you have rho ui dot – i dA, this integral over Ai + rho ui, now what is for Ae; n cap, it is I, so this dot i dA for Ae = 0, so you can write integral of rho u dA, so what does it say physically? It says that at for steady state whatever is the mass flow rate across this section the same must be the mass flow rate out across this section that is what it is physically saying mathematically the statement is straightforward.

Now, it is many times convenient to express this in terms of the average velocity because it might so happen that u is a function of the transverse coordinates, let us say we have transverse coordinate as maybe say r or y that type of a coordinate and it is possible and it is almost always likely that u will be the function of a transverse coordinate because u will be 0 at the walls by no slip condition.

And then you will change may be maximum at the center line, so it is expected that along the transverse direction, u will vary, so it is not that we are talking about a cross-sectionally constant u, it is rather a cross-sectionally variable u. Now, if we assume that rho is not varying across the section as an example, so let us take an example, where rho does not vary over a given section but it may vary from one section to the other.

So, rho does not vary over a given section that means, you can take that rho out of the integral and you are left with these types of terms. For example, for the left hand side, it will be rho at the inlet surface; inlet section times this integral. Now, there is a definition which is called as average velocity, so how the average velocity is defined? The average velocity is a cross-sectionally average velocity.

So, for a one dimensional flow, it is like u average, so we call; we give a notation u bar to indicate that it is an average, so it is basically integral of u dA/A. What is the physical meaning of this? Physical meaning is; see, the entire section has a variable velocity with that variable velocity, it has a flow rate; volume flow rate. Now, if you have the same volume flow rate with an equivalent velocity that would have been uniform throughout.

Then that uniform equivalent velocity is the average velocity, so what we are basically doing; we are equating the volume flow rate, in one case it is a variable velocity; the real case, in the other case, it is an equivalent idealistic case, where it is a uniform velocity but the end effect the mass the volume flow rate is the same and then that equivalent velocity; equivalent uniform velocity over that section is known as the average velocity, okay.

So, we can replace these integral or this term by what; we can write this as rho i * ui average * Ai, therefore we can clearly say that this equation; this boils down to a very simple form rho i Ai ui average is = rho e Ae ue average. If the densities are not varying spatially, then rho i and rho e get cancelled out, so you get Ai ui average is = Ae Ue average, which is like a A1 V1 = A2 V2, these types of equations we have used earlier for solving simple problems.

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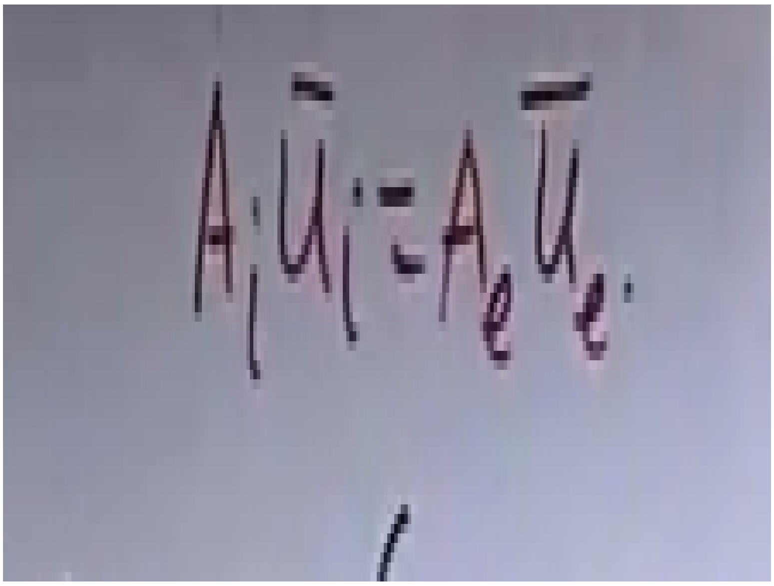

Now, what you realize here? See, whenever you come up with an equation again, I am saying you have to keep in mind that what are the assumptions. So, if you have say an equation like this Ai ui is = Ae ue. So, what are the assumptions under which it is valid? See, we have of course, we may go on deeper and deeper into the assumptions but let us talk about only the major assumptions. What are the major assumptions?

First, rho is a constant, the density is a constant then we are talking about these velocities not local velocity at a point what cross-sectionally average velocity. If it is an ideal fluid flow, then it is possible that local velocity is same everywhere because the velocity gradient is created by viscosity. So, if you have no viscous effect, then the effect of the wall is not propagated into the fluid and it is possible that there is a uniform velocity profile.

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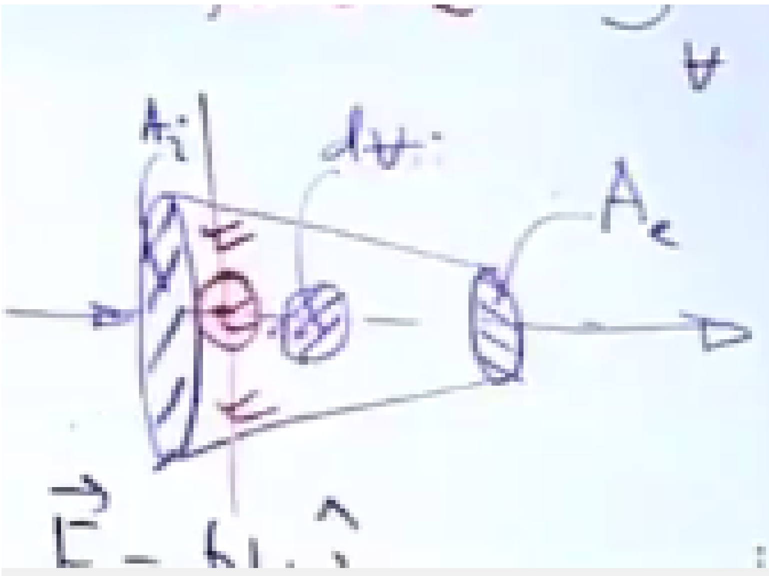

So, then the average velocity and the local velocity may be the same, so if you consider, say a cross section like this. Now, let us identify 3 different points, say these 3 different locations, at these 3 different locations, the velocities are different. So, when you write say, Ai ui bar may be sometimes in your previous studies, you have written it as the velocity at this point, right, fundamentally that is incorrect that is wrong.

When you get rid of that wrongness by only one thing either you are writing; you are although, you are thinking that you are writing velocity at that point actually, you are writing the average velocity over the section or even if it is velocity at this point that may be okay, if the velocity is not varying over the section that is a uniform velocity profile if it is there across the section.

That means, you are implicitly treating it as an inviscid flow, so these are some subtle important concepts that go into the equation that is why I always say that try to get rid of whatever you have learnt for the entrance exam preparations because you know the end formula but many times, you do not know that what are the restrictions under which you are using that end formula.

And that may be dangerous that will be more dangerous I would say it is worse than not knowing the formula, so, let us keep that in mind. Now, next let us look into another issue that we have discussed about the continuity equation in a general vector form but we have not looked into the other coordinate systems. We have looked into the Cartesian coordinate system.

But let us say that we are also interested about the cylindrical polar coordinate system that coordinate system many times is important, if you have something of say cylindrical symmetry; some body of cylindrical symmetry. We will not go into the detailed derivation of the continuity equation for a cylindrical coordinate but I will tell you how to do it and I will leave it on you as an exercise to complete it.

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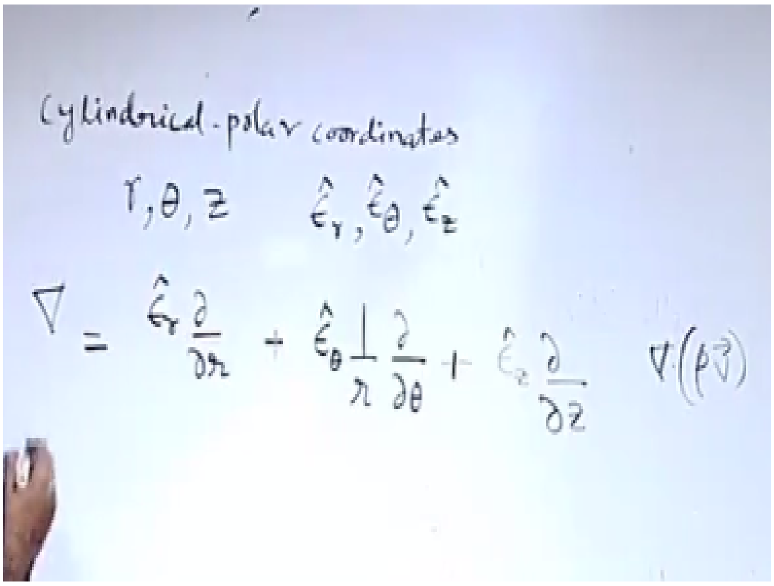

So, if you consider say, cylindrical coordinates; cylindrical polar coordinates. So, in the cylindrical polar coordinate, you have a polar nature that means you have the r theta coordinate system just like the polar coordinate and you also have a 3 dimensionality, so you have the axial coordinate system given by the z. So, t theta z, coordinate system and let us say that these have their unit vectors as given by epsilon r epsilon theta and epsilon z, just like i, j, k.

Now, we can; so we have to see that; what are the differences in the Cartesian system and in this system. So, first we have to know what the del operator in the system, so first just like the i, j and the k here also, you will have the corresponding epsilon r epsilon theta epsilon z, so for the first component that is for the component along r, it is just like component along x for the Cartesian coordinate.

For the component along theta, it will be this one because they have to keep in mind that the line element along theta is like rd theta that is the length element not only that even if you forget about that fundamental consideration, just look into the dimensionality i, it is 1/length, so it has to be that this unit should be length. So, when you write that theta derivative, theta is like it does not have a dimension.

So, you have to adjust it with a linear dimension, so just from the dimensional arguments also like these things I am telling because if you are confused that you are not being reminded that what should you write at least, these common sense things should guide you that what should be the correct way of writing this and then this. So, in the continuity equation see the first term is the time derivative.

So, the time derivative you do not care much you know that like it is not dependent on these operators; the time derivative of the density but the next term is you have the divergence of rho v. So, their calculation with a del operator will be important. Can you tell where does it fundamentally differ from what you do in the Cartesian system? Yes, there is only one fundamental difference and if you keep that difference in mind, it is just a straightforward exercise.

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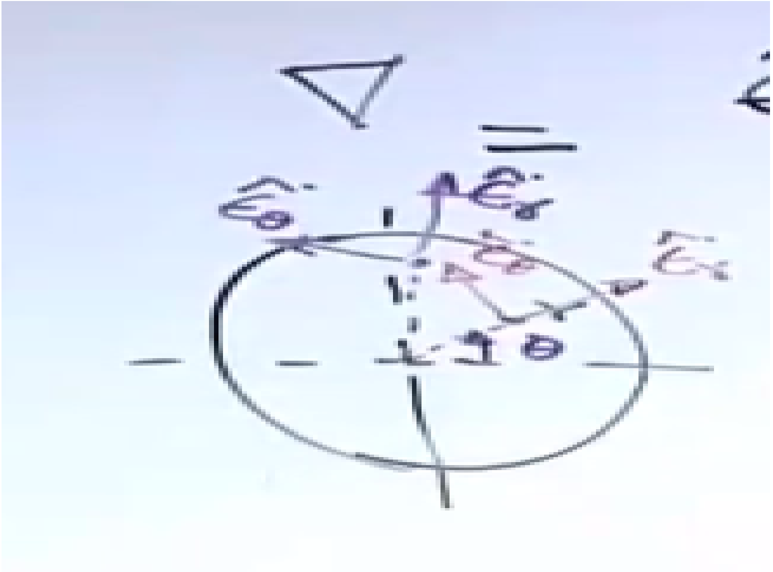

In the Cartesian system, when you have i, j, k those are invariant in direction whereas, when you have epsilon r epsilon theta, these are not invariant in directions. So, if you consider say a point, which is located at a position r, so how do you write it; epsilon r and epsilon theta, so this is the radial direction, so this will be the epsilon r and perpendicular to that will be epsilon theta you go to a now a different point.

Let us say you go to this r, even if you keep the radial magnitude same, now what will be your epsilon r; your epsilon r will be this and your epsilon theta will be again perpendicular to that. So, if i and j, they are invariant but epsilon r and epsilon theta they actually vary with theta, where theta is the angular coordinate. So, when you differentiate; so this del operator is basically for differentiation.

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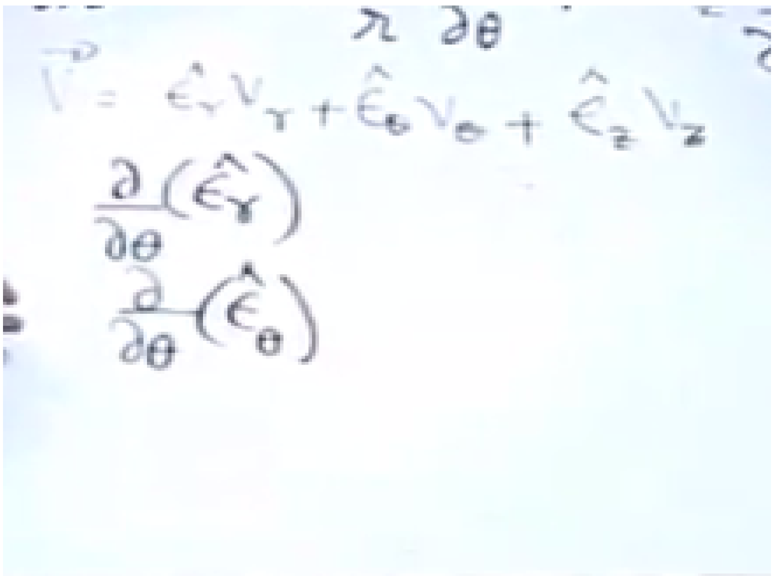

And when you differentiate, you have to keep one thing in mind. What you have to keep in mind? So, when you write v, you also write v in terms of its r theta and z components, so when you write v, it is epsilon r vr + epsilon theta v theta + epsilon z vz. So, when you are differentiating, it is possible that you have to find out these quantities and these ones. These kinds of quantities, they were not relevant for Cartesian system.

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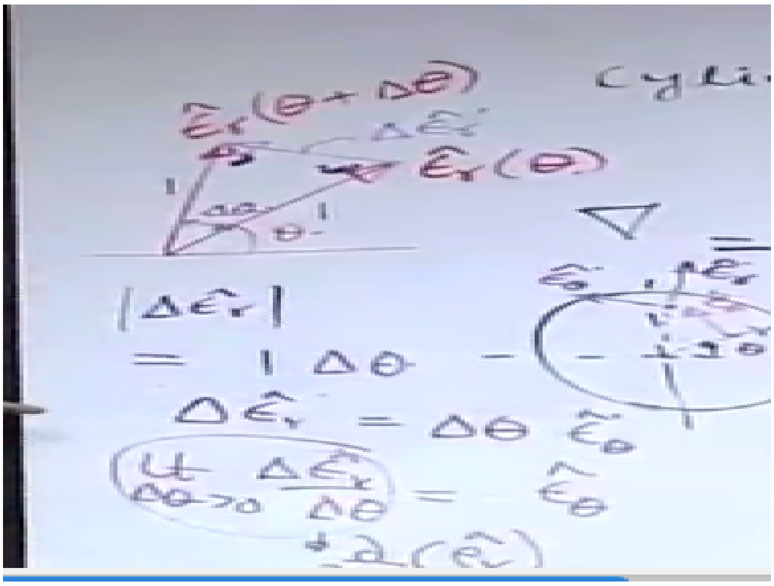

Because invariant direction unit vectors, so we will find out one, let us say you want to find out the derivative of epsilon r. So, how do you look into that; let us say that you have epsilon r for a particular angle theta as this one and let us say that epsilon r has changed with the angle theta + d theta, so let us say this is epsilon r for theta and let us say this is epsilon r for theta +; we can write d theta straight away or if you want to be more fundamental, let us consider it as delta theta in the limit as delta theta tends to 0.

So, let us say that this is at theta + delta theta, where theta + delta theta is the corresponding angular position. So, you can see that in a scalar form actually magnitudes of these are the same both are unit vectors, so length of this is 1, length of this is 1, what has change is the directionality. So, what is the delta epsilon r, what is the change in the epsilon r? That is nothing but delta epsilon r, the change in epsilon.

What is the magnitude of this delta epsilon r? So, if this angle is delta theta, so magnitude of delta theta epsilon r is like; it is like 1 * delta theta. So, for small delta theta, it is just like arc of a circle, so it is 1 * delta theta. What is its direction? You have to keep in mind that we are talking about the limit as delta theta tends to 0, sum of the 3 angles of this triangle is 180 degree, this is an isosceles triangle, so these 2 angles should be equal.

So, when this tends to 0, these 2 tend to 90 degree almost that means, delta epsilon r has a direction epsilon theta that is perpendicular to epsilon r, so we can say that delta epsilon r is like delta theta, now you give the directionality epsilon theta sorry; now you get rid of the magnitude. Now, on the basis of this, you can find out what is delta epsilon r delta theta that is you can take limit as delta theta tends to 0, delta epsilon r delta theta that is epsilon theta.

So, this is nothing but okay, similarly if you work this out, it will be - epsilon r; very same; I mean I need not work it out again because it is just very, very similar. So, now when you plug in those considerations in this del operator, in this divergence operation by considering this as the del operator for the cylindrical polar coordinate system, you will get a form of the continuity equation in r theta z system.

I would advise you that you complete that exercise and then in your book you will see that the final form is written in the r theta z system, so you check or verify your final expression with that that will give you a confidence that you have done it correctly in terms of the cylindrical polar coordinate systems, okay. So, we have got some preliminary understanding of the use of the continuity equation, whether the understanding is good enough.

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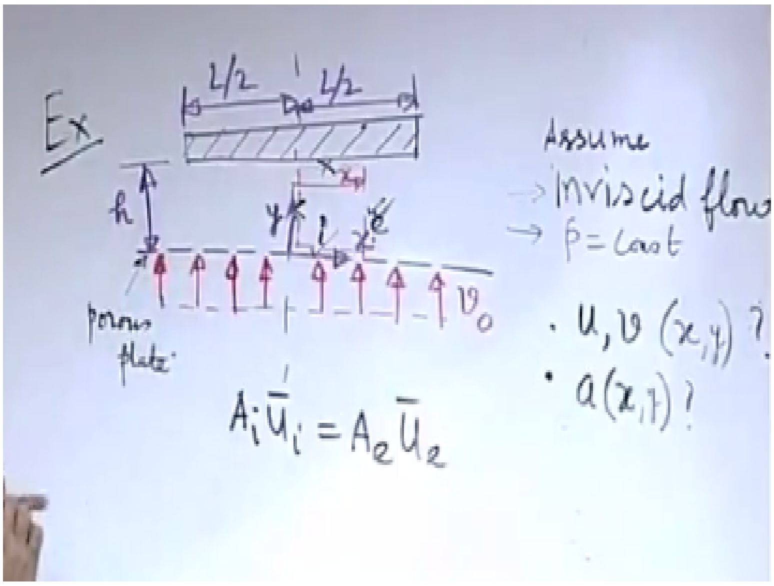

Let us try to work out a problem and see, let us say that you have a plate like this, you also have a bottom plate say, the top plate is a rectangular plate with the dotted line representing the axis of symmetry. The bottom plate is a very special plate it has some holes, we call it a porous plate, so it has some pores or holes and the idea of keeping these holes is to blow some fluid.

We will see that it is not just a mathematically defined idealistic problem many times; it is something which is followed in technology. I can give you one example, let us say that this is a heated electronic chip and you want to cool it, so it is possible that you blow air through porous plate, which goes into the chip and tries to keep it cooled. So, it is not a very hypothetical type of a situation.

But the way we will look into this particular problem is abstracted from that any specific application but more into the fundamental that what goes behind this. Let us say that this is a uniform velocity with which it enters say, we call it v0, so this is a porous plate. The gap between these 2 say is h, the length maybe this is l/2 and this is l/2 and let us say that we have x coordinate like this and y coordinate like this.

We make an assumption that it is an inviscid flow that is given, assume inviscid flow. What is your objective? Your objective is to find out the velocity components u and v as functions of x that is number 1 and number 2, what is the acceleration of fluid at a given x or maybe at a given x, y. So, when we say x may be; let us make it generalized say it could be x as well as y, okay.

So, let us first physically try to understand that what is happening, whenever you are solving a problem we can of course, start putting equations but that is not always necessary, first you have to understand. So, what is happening; some fluid is entering, now the fluid cannot leave through the top because of what; is it because of no slip condition? No, because no slip end ensures that it has no tangential component.

But it simply it is a no penetration condition because it cannot just penetrate through the wall and go out because it is just a fully covered solid wall, so no matter whether it is sleep or no sleep, you cannot actually penetrate it and go out along y, so only way this fluid can move in a steady condition is it can move sideways, so whatever fluid enters now maybe half enters right and half enters the left, okay.

So, what we can say is that we may write a gross overall mass balance, so when you write a gross overall mass balance what we have to keep in mind that very simple equation just like rho i * Ai * ui average is = rho e * Ae * ue average, so here let us say that rho is a constant, so that is another assumption that we make rho = constant. So, when we make the assumption of rho = constant, it is; we have to just consider the area times the average velocity is same as what enters is same as what leaves.

So, we have to fix up a control volume to write that expression because you require specific surfaces, so what control volume we may choose; let us say that we choose this control volume, see it is symmetrical with respect to the y axis so I mean, if we consider only one part of the domain, half of the domain to the right of y axis, the same happens to the left. Let us say that we consider a control volume like this, which is say located in phase is located at a distance x, so it is local x coordinate is x.

So, with respect to this control volume, what are the phases across which fluid flows, can you tell? One is the most straightforward is the bottom phase yes, through these fluid flows, another straightforward is the top phase through which fluid does not flow. This one; yes or no, how many will say yes and how many no? Think again; again think physically, see the fluid enters here, fluid does not know whether to go to the left or to the right, so it is equally flowable.

And it is actually equally so that maybe half will try to go to the right and half will try to go to the left because it is perfectly symmetrical. So, if it tends to go to the right with a velocity say, +u, similarly it will have tendency some fluid particle locates at the same position to go to the left with – u, net effect is that at this axis of symmetry, you have no u; u is 0 because it is just like balanced from what; so, whatever enters it has a balancing effect of going to the right and to the left.

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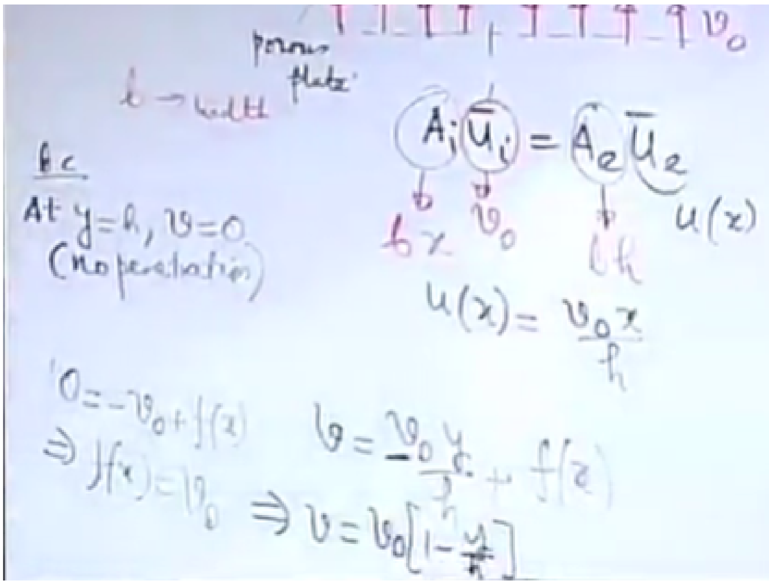

So, there is no net flow across this, so when you have there is no net flow across this, there is only; so this one has a net flow. So, when we say i and e, maybe this is the surface i and this is the surface e, so for the surface i what is; so we can write again; so here rho is a constant, so we can write, Ai ui is = Ae ue. So, what is Ai? Let us say that the length; that the width of the plate perpendicular to the plane of the figure is b that is the width.

So, what is Ai, what is this Ai? B * x, what is this ui average? It is v0 because it is a uniform, so average and local everything is same. Now, come to these ones, what is Ae? Yes, so h 8 b, so this is; and what is uv average? See, for that this assumption of inviscid flow is important, so if you do not consider inviscid flow then you have to know what is the velocity profile in between, you have to integrate that to get the average velocity.

But when you are given that is inviscid flow, your inherent assumption is that it is a uniform velocity profile like this, so u is not changing with y, it is locally changing with x but along y, it is uniform because it is inviscid, you can see that with inviscid flow, you cannot impose no slip boundary condition because if it is; if it has to be uniform suddenly, it cannot go to 0 at the wall, so it is not no slip here at the wall.

But no penetration at the wall that is sufficient for solving this problem, so no sleep is not a necessary condition here and in fact it will contradict, if you say that it is an inviscid flow, you cannot have sleep and inviscid; no sleep and inviscid simultaneously, so if there is some sleep. So, this velocity; so it does not vary with y, so we can say that it is like u just a function of x from the inviscid flow consideration.

So, this is like that u, which is a function of x, so no more it is a function of y from the inviscid flow consideration. So, from here, you can say that what is u as a function of x, it is v0x/h, which point? Inviscid flow; see when you are considering inviscid flow; see, what does a viscous flow do; see, we have earlier discussed always keep this qualitative concept in mind, what does viscosity do?

It propagates the effect of a momentum disturbance, so here you have momentum disturbance imposed by the wall, so if the fluid has a viscosity that will be propagated from the wall to the inside and it will in effect try to slow down the fluid elements, which are close to the wall and as you go more away and more and more away from the wall, the velocity will be more and more.

So, the velocity profile in that case will have like a 0 value at the wall and then increasing away from the wall but if you have no viscous effect, then the effect of wall is not propagated in the fluid; fluid does not know that there is a wall and therefore, it tends to maintain a uniform velocity and that is why for an inviscid flow, you have such a kind of uniform velocity profile.

So, when you have such a kind of a profile, so you have this; like you have u only function of x but not function of y, very idealistic situation but like simple one to begin with. Now, how do you find out v? So, you know u, how do you find out v, so you have the continuity equation that relates u with v, so what is the special form of the continuity equation with rho = constant and it is a 2 dimensional flow.

So, when rho is a constant, the first term the time derivative is 0, the next one is if rho is a constant, so you have like this plus this that is = 0 for a 2 dimensional flow, so w is not there, rho being a constant, it will come out of the derivative, so it will be this = 0, okay. Now, so you know, what is u as a function of x, so you can write the partial derivative of u with respect to x, what is that? V0/h.

So, now you can integrate this with respect to y, to find out how v varies with y. So, if you integrate it, how will you, what will you get? V = v0y/h + what? Plus, or some maybe some function of x also, it is a constant of integration but since it is a partial derivative, when you are integrating it you are integrating it partially with respect to y with respect to that integral, x is like a constant.

So, you could have in general a function of x, within that there may be a constant; it may itself be a constant but for generality, it is better to write it in this way. Now, you can find it; okay minus, yes, this is minus and then this one. Now, you can find out this with a boundary condition, so what boundary condition is there? At y = h, you must have v = 0, this is no penetration boundary condition not no sleep, again I am repeating.

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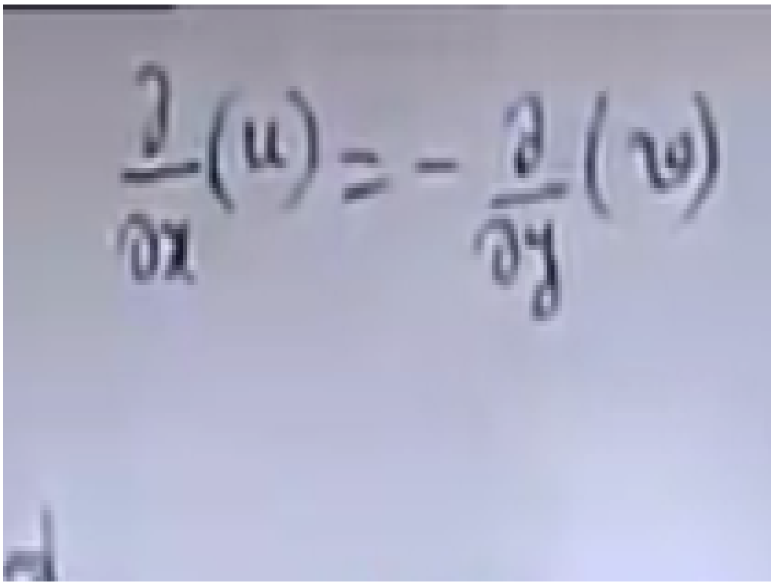

Because it cannot penetrate physically through the boundary, so that if you substitute, so minus v0 + fx, therefore fx becomes = v0, this means that you have v is = v0 * 1 – y/h, so you can see here that u is a function of x, v is a function of y only and see look at this equation, you can do a mathematical; simple mathematical jugglery with it, so the left hand side u is a function of x, so this is a function of x only.

V is a function of y, so this is expected to be a function of y only and ironically, you are getting a situation where left hand side is a function of x only, right hand side is a function of y only, it is possible only when each is equal to a constant. Otherwise, you do not have a way by which you can cancel x and y from both sides, so only way is you may get it as a constant. So, these are just ways of views of looking into a problem, it is not just a matter of solving a problem, there are nice interesting concepts and remarks that you can get from a problem.

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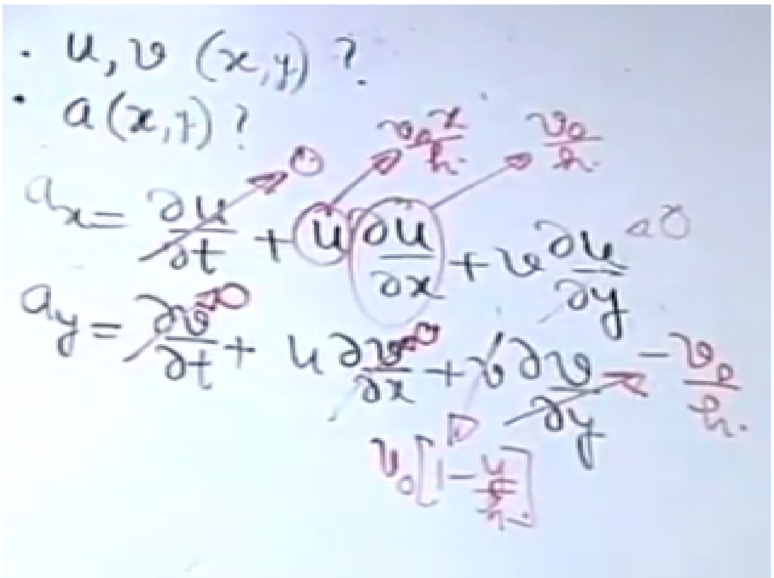

Now, the next is; so, we have got u and v, acceleration is very straight forward. So, how do you calculate the acceleration? Let us do that so what is acceleration along x? This is acceleration along x, this we derived earlier, so it is not a 3 dimensional case, so the w term is not there. So, first of all its u is not a function of time, so this is 0, then you can just substitute what is u here, v0x/h and so this will become v0/h.

The other term will be 0 because u does not vary with y, similarly let us write the y component of acceleration, again because v is not a function of time, this is 0, then v is not a function of x, so this is 0 and here you have; so v as a function of y, so what is v as a function of y; you have v0 * 1 – y/h and what is dv dy; - v0 yh. So, you have final concrete expression for acceleration components along x and y.

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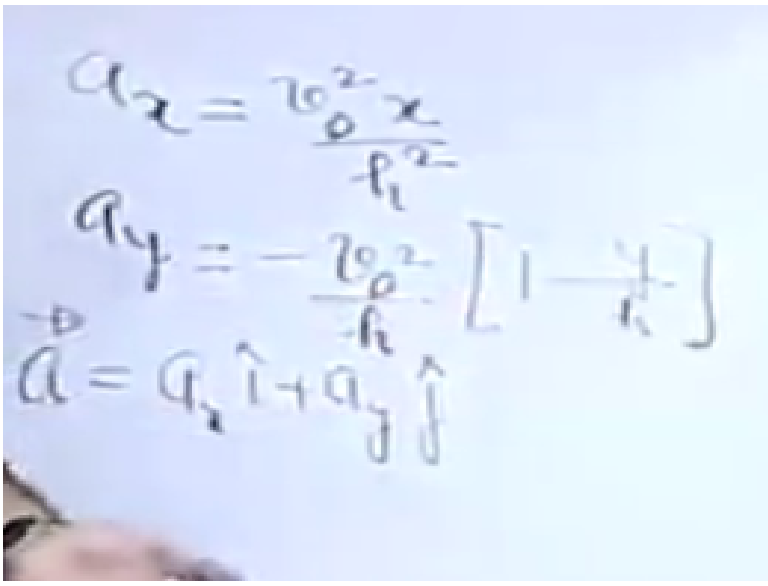

So, you have acceleration component along x as v0 square x/h square acceleration along y – v0 square/ h * 1- y/h and the acceleration vector is axi + ayj, which is a function of both x and y, so this is the acceleration at a point so if there is a fluid particle located at some x, y that will locally be subjected to that acceleration because locally, the fluid particle and the flow behaviour is identical.

So, you can clearly see that although, the velocity components are not functions of time, still you get an acceleration that is what is the important implication that we get from an Eulerian approach because the entire acceleration component has arisen because of the convective component of acceleration because of the variation of velocity with respect to position, it is not because of change in velocity due to change in time okay. So, we stop here today, we will continue in the next class.