Introduction to Fluid Mechanics and Fluid Engineering

Prof. Suman Chakraborty
Department of Mechanical Engineering
Indian Institute of Technology – Kharagpur

Lecture – 10
Fluid Statics (Contd.) and Fluid Under Rigid Body Motion

We were discussing earlier about the stability of floating bodies and we will continue with that. So, if you recall the sketch which we were discussing in the previous class. We identified a point know as metacentre and the motivation behind identifying this point is that when it is a floating body and it gets displaced and its submerged within the water changes its configuration the center of buoyancy does not remain fixed at its own position.

So, referring to a fixed center of buoyancy might not work as stability criteria. Now, we will try to find out as what we have done for the case of totally submerged bodies that how we can find out stability criteria for floating bodies. As you can see the presence of or the existence of these metacentre is something which is equivalent to the presence or existence of the center of buoyancy for a case when the center of buoyancy remains at its own position.

So, the metacentre somehow reflects the location of the center of buoyancy with respect to the axis of symmetry of the body. So, it has some relationship with the center of buoyancy definitely. But by specifying the metacentre or by specifying the center of buoyancy just as their absolute locations. It is not just possible to talk about the stability criteria. So, we have to look into more details. To look into more details, we will set up some coordinate axis.

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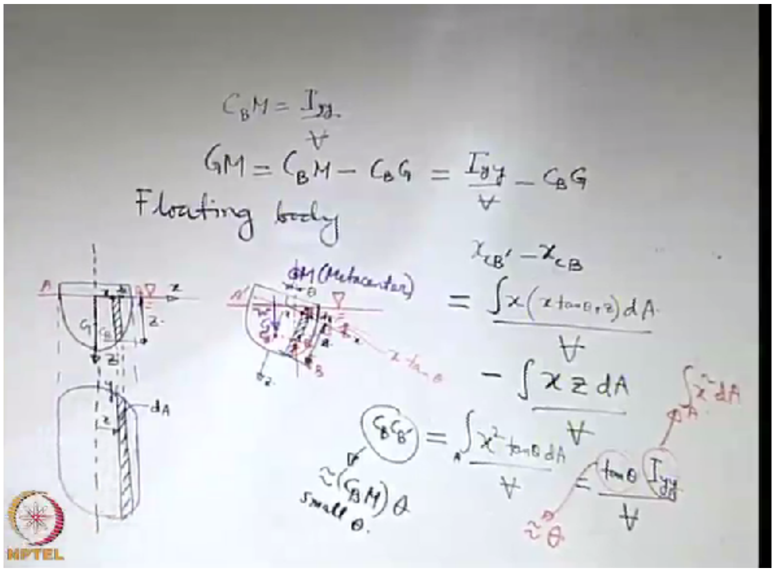

Say we have x-axis like this say we have G axis like and the y axis is perpendicular to the plain of the figure. So, the y axis is this dotted line which represents an axis in the other view. Now, if consider that how we calculate the submerged the volume the volume of the solid which there within the fluid. Let us say at a distance of x we take a small element of width Tx and this element has a depth of z.

So, this element if you look into the other view will be like this which is basically located at a distance x from the y axis. Let us say that this shaded area is the dA. So, you can say z*dA is the representation of the elemental volume of this shaded portion. Now, when it is tilted even when it is tilted we fix the x and z axis with respect to the body and we keep them as same. So, we still have this as the x axis and may be this as the z axis.

These are fixed relative to the body. Although, the body has tilted we are using a body fitted set of axis. Now, let us say that the angle of tilt is theta if the angle of tilt is theta then what happens? We can find out that what is now the displaced volume. So, if you consider again say at a distance x from here some strip width dx. Now, if you consider the displaced volume the displaced volume is of course corresponding to this part.

This part is like z same as this one because it is the same axis that has got tilted so if this is z, this displaced part is also z. But you have the additional displaced part which may be let us mark with the different colour. So, this is an additional displaced volume. So, this additional displaced part will correspond to a particular length along the z axis and what is this? Remember that if this angle is theta then the angle of tilt of the x axis with respect to the horizontal is theta.

So, what is this dimension? You know that this is x so that is x tan theta. So, if we calculate what is the displacement in centroid of the displaced volume? That is what is the difference between the x coordinate of say the original center of buoyancy let us give it a name that there was original center of buoyancy say CB and now the new center of buoyancy CB prime. So, the difference in the x coordinate of CB prime and CB.

That is what we are interested to find out. What is that? Center of buoyancy you can calculate by utilizing the formula for centroid of a volume which is here the displaced volume. So, this will be integral of x DB. So, this x what is DB so you have this total height as x ten theta + z dA, dA is the area in the other view. So, the depth times the dA is the volume. So, this divided by the volume of the immersed part.

Because of the symmetry the volume of the total immersed volume does not change whatever comes down the same volume goes up above the water. So, the total displace volume does not change minus what was the original one? Integral of x z dA/v. It may be easier if we mark the original CB and the displaced CB here. So, there has been a displacement from CB to CB prime. The location of the center of buoyancy has got shifted.

So, this is nothing but = CB, CB prime that is the length that we are talking about. And this is integral of x square tan theta dA/ (()) (8:13). Theta being like a constant parameter tan theta you can take out of the integral and this area integral is carried out over the shaded area dA. So, what does it represent if you take tan theta out of the integral? Second moment of area with respect of which axis?

You now refer to this plan view, integral of x square dA is the second movement of area with respect to the y axis. So, it is tan theta * Iyy/ v where what is Iyy? It is integral of x square dA. Now referring to this figure you can say that for a small angle theta CB, CB prime this particular small distance is a small arc of may be a circle. So, you can write this as approximately

CBM*theta, for small theta. It is just like S =r theta for a small part of an arc of a circle.

Because this is very small we are assuming a small displacement we test stability not with the large displacement but with the small displacement. M is the metacentre. Now, for the same smallness tan theta will be approximately =theta of course you are expressing theta in radian that is understood implicitly. So, then if you equate the 2 parts what you will get CBM=Iyy/v. Now, look into this figure. In this figure you see that M is above G.

When M is above G the couple movement created by the forces is trying to restore the body to its original configuration. However, if G was above M it would have been just the opposite case that you can clearly visualize. So, what is important here is the location of M relative to G just like for a totally submerged body it was location of B relative to or center of buoyancy relative to G. So, what we are interested to find out is what is the height GM?

So, GM you can write it as CBM –CBG=Tyy/v –CBG. See why we have written the formula in this way? In the right hand side, you have terms which are independent of the deformed configuration or deflected condition. So, if you see a Iyy V these can be calculated based on what is the original configuration and CB is the location of original center of buoyancy relative to the body. G is the location of center of gravity relative to the body.

And distance between those 2 that does not change with deflection. So, we are able to express something which should be a function of deflection in terms of certain things which are not function of deflection. Therefore, this is also not a function of deflection and m remains sort of fixed. But you have to keep in mind that there are certain assumptions that go behind this. What are the important assumptions?

Theta is small so this analysis is valid only when you are having a small theta not only that we have implicitly assumed that the z axis is the axis of symmetry for calculating or for coming up with expression. So, we are essentially dealing with the simplified case with symmetric bodies that we have to also keep in mind. Let us look into an example where we illustrate the use this expression for finding out the stability criteria.

So what is the stability criteria here? Let us summarize.

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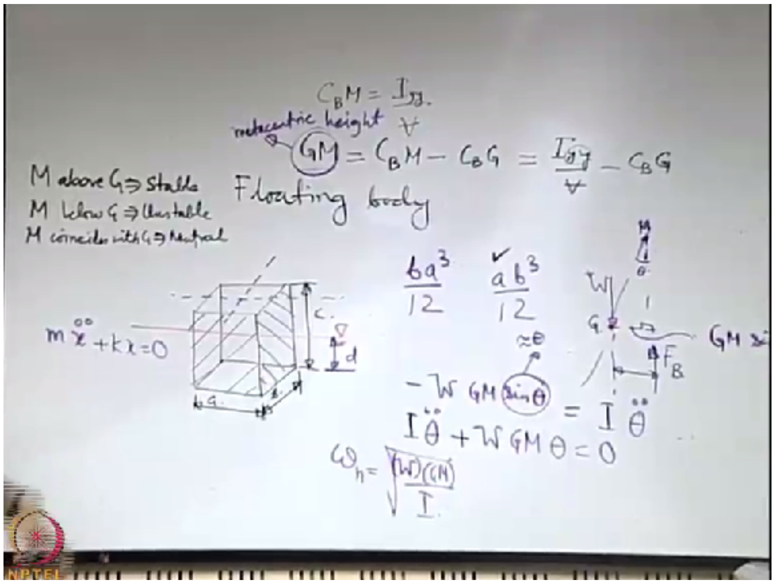

If M is above G it is stable. If M is below G it is unstable and obviously if M coincides with G it is neutral. So, you can clearly understand that M plays the equivalent role of center of buoyancy in this case. But it is not exactly the center of buoyancy. We consider a very simple example and we will illustrate the use of this through that example. Let us say that you have a body of this shape rectangle parallel perpendicular shape and you can give dimensions.

Let us say A, this is B and let us say this is C. Say this is partially immersed in a fluid so it is like a floating body. Now, let us try to see that how we can apply this stability criteria here and what are the important issues. First important issue is that when you apply this criteria what should be the y axis with respect to which you are considering the second moment of area. If you look into this problem very critically you will see that there are certain non-trivial issues.

Like you can consider may be this as one of the frontal surfaces. You might consider this as one of the frontal surface or even the third one. So, accordingly it is not very straightforward to say so when we drew the picture of the boat see it could be this front part. It would be the side part both are exposed to the water. I mean both are within the water. So, if you could say that what should be the corresponding axis that you need to consider?

Or does the choice of the axis change if you shift your attention from the frontal area to the side area. At the end, what you are bothered with? You are bothered with the plan view so when you have the plan view which is like the top view of this one. What is the plan view? It is the intersection of the body with the free water surface, free fluid surface so that particular view it boils down to the same when you consider this surface or this surface.

But again when you look from the side and when you look from this one may be in one case you are considering this as the y axis. In another case you are considering this as the y axis. Both axis are relative to the top. So, the question is with respect to which axis you should evaluate this. See, this is what? This is like evaluation of a safety criteria that it will be stable. So, when you want that it should be stable what should be the guideline?

The guideline should be that like you want this to be positive. That means this height is greater than this height. Then like M is above G so the way in which this is written the right hand side has to be positive. So, more positive means so to say it more stable so to say. So, if we take the least of this one, least possible value of this one and still find that the right hand side is positive then for all other cases it should be positive.

So, it is like a safety design. So, you look into the most adverse condition make sure that it is positive even for that. So, for better conditions it would always be better. So, out of these 2 axis for one axis, what will be the I? It will be Ba cube/12 another will be ab cube/ 12. So, you take the smaller one that means if B is smaller than a as an example you take this one. So, then when you substitute that in this expression and still you calculate this as positive.

You are assured that it is safe. Because with respect to other axis when you calculate the metacentric height it will definitely be greater. This is known as metacentric height. And really when you calculate different metacentric height based on different views these represent different types of angular motion like rolling, pitching these are different technical names based on with respect to what type of axis it is tilting.

So, but just for simple design you can consider the smaller one and you divide it by the volume, not the total volume but the volume of the submerged part. So, that you can easily calculate based on what part is submerged? Let us say that d is the submerged depth. It is easy to calculate this because you can use the equilibrium that for equilibrium the buoyancy force must be equal to the weight.

So, from that if you know the density of fluid and density of the solid body you can come up with what should be the equilibrium d by very simple equating of the 2 forces. So, you can calculate what is the volume which is immersed in the fluid, center of buoyancy location you can get from the centroid of the displaced volume. Center of gravity also has a fix position with respect to the body.

So, you can clearly substitute these dimensions and find out what is the metacentric height. So this effectively requires only the calculation of the immersed volume, the calculation of the second moment of area with respect to these axis and specification of the locations of the center of buoyancy and center of gravity with respect to the axis fixed on the body. So, that is a sufficient information to calculate the metacentric height.

So, what we can clearly see is that if M is above G that makes it stable. That mens lower the location of the center of gravity it is having a greater chance that it will be stable. Because then it's a greater chance that M is above G, lower the location of G. So, location of G if it goes higher and higher it might make a previously stable system or convert it into an unstable (()) (21:34). Let us look into an animated example to consider this case.

So, in this animated example what we will see. We will see that how the stability may be disturbed because of the shifting of the distribution of the weight of the body. So, just look into it carefully. So, there is a body which has given a slight displacement and you see that for a small displacement it oscillates like a pendulum. We may easily derive what is the time period of it. Now, you see that the distribution of the weight is being altered.

So the center of gravity is being shifted higher and higher by putting the load more and more towards the top and you see that it topples. So, obviously this is clear illustration of this concept that we have learnt in this example that if you have metacentre above the center of gravity it makes it more stable and otherwise it is not so. So, now for small oscillations or for small deflection you could see that it oscillates like a pendulum.

So, when it oscillates like a pendulum it has a time period and that may be calculated by calculating the moment of the resultant force with respect to the axis. So, if you have a tilted axis like this and if you have a force FB which is the buoyancy force. This force has a movement with respect to the axis of the body. So what is the movement of this force with respect to the axis of the body you can just find out the perpendicular distance of this force from the axis of the body.

What will be the perpendicular distance of this buoyancy force, line of action of the buoyancy force? I mean you can see that it eventually passes through the axis of the body, right. So, this force individually which is important it is the couple moment that you are having the so called the restoring couple. So, you have also the W and it is basically the couple moment of these forces that you need to consider. So, the perpendicular distance between these 2.

In terms of the metacentric height would you express this so if you have this as the metacentre M. So, we are interested about these distance so it is possible to express it in terms of the metacentric height. So, this angle being theta, what is this perpendicular distance? GM sin theta. So, the moment of this force is W GM sin theta and the resultant moment of all the forces is nothing but if it is like rotation of a rigid body with respect to a fixed axis.

It is a restoring moment so you should have a – sign associated with one. That means whatever is the chosen positive direction of theta, theta dot theta double dot this has a direction opposite to that bring it to its original configuration. So, you can write this as so I is what now? I is the mass moment of inertia. It is not the same I that we were talking about it is the real mass moment of inertia with respect to the axis of the body relative to which it is tilted.

So, this plus for small theta again this is approximately = theta that is = 0. So, it is just like equation of a spring mass system mx double dot + kx = 0. So, what is the natural frequency of oscillation of this system? Root over equivalent stiffness/ equivalent mass. So, W* GM/I that is the natural frequency of oscillation of the system. It is angular oscillation not a linear one so you can clearly see that greater the metacentric height, greater will be the frequency of oscillation.

So, if it is a ship it will be more uncomfortable to the passenger. So, these are 2 conflicting designs. See greater the metacentric height you expect it to be safe in terms of stability but it will have more oscillation within that stability regime. That means for a passenger it may be quite uncomfortable at the same time if it is used for a particular say critical purpose like warfare and so on.

So, there stability of ship more than the comfort and they are obviously it is the stability that should be the driving factor for designs. So, when a ship is designed you have 2 conflicting things. One is the comfort another is the stability and comfort factor comes from this high frequency of oscillation and the stability comes from the metacentric height or location of metacentre relative to the center of that.

Now, that we have seen the stability criteria and so on we will come to our final topic fluid statics which ironically is not fluid statics but fluid with rigid body motion. And as we discuss earlier that we are going to address this issue within the purview of fluid statics for a very simple reason that when you have fluid under rigid body motion it is still fluid element without any shear.

So, when it is without any shear it is just the normal force which is acting on the surface so the distribution of force in terms of pressure on the surface remains unaltered. No matter whether it is at rest or under rigid body motion when the fluid elements are deforming then only you have the shear. So, let us take a simple example for fluid under rigid body motion.

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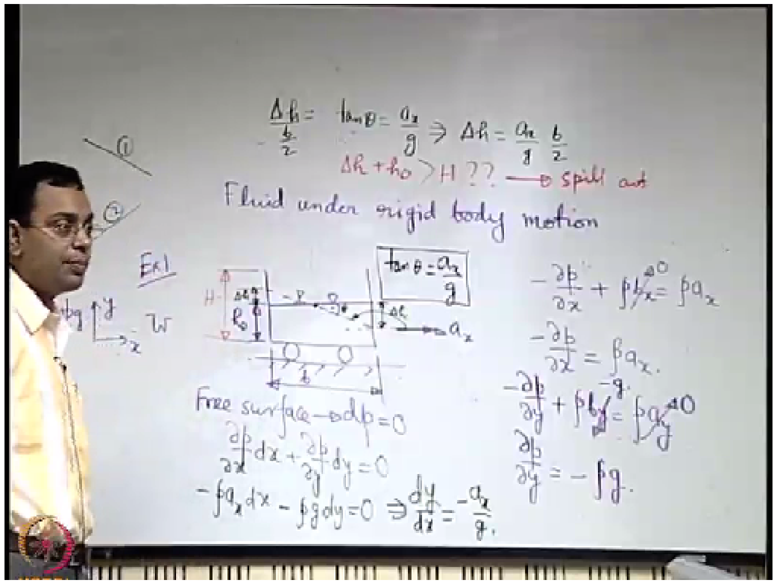

Let us say that we have a tank partly filled up with water and the tank is accelerating towards the right with an acceleration of ax along x, fixed acceleration and neglect the deformation of the water that is there in the tank. In reality that deformation is there so whatever analysis we are presenting here is not perfectly correct. Because in reality there will be shear and deformation and so on.

But just as an idealization just like many we discussed about frictionless surfaces not that they are there but through this idealization we learnt certain concepts. So, let us say that there is no deformation. So, the water which is there within the tank just get deflected in its configuration in terms of the free surface like a rigid body. So, how we calculate that what should be the new location of the free surface because of this acceleration that is what we want to see.

Say initially the height of water in the tank was h0, may be let us specify the width of the tank as b and may be the width perpendicular to the plain of the board as W. Let us say that this is rectangular plain. Now, recall that we derive the sudden expressions with regard to distribution of pressure in presence of body forces. So, far as I remember this is the expression that we derive some time back when we were actually starting with our discussion on fluid statics.

So, we will try to use this expression here. So, along x we have minus because there is no body force acting along x. Let us say that the y direction so we have x and y as our chosen directions.

Let us say that the y direction is the direction opposite to the acceleration due to gravity. So, for y direction what you can write? So, what is ay, ay is 0 it is accelerating along x. So, ay is 0. What is by-g?

So, you can get –rho g. Now when you have a free surface, the free surface is characterized by what? The free surface has the same pressure throughout because it is exposed to the atmosphere which has same pressure throughout. So, there is a pressure equilibrium between that and the atmosphere. So for the free surface you must have dp =0 that should be the governing parameter for locating the free

So, when you have dp =0 remember that now p is a function of both x and y. So, you can write dp as this one. So, you can substitute in place of the partial derivative with respect to x as –rho ax dx and this one is –rho gdy =0 that means what is dy/dx that is = -ax/g. What does this dydx represent? It represents the slop of the free surface. So, can you tell now weather the slop of this surface will be like this or like this. 1 or 2?

1 because you can clearly see that this will represent a kind of tilt like this so this will become the new free surface and the angle that you are considering for the slop is basically this one. Because this is negative ax is positive and g is positive so this is negative. So, it must be an (()) (33:58). So, in the direction in which the tank is accelerating the liquid will be more down and in the other direction it will be more up.

And if you specify this angle as theta then that theta is nothing but 180 degree – this slope angle. So, you can say that tan theta is nothing but = ax/g. Because theta is nothing but 180 degree minus this. So, you can find out that what is the extent to which the water level will rise on one side and may be fall on the other side. Let us say that this rise is delta h on one side because of symmetry it will be a fall of equivalent delta h on the other side.

So, it will be as if (()) (35:00) with respect to the center. So, we can calculate what is delta h. So, if you calculate delta h what will be that? It is nothing but delta h/p/2 is tan theta that is ax/ G. So, from here you can calculate what is delta h? ax/g. Now, let us come to a critical condition if we are happy with this sometimes we are deceived. How? Let us say that the total height of the tank is H if it so happens that let us say from calculation we get delta h + h0 > H practically that is not possible.

Because the liquid cannot occupy a height which is greater than that is provided by the tank. So, what this will mean? This will mean some water has spilled out. So, this is a condition from which you can say that it has actually spilled out. When it has spilled out it is more dis-configuration. So, when it has spilled out what will happen? What type of configuration you expect?

So before spilling out it tried it best to climb up to the top most level and then it spilled out. So, this will be one end of the surface and may be the other end is like this. So, with spill out may be this will spill out. Irrespective of whether it has spilled out or not you still have this expression applicable. So, now this will be the angle theta you can find out that what should be this length let us say d1, what should be this length d1.

Because you can say H/b1 =tan theta which is ax/g. H being the height of the tank know so from there you can find out what is b1 and Therefore you can calculate what is the volume which is there now within the tank. And the difference between the original volume and that volume will give you what is the volume that has got spilled. So, you see that it is not just like your solution should not be driven by a magic formula.

But based on the numerical data given you have to come to a decision whether the water is there inside or it has got spilled or so on. Let us take a variant of this example. So, the variant of the example is that now we have the tank located on an inclined plain.

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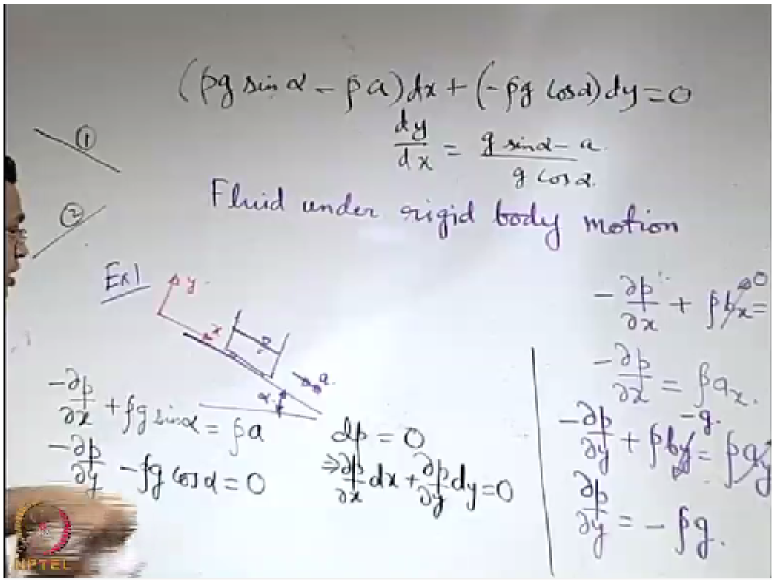

With an angle of inclination alpha tank is there on the plain and it is accelerating say downwards with an acceleration of a, which is a uniform acceleration may be because of a resultant force which act along that direction. So, in this case it may be more convenient it you fix up your coordinate axis relating to the inclined plain say x and y. So, the similar equation will be applied and let us just do it very quickly.

Because it is very straight forward. So, you will have minus partial derivative of p with respect to x + what will be px now? bx is g sin theta + rho g sin theta =rho here it is not theta we have given a name alpha. So rho g sin alpha = rho a, ax is a. What about y? So, -this so y will be –g cos theta so –rho g cos alpha =0. So, from here you can find out when you have dp=0 that means you can substitute the expression so it will be.

Let us write it at the top the expression now becomes in place of partial derivative with respect of x you can substitute rho g sin alpha – rho a dx + -rho g cos alpha dy. So, dy/dx =g sin alpha – a/ g cos alpha and this is now = tan theta where theta is the angle relative to the original location of the free surface or like assumed x direction. So, you can see now that there is depending on the magnitude of a this may be positive or negative.

So, you may have a case when sin alpha is > a or g sin alpha < a. So, here you cannot (()) (41:34) say that whether it is 1 or 2, case 1 or case 2. Again you see that it is not a magic formula that should drive your decision it depends on what is the physical situation that is prevailing. Let us consider a 3 example. **"Professor - student conversation starts"** Surface should always be normal on the resultant.

See here what we are doing here we are implicitly applying the same condition. You see that we are having one very important assumption. The assumption is that you do not have an oscillation in the surface. So sometime because of this displacement the surface oscillates it becomes like a wavy situation and that is known as sloshing of tanks. So, we are not going into that details so we are assuming that the surface remains flat.

And when the surface remains flat and under these conditions when you have dp=0 that is exactly the same condition. See vector analysis is not, it is also mathematics right when you say that when you are dealing with mathematical analysis I feel this is too mathematical I do not see any difference. Like if you have vector analysis this is also vector analysis just dealing with scalar component.

So, it is better to be habituated with these because again I am telling you there are situations when it would not be as straight forward as this. So, we have to use this fundamental equation. So, whenever you are solving a problem try to adhere to the fundamental situations. I am getting your point why you are telling this because you have been habituated in solving problems in that way through your entrance exam.

But we will be encountering more challenging problems then what you have solved earlier through that type of magic situations, we try to avoid that. So, our basic intention will be that we have this basic equation this should solely guide us for solving whatever complicated problems that we are having of these type. **"Professor – student conversation ends"**. Let us consider another one.

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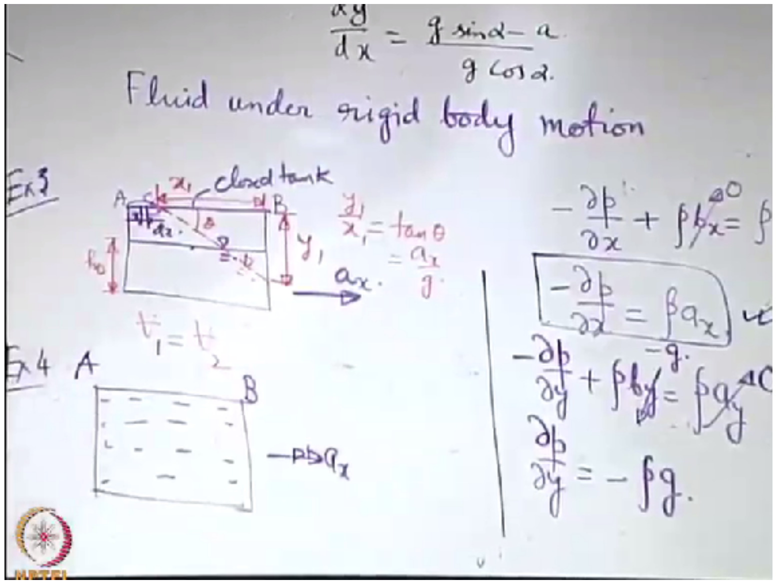

Say example. Now let us say our close tank, close tank partially filled with water this is closed. Again you are doing the same thing accelerating it along x. What will happen to the free surface? Now there cannot be any spilling. So, when there cannot be any spilling there is even a chance that the free surface is like this. So, the symmetry with respect to the central line which was there for the previous case without spilling is now broken.

That symmetry is not guaranteed because it may try to escape and since it is not finding an escape route it will break the symmetry and get distributed in what way? In such a way that the volume of the liquid now is conserved because it cannot go out of the tank. So, if you say that this dimension is y1 and this is x1 then you have y1/x1 as tan theta. And that is =ax/g and you can calculate y1 and x1.

By considering that the volume which was there originally is the same as the volume of water after it is tilting of the intervals. It is the new interface. So, this will give you another relationship involving x1 and y1 and you can solve for x1 and y1. Now, let us say that we are interested to find out what is the total force acting on the top surface or the (()) (46:14) container. How will you find it out? So, let us say this point is c.

So, you have to keep in mind that up to AC it is in contact with a fluid. And you can use this one. So, you can find out the pressure distribution as a function of x from a to A to C. Take a small element at a distance x from A of width dx so the force acting on that is b*dx* the third dimension integrate it from A to C and AC you can find out from these geometrical considerations. So, that will give you the total force.

Let us consider may be a 4 variant which I will not solve but just tell you that such a variant is also possible. Say you have tank now it is completely filled with water and it is closed. Which p will be p0 you assume any one of the point say A as a reference pressure say B, some reference B because always when you have pressure it is relative to some point and so you can express the pressure distribution in terms of the pressure at the point A and then integrate it.

So, next example is you are this tank accelerating towards the right. But completely filled with water but closed so water cannot escape. So, if I tell you find out the total force on the (()) (48:14) AB how will you do it? I will not do it myself leave it on you as an exercise. And I can only tell you that this is the simplest of all the cases that we have considered. But you have to keep in mind that same consideration as this also should work.

That will give you a natural pressure distribution from A. And similar to this you can just integrate from A to B to find out here you do not have the botheration of finding out what is the portion that is exposed with the liquid because if it is completely filled it should be completely exposed. Now when we consider the rigid body motion it is not always just translation it may also be rotation. So, let us consider a rigid body rotation example.

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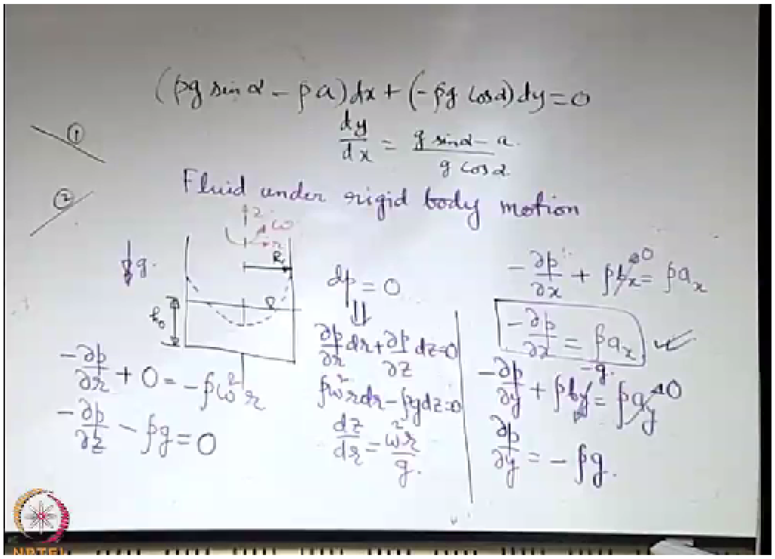

These types of examples you have seen earlier that you have a tank filled up with water to some height h0. It is a cylindrical tank of radius R. So, are using a RG coordinate system, axis symmetric coordinate system so this is r coordinate and along this there is z coordinate. So, this tank is rotated with respect to its axis within angular velocity momentum. So, when it is rotated you already know it that it will come to its free surface.

Will come to a deform or deflected shape like this which is a paraboloid of revolution. Let us quickly see that how we can derive that it should be a paraboloid of revolution from these fundamental principles not from any magic formula. So, let us see you start with here so there are 2 directions r and g. So, you have - dp/dr + what is the body force along r – omega square r. See these we are writing in the original form of the Newton second low of motion.

Now, with respect to an accelerating difference. So, no question of centrifugal. Now, you tell when it is no question of centrifugal whether the body force is there or not. Yes, or no. Say you are standing on a platform and you are looking into it from the inertial reference frame this is not an external force that is applied that you can see only when you have a rotating reference frame that is attached to the platform that is having angular promotion.

So, obviously there will be no br that you have to keep in mind. This is the form of the Newton second law in an inertial reference. With a non-inertial reference frame you have to use a pseudo force for the inertia force but then you do not write equation of dynamic but equivalent of equation of static equilibrium. You convert that in an equivalent static equation through D'Alembert's principles.

But here we are not taking about that we are talking about the proper acceleration. So, you have no body force but you have acceleration. What is the acceleration? It is nothing but the centripetal acceleration. So, -rho omega square r. See, if you had considered within a rotating reference frame the right hand side would be 0 because you are writing static equilibrium this would be represented by the pseudo force.

So, the final equation would eventually be the same. It is just a matter of the reference frame with respect to which you are writing. But this is read to with respect to the inertial reference. Then when you come to the z direction – dp/dz then let us say this is the acceleration due to gravity direction –rho g = there is no acceleration it is not vertically moving again like whenever it is vertically moving and so you can substitute.

So, start with this basic equation depending on whatever information is given in the problem you try to use it. So, for the free surface you have dp=0. So, when you have dp =0 you have this = 0. So, this is rho omega square r. This is – rho g so you have dg/dr =omega square r/g. You can now integrate this with respect to r. So, on integration what follows let us just complete that.

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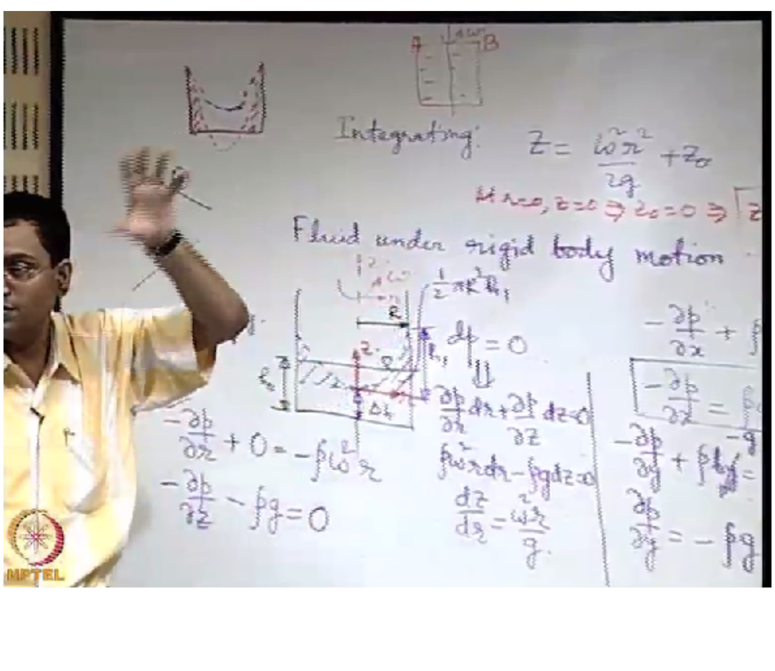

Integrating z = omega square r square/2g + constant of integration Z0. You can set of the constant of integration z0by choosing a reference such that when r=0, z=0 so if we choose this as our origin of the coordinate so this is r and this is z. So, if you have at r =0, z=0 then z0=0, z will be omega square r square. This formula you have encountered earlier. So, we can clearly see that it is an equation like a of a parabola.

So, it's a parabolic of evolution. It is a 3 dimensional situation. And you can calculate the other things just similar to what we did in the previous case with an understanding of what that again if it does not spill whatever was the volume that should be conserved. So, how can you calculate the volume? So, initial volume you can calculate, initial volume is pi R square *h0. What is the final volume?

Final volume is whatever is the depression say delta h + the volume of the shaded parabola (()) (55:15) and I leave it on you as an exercise if you calculate this shaded volume you will see that it will just be half of the volume of the circumscribing cylinder. That means if this height is h1, then the shaded volume will be half of pi r square h1. By simple integration you can find out this volume.

So, by equating the initial and the final volume you can find out totally the deflected configuration. Again you may have to check that h1 + delta h if it becomes greater than the height of the original tank then it will spill and spilling again may have 2 different cases. You may have in one case the cylinder is rotating in such a way that you have spilling but still it is having an interface shape like this.

Again it may so happen that this rotating so fast that it is spilling but only the part of the parabolic of revolution is within the cylinder. So then it is an imaginary parabolic revolution even outside that you have to consider. Find out what is the volume that is there inside. So, it all depends on the rotational speed the given dimension and so on. So, it is not just like a fixed formula but you know what is the basic principle.

We have discussed enough numbers of examples to see that what is the basic principle and that basic principle should guide you to find out that what is the case. Now, if it is totally closed cylinder as a final example say we have cylinder that is totally closed and filled up with liquid and rotated with respect to this axis. What is the total force on the top (()) (57:07) AB. Again the basic principle is the same.

You just use this dp/ dr formula to find out how pressure varies with r. Of course with the reference say r=0, p=p0 with the reference because pressure you always calculate with the reference. So, you know how p varies with r by integrating this with respect to r. When we integrate with respect to r, g is fixed. So, then you can find out the total force by taking an element. Here an element will be 2 pi r dr.

So, integrating over that you can find out the total force of the top surface. Total force on the bottom surface will be that plus the weight of the fluid which is there. So, we will close this discussion by seeing just one may be example where we will see that how this type of vertex motion is generated in practice. You see that this type of a vertex motion that is there I means it is not exactly a paraboloid of revolution. But it is by rotating the fluid in a container.

Why it is not exactly a paraboloid of revolution is because we have neglected here the viscous effects. The shear between various fluid layers we have assumed that the fluid rotates like a rigid body. In reality that is not the case and we will look into these situations more emphatically whenever we are discussing viscous motion. So, we close our discussion on fluid statics with this. Thank you very much.