Introduction to Fluid Mechanics and Fluid Engineering
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Lecture – 11
Fluid Kinematics

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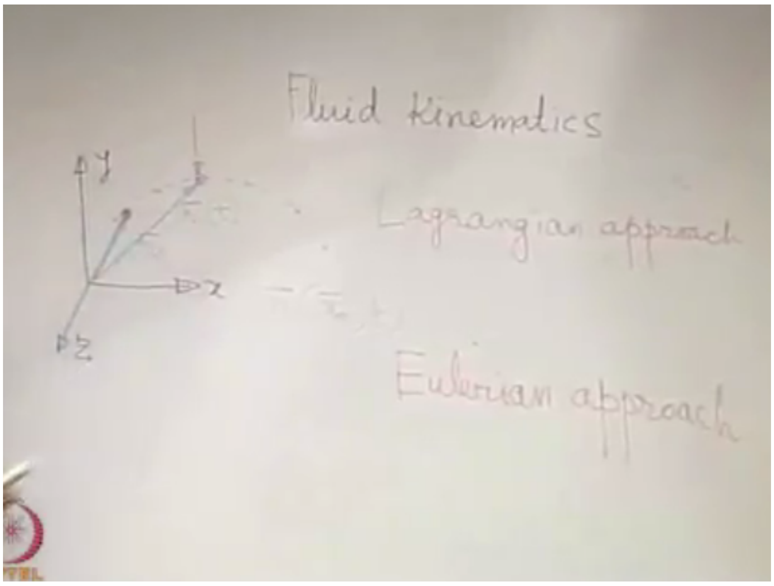

We will start today with the discussions on fluid kinematics, as you understand, when we will be concentrating on fluid kinematics will be somewhat abstracted from whatever the; are the forces, which are giving rise to the motion but primarily, we will be now concentrating on the nature of motion that means displacement, velocity, acceleration. When we say motion, we also mean angular motion, deformation.

We have earlier seen that deformation is one of the very important criteria for demarcating a fluid as compared to that of a solid. So, we will try to see that how we can characterize the kinematic features of a fluid motion. To do that we have to go through certain formalities because we need to first appreciate that there is a necessity of describing the fluid motion in a manner different from that of a solid.

Usually, whenever we are dealing with particle mechanics, what we are trying to do; most of the times, you have like a particle, you are trying to find the evolution or locus of the particle as it moves with time. So, you have an identified particle which is tagged like this, this particle at different instants of time comes to different positions and the locus of the particle can be obtained by joining those positions successively.

This type of approach is known as Lagrangian approach, so Lagrangian approach is nothing but tracking of individual particles, so you identify particles and you tag those particles and track the particle so to say, as they evolve. This Lagrangian approach is good for particle mechanics but when you come to fluid mechanics, you see that it has certain limitations, not that it cannot be employed in principle but when it comes to practice.

Because fluids have numerous particles well, solids also have numerous particles but the difference is that fluid particles when they are in motion, they are continuously deforming, so it is very difficult to keep track of individual particles and therefore this type of description of motion is not so convenient. If it was convenient, how would we have described that motion? So, let us say that we would have set up a coordinate axis like say x, y, z.

And let us say that this was the initial position of the particle which we try to designate by some position vector, say r0, so the position vector at any other instant of time, say the particle is at this location at an instant of time T and the position vector is r, so r at time T, we can say in this approach is a function of what? It is a function of the initial position and the time that has elapsed.

So, when you describe the displacement in this way or the position vector rather, then this is a Lagrangian description. As we have discussed that for fluid it is difficult because you have numerous particles, which are continuously evolving over space and time, so for fluid there is a more convenient approach which is known as the Eulerian approach. So, in the Eulerian approach, what we are doing?

You are not tracking individual particles, instead of tracking individual particles, what you are trying to do; you are trying to focus your attention not on a particle but on a position. So, let us say that you are focusing your attention on this position. Now, what is happening? There will be many particles which had come to this position and had left, you are not bothered about which particle has come from where and which particle is going where.

As if you were sitting with a camera and the camera is focusing on a position you are bothered about what is the change that is taking place across this position, so you are; what you are bothered about is that maybe what is the velocity at this point, so velocity at this point is nothing but the velocity of a fluid particle at that instant, which is passing through that point. So, what is the fluid particle?

I mean there are involved concepts associated with it but in a very simple term it is just like an inert tracer particle moving with the flow, so it does not interact with the flow, it does not have any density difference with respect to the flow, so it is just a passive inert particle moving with the flow. So, if such was a particle then at a particular instant, the velocity of the particle which is seen by an observer, who is focusing the attention here is known as the; that velocity is known to be a velocity that is obtained by using Eulerian approach.

Again that velocity would be same as the velocity of a tracer particle that at that instant of time was passing through that position, right. So, what you can say from here is that we can write the Eulerian description in the following way, so we can write just like in this way we could write the Lagrangian approach, Eulerian approach, we can say that the velocity at a particular position is function of what?

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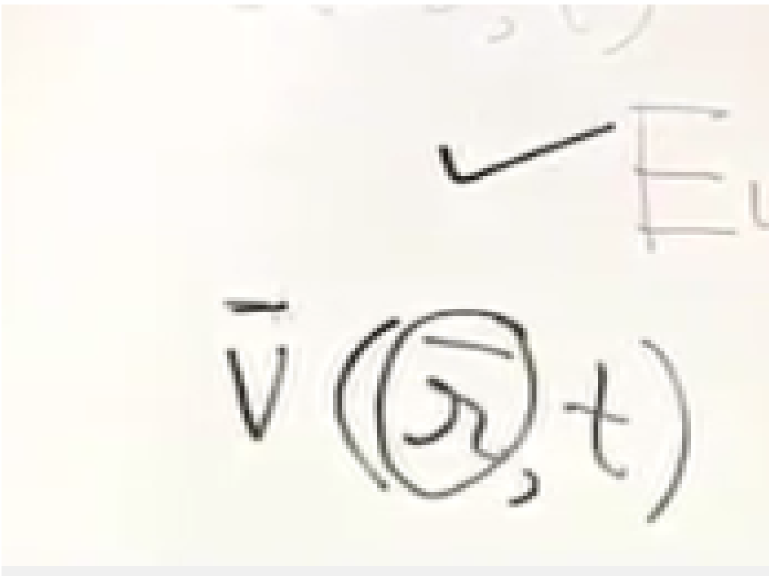

It is a function of the position r and the time t because it also varies with time, so the key difference is here you focus your attention on a specified r across which you are trying to see the changes in a fluid property. So, here velocity is the fluid property that you are looking for as an example. Regarding the fluid particle, one important concept is like this that all of us know that fluids are comprising of molecules at the end.

So, were from the concept of particle comes; if you try to extrapolate the concept of molecules to particles and it can be thought of in this way, so there may be a collection of molecules in the fluid, which on a statistically average sense have a common group behaviour and that collection may be conceived as a fluid particle and the behaviour of that is equivalent to the behaviour of an inert tracer particle put in a fluid and it just moves with the flow.

So, it does not do anything, does not interact with the flow, it is so passive that wherever flow takes it, it just goes there and it does not have any density difference, so it does not have any buoyancy force also because of its existence on the flow and such a particle which has the density same as that of the fluid conceptually is known as a neutrally buoyant particle that means a particle, which does not have any resultant buoyancy effect.

Because of its interaction with the flow that means, the densities of the fluid and densities of the particles are considered to be the same. Clearly, the Eulerian approach is easier for implementing for fluids because when you are implementing that you are abstracted of individual particles, you are only concerned on a particular position and you see that some particle is coming and some particle is going.

You are not bothered about the identity of the particles, which are coming there and leaving that place and that makes it more convenient for employment for analysis of fluid flows, so we will mostly be bothering about the Eulerian approach. Keep in mind that the basic laws of Newtonian mechanics were originally derived by using Lagrangian approach, so a major emphasis on fluid mechanics will be to convert those expressions into equivalent forms which are implementable using the Eulerian approach.

So, the expressions that we will be writing say equivalent to Newton's second law of motion, in terms of Eulerian approach will not exactly right in the same; will not exactly look in the same form as we are familiar with for simple particle mechanics for; I mean for solid mechanics say but we will see at the end that these approaches are inter convertible and it is possible to derive one approach or expressions in perspective of one approach from the expressions of the other and that we will take up in a later chapter.

But we will keep in mind that Eulerian approach is something that we will be trying to follow for most of the problems in fluid mechanics. Now, regarding the use of the Eulerian approach, we will try to understand or we will try to learn some basic terminologies.

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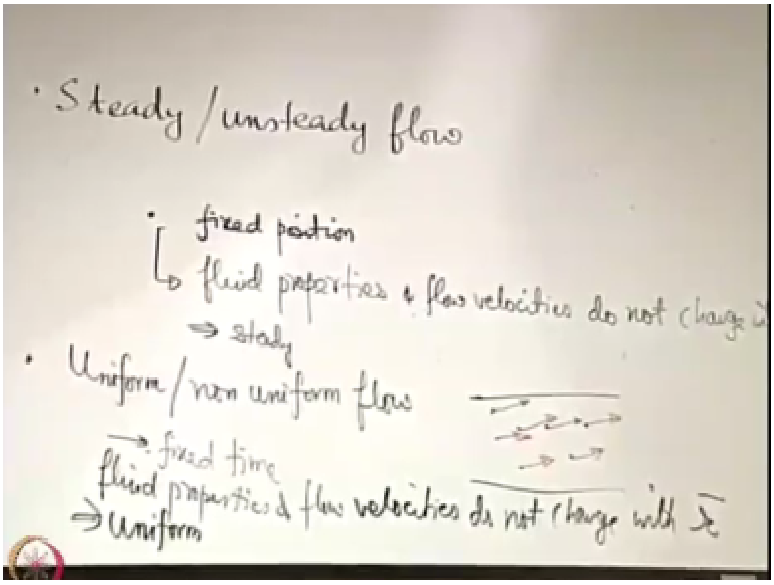

The basic terminologies are as follows; first concept is steady and unsteady flow. When do we say that a flow is steady? Whenever we are trying to focus our attention at a point say, by following the Eulerian approach, we are interested to see that the flow velocity or fluid properties at that particular point are changing with time or not. So, if at a specified location, the fluid properties or the flow velocity are changing with time, then we call it an unsteady flow.

But if that is not changing with time, then we call it a steady flow, so what are the keywords here that we are having a fixed position and what we are trying to see is the fluid properties, so when we say fluid properties we have discussed mini fluid properties may be density, may be viscosity whatever fluid properties and flow velocities do not change with time. If that is the case at a fixed position, then we say that it is a steady flow.

If that is not satisfied as obvious that is unsteady flow, we will learn a related but not the same concept that is uniform and non-uniform flow. What is that? Just like steadiness discusses about change with respect to time at fixed position uniformity discusses about change with respect to position at a fixed time that means, let us say that you have a domain in which flow is taking place.

In technical terminology sometimes, we call it a flow field. What is the field? Field is a domain over which some influence is felt, so when you have a flow field it is basically a domain over which the influence of the flow is felt, so you are having some velocities at different locations. So, in the flow field at different points, let us identify different points, let us say that these different points have different velocities.

So, let us try to mark this different velocity, say the different velocities are like this, if they are such that at a given instant of time they are same at all points, then we say that the flow field is uniform, so not only the velocities but also again the fluid properties, so fluid properties and flow velocities, so what we are doing now; we are fixing the time, so fixed time that is our concern.

At a fixed time, we are trying to see that fluid properties and flow velocities are invariant with position do not change with position. So, if that is the case that at a fixed time we are finding that fluid properties and flow velocities do not change with r, we call it a uniform flow field, so the distinction between steadiness and uniformity is quite clear. Let us ask ourselves certain questions related to this.

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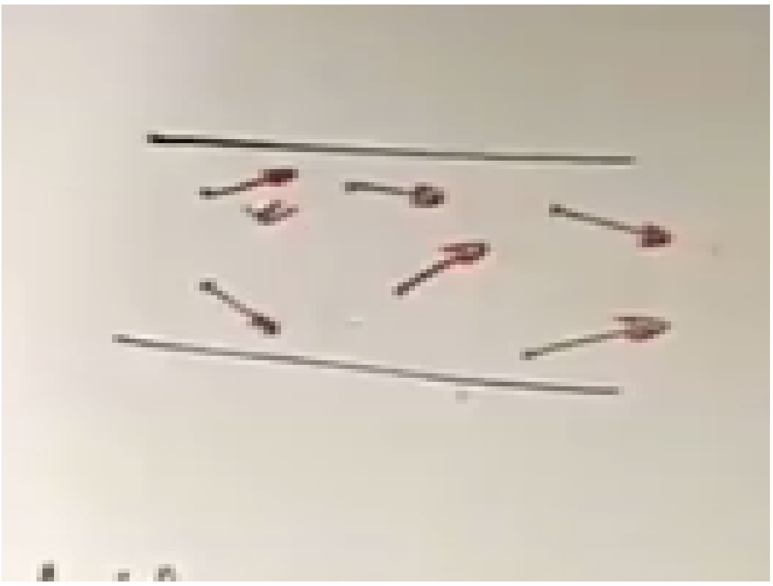

If the flow is steady could it be non-uniform that is very much possible, so if you have a domain like this with different points, so you have velocity at different points different but whatever those are, those are not changing with time, so it is possible to have a steady but non uniform flow. Is it possible to have a uniform but unsteady flow? So, 50% of you are saying yes and 50%, no.

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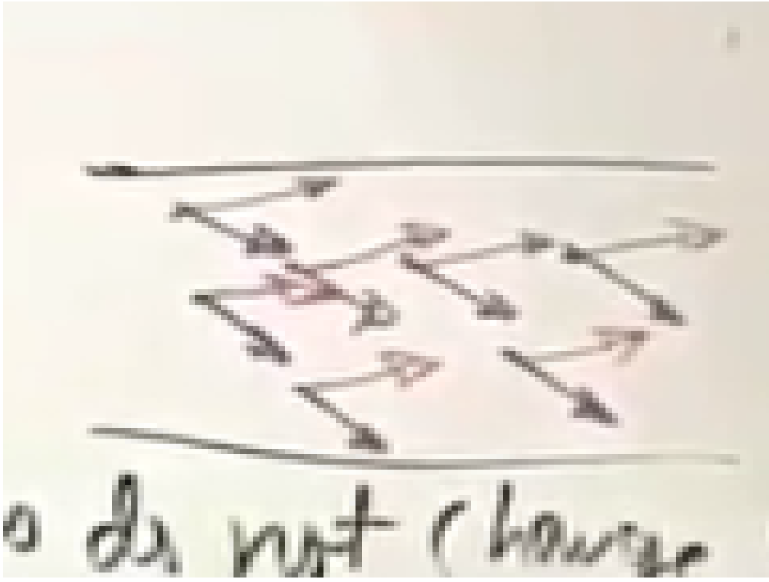

Let us look into this example, so let us say at some time instant, these are the velocity vectors. Let us say in the next time instant, you have the velocity vectors like this, assume that they are same. So, with change in time at a given point if you focus your attention, you can see that the velocity has changed, so it is unsteady but at a given instant of time, it is same at all locations, so it is uniform.

So, it is possible that you may have unsteady but uniform flow just like it is possible to have steady and non-uniform flow, so the steadiness and uniformity should not be confused as equivalent concepts; they relate to 2 different things of course there may be relationships between these 2 in very special cases. The other important remark is that whenever we are talking about the steady and unsteady flow, we have to keep in mind that steadiness or unsteadiness is not something which is absolute.

It depends on the choice of the reference frame, with respect to which you are analysing the flow and this type of choice of reference frame is very important in mechanics; fluid mechanics is of no exception. Let us try to understand this through an example again. Let us say that there is a river, on the top of the river there is a bridge and there is an observer standing on the bridge, the whole idea is to observe; just qualitatively observe how the fluid flow is taking place below the bridge on the river.

Now, suddenly a boat comes; the boat approaches the breach and it just crosses the breach below it, so what happens to the water, the water was say earlier stagnant, now the boat has come, so it has disturbed the water and the boat has gone again after some time, the water will come back to its original state. So, to the observer, who is standing on the bridge how the flow will appear, it will appear to be changing with time.

So, initially; so, he is focusing his attention on a particular location, he is finding initially it is stagnant then velocity is changing because of disturbance created by the boat and then again the velocity is coming back to its original state, so it is a strongly unsteady flow. Let us take the same example in a different perspective say the boat is moving relative to the river at a constant velocity and a person sitting on the boat is trying to observe the flow.

So, when the person sitting on the boat is trying to observe the flow of the same river, what is the interpretation because the velocity of the water relative to the boat is not changing and the person is only observing that relative velocity for that person, the same flow of the river water is appearing to be steady not changing with time, so the same flow may appear to be steady or unsteady depending on the choice of the reference frame.

So, these 2 were 2 special reference frames; one is a fixed reference frame just like the bridge, another is a reference frame that is moving with a boat like moving with something at moving with a reference at constant velocity and you can see that with respect to the reference frame moving at a constant velocity, it is possible to analyse a flow which is otherwise unsteady in terms of a steady concept.

That means, it is possible to convert the notion or transform the notion from an unsteady to a steady flow by such a transformation of reference and this in mechanics is known as Galilean transformation. So, we will try to maybe have a visual example of what type of flow takes place as a boat is moving, I cannot show you the bridge in this example but at least, I can show the boat.

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So, we will try to follow very clearly that what is happening across the boat, as the boat is moving, it is an artificial tunnel, this tunnel is not very wide and just see that what is happening to the water; water is not of course, very clean but it will help you to visualize it better. See the boat was moving relative to the water at a uniform speed but what you can see is that after the boat has gone, there is a change in a velocity pattern.

We are now; observers with respect to a fixed reference frame and we are observing it to be changing with time. See one important thing is we are not talking about inertial and non-inertial frames both are inertial frames because when something is moving with a uniform linear velocity that is still an inertial frame, so do not confuse these with the concept of inertial and non-inertial frame.

So, Galilean frame is still an inertial frame because it is moving with a uniform velocity, it is not an accelerating reference frame that we need to understand. So, with this background what we will do next, we will try to see that we have got a basic concept of what should be the description of a flow in terms of change in position, change in time and the description of the flow velocities, the flow velocities as you can see that the velocity vector V, these may have their individual components like components along x, y and z.

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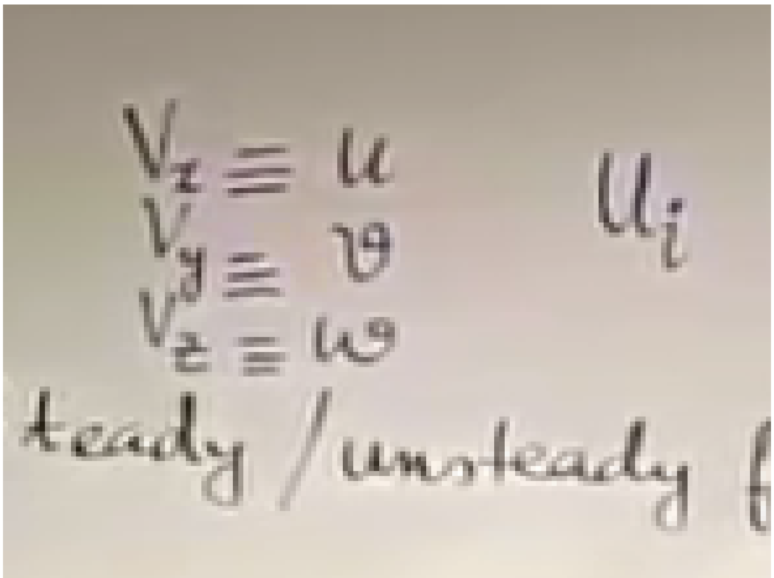

In fluid mechanics, we give them certain common notations, the velocity component along velocity component along x, we give a name commonly as u, velocity component along y, we give a common name v and velocity component along z, as w, so these are just names, I mean just notation so to say commonly used in textbooks. In the index notation of course, you can use ui, where i = 1, we will imply Vx, i = 2, will imply Vy and i = 3 will imply Vz.

Because it is a vector so we have seen how to write vectors in index notations, velocity is of no exception. Now, whenever a flow field is having a velocity, it is not so easy to map the velocity vectors in a flow, so there should be some mechanisms by which we are trying to develop a feel of how we visualize a flow and for that we have to consider certain imaginary lines in the flow, just like you have lines of force in a magnetic field by which you try to visualize how the; how the magnetic forces are acting.

Similarly, whenever you have a flow field, you must develop some concept of imaginary lines, these lines are not exactly existing in a flow but as if these lines were there to aid you to visualize the flow both qualitatively as well as quantitatively, so we have earlier seen some examples of flow visualization may be what we will do is; we will try to see some more examples to see different methods of flow visualization.

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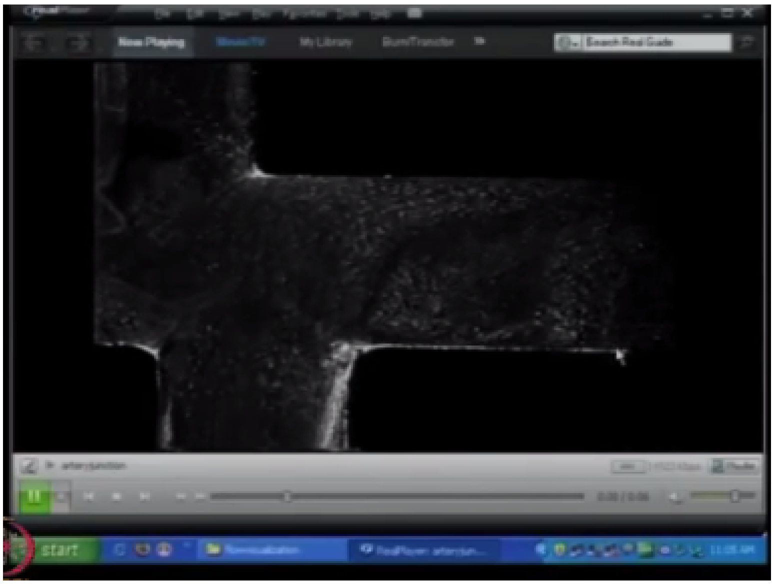

See this first example, so in this example, it is like flow in an artery junction, so there are certain tracer particles, so how this flow is visualized? You have tracer particles, so these particles may be very small beads of maybe nano meter size or few micron size and these beads are put in the flow and the flow is illuminated by say a laser source, so in that illuminated condition when the particles are moving, it is possible to not track individual particles all particles as such.

But it is possible to statistically track them that means, it is possible to get a statistical picture of the displacement and velocities of these particles and from that it is possible to have a post processing and map the velocity field also by a statistical operation on the motion of selected particles and that principle is followed in one of the devices, which we commonly used for flow visualization in advanced research applications known as particle image velocimetry or PIV.

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Now, we will not go into such analysis for this elementary course but if you just try to look into some other examples, this is particle visualization, this is also particle based visualization but visualization through smoke emission, so you can clearly see basically a wing of aircraft has passed and when it has passed, it has created a vortex, see the rotations and by the smoke emission that is being visualized.

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So, by illuminating with a smoke; smoke is the natural emission and it is possible to visualize the flow at least qualitatively by using that. The same type of thing that is possible to be observed in the chimney of a power plant and we have seen this example earlier, when we were discussing about the introductory concepts and you can see that that also gives rise to a good visualization of the flow field.

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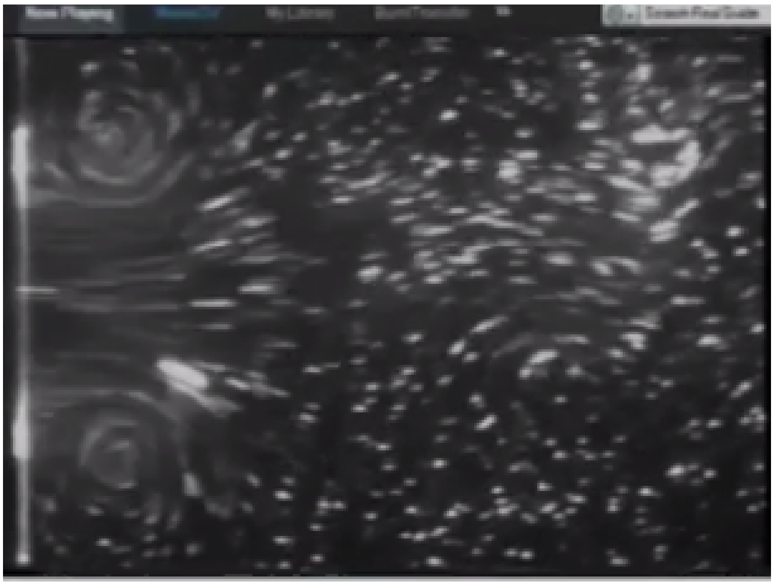

When you have these particles put in the flow and those particles are in forms of beads, so you can see that nice rotating structures or vortices can be observed with these particles, so these are called as streaks and we will see that why these are called as streaks.

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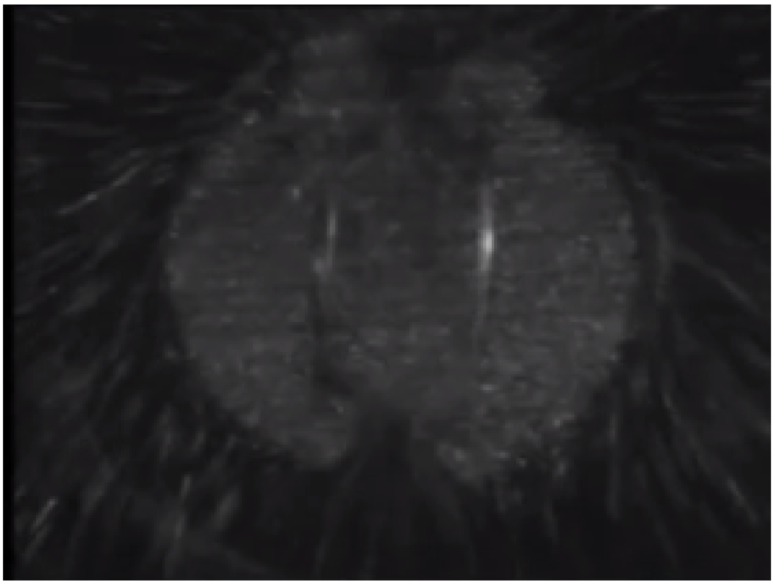

We will look into one or 2 more visualizations with the streaks and then we will try to formalize that how to mathematically write or express these concepts of visualization.

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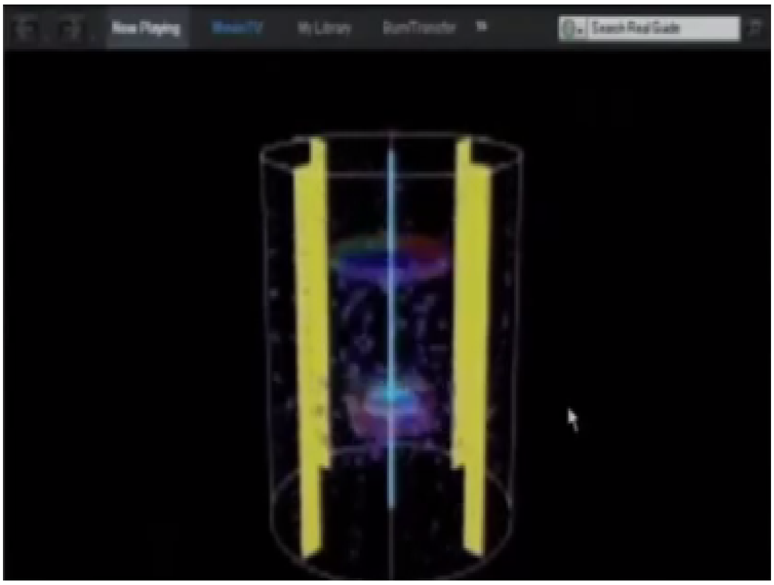

This is a numerically or computationally simulated flow with the help of synthetic particles, so these are not really particles in the physical sense but this is just the entire flow is simulated in computer to have this kind of visualization. Now, what we will do; we will try to see that; what are the conceptual lines, which are important for quantifying these visualizations and for that we will learn certain concepts.

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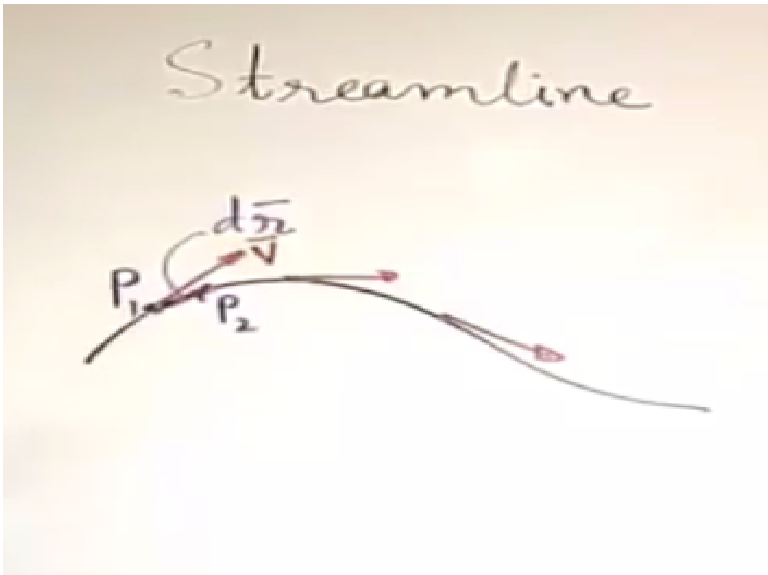

So, the first concept that we will learn is something which you have heard of earlier and that is the concept of a streamline. So, we are discussing about some conceptual paradigms, which help us in visualizing fluid flow. So, when we say streamlines, how do we define streamlines? Streamlines are imaginary lines in the flow field, these are not existing in reality, so imaginary lines. What type of imaginary lines?

These are such lines that at an instant of time, tangent to the stream line at any point represents the velocity vector at that point. So, if you have a stream line like this say, so when you have a stream line like this, you may have tangent to it at different points and these tangents are representatives of the velocity vector at these points. One important concept that we mean, many times is that it is defined at a particular instant of time.

That means at different times, you make a different stream lines, only when the flow field is not changing with time that is a steady flow, you get same stream lines at all instants of time, otherwise you may get different stream lines. Now, to express the stream line in terms of some equation, so this is a line, this is the locus, so it should be expressible in terms of certain equations.

Let us try to see that how we can express that. Towards that we will first recognize that let us say that there is a point P or P1 located on the stream line, the fluid particle at a particular time; the fluid particle is located also here, it is coincident with this point. When the fluid particle is coincident with this point then after some time, the fluid particle has come to a different point and so on.

We are not bothering about the motion of the fluid particle, we are just bothered about say, 2 points which are located on the streamline, which are quite close to each other, say P1 and P2, not that P2 represents the location of the fluid particle at a different time not like that just it is another point on the stream line, which is very close to the point p1. The vector P1 P2, let us say, we denote it by a change in position vector dr.

And let us say that V is the velocity at the particular point. When we give it a name dr? we have to keep in mind that it is very small, right and it is differentially small, it is as good as writing delta r as delta r tends to 0. So, when delta r tends to 0, then what is the status of the points P1 and P2, they are almost coincident. When they are almost coincident that means, P1, P2 then represents tangent to the stream line at the point P1.

So, what is the tangent? Tangent is the limit of a chord in the limit as that gap the distance between the 2 points becomes infinitesimal, so in the limit P1, P2 becomes tangent to the stream line at the point P1.

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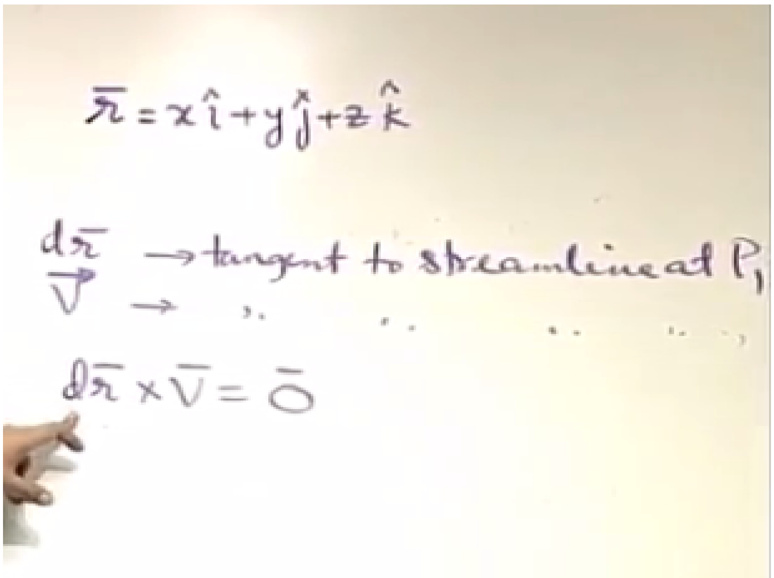

So, dr in that differential limit is tangent to the stream line at which point at P1. By definition, V is also tangent to the stream line at P1 that is the definition of the stream line that means, these 2 are parallel vectors. If these 2 are parallel vectors, their cross product should be a null vector. So, you have dr cross V is a null vector, so we can write dr and V in terms of components, you have r as; how do you write r? xi, yj and zk where i, j, k are the unit vectors along x, y, z.

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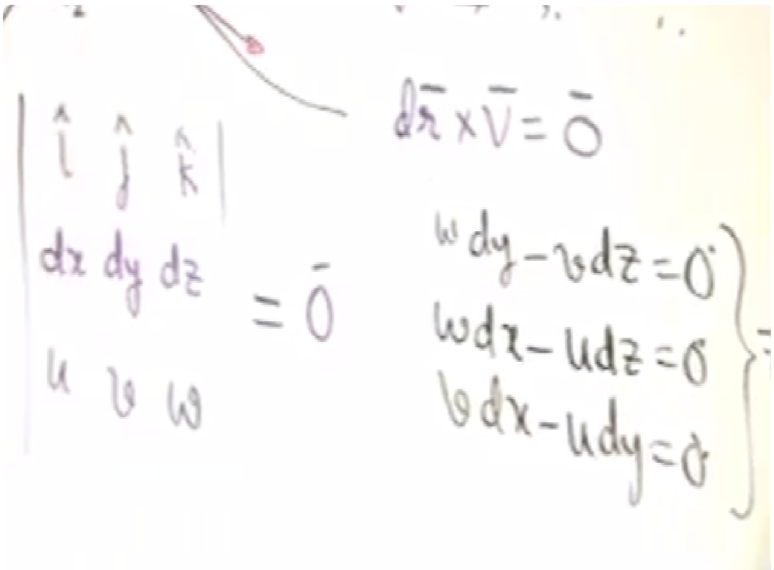

So, you can write dr as dxi + dyj + dzk, so when you write this cross product, it is possible to write it in a determinant form, so let us write that i, j, k then components of dr, dx, dy, dz and components of the velocity vector u, v, w that is equal to a null vector. We can easily see that it boils down to 3 scalar equations for each for the x y and z components. So, what are these scalar equations? wdy - vdz = 0.

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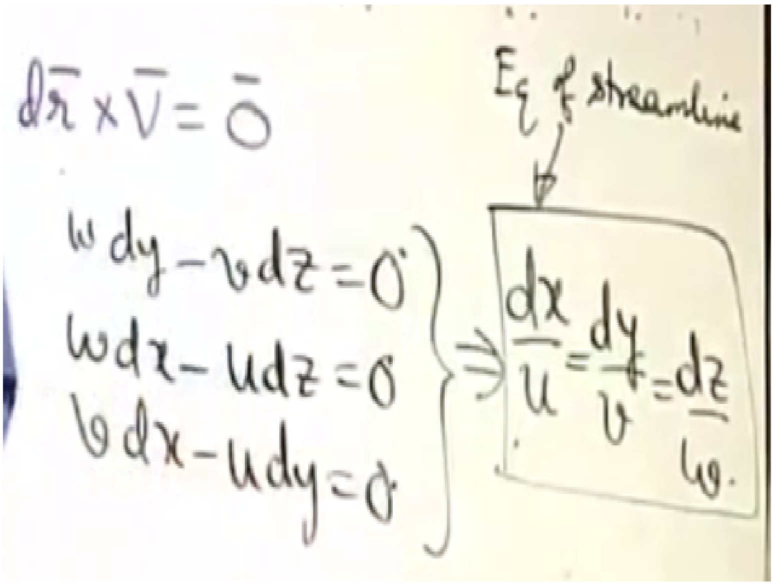

Then, wdx – udz = 0 and vdx - udy = 0. So, if you combine these 3 together, you can get a compact expression dx/u = dy/ v = dz/w, which is nothing but the equation of the streamline. these are locus that we are looking for, you can easily obtain the locus by keeping in mind that u, v, w are functions of position like x, y, z and also time but when you are considering a streamline, you are freezing the time.

So, at a given instant of time so that does not become a variable in this case, so x, y, z are the variable, so if these are substituted as functions of x, y, z, you may integrate these to find out the locus that is very straight forward. Later on, we will work out some examples to illustrate that how we can do that so, this is the concept of a streamline. Now, related to this concept, there are certain other terminologies again.

Sometimes, they are confusing because streamline is more commonly used, those are not very commonly used but those are sometimes more fundamental and more relevant than the streamline.

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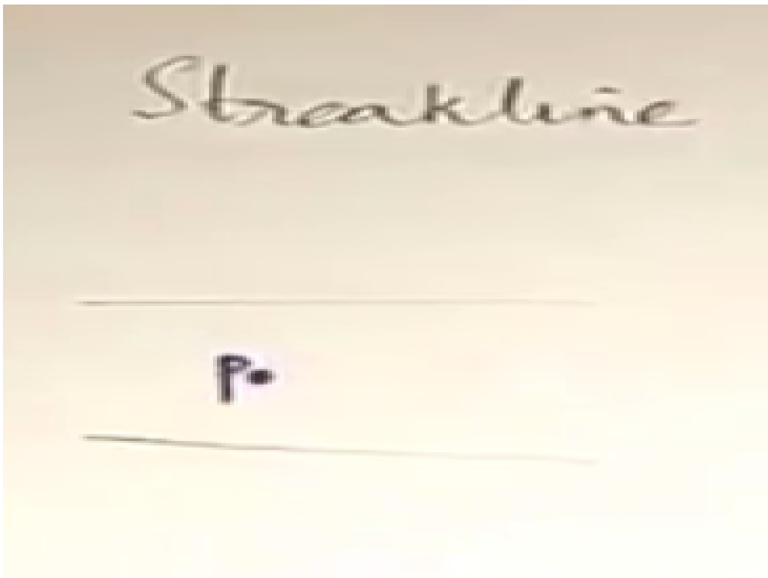

So, we will see the next example that is called as a streak line. So, what is the streak line? Let us say, you want to visualize the flow, we will try to identify the concept from where it has come. So, when you want to visualize the flows, this is the flow field you want to visualize a flow. A very common technique is what; you take an injection syringe, in that injection syringe you take some coloured dye, say a blue coloured dye.

A common name of a blue colour dyes called as thymol blue, so you have taken a thymol blue looks like the ink, so when you have taken that blue colour dye and say that you are trying to put that blue colour dye, inject that blue colour dye through this point P, so the blue colour dye is coming here through an injection syringe. So, now you are going on injecting the dye here, so what is happening?

Whatever, fluid molecules or fluid particles which are passing through this point at different instants of time, they are illuminated by the dye and so, wherever they go that tag of illumination remains. So, when you get an illuminated line, it is at a particular time then what does it represent; it essentially represents locus of all fluid particles, which at some earlier instant of time passed through this point of dye injection that is how they were coloured by the dye.

So, when you see a colourful line in the flow field, it represents that locus and that locus is called as a streak line. So, what is the streak line? Streak line is the locus of all fluid particles, which at some instant of time; at some prior instant of time all of which had passed through a common point. Mathematically, we stop here but physically we understand that common point is the point of dye injection, okay, so that is called as a streak line.

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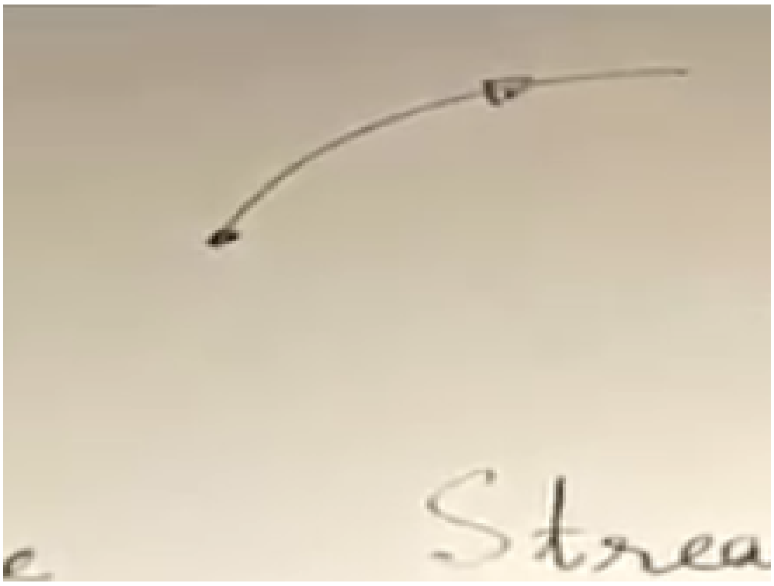

Let us try to conceptually draw a streak line, let us say that this is a point at which dye is injected, so when a fluid particle passes through these points say, at time = t0, so when at time = t0, the fluid particle passes through this point, the fluid particle then undergoes a locus. So, what is this locus?

We introduce a third line, which is called as a path line; path line is the simplest and the most trivial concept to understand, it is the description of a flow from a Lagrangian viewpoint, so it is the locus of an identified fluid particle. So, if you identify a fluid particle, how it moves; the path traced by that that is called as a path line that is very simple and trivial requires no explanation.

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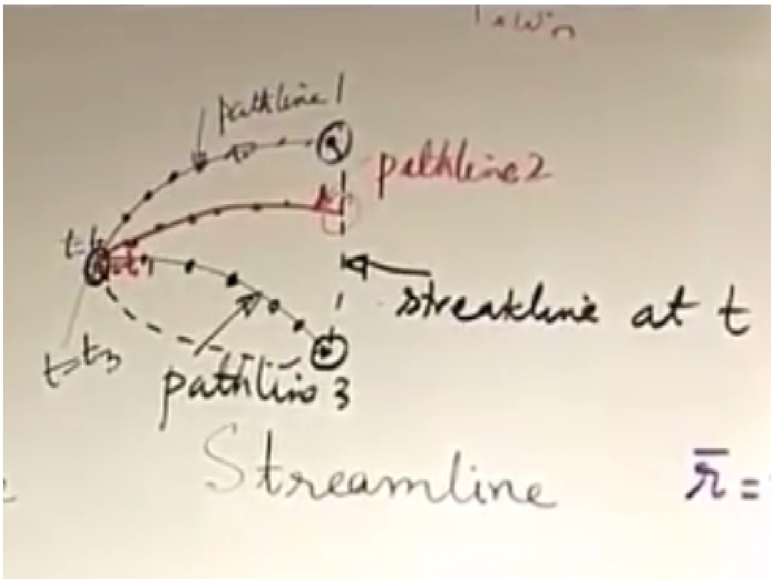

So, when we want to draw different path lines see at say time t = t0, you have one fluid particle which pass through this point of dye injection that fluid particle at subsequent instants of time, it is passing through different points, so this is the path line 1 say, what is that path line 1? Path line 1 is the path line of a fluid particle, which pass through the point of dye injection at time = t0.

Let us say there is another fluid particle, which has passed through this at time = t1 and let us say that this red line represents it locus, so this is something which was injected at t = t1, the path may be different because it may be an unsteady flow, so it is possible that the velocity field has changed with time. So, when just change with time, the particle may be forced to move along a different locus, so this is path line 2.

Let us consider the third path line maybe for completeness, let us say that we have a third path line like this, which corresponds to that injection here at time = t3 and again the path line is different because the velocity might have change at different points with time, so this is path line 3. Now, say we are bothered about at time = t, say now, so at time = t that particle, which passed earlier through this point at t0, say now it is here.

That particle, which pass through this point at t1 is now at here and that particle, which pass through this point at say, t3 is here, right now at time = t, say there is one more fluid particle that is injected just here, if it is a continuous process. So, the locus of all these, which at some earlier instants of time pass through the point of dye injection that locus is now the streak line at time t.

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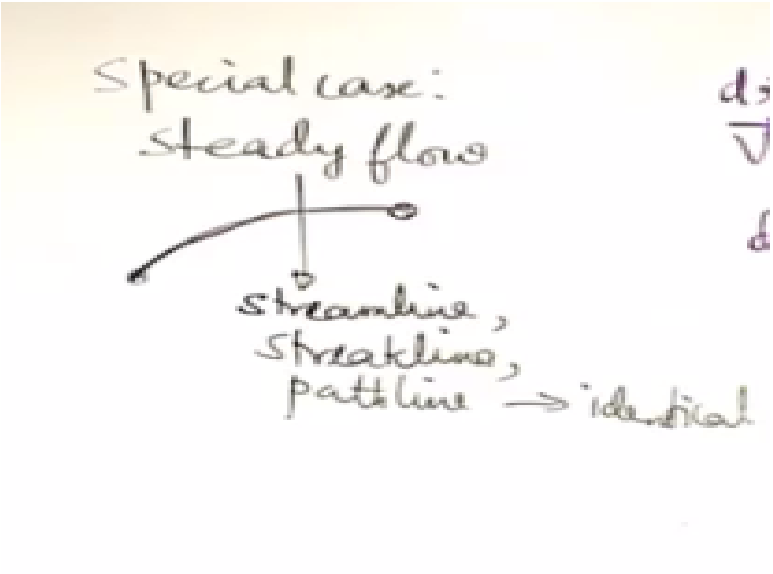

So, you can clearly demarcate between streak line and path line. Let us take the example of a very special case but a very interesting case. What is that special case? That special case is for a steady flow, so when you have a steady flow, then let us try to draw these path lines. So, first let us draw the path line of that particle, which pass through this point of injection at t = t0. So, let us say that that path line is this one.

Now, another particle which pass through this at t = t1 that will also follow this line because with time, the velocity field has not changed, so it will be constrained to follow the same line. So, it will follow this line similarly, the third one that is the one injected at t = t3 that will also be constrained to follow this line and what is the speciality of this line? This line is the locus of the fluid particle, so that means at some time, tangent to this line represents the direction of the velocity vector.

So, we can understand that this is also a stream line, this is also the path line and again this is also the streak line because whatever are the locations of fluid particles, those are always constrained within this line, so we can say that for steady flow stream line, streak line and path line are identical that is one very important concept that we should remember that in this case, you have stream line, streak line and path line are identical, they are identical.

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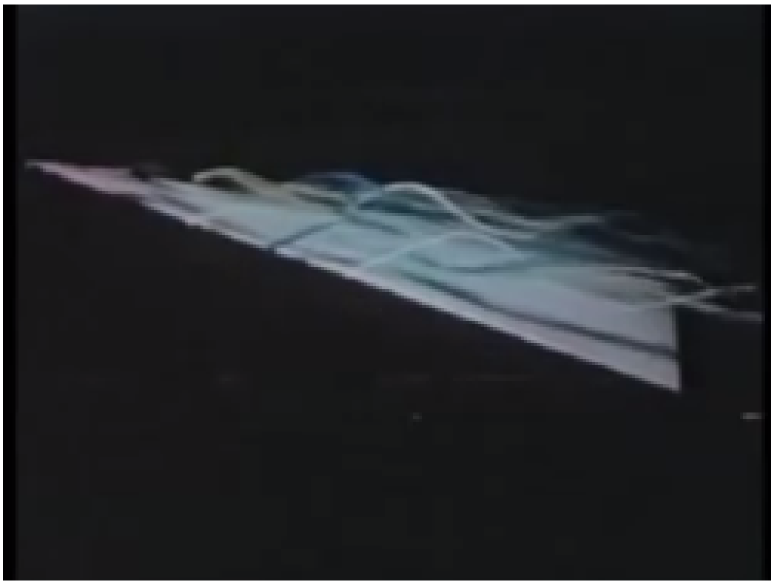

So, whenever we visualize the flow, let us look into some example maybe some images through stream lines or streak lines, so let us say that we see first this example, you can see these are streak lines, so dye streaks which are injected at the tip, you can see that now at different instants of time, they are forming different coloured images and if you track individual one, then it is like a path line.

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Now, if you see; let us look into a path line example, see this is like a lawn sprinkler, so many times it is used to sprinkle water in a garden, you can see that if you track the water droplets, you can clearly see that the path what they are following, so it is something like a visual representation of a path line.

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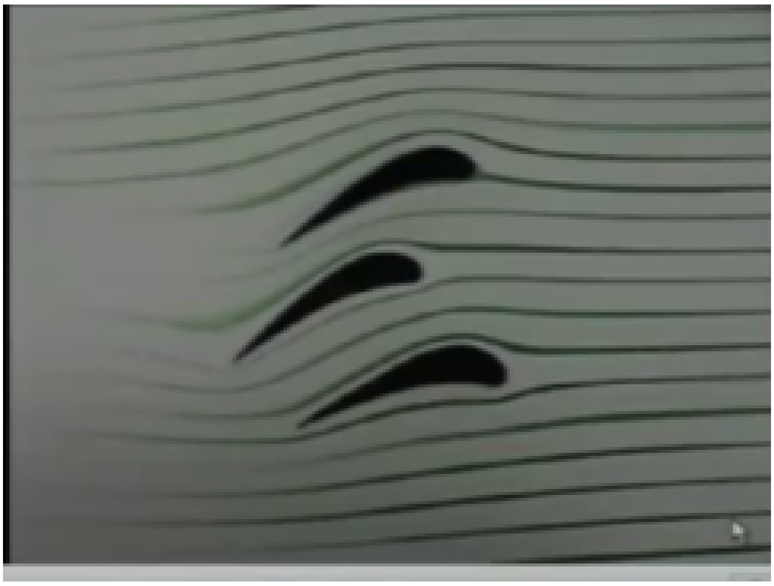

Let us look into a stream line example, so these are 3 cylinders in a steady flow and you can see that this green colour dye is giving an appearance of a stream line. Fundamentally, this is actually a streak line but because it is a steady flow, stream line, streak line, path line these are all identical, so these represent different streak lines or different stream lines or different path lines whatever you say, if it is a steady flow.

If it is an unsteady flow, these will represent streak lines rather than stream lines, so you can clearly see that these visualizations of fluid flow that we have seen as concepts, these may also be visualized in experiments.

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Many times, we are interested to construct the velocity vector, so see the example of the boat that we saw earlier and you may construct such velocity vector, so it is not a direct visualization but you may do it in 2 ways by post processing visualization of the particles, which are injected into the fluid or by doing a computer simulation and sometimes these may be equivalently compared.

Computer simulation of course, then idealization because you are using certain boundary conditions, certain properties which might not exactly prevail in reality but sometimes it gives a very important idea of how the fluid flow is taking place and it is used for advanced designs also, so this is known as computational fluid dynamics or CFD, so that is a separate area of research altogether, where the whole idea is to computationally solve the equations of fluid flow to get a picture of the velocity field.

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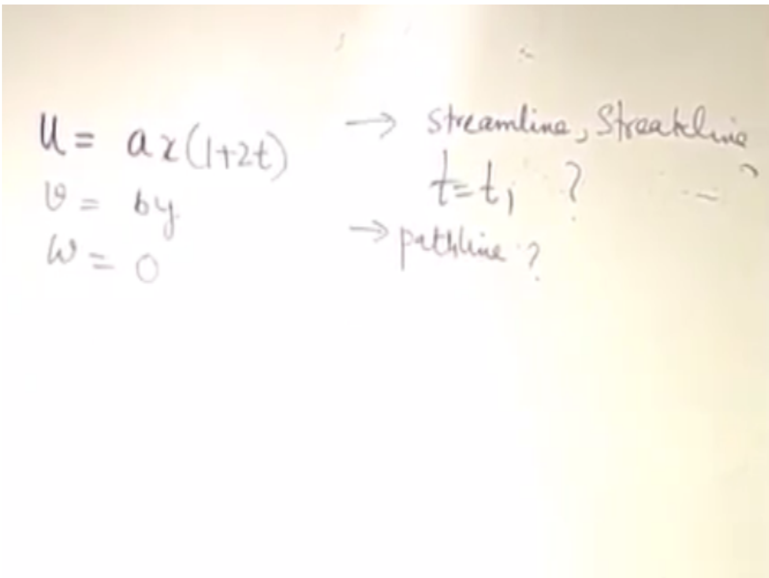

With this understanding, we will try to quickly work out an example to illustrate the concept of these lines; streamline, streak line and path line. So, an example; we will consider a 2 dimensional velocity field as an example, so let us say that a velocity field has these types of components; u is given by this ax * 1 + 2t, v is given by by and w is 0, so it is a 2 dimensional field.

Usually, whenever you have a velocity field, we call it 1 dimensional, 2 dimensional or 3 dimensional depending on the number of independent velocity components that you are having. So, if you are having 2 independent velocity components, it is a 2 dimensional velocity field, a and b are some parameters and x, y are the coordinates, t is the time and so, a and b have certain dimensions with adjust this, so that you get the dimensions of velocities at the end, so these are dimensional parameters but constants.

Let us say we are interested to find out the equation of stream line at a time say, t = t1, we are interested to find out stream line and streak line at time = t1 that is one objective. The other objective is to find out equation of path line. So, to find out equation of stream line that is the easiest part, let us do it first.

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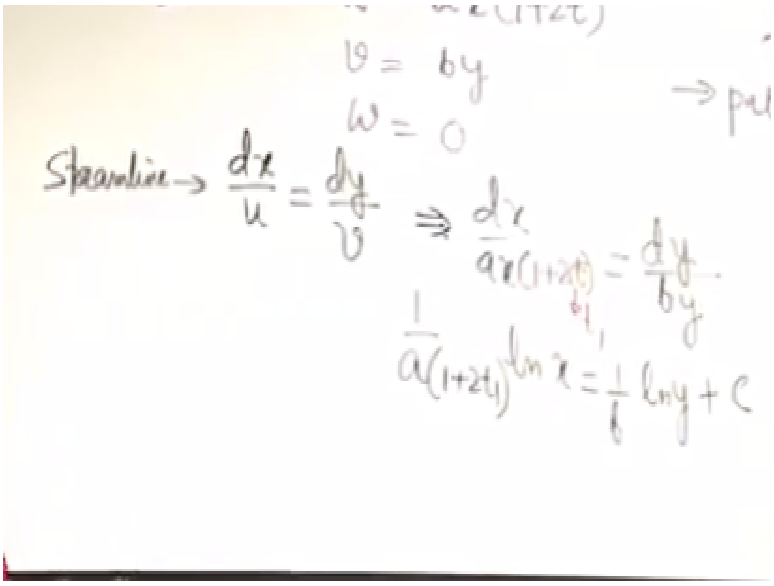

So, you have dx/u = dy/v, dz/w is not relevant because it is a 2 dimensional flow, so dx; so we are talking about first streamline, so dx/ax * 1 + 2t is = dy/by. At what time, we are focusing our attention? At time = t1, so you replace this t with t1, so when you are considering a streamline, you are freezing the time at the instant that you are considering, it is clearly an unsteady flow, so at different times you will get different streamlines.

So, you can integrate this and what you will get is; 1/ a * 1 + 2t1 ln of x is = 1/b ln of y +, say some constant of integration, C. How do you evaluate the constant of integration? You must be given a point on the streamline, right say; the stream line passes through some point. So, when you are given that the stream line passes through that point from that you can find out C that is if you know that at time = t1, whatever stream line you are drawing, there is one point on it.

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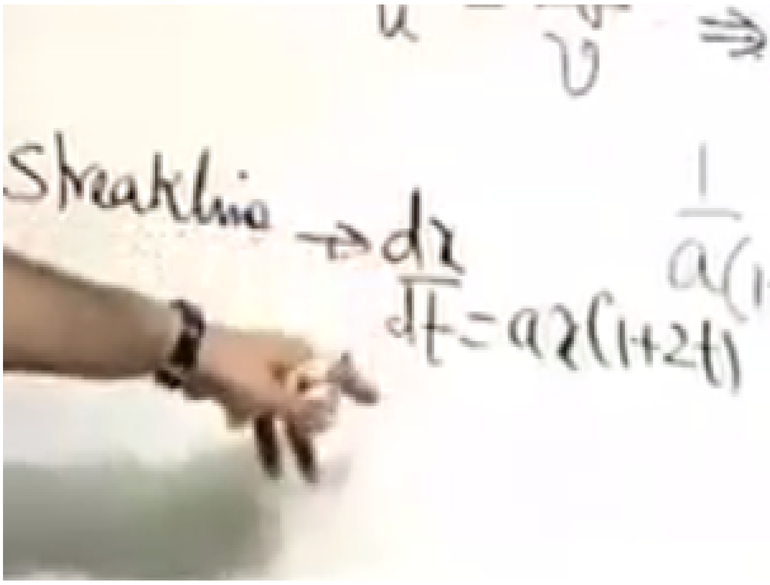

So, that point when substituted x say, x1 and y = y1 will give the value of C, so that will give the equation of the streamline if you arrange it properly and in a compact form. Let us consider the streak line, to consider the streak line; you have to remember one thing that this is a velocity that means, if a fluid particle is injected at a point, it will also represent its rate of change of position.

That means, you will have dx/dt is = ax * 1 + 2t, where x represents the x component of displacement of a fluid particle, which is subjected to this velocity field. Remember fluid particle is inert to the velocity field, so whatever the velocity field is imposing on it to do, it will do that so, this is what it is imposing. Now, it is possible to find out how x changes with time, it is straightforward but conceptually not that straightforward.

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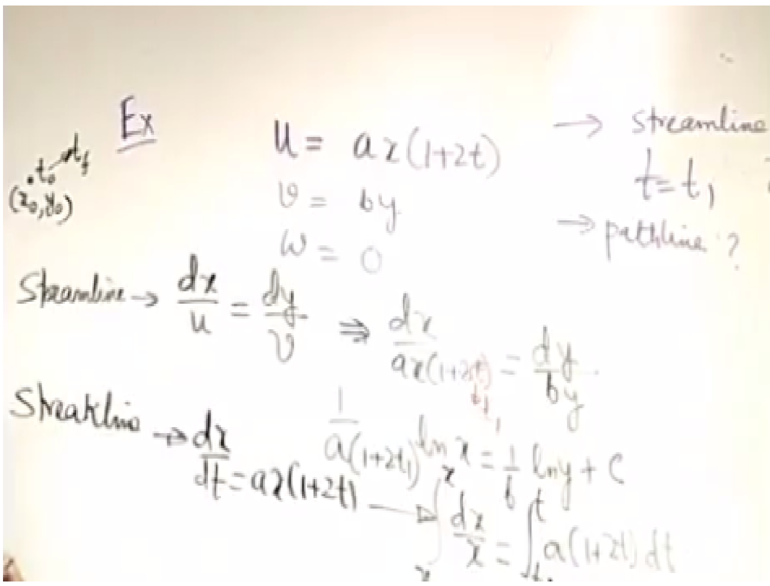

To understand, why it is conceptually a bit more involved, we will parallely write the equation of the path line, so let us write the path line, this streak line is not yet complete but we will draw a parallel analogy with the path line and see where is the difference. So, for the path line again you see path line is what; it is the locus of the fluid particle, so for path line also there is no need to believe that it should be something different than this one.

Similarly, dy/ dt, now the difference in approach comes in the concept by which we integrate these 2, so when we integrate this one, say we integrate the equation of the streak line. So, how we do it, we write dx/x is = a * 1 + 2t dt, similarly here also we write the same thing, dx/x = a * 1 + 2t dt, so in the left there is in; there are going to be limits of x and the right, limits of t, here also same. Now, what limits we will put?

Let us say that you are injecting the dye at some point x_0, y_0 this is a point of dye injection, the dye injection starts at t_0 and the time the dye injection ends at t_f, the final time, so this is the interval over which the dye injection takes place okay and the time that we are bothering about t_1 is something in between t_0 and t_f, so when we write the integration for the streak line, what we will do; we will integrate at time at some time say, t_i, $x = x_0$ and say at some time t = whatever say, t_1 $x = x$.

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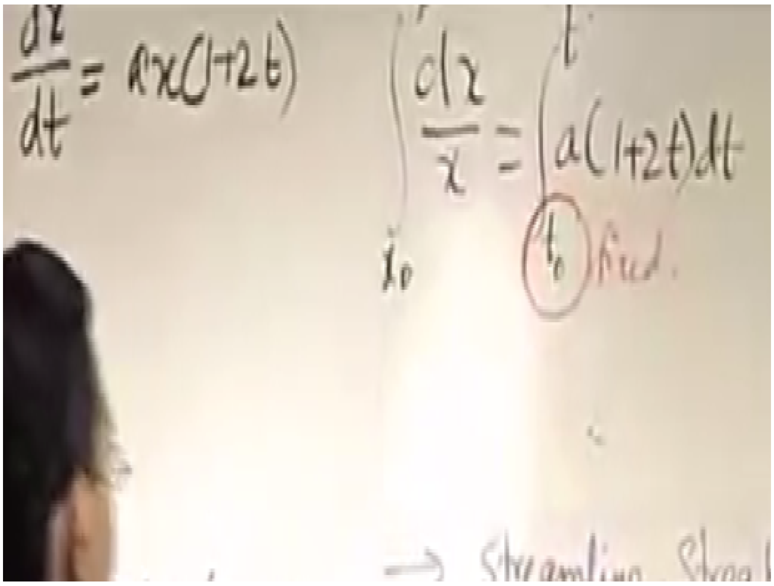

When you are considering the streak line, you have to keep in mind one important thing that importance will be clear when you write the integration here, so when you write the integration here at time =; now, here the time = t_0 because you are finding the locus of the particle, when we are talking about the path line, so when at time = t0, x = x0, at time = t, x = x, what is the difference between these 2?

See, look carefully into the limits of t, so when you say this is to, this is the fixed t0 that means, you are finding the locus of a particle which at time = t0 passed through the point of dye injection. Here, you are dealing with a variable t0 because you are dealing with locus of all particles, which at different instants of time pass through the point of dye injection, so ti is a variable, which may be anything between t0 and tf, okay, so this is a variable limit.

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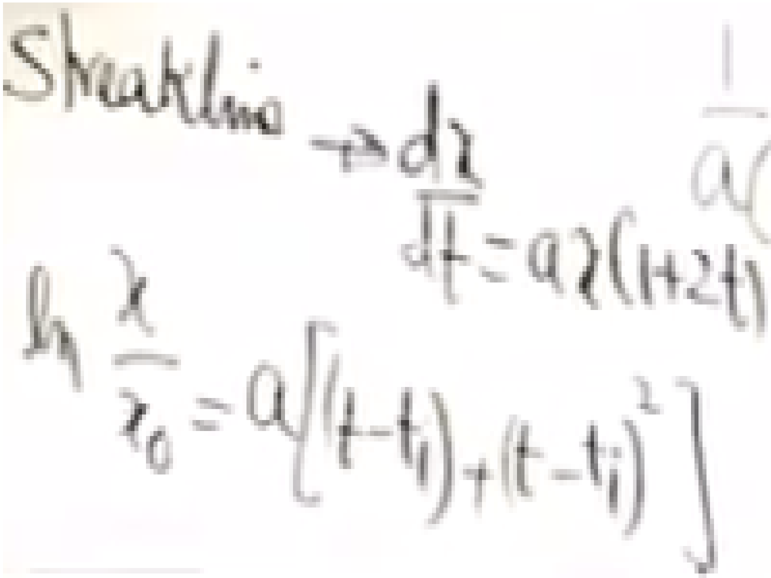

So, this actually needs to be eliminated, we do not know what is this, only what we know is that ti has to be between t0 and tf but it is not a specified time, this is a fixed specified time and that is how you are going to find the path line so, similarly when you so; when you write this in terms of x, so you have ln x/ x0 is = a * t - ti + t - ti square, where ti is a variable. Fortunately, you will also get another equation involving y.

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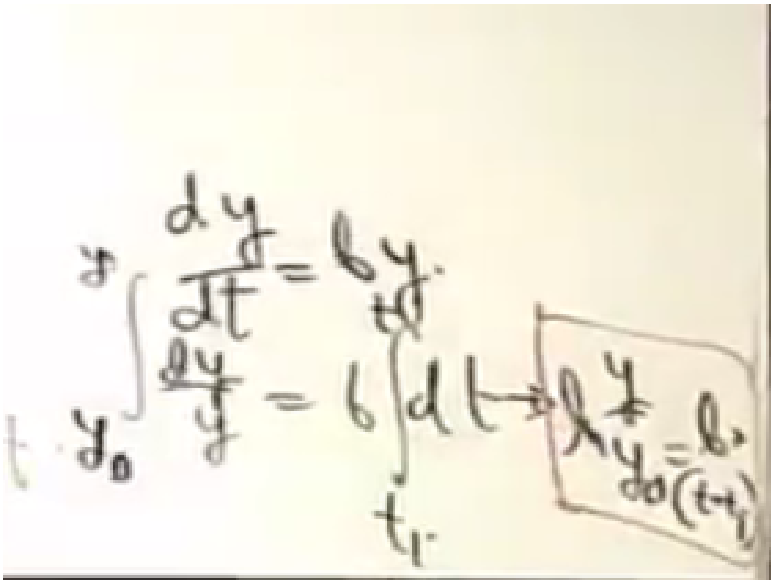

So, you can write similarly that dy/ dt is = by, so dy/y is = b dt, again you integrate with respect to the same limits, x0 to x and ti to t; t square - ti square; very good, so this is t square - ti square, right. So, when you do this, this will be not x but y, these limits are y0 to y, so that will give you what; ln of y/y0 is = b * t – ti, so you have an expression here, which involves ti, you have another expression here, which involves ti and you can eliminate ti from these 2 to find out the relationship between x and y.

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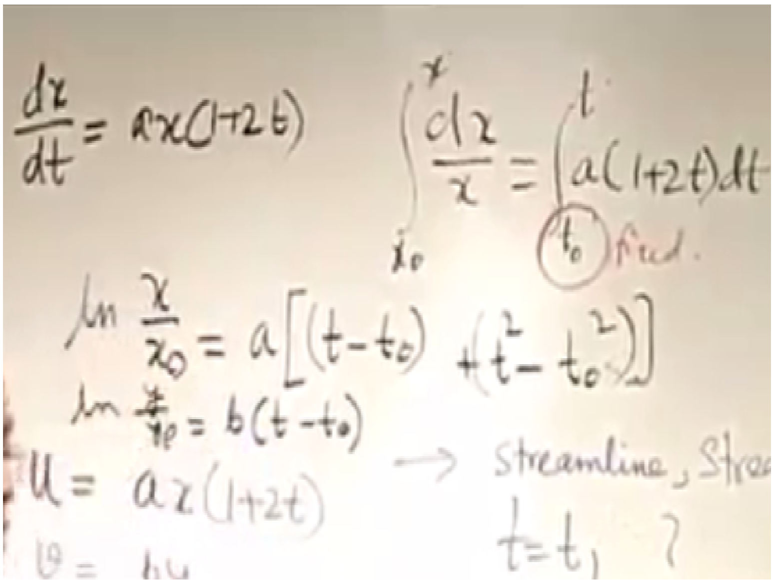

Here, that is not necessary because here you can straight forward write this, so you can write ln x/ x0 is = a * t – t0 + t square – t0 square and ln y/ y0 is = b * t – t0, here what is the variable? Here, actually t is a variable parameter because at different time, it will have different position to get that locus, it is it may be convenient even you may write it in a parametric form but conceptually, you may eliminate t.

To write the locus y as a function of x, whereas to write this, you have to eliminate ti, so here ti is a variable, whereas here t is a variable, so conceptually it looks very similar but there is a subtle difference and that subtle difference needs to be appreciated in the context of streak line and path line, stream line becomes more or less straightforward. So, I hope you get the distinctive concepts between stream line, streak line and path line and how to find out their equations given a flow field, okay. We stop here today, thank you very much.