Introduction to Fluid Machines and Compressible Flow
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Lecture - 07
Analysis of Force on the Bucket of Pelton Wheel and Power Generation

Good afternoon, I welcome you to this session, where we will discuss today the analysis of force on the bucket of pelton wheel and the power generation. So, let us make a recapitulation of this pelton wheel, the first part of the pelton wheel, which we discussed in the last class.

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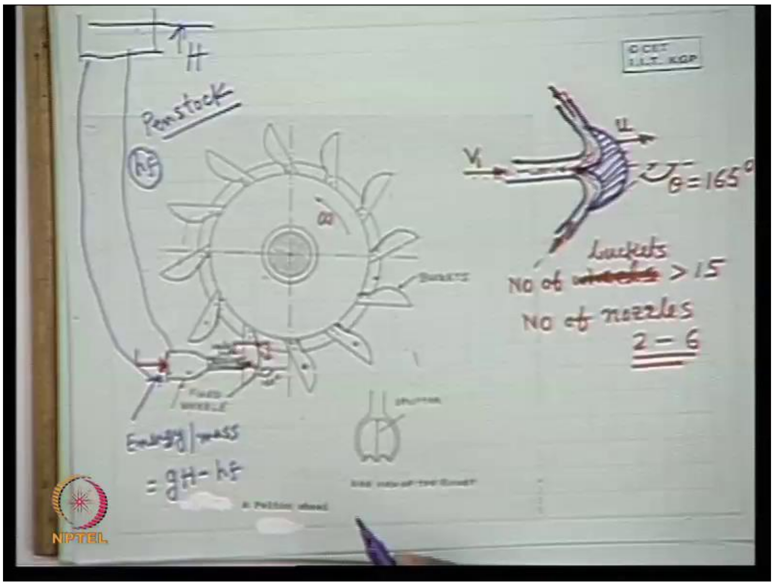

That pelton wheel is an impulse hydraulic turbine, where a number of spoon shaped buckets are mounted on a wheel of disks, which rotates. The rotating shaft is mounted on the center of the disk, and a number of fixed nozzles, which converts the pressure of the liquid that is entrance to a high velocity jet. Directs the jet at the center of the bucket tangentially thism we recognized last time how that if we see this sectional plan wheel the spoon shape bucket is like this. There are here two halves two symmetric halves and the jet strikes the bucket tangentially that means, that direction of the incoming jet velocity is at tangent is in the direction of the tangent to a circle drawn through the centers of this bucket. This circle is known as the pitch circle.

And the water jet glides along the blade flows along the blade or bucket and then comes out. We also recognized that these fluid should come relative to the bucket in a direction exactly opposite to the incoming velocity to have a maximum change in the momentum by the fluid, so that a maximum force can be exerted on the bucket. But it is not done in practice, in practice a deflection of theta is equal to one sixty five degree; that means, the direction of the relative velocity of the jet with respect to the bucket is made. This is because that the fluid, otherwise will strike the back of the wing following the one.

Now the vane is also moving with a velocity u. So, we also discussed that number of buckets are usually more than fifteen and number of fixed nozzles which are the stators at the fixed part of the machines varies from two to six. We have also recognized that there is a splitter read which divides the water into equal parts to this two halves of the bucket or vane or blade whatever you may say. Now we come to the force analysis, now for force analysis what we have to do, we have to make the velocity triangle; we will have to draw the velocity triangle diagram at the inlet and outlet. Let us do that now.

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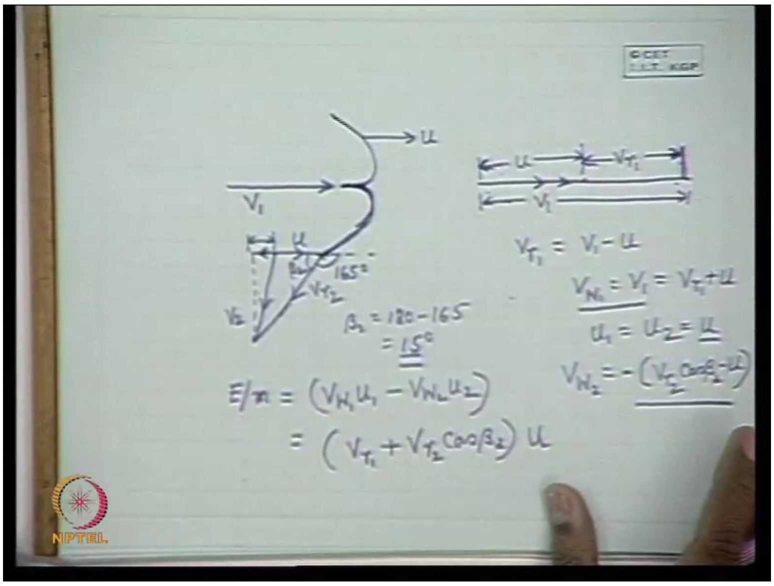

Now, let us draw the vane again in this fashion, this is the vane. Now this is the inlet velocity v 1, now this is the look here, this is the velocity u with, which it is moving. Now at the inlet here incoming water velocity and the bucket velocity is in the same line they are collinear. So, therefore, the inlet velocity diagram is like that if this is the v 1; that means, if this is v 1 - the inlet velocity of water. Let u is represented by this, this is u same direction, all are in the same direction. So, therefore, this will be the relative velocity at the inlet of the flow. So, therefore, the velocity triangle or the vector is a line; that means, the vector diagram is simply a line, this is the v r one, simply a straight line because velocities are collinear the same direction.

So, therefore, we can write the relative velocity v r 1 is equal to v 1 minus u. Here one thing you have to recognize that the tangential velocity of the bucket at inlet and outlet is same; that means, if you see this, the tangential velocity; this is the same plane, but the liquid enters and comes out. So, therefore, the radial location of the inlet and outlet from the axis of rotation remains the same.

Now, if we draw the outlet velocity triangle, the outlet velocity triangle will look like that this is the v r 2 that is the relative velocity with respect to this and this is v 2. We can make this component. So here you see the outlet velocity triangle shows that the relative velocity of the fluid with respect to the bucket is such that it coincides with the angle at outlet of the bucket. This is for the smooth flow of the water to the bucket. You know that for smooth entry and smooth discharge of the water, the velocity of the fluid with relative to the water should glide along the velocity of the fluid, it relative to the bucket should glide along the bucket or vane. That means, the outlet the angle of the relative velocity, that is the relative velocity with respect to the bucket should make the same angle with that of the bucket, that means, this is nothing, but this angle we recognized as one sixty five degree it is made. This angle we call it as beta 2.

So, therefore, beta two is usually 180 degree, 180 minus 165 that is 15 degree usually. It may be of differ in value; that means, the angle of blade at outlet is equal to the angle of the relative velocity at outlet. And inlet also the angle of blade, this is zero, so inlet velocity is having a zero angle with the tangential direction. That means, that both inlet and outlet the relative velocity should match the blade angle this I discussed earlier also. Now, we know the energy, we know that energy transferred per unit mass between the fluid and the rotor, if I write the energy giving by the fluid per unit mass to this bucket will be equal to v w 1 u 1 minus v w 2 u 2 from the general Euler's equation for fluid machines.

Nomenclatures are we will explain that v w 1 is the tangential component of the inlet velocity. What is the value of v w 1 here, here v w 1 is equal to v 1, because v 1 itself in the tangential direction. And u 1 is equal to u 2, here what is the value of v w 2, v w 2 can be found out from that means, it is in the opposite direction to that of the v 1 or v w 1 and this is this much from the velocity triangle. So, with a negative sign, because the velocity is in the opposite direction to the incoming velocity, it can be written as v r 2 cos beta 2, this much cos of beta two minus u.

So, u 1 is equal to u 2 is equal to u here. So, if I substitute these values of v w 1, v w 2 and u as the tangential velocity of the rotor that is u 1 u 2 then we get e by m is equal to… Again V 1 can be written in terms on u as v r 1 plus u, because from the inlet velocity vector diagram, we see that V 1 is equal to v w 1 that that is equal to v r 1 plus u. So, therefore, we get v r 1 plus v r 2 cos of beta 2 into u, clear v r 1 plus v r 2 cos of beta 2 into u.

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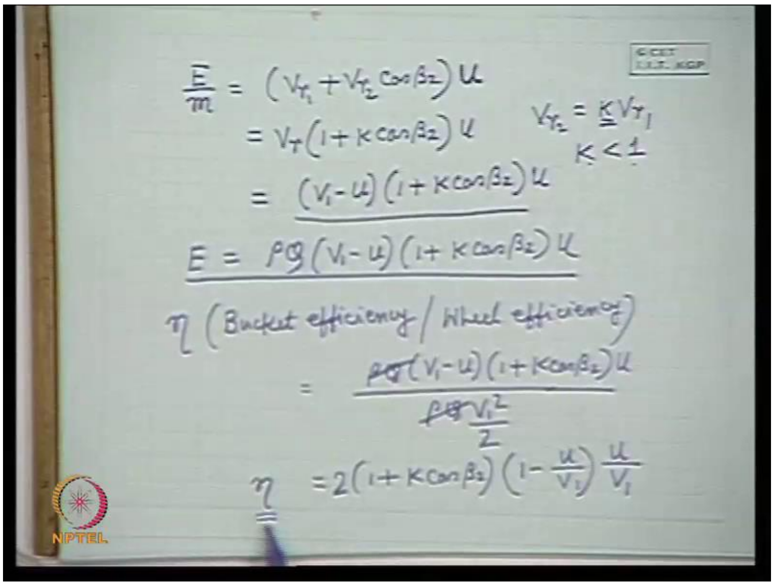

We can write it again that e by m per unit mass. Again I am writing v r 1 plus v r 2 cos of beta 2 into u, where v r 1 is the relative velocity of the fluid at inlet, which is simply the difference between the inlet velocity and the tangential velocity, because they are in the same line. V r 2 is the relative velocity of fluid at outlet, which we can find from the outlet velocity triangle, and beta two is the angle of the bucket at the outlet, and u is the tangential velocity of the bucket at its center, where the jet strikes.

Now, you see that in the bucket, if you see this figure why the relative velocity at the inlet and outlet changes. Why the relative velocity at the outlet changes from that at the inlet, why? You please recall our earlier discussion that the relative velocity at outlet will be differing from relative velocity of inlet for what reasons, what are the reasons for which the relative velocity of fluid at outlet will be different from that at the inlet. This is because of what?

Here the fluid is throughout at atmospheric pressure, fluid is open jet. So, there is no question of change in pressure. So, pressure remains constant throughout the flow through the bucket. So, why the relative velocity will change, this is only because of friction, only because of friction. You consider the relative velocity in this way then as if the blade is fixed and fluid flows positive, that means, fluid flowing positive is fixed blade or a fixed surface, then why the velocity will change only because of friction.

If the pressure remains constant, so that is only way that velocity can change is the friction. If the pressure remains constant that is an another important thing. So, therefore, v r two changes from that of v r 1, the value of v r 2 changes from that of v r one, because of friction, and the role of friction is to reduce the value of v r two from v r one. So, therefore, we can consider or we can write v r two in this fashion a some coefficient k times v r one where k is less than one this takes care of the friction; that means, v r two is reduced from v r one by a factor less than one. So, that we can express v r two in this fashion where k is the factor less than one and takes care of the friction in the bucket.

So, therefore, we can write v r one into one plus k cos of beta two into u well and again I just substitute v r one as v one minus u from the inlet velocity triangle you can recall cos beta two into u. So, this is the energy per unit mass that is being transferred or given by the fluid to the bucket I think there is no difficulty its extremely simple. Now the total energy given per unit time that is the rate of energy, that is being transferred by the fluid to the rotor will be this multiplied by the mass flow rate. Because this is the energy given per unit mass if we multiply this if we multiply this with the mass flow rate of the fluid then you will get the rate of energy transferred by the fluid to the rotor.

Now, what is the mass flow rate of the fluid it is density rho and the q is the volume flow rate. So, this is the mass into v one minus u into one plus k cos beta two into u. So, this is the expression for the rate of energy that is given by the fluid to the bucket now we define a efficiency eta known as bucket efficiency which is very important bucket efficiency or sometimes we define it as wheel efficiency wheel efficiency because buckets are mounted on wheels wheel efficiency how is it defined.

Now, the power that is being developed by the wheel due to the motion of the fluid is this much. So, this comes as the useful work or useful energy from the bucket cos beta two into u. What is the input to the bucket, what does bucket receive as the input energy you see here the bucket receives the input energy in the form of kinetic energy of the fluid; that means, it is the kinetic energy of the fluid total kinetic energy of the fluid; that means, mass times v one square by two. So, it is the rate at which the energy is received by the bucket at its inlet that is the rate of kinetic energy that is being received by the bucket at its inlet due to the incoming flow of water and the numerator is the power developed by the bucket per unit time.

So, therefore, we get an expression from this about this eta rho q rho q cancels out. So, you can write one plus k cos beta two into you write in a fashion it is twice well. So, this will be well u square minus v one square. So, u by v one u by v one into one minus u by v one of course, a two will be there well. So, this is the expression for the bucket efficiency or wheel efficiency now wheel efficiency efficiency is the dimensionless parameter. So, it is expressed in terms of the dimensionless parameter.

Now, what is that physical significance of this efficiency, physical significance of this efficiency is that how effectively the bucket can convert the incoming kinetic energy which is receives in the form of power developed. Now you can ask me sir why is the discrepancy is it due to friction, no, even if you make k is equal to one. You see this the value of eta is not always one; that means, it is not the friction it is not the friction, but it is it depends upon the deflection of the jet by the bucket. So, that the absolute kinetic energy at the outlet or the discharge should be minimum because the numerator is equal to the change in the kinetic energy of the jet; that means, some energy in the form of kinetic energy is always rejected at the discharge.

So, how effectively it can utilize its kinetic energy in the form of work. So, that the discharged kinetic energy which is loss is which is a loss is minimum, so that is the physical concept of the efficiency. And it becomes a function, function of partition dimensionless parameter one is the k that is the friction with a scare of the friction another is the bucket outlet angle and the ratio of u by v one; that means, the speed of the bucket and the speed of the incoming water jet or the inlet velocity of the water jet.

Now, for a fixed bucket bucket with a fixed geometry k and beta two are fixed. So, therefore, the efficiency of the bucket is a function of the ratio u by v one which becomes a partition dimensionless parameters representing the operating conditions; that means, it is a function of u by v one

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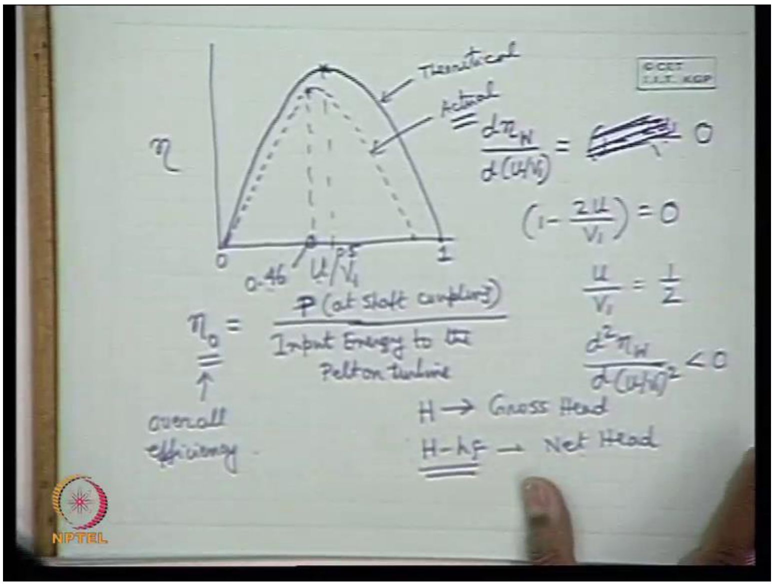

Now, if you plot these functions you will see you will get a curve like this. If you plot this function eta versus u by v one correct it is a parabola. You get an exactly the maximum efficiency is attained when this value is zero point five. And this become equal to zero, when u by v one is equal to one, it is very simple that if you make here sorry I can give a suffix or subscript w is value giving to denote that it is the efficiency of the wheel d eta w d u by v one. That means, if I differentiate this eta with respect to u by v one for a fixed blade fix geometry of the blade, we get that it is one minus two u by v one. I should not write that if I make this is equal to zero, it becomes one minus two u by v one is equal to zero, because the expression of this is not one minus two u by v one, it is multiplied by with a constant factor two into one plus k cos beta two if we make this is equal to zero.

So, it is obvious that u by v one is equal to half and we can check the maximum condition analytically by checking that or noticing this fact that d u by that second derivative is less than zero or simply by plotting the curve for the expression we will see this expression has a maximum. So, therefore, we see when u by v one reaches half the blade efficiency is maximum again when u by v one is equal to one blade efficiency is zero no power will be developed as you know that when the blade velocity is exactly equal to the incoming velocity of the water water cannot reach the blade. So, flow will be zero and more than that that blade will receive away from the jet before it strikes the blade. So, therefore, it again falls at one. So, this is the maximum value.

Now, in actual cases we define a overall efficiency eta o which is equal to the power that is being delivered at the shaft divided by the input power input energy rather input energy to the wheel input energy to the turbine pelton turbine. Before coming to this, I like to tell you that the actual power developed by the wheel depends upon friction not only the fluid to blade friction. There are other frictions in the bearing in the mechanical system and the windage losses that is the friction with the air and the parts of the fluid machines and these losses also increase with the increase in speeds.

So, therefore, it has been found practically that the maximum efficiency becomes lower than the theoretical efficiency calculated and it attains the maximum value always lower than that and also becomes zero. Even if the bucket velocity is less than v one and maximum is also attained at a value which is less than point five the value is point four six. So, this is the theoretical curve theoretical curve and this is the actual (( )); that means, in actual case considering the frictional losses we see that the maximum efficiency occurs at a value of u by v one that is the ratio of the bucket speed to the incoming water speed equal to point four six.

Now, we come to the definition of eta o that is overall efficiency it is overall efficiency how you define now overall efficiency overall efficiency of a turbine as you know that is the p at shaft coupling; that means, this is the shaft power final power. So, this is less than this quantity; obviously, because this is the power developed by the rotor. So, this gets reduced by the mechanical losses that is losses in the mechanical system. So, that at finally, at the shafting the shaft coupling we get this power which is the numerator of the overall efficiency expression.

Now, input energy to pelton turbine. Now you tell me which should what should be the input energy to a pelton turbine what should be the input energy. You see this figure, what is the input energy to the turbine. If we consider the nozzles at the part of the turbine or the fluid machines which are the fixed parts that are known as stator and this is this wheel with the buckets at the moving parts known as rotor and both the nozzles and the moving wheels with the buckets form the fluid machines. So, what is the input energy to this machine input energy is what where is the input to the machine at entrance to the nozzle which means the energy at entrance to the nozzle.

So, what should be the energy at entrance to the nozzle energy here this is the pressure energy of the fluid. So, this we can calculate like this, if we consider that if what happens in practice that the liquid from a very high head, it is stored in a high head the value of which is h from a fixed reference datum then it comes to the nozzle. And if we consider that the friction loss is h f while flowing through this this line this pipeline which is known as penstock in our terminology of fluid machine penstock if h f is the loss of energy per unit mass. So, therefore, energy for unit mass which comes at the inlet to the nozzle; that means, this is the input point at inlet to the machine is equal to h minus g h minus h f per in a energy per unit mass energy per unit mass g h minus h f.

Or you can consider in terms of h that is h or rather you consider h f is the loss of it energy per unit to it. So, this will be g into h minus h f and h minus h f is the energy per unit weight; that means, the head; that means, this is the head at inlet to the machine and this is the head available at the reservoir at a height. So, these are known as gross and net head. So, the head this h which is the energy of the fluid at the reservoir at a great height is known as the gross head, which is the gross energy per unit weight. And this energy is purely in the form of potential energy these are the terminology these are extremely simple. But I tell it with a categorically because these are the terminology used and this h minus h f; that means, the energy per unit weight at entrance to the fluid machines; that means, that inlet to the nozzle is known as the net head.

So, net head; that means, energy per unit weight available at the inlet to the nozzle which is in the form of both pressure and velocity energy because the flow velocity is there through the pipeline leading from the reservoir to the nozzle of the machine. So, therefore, this is the main energy main energy per unit weight coming to that. So, therefore, the input energy will be this one. So, therefore, if we write the overall energy eta o

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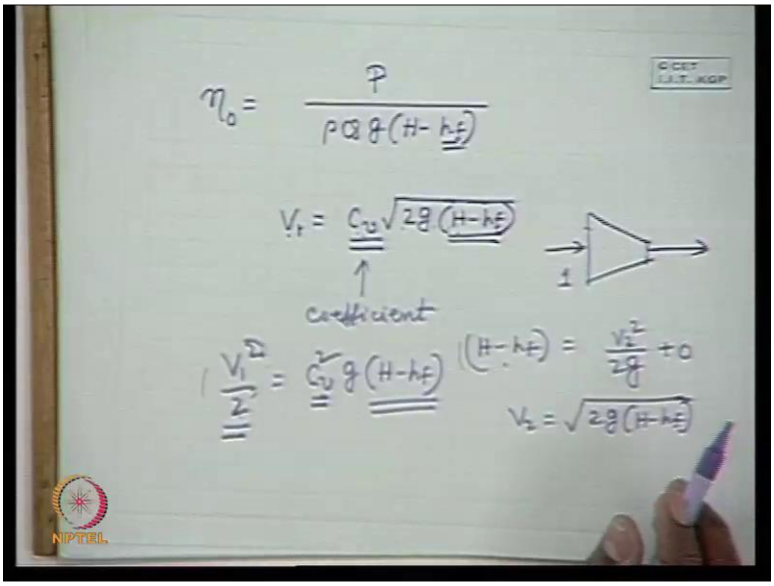

So, eta o will be the power total power at the shaft coupling and the input energy; that means, the rho q g into h minus h f where h f is the friction loss in the pipeline leading from the reservoir to the nozzle inlet; that means, the inlet to the fluid machines.

Now, nozzle is a part of the fluid machine, which converts the energy into kinetic energy. So, therefore, if I want to find out the kinetic energy at the inlet to the bucket at the inlet to the bucket if I want to find out the if I have to find out the kinetic energy at inlet to the bucket. Then this kinetic energy can be found out by application of the Bernoulli's equation between the nozzle inlet and outlet. That means, if I apply the Bernoulli's equation between the nozzle inlet and outlet simply per unit mass the total energy here or per unit weight total head here is h minus h f and that becomes is equal to v square by two g.

If I write the Bernoulli's equation for an implicit fluid this total head is the head above the atmospheric pressure. So, therefore, here the pressure is zero and we take the datum as the to the central line of the buckets; that means, the plane at where the jet is striking the buckets. So, therefore, it is very simple that v two becomes equal to two g; that means, the net energy at the inlet is exploited fully in terms of the velocity, but it is multiplied with a term known as coefficient of velocity which is same as that term k.

We discussed regarding the flow through flow of the liquid through the bucket which took care of the friction between the liquid and the bucket surface. Here also c v takes care of the friction in the nozzle, which reduces the velocity from that of two g h minus h f which has been deduced by considering the fluid to be inviscid, and by the application of Bernoulli's equation.

So, therefore, you will have to know very carefully that v one is c v two g h minus h f. So, therefore, v one square by two is equal to c v into c v square rather into g into h minus h f. So, you see there is the relationship between the kinetic energy at the inlet to the bucket and the energy at the entrance to the nozzle. So, this is they are not equal because of this c v quantity c v square because of the friction. So, these are the stages. So, initially, we get the energy at the entrance to the nozzle or entrance to the machine in the form of h minus h f for unit weight. This is being converted to kinetic energy v one square by two; that means, kinetic energy per unit weight; that means, kinetic head or the dynamic head through the expansion of the flow or the to the flow of the liquid through the nozzle. And we get this head which is less than the head at the entrance by the factor c v square which takes care of the friction in the nozzle.

And the shaft power is the power which is developed at the shaft coupling. So, you see this is the power that is developed by the fluid to the rotor the rotor extract this power because of the deflection of the fluid flow through it and this power is getting reduced to p because of mechanical friction. So, these are the stages how the energy is transferred from the inlet to the fluid machines inlet to the pelton wheel down to the shaft coupling final power. So, I think today I will finish here stop here. So, next class I will discuss the specific speed.

Thank you.