Introduction to Fluid Mechanics and Fluid Engineering
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Lecture – 12
Fluid Kinematics (Contd.)

We were discussing about different flow visualization lines in our last class and we will discuss one more flow visualization line, which is called as time line. So, what is the time line? If you have a snapshot at a particular time in the flow field where you mark nearby particles, so nearby fluid particles, which are located in the flow field at a given instant of time, if you somehow mark those particles by some way.

Then if you now get the snapshot at different times, it will give a picture of evolution of the flow field as a function of time and that is known as a time line, so it is nothing but like snapshot of nearby fluid particles at a given instant of time that is called a time line. So, let us look into a small movie to see that what we mean by a time line.

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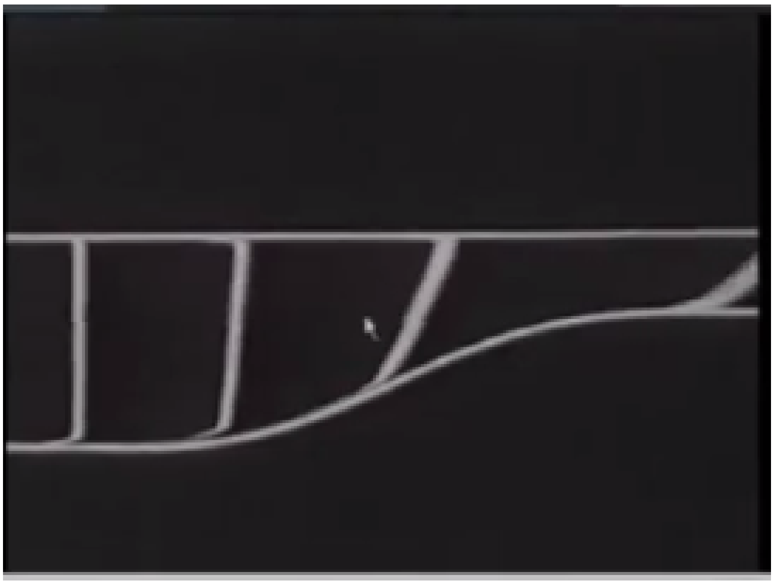

So, if you see now this gives snapshot at different instants of time of nearby fluid particles and in a way, it gives a sense of the velocity profiles at different instants of time you can see that in this example, the flow passage is narrowing and as the flow passage is narrowing, the fluid is moving faster to make sure that the mass flow rate is conserved. We will see later on that formally this is described by the continuity equation and in maybe a differential form or an integral form.

But at least, this gives us a visual idea of what the time line is all about, now with this background on the flow visualization lines, we have now understood that how we can visualize the fluid flow in terms of some imaginary description like through the streamline, streak line path line or maybe the time line. Next, we will go into the description of acceleration of fluid flow.

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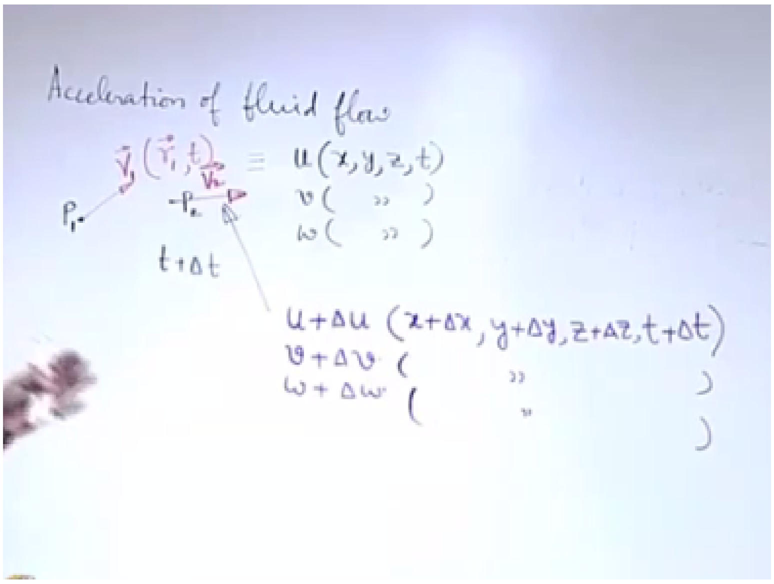

So, we have discussed about the velocity, the next target is the acceleration. Let us say that you have a fluid particle located at a position P at specifically the location P1 at time = t and how the velocity is described here; the velocity is described here through a velocity vector v, which is a function of r1 that is the position vector of the point P1 and the time t, this is nothing but the Eulerian description.

If you write it in terms of components, you can write an equivalent scalar component description that you have u as a function of x, y, z and t, v as a function of x, y, z, t and w, as another function of x, y, z, t. So, we are trying to describe it in terms of Cartesian coordinates, it is not always necessary to do that but it may be a simple way to demonstrate, one may use other coordinate systems as well.

So, if you are using a Cartesian coordinate system, 3 independent coordinates; space coordinates plus time coordinate that together give the velocity at a particular point, so if the fluid particle is located at P1, the velocity at that point is basically the velocity of a fluid particle located at that point and that is given by these components. Now, let us say that at a time of t + delta t, these things get changed.

Now, at a time t + delta t, what happens; this fluid particle is no more located at this point, the fluid particle is located at a different point, so let us say that the fluid particle is located at a point P2. So, at the point P2, now let us say that the velocity is whatever at some arbitrary velocity, so initially it may be velocity at the point 1, say v1, now it is v2, which is again a function of its local position and time.

So, you have this v2, this one a function of what; so, let us say that it is given by its components u + delta u, v + delta v, w + delta w, these are functions of what? These are functions of the new position vector, the new position vector say is r1 + delta r1, so in terms of scalar components, it may be x + delta x, y + delta y, z + delta z and the time has also now changed, it has become t + delta t.

So, we are thinking about a small interval of time delta T, over which the fluid particle has undergone some displacement, which is a change in position vector having components delta x, delta y and delta z that is what we are trying to understand. So, we can clearly see that there is an original velocity in terms of its 3 components, there is a change velocity in terms of which 3 components.

And if we want to find out the acceleration; see the basic definition of acceleration is based on a Lagrangian reference frame that is the rate of; time rate of change of velocity in a Lagrangian frame not in a Eulerian frame, all the basic definitions in Newtonian mechanics that we have learnt earlier are based on Lagrangian mechanics. So, when you say that it is a rate of; time rate of change of velocity.

Then that has to deal with the time rate of change of velocity of maybe an identified fluid particle, which earlier was at P1, now is at P2. So, if we want to find out the change, so you can write of course, you can write it in terms of the 3 different components but just for simplicity, let us write for the x component, similar things will be there for y and z component. So, how can you write u + delta u as a function of u.

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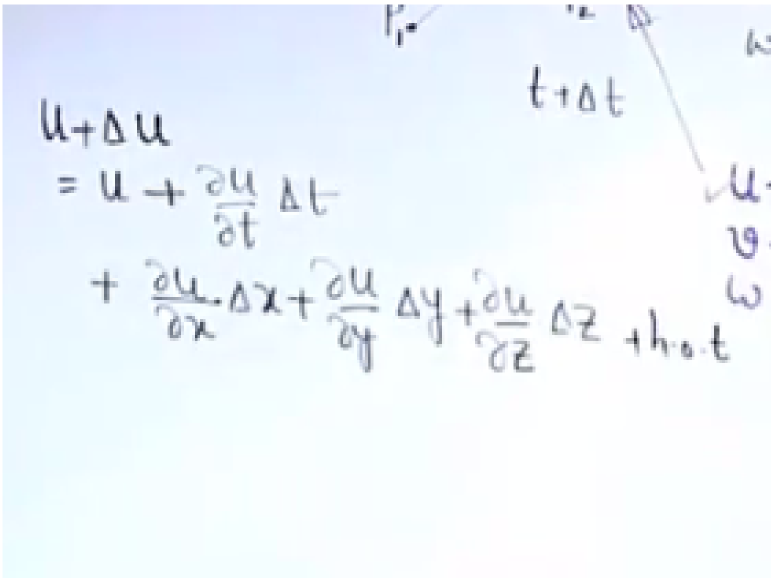

So, u + delta u is now dependent on the local position of the particle and the time that has elapsed, so it is a function of; it depends on what; it depends on the original u plus the change. So, what was the original u? That was u +; see, it is a function of 4 variables, so you again it is a same mathematical problem that there is a function of 4 variables, it is known at a given condition, now you make a small change in each of these variables and you want to find out the new function.

Again, you can express it through a Taylor series expansion, now it is a function of multiple variables instead of a single variable. So, we will use the Taylor series expansion, you have to keep in mind that now you are having 4 variables. So, let us first consider the time variable may be because it is bit different in characteristic than the earlier one, so this is with regard to the time, then with regard to that space okay, plus higher order terms.

This we have just written the first order term in the Taylor series, since it is a function of 4 variables, you have 4 first order derivative terms, similarly you will be getting second order derivative terms and so on but we will neglect the higher order terms by considering that these delta x, delta y, delta z, delta t are very small, so we have to keep in mind that all these are tending to 0.

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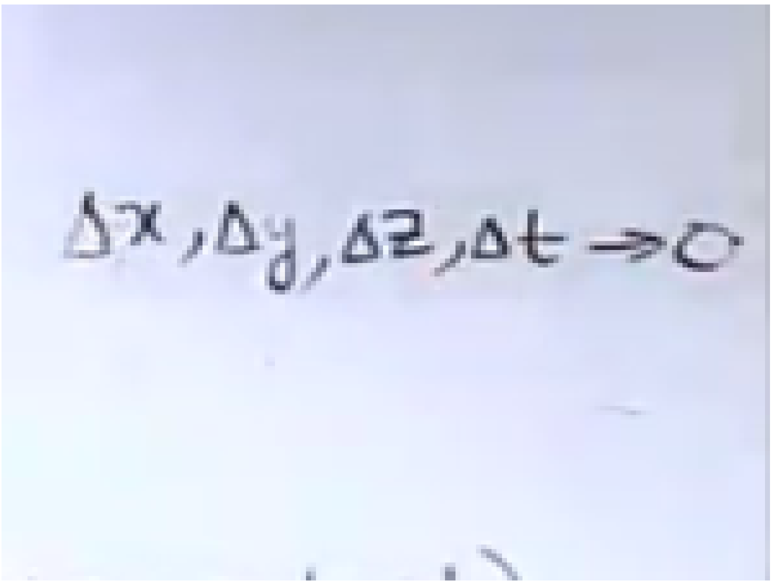

And because all they are tending to 0, we are neglecting their higher orders, so you can first think what you can do, you can cancel u from both sides and what is the definition of acceleration along x from a particle mechanics viewpoint or a Lagrangian viewpoint?

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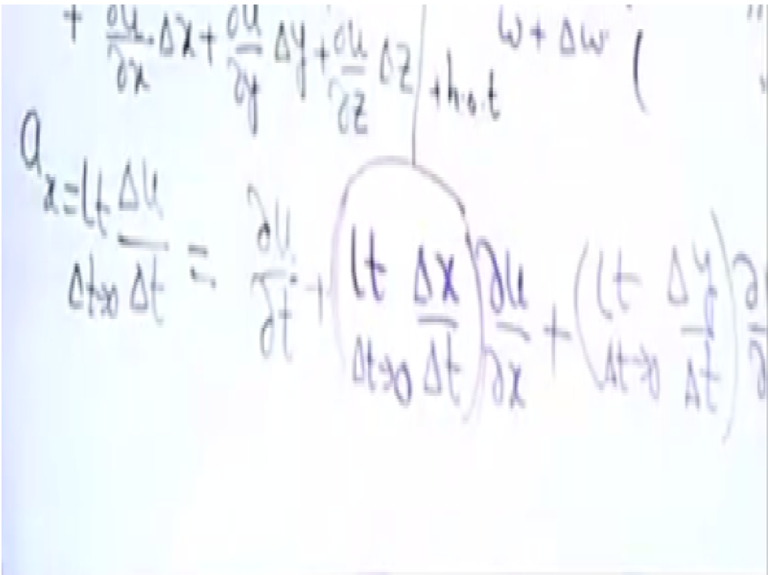

So, you have to find out the change in velocity; x component of velocity because we are writing acceleration along x divided by the time delta t in the limit as delta t tends to 0, very simple straightforward Lagrangian description. So, when you do that basically what we are doing; we are dividing the left hand side by delta t, so right hand side is also divided by delta t and a limit is taken as delta t tends to 0.

So, the first term is straightforward, let us look into the next terms, so first we will evaluate the limit; limit as delta t tends to 0, delta x divided by delta t that multiplied by the derivative with respect to x, similarly the other terms, let us just complete it. So, what we are doing is; we are trying to find out that because of the changes in velocity component along different directions, what is the net effect in acceleration?

And these terms are basically representatives of that we will formally see that how they represent such a situation. So, now let us concentrate on these limiting terms say, the first limiting term. What it is representing? It is representing the time rate of change of displacement along x of the fluid particle over the period delta t. Now, you have to keep in mind that we are thinking about a limit as delta t tends to 0, this is a very important thing.

What is the significance of this limit as delta t tends to 0, when delta t tends to 0, P1 and P2 are almost coincident, right that means, let us say that P1, P2 all those converts to some point P and that point is a point at which say we are focusing our attention to find out what is the change of velocity that is taking place, so when in the limit delta t tends to 0? we are considering the Eulerian and Lagrangian descriptions merge.

This is very very important, so we are trying to see what is our motivation; we know something and we are trying to express something in terms of what we know; what we know, we know the straightforward Lagrangian description of acceleration, we are trying to extrapolate that with respect to an Eulerian frame. To do that we must have a Eulerian, Lagrangian transformation and essentially, we are trying to achieve that transformation in a very simple way that as the delta t tends to 0, Eulerian and Lagrangian description should coincide.

And then what does it represent; it represents the instantaneous velocity; x component of the instantaneous velocity of the fluid particle located at P that means, it represents the x component of the fluid particle located at P, since you are focusing our attention on P itself and the velocity of the fluid particle, if it is neutrally buoyant is same as the velocity of flow, we can write that this is same as what; this is same as u at the point P.

See, writing this as u is very straightforward understanding it conceptually is not that trivial and straightforward, if the Eulerian and Lagrangian descriptions did not merge, we could not have been able to write this because this is on the basis of a Lagrangian description and this is Eulerian velocity field. How these 2 can be same? They can be same only when we are considering a particular case, when in the limit as delta t tends to 0.

So, wherever we are focusing our attention at that particular point, this replaces the velocity of the fluid particle, if the fluid particle is neutrally buoyant with the flow, then it is like an inert tracer particle moving with the flow and then it would have the same velocity as that of the flow at that point; at that point, at that instant. However, if the fluid particle has a different density than that of the flow, then this would be u of the fluid particle.

So, fundamentally this is u of the fluid particle not u of the flow field, if it is neutrally buoyant then it becomes same as u of the flow field. If it is an inert tracer particle in the flow, which is the definition of the fluid particle then it is definitely same as u at that point but if it is a fluid; if it is a particle of a different characteristic, different density characteristic than that of the flow, it may be different from that of the velocity field at that point, so that we have to keep in mind.

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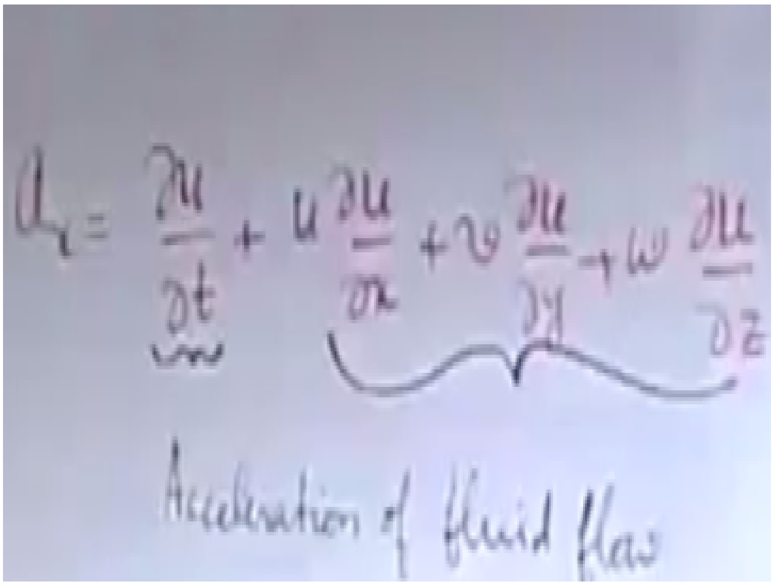

So, if you complete this description of this term what we will get; you will get ax is =; that is the straightforward follow up of this expression because the other limits, you can express in terms of v and w, again with the same understanding as we expressed as we use for expressing the first term. Now, if you clearly look into this acceleration expression, there are 2 different types of terms; one is this type of term which gives the time derivative, another gives the spatial derivative.

You will see that this expression will give you a first demarcating look of how the expression is different in terms of what we express in a Lagrangian mechanics. In a Lagrangian mechanics, it is just the time derivative that comes into the picture, here you also have a positional derivative and what do these terms represent; we will give a formal name to these terms but before that first let us understand that what these 2 terms represent.

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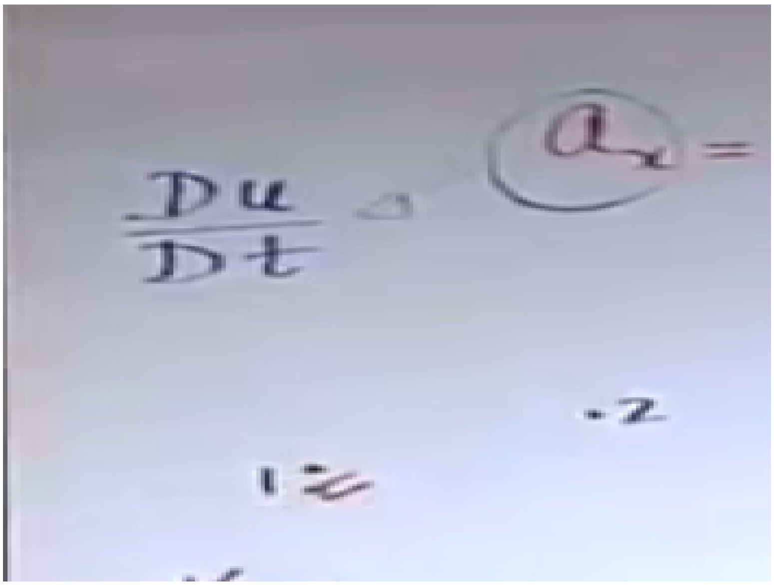

Say, you are located at a point 1, now you go to a point 2 in the flow field, so when you go there, there are 2 ways by which your velocity gets changed, how? One is maybe from 1 to 2 when you go, you have a change in time and because of a change and you also have a change in position, you have a change in velocity and that is a solely time dependent phenomenon. How can you understand what is the component of the time dependent phenomenon?

If you did not move to 2 but say you confine yourself to 1, say you are not moving with the flow field, you are confining yourself with 1, then you are freezing your position but still at the point 1, there may be a change in velocity because of change in time, if it is an unsteady flow field. So, because of that it might be having an acceleration, so the acceleration; that acceleration component is because of what; the time rate of change of velocity at a given point at a given location, so that is reflected by this one.

But by the time, when you are making the analysis the fluid particle might have gone to a different point even if it is local velocity that is velocity at a point is not changing with time, it has gone to a different point, there it encounters a different velocity field. So, here it was encountering a particular velocity field because of its change in position, so what it has done; it has got advected with the flow.

It has moved with the flow and it has come to a new location, where it is encountering a different u, v, w, so because of the change in u, v, w with a change in position, it might be having an acceleration, so that acceleration is not directly because of the time rate of change of velocity at a given point but because of the spatial change, since the particle; the fluid particle by the time has travels to a different location, where it finds a different flow field.

And since, we are considering that it is an inner tracer particle, it has to have the same velocity locally as that is there in a new position, so because; so the next combination of terms, it represents the change in velocity solely due to change in position, so the total the net change is because of 2 things; one is if you keep position fixed and you just change time because of unsteadiness in the flow field, there may be an acceleration.

The other part is even if the flow field is steady but you go to a different point because of non-uniformity because of a change in velocity; due to change in position, the fluid particle might have a change in velocity, so the change in velocity in the fluid particle may be because of 2 reasons; one is because of the change in velocity due to change in time, even if it were located at the same position as that of the original on.

And the other one is not because of change in time but because of change in position as it has gone to a different position because of non-uniformity in the flow field, it could encounter a different velocity and the resultant acceleration is the combination of these 2. So, let us; let us take a very simple example to understand it. Say, you are traveling by flight from Calcutta to Bombay.

So, when you are taking the flight; before taking the flight, you see that it is raining very, very heavily and then say you take 2, 2 and 1/2 hours you reach Bombay and you find that it is a very sunny weather. So, the question is now if you want to ask yourself a question, does it mean that when you departed from Calcutta it was raining in Bombay or when you departed from Calcutta, it was sunny at Bombay or when you have reached Bombay, is it still raining at Calcutta or is it still sunny at Calcutta.

It is not possible to give an answer to any one of these because the net effect that you have seen is a combination of 2 things, you have traversed with respect to time, so you have elapsed certain time by which maybe it was raining at Calcutta but right now, it is not raining at

Calcutta, maybe it was sunny at Calcutta and right now it has started raining, so it is like at a particular location, the weather has changed because of change in time.

But the other effect is that you have migrated to a different location and because of the change in location maybe it was before 2 years raining at Bombay, now it is sunny or it might so happen that it was sunny 2 hours back in Bombay, still it is sunny, so you can see that individual effects, you can maybe try to isolate but what is the net combination of changing with respect to position in time that is the net effect of this.

And it might not be possible to isolate these effects, so when you think about the total acceleration, so it is just like a total change. So, when you have the total change, it is a change because of position and because of time and that is why, this ax or maybe ay or az, this is called as the total derivative of velocity. So, it is given a special symbol capital D Dt.

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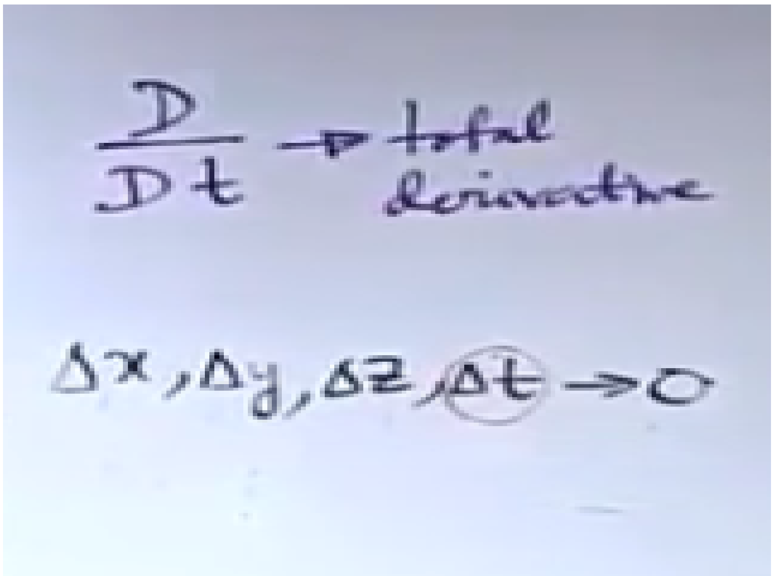

So, capital D Dt has a special meaning, it is called as total derivative it is to emphasize that it is a resultant change because of change in position and change in time, so with respect to change in time, if you have a change then it is called as a temporal component of acceleration. Temporal stands for time; temporal or transient or local, so these are certain names, which are given.

Again by the name, local it is clear, local means, confined to a particular position only with respect to change in time and this is known as the convective component. So, convective component is because of the change in position from one point to the other and this therefore is the total or sometimes known as substantial. So, the total derivative is a very important concept mathematically here, we are trying to understand this concept physically.

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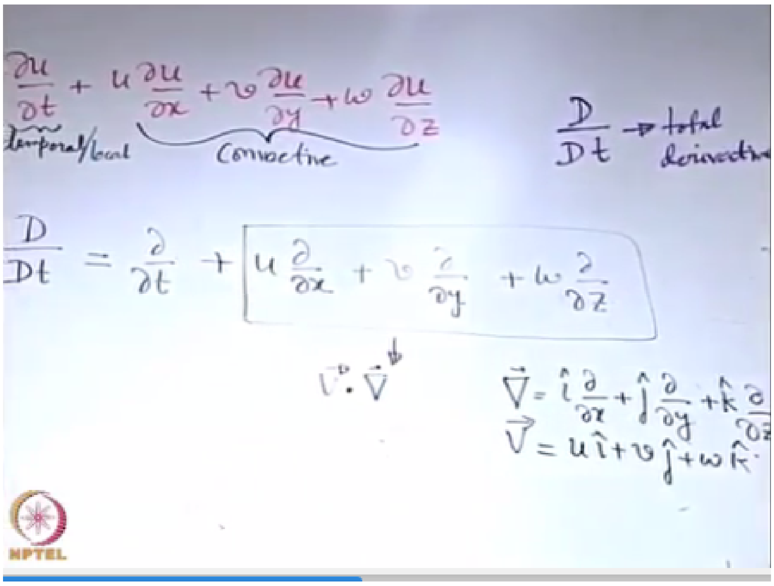

But it is not just restricted to the concept of acceleration of fluid flow, it is applicable in any context. In any context, where you are having an Eulerian type of description and it is therefore possible to write the general form of the total derivative as this way, where it has a local component and a convective component. So, we can try to answer some interesting and simple questions and see and get a feel of the difference of these with again the Lagrangian mechanics.

So, if we ask a question, is it possible that there is an acceleration of flow in an in a steady flow field that the flow field is steady but there is an acceleration, it is very much possible because if it is steady only the first term will be 0 but if the velocity components change with position, then the remaining terms may not be 0. So, this is of like, these are certain contradictions that you will first face, when you compare it with Lagrangian mechanics.

In Lagrangian mechanics, if there is something which does not change with time, its time derivative is obviously 0 but here even if it does not change with time, the total derivative is; it may not be 0. On the other hand, it may be possible that it is changing with time at a given location but acceleration is 0 because I mean in a very hypothetical case, it may so happen that the local component of acceleration say it is 10 meter per second square, convective component is - 10 meter per second square so the sum of that 2 is 0.

But individually, each are not 0 that means, it is possible to have a time dependent velocity field but zero acceleration and it is possible to have a non-zero acceleration even if you have a time independent velocity field, so these are certain contrasting observations from the straightforward Lagrangian description. So, you can write the x component of acceleration in this way and I believe it will be possible for you to write the y and z components, which are very straightforward.

And you have to keep in mind that when you write y component this D Dt operator will act on v and when you write the z component, it will act on w, so you can write the individual components of acceleration vector and the vector sum will give the resultant acceleration. Now, you can write these terms in a somewhat compact form, so this you can also write as v dot del, where del is the operator given by; okay.

And v is the velocity vector you know that is ui + vj + wk, so if you clearly make a dot product of these 2, you will see that this expression will fall. So, it is a compact vector calculus notation of writing the convective component of the derivative okay. So, we have got a picture of what is the acceleration of flow, how we describe acceleration of flow in terms of expressions through simple Cartesian notations and maybe also through vector notations.

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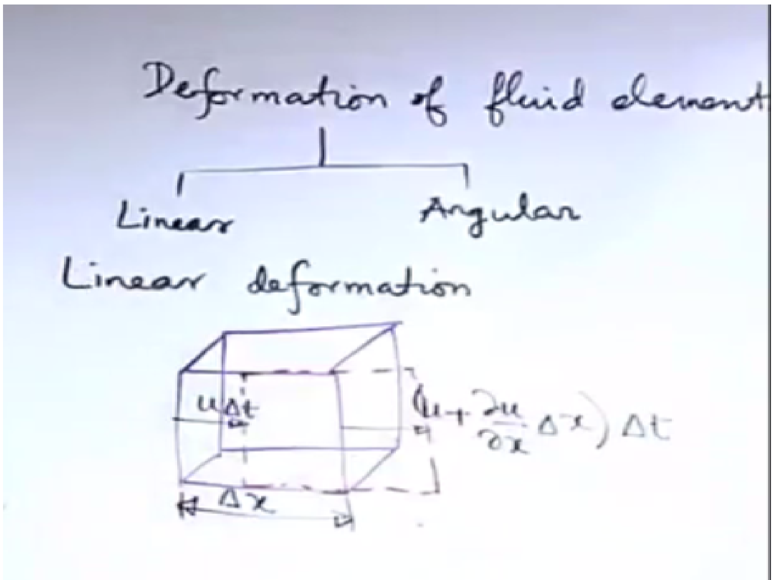

Next, what we will do; we will start analysing the deformation of fluid elements. Why this is very important? Because we have seen that fluids are characterized by deformation, they undergo continuous deformation on the under the action of even a very small shear force and the relationship between the shear force and or the shear stress and the rate of deformation is something, which is unique to the constitutive behaviour of different fluids.

So, we must first understand that how to characterize deformation of fluid elements in terms of the velocity components. Once, we understand that it will be possible for us to mathematically express different types of deformations in terms of the velocity components the uv and w. When we do that we have to keep in mind that we will be essentially bothering about 2 types of deformations; one is the linear another is the angular deformation.

When we talk about the linear deformation, it may eventually give rise to a changing volume of the fluid element also because if you have a length element and the length element gets changed, a volume element is comprising of several such length elements, so if length linear dimension gets change, the volume is also likely to get changed. Initially, we will think of how we can say, estimate the linear deformation.

So, we will start with the linear deformation, to understand or to get a visual feel, we will consider a fluid element like this, maybe we may consider even a 3 dimensional fluid element, if you want but that will not make the thing more complicated because at the end, we will be dealing with linear deformations in individual directions. See, why we use a coordinate system for analysing a problem?

The reason is like say, when you think of x, y, z the Cartesian coordinate systems, these are independent coordinates, a combination of which describe the total effect in the system. So, when you are thinking of a linear deformation along x, you may be decoupled from what is the linear deformation along y and z and these individual effects, you can superimpose because you are dealing with linearly independent components.

And these vector components actually give you linearly independent basis vectors like components along x, y or z. So, similar concept whenever, we are considering a change along x, maybe we are bothered only with respect to; like what is the change in the linear dimension along x, disregarding what happens along y and z. So, let us keep that target, let us say that delta x is the length of the fluid element, which is originally there.

And now, what is happening; now, we are having a change in time and because of a change in time now, you see that let us, consider the front phase of this cuboid, so this left phase over a time interval of delta t will traverse a displacement; will undergo a displacement. What is the displacement? If u is the velocity at this location simply, u * delta t, we are considering the time interval delta t to be very small, so it is like just a product of the velocity into delta t.

The right phase will also undergo some displacement, what is that? So, if this is the x direction, the new u here is not the same as the u at the left phase but this is because of change in u due to change in x, so this is the new u times delta t. So, if you consider only the front phase and only subjected to this motion, say we freeze all other events just for a clear picture, so maybe now, it is having a new configuration shown by this dotted line.

So, what is the change in its length along x; that is the final length minus the original length, so what is the final length, so what is the net change? See, the right hand phase has got displaced by this amount, the left hand phase has got displaced by this amount, so the net displacement is the difference between these 2, so what is that change in length?

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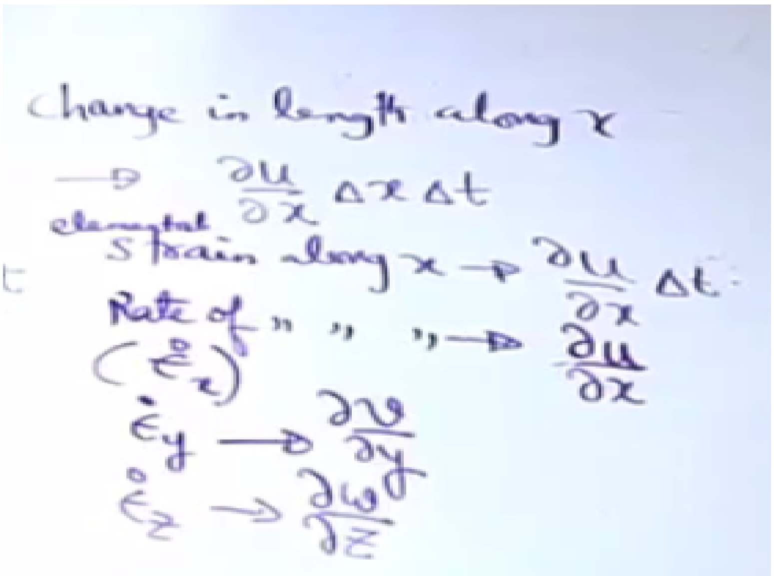

So, change in length along x, this is the change in length along x. What is therefore the strain along x? The change in length per unit length, so the strain along x; this is the elemental strain because we have considered only a small part of the fluid, which is having an extent of delta x, so this is elemental strain along x. As we have discussed earlier, we are not just interested about the strain for a fluid.

Because if you allow it to grow in time, the strain will be more, see if this delta t is larger and larger and you integrate it over a large interval of time, this will be trivially more and more, so measuring strain in a fluid is nothing that is important, it is just a function of the time that is elapsed. What is more important is the rate of deformation or the rate of strain. So, the rate of strain along x, what is that; it is basically this divided by delta t as delta t tends to 0.

So, when we say rate that means, the time rate, we always implicitly mean that so that will be simply the partial derivative of u with respect to x. We may give it a shorthand notation say, epsilon dot x, similarly just from your common sense, you can say what will be epsilon dot y and what will be epsilon dot z. So, what is epsilon dot y and epsilon dot z is this, so we have been successful in finding out a very simple thing, what is the rate of linear deformation along x y and z in terms of the velocity component.

So, if you are given u as a function of position, v as a function of position and w as a function of position by simple partial differentiation, it will be possible to find out the rates of change. Now, we are interested not only just in terms of the rate of change in the linear dimension but maybe rate of change in the volume. So, to understand that what is the rate of change in the volume; let us say, that we are having this fluid element, which has dimensions along x, y, z as delta x, delta y and delta z.

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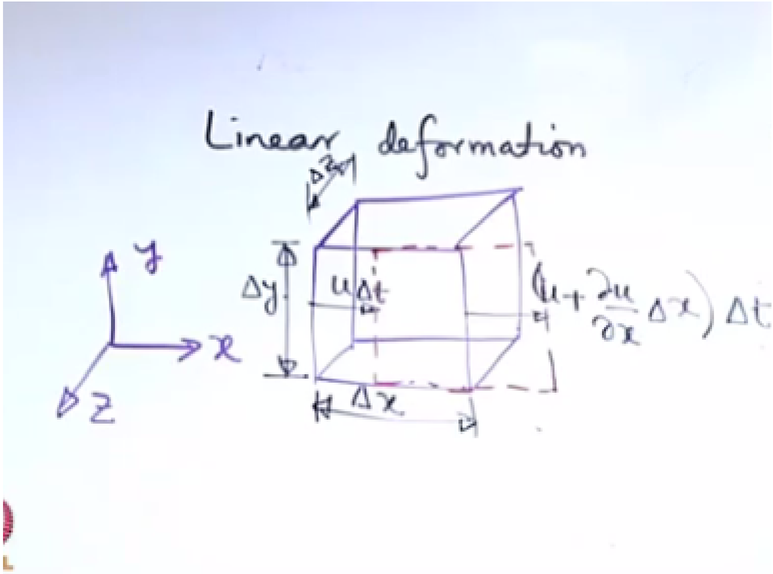

So, we set up coordinate axis as this is x, this is y and this is z, so delta x, delta y and delta z. Now, what is the new length? So, we are interested to get the new volume, so what is the new volume? The new volume is new length along x * new length along y * new length along z.

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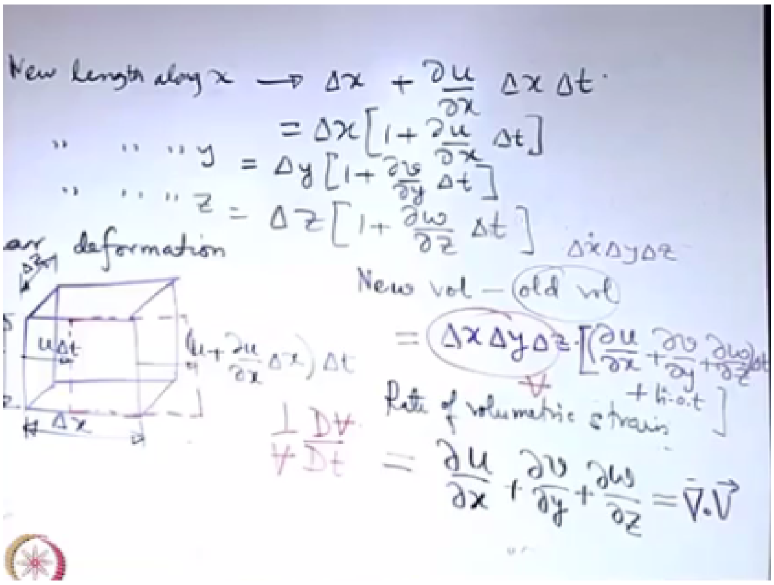

So, what is the new length along x? That is the old length plus the change. So, the old length is delta x + the change is this one, so we can take delta x common and write this one. Similarly, it is possible to write what is new length along y and new length along z, so let us complete those expressions. So, the new volume is a product of these 3, so what is the new volume; minus the old volume, yes.

So, what you have to do; you have to find out the product of this then subtract the old volume that is delta x * delta y * delta z, you will see that the first the delta x * delta y * delta z that term will go away, will get cancelled, then out of the remaining terms you have to neglect the terms of maybe higher order in delta, so like if you have products like delta t square or delta x * delta t square that type of term you tend to neglect.

Because those are higher order terms, so retain only the leading order terms because you have to keep in mind that you are dealing with a situation again as delta x, delta y, delta z, delta t, all tending to 0 and then what will be the term that is remaining here, yes, so obviously a product of these 3 is there then then, what is the remaining term; these plus higher order terms will be there that into delta t, other terms will be of order higher than delta t.

So, what is the volumetric strain; the rate of volumetric strain? So, the rate of volumetric strain is the change in volume per unit volume per unit time just like what we found out for the linear strain. So, when you say find out per unit volume, you are basically dividing it by this delta x, delta y * delta z, so this is like the original volume, let us give it a symbol V with a strike through, so that is the original volume.

So, what is the rate of volumetric strain? This change in volume divided by volume divided by delta t and take the limit as delta t tends to 0, when the all other higher order terms in the limit will be 0, so it is not that we are neglecting. The one delta t here will remain even after division by delta t that will be tending to 0 as in the limit delta t tends to 0. So, then what will be the final expression of this?

Some of these 3; so we may write it in terms of the total derivative, see the volumetric strain it may be due to many things; change in time, change in position and a combination, so we are not bothered about that what is the individual effect, we are bothered about the total effect, what is the net change in the fluid element volume because of this, so this should be expressible in terms of the total derivative.

So, it is capital D Dt of the volume with per unit volume, so this is the rate of volumetric strain and in terms of the vector calculus notation, you can also write it as the divergence of the velocity vector. This leads to a very important definition; the definition is with regard to incompressible flow. So, when we say that a flow is incompressible, so incompressible flow by the name it is clear that we are looking for a case, when the fluid element does not change in volume.

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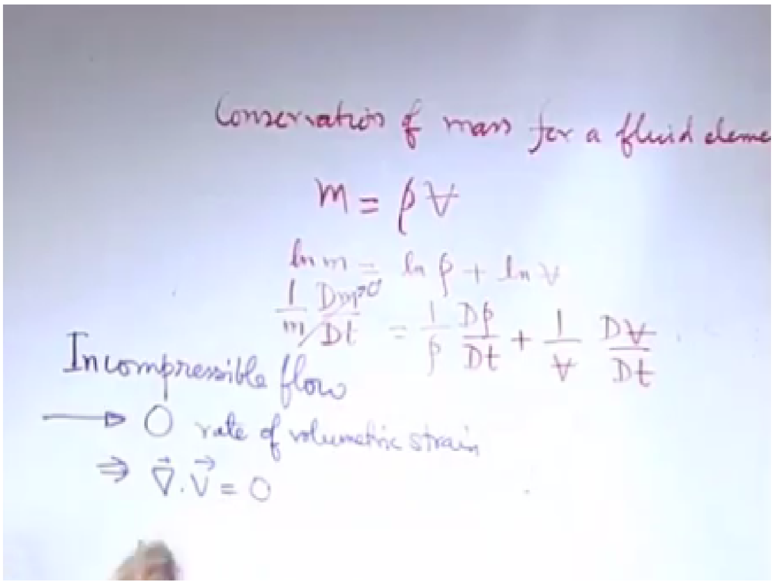

So, incompressible flow it will have what signature; one and only important signature, zero rate of volumetric strain because the fluid element may not be changing its volume that is the meaning of; that is even the literal meaning of incompressible that you cannot really compress it. So, zero rate of volumetric strain and that boils down to the divergence of the velocity vector is = 0.

So, if you are given a velocity field and you are asked to check whether it is compressible or incompressible flow, then it is possible to check by looking into the fact, whether it is satisfying this equation or not. If it satisfies this equation, we say that it is an incompressible flow, keep in mind the distinction between this definition and incompressible fluid definition. So, earlier we also introduced the concept of incompressible fluid.

And we said that a fluid is incompressible, if its density does not significantly change with change in pressure, so that is incompressible fluid. Now, we are talking about incompressible flow and these 2 are again related but different concepts that we have to keep in mind. So, when you are having an incompressible flow, it is possible to characterize the particular flow in terms of its mechanism by which it satisfies the overall conservation of mass.

To understand, how it does let us try to write an expression for conservation of mass of the fluid element. So, we will now write conservation of mass for fluid for a fluid element, let us say that m is the mass of a fluid element, you can express it in terms of the density and the volume, let us say that rho is the density and v is the volume. Since the mass is conserved of a fluid element, so there will be zero rate of change of mass.

So, the; since we know already the expression for the volumetric strain and in that volumetric strain 1/v appears, it may be useful to utilize that expression by taking log of both sides and then differentiating because then 1/v will automatically come out. So, let us take the log of both sides and then differentiate with respect to time, when we say we want to differentiate with respect to time, it has to be a total derivative.

So, because it is a fluid element now, it may have change with respect to change in position, time whatever, we are bothered about now the total effect because the conservation of mass is not for individual effects, it is a combination of total effects that gives rise to a mass of a fluid element is conserved. So, when we write say, when we differentiate it with respect to time by keeping that in mind, we have the left hand side like this, which again becomes 0.

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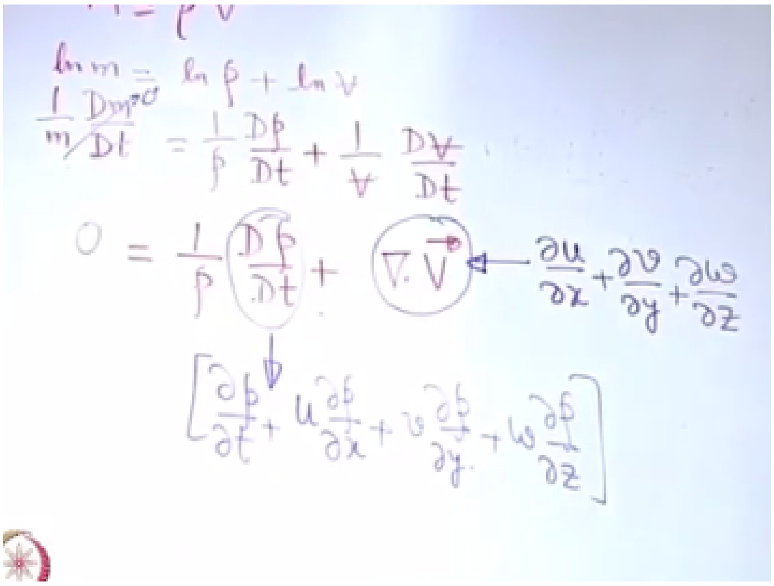

Because the mass of the fluid element is conserved, the right hand side 1/rho, so you have 0 = 1/rho d rho Dt + 1/ v dv Dt is what; is the divergence of the velocity vector or if you write in terms of the Cartesian coordinates, when it; till you write it in a vector form it is coordinate system independent but when you write its corresponding say, components then the components depend on how you take your reference.

So, in a Cartesian reference it is this, now let us write this one, what will be this; use the definition of the total derivative, okay, now you can multiply both sides by rho because density of the fluid is not 0, so you can multiply both sides by rho and then if you multiply both sides by rho, what you get?

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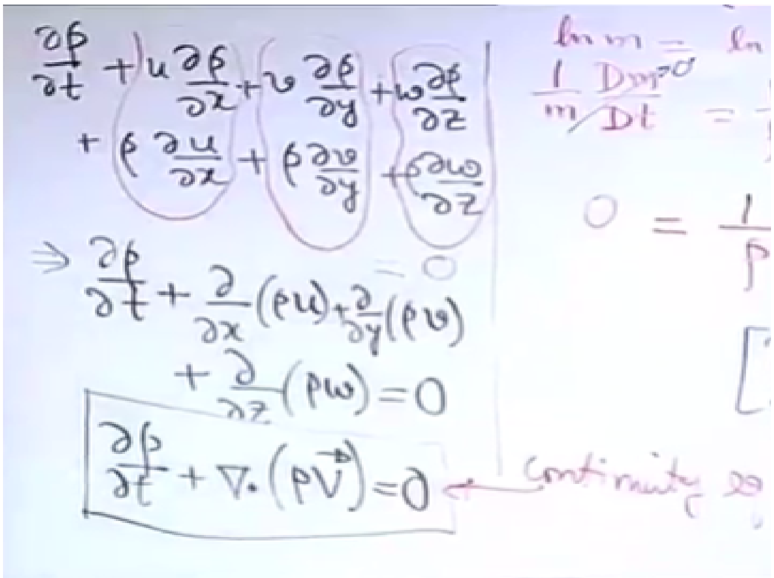

You get okay; now, you can combine these types of terms and write them in a compact form by using the product rule of differentiation, so this = 0, will imply; this = 0, okay, just by using the product rule of differentiation. It is again possible to write it in a compact vector notation, this becomes divergence of rho * velocity vector that is = 0, this equation in a general understanding is supposed to be the most fundamental differential equation in fluid mechanics.

Because no matter how complex or how simple the flow field is, it should satisfy the law of conservation of mass, so this is a differential equation expressing the law of conservation of mass for a fluid element and this is known as continuity equation. So, if you are given a velocity field, you must first check whether it is satisfying the continuity equation, if it does not satisfy the continuity equation, it is an absurd velocity field.

It may be mathematically something but it does not physically make any sense because it has to satisfy the mass conservation. Now, briefly let us look into certain special cases of these, so what are the special cases?

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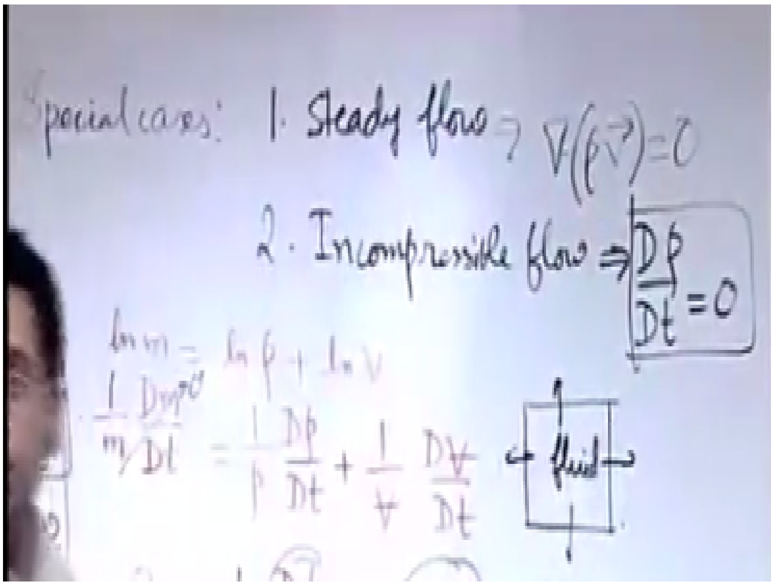

The first special case, we consider as steady flow. So, when you consider a steady flow, then how this equation gets simplified to; so steady flow means the first term at a given; remember what is the definition of steady, at a given position, any fluid property will not change with time, so at a given position that is why the partial derivative with respect to time that means, keeping the position frozen you are trying to find out the change in density with respect to time.

So, that is 0, if it is a steady flow, so steady flow will have that term = 0, so that will boil down to; but that does not ensure that rho is a constant; rho might not be a function of time but might be a function of position, so still rho remains inside the derivative, it does not get disturbed. Let us consider a second case, incompressible flow. We have to keep in mind that there is a very big misconcept that we should try to avoid.

What is that? Many times, we loosely saying incompressible means density is a constant, it is a special case of incompressible flow but it is not a general case of incompressible flow because general case of incompressible flow is what; the divergence of velocity vector = 0 that is the definition. Now, where does it ensure that rho is a constant that basic definition never ensures that rho is a constant?

At the same time, it can be shown that if rho is a constant then this will be satisfied, so the converse is true that means, rho is a constant is a special case of incompressible flow but it is not a general case. What is the general case? let us look into that. So, when you are looking about the general case, you have to see the continuity equation. So, when you look at the continuity equation, look into this primitive form that is not the compact form but this form before that.

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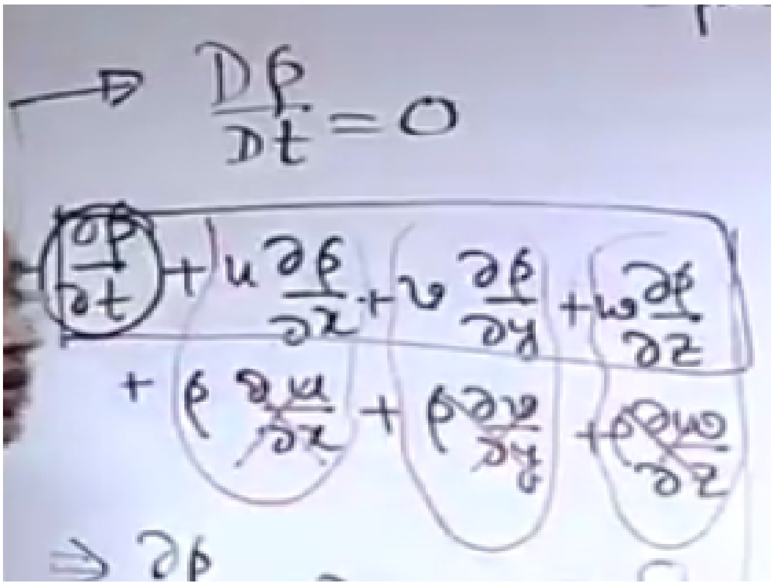

So, here when you have an incompressible flow; which terms will go away; these terms will go away because divergence of the velocity vector is 0, so then what you are left with; the total derivative with respect to; total derivative of density with respect with time is 0, so that means incompressible flow means D Rho Dt; capital D Rho Dt = 0. See, a very interesting thing, it does not mean that rho is a constant.

Because rho might be a function of position and time in such a way that this collection of terms eventually gives rise to 0, if rho is a constant, this collection will definitely give it to be 0 but that is a trivial solution; that is a trivial solution to the case that is rho is a constant therefore, any derivative of rho with respect to time or position is 0 but even if any derivative of rho with respect to position and time is not 0, still the net effect may be 0.

And then even though rho is a variable, we will say that the flow is incompressible, so a variable density flow may also be an incompressible flow, this is a very important concept. So, incompressible flow need not always be a constant density flow. So, a typical example is; let us say that you have a domain like this, within this there, is a fluid. Now, this fluid changes its phase, say it was in a particular phase, it was in liquid phase, now it becomes a vapour phase.

So, when it becomes vapour phase, it becomes lighter, so the same mass now cannot occupy this volume, so there is; so it wants to occupy an extra volume but given a particular volume what it will do; some extra mass we leave because you are constraining the volume and if you are having a change in density, you must have a flow to accommodate a change in density, so that whatever fluid is there now is accommodated within the volume that was given to you.

So, you can see that you might have a change in density at a fixed position with time because maybe with time the phase change has triggered, so with time the density has changed, so this has to be now adjusted with some u, v, w, so that the net effect may still be 0, so it might so happen that now here, the net effect it may be 0, it may not be 0, so let us take an example, where the net effect is 0.

What is that example? Maybe there was a fluid, now it is getting frozen and because of freezing its volume gets change, so it is possible, so its density has got changed but we do not call say liquids or solids as compressible fluids, so what has happened because if with freezing, there is a shrinkage then there will be a deficit in volume here, maybe to satisfy the deficit in volume there might be a material supply from all sides.

So, it is possible that to make a balance of what is happening locally and what is happening over the volume element you might have to adjust these things with velocity across the different phases of the element. So, in summary we can say that incompressible flow definition is the total derivative of rho with respect to t is 0 but not just rho is a constant, okay. We will stop here today; we have just seen one way of deriving the continuity equation.

But we will learn more by having different ways of deriving the continuity equation and that we will do in the next class. Thank you.