Introduction to Fluid Machines and Compressible Flow
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Lecture - 05
Principles of Similarity in Fluid Machines

Good morning, I welcome into this session. Today we will discuss about the principles of physical similarity as applied to fluid machines. At the outset, I will discuss in a brief the principle of similarity in general applied to any fluid flow problem. As you know the solutions to engineering problems a determining mostly from experiments; and due to certain reasons for example, economic reasons, due to economic conditions, saving of time and is of investigations, it is not possible in a number of instances to perform the laboratory experiments under the identical conditions of the operating variables that exists in practice.

So, what happens is that we have to perform the experiments in out laboratory under altered state of conditions from the actual problems existing practice. This say conditions or the operating variables refer in case of fluid flow problems are geometrical dimensions, pressures, flow velocity like that. You know that geometrical dimensions for example, we may not perform the experiments in laboratory of the full scale system as existing in practice, because of the availability in floor space. Sometimes we may not cover the range of pressure or flow velocities as happened in practice, because of the restrictions in the laboratory experiments. Similar may be the cases for using the particular fluid in actual using the particular fluid, which is actually used in practice.

So, therefore, we see that the laboratory tests are always performed under altered conditions of the operating variable that exists in practice. Now the two partition question, now come or arise out of this situations. What are those questions? Number one is that how can we apply the results or the test results from laboratory experiments to the actual problems at a different set of conditions. Number two is that if the performance of a system is governing by a number of independent operating parameters then a huge experiments, number of experiments are required to find out the influence of each and every independent of operating parameters on the performance of the system.

Now, is it possible by any way to reduce this number of experiments to a lesser one? For example, we can vary one or two independent of operating parameters to predict the influence of all the operating parameters on the performance of the system to save huge time energy and money. So, a positive clue in answering these two questions lies in the principles of physical similarity. So, it is the principles of physical similarity, which makes it possible and justifiable to apply the test results from laboratory under altered state of conditions to the actual problem in practice at a different set of conditions. And also to perform a lesser number of experiments with the variation of a lesser number of independent operating parameters to predict the influence of a large number of independent operating parameters on the performance of a system.

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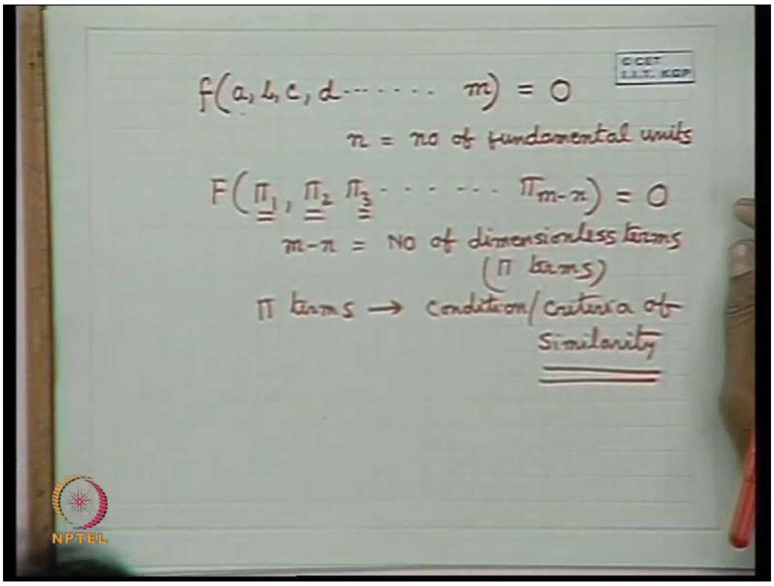

So, how it is done, let us see that if you process, for example, governed by a number of variables… Let we express a process as a functional relationship of all the variables. Let a process is denoted or a process is expressed by m variables, m physical variables describe a process. Therefore, the process can be expressed as a functional as an implicit functional relationship like that. If this variables are expressed by n fundamental units; n is the number of fundamental units. Then we know that by use of dimension analysis, it can be proved that the functional relationship of all the dimensional variables of the problem can be expressed by m minus n number of dimensionless terms known as pi times. This you know m minus n; that means, we get m minus n is equal to the number of dimensionless terms known as pi terms.

So, you see the number of independent variables where reduced from m to m minus n where the variables are dimensionless variables and known as pi terms. So, these variables are a combination some specific combinations of the dimensional variable; in a sense that each and every dimensionless variables which describes the problem. Now, can be made like this which describes the problem is now dimensionless. So, therefore, we see that this dimensionless numbers as independent variables are much less as compared to the number of dimensional variables. And this pi terms each pi terms, all the pi terms rather represents the conditions similarity represents the condition or criteria of similarity.

What is meant by similarity? Now you see what is meant by similarity, as I told earlier that similarity is that clue which gives a positive answer to the question that how can you apply the test results under altered set of conditions to the actual problem in practice. This can be done, if for a particular problem; that means, when the physics of the problem is fixed; that means, for a particular problem, if we maintain the similarity conditions. How it can be maintain that if a particular class of problem its started under a conditions where the entire physical similarity is maintained, then we tell that the physical similarity between the problems are maintained.

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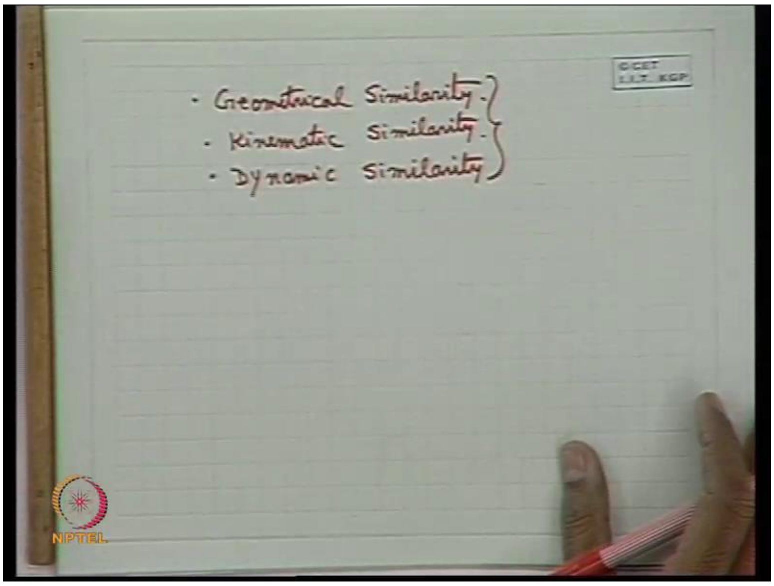

What are those similarity, let us see. These are geometrical similarity, kinematic similarity, and dynamic similarity. Now if we identify a particular class of problem then the geometrical similarity between two problems of the same class is attained, when the ratio of one length of a system to the corresponding length of the other systems be a fixed ratio. Similarly, the kinematic similarity is attained, when the motion or the velocity of a particle or a point of a one system corresponding to the velocity of the same point of the other system or the corresponding point to the other system, we have fix the ratio. Similarly, dynamic similarity is the similarity of the force when the force in one system the ratio of force in one system to the corresponding force or force to the corresponding point of the other system we are say fixed ratio.

That means, this means the geometrical similarity is similarity of shape; that means, they are proportionality against shape. Kinematics similarity is the similarity of the motion; that means, when the motions of the corresponding points between the two systems are same. Similarly, the ratio of the forces at the corresponding points between the two systems is same, the similarity is known as dynamic similarity.

Now, the question is that how do you know that this similarity is between the two systems of the same type of problem, when this dimensionless term remain the same. Now you see that the operating variables may vary, but the dimensionless terms for the two systems remains same. Then we ensure that the similarity is obtained and then the laboratory tests can be used to predict the actual performance and we can reduce the number of experiments to predict the influence of a large number of parameters.

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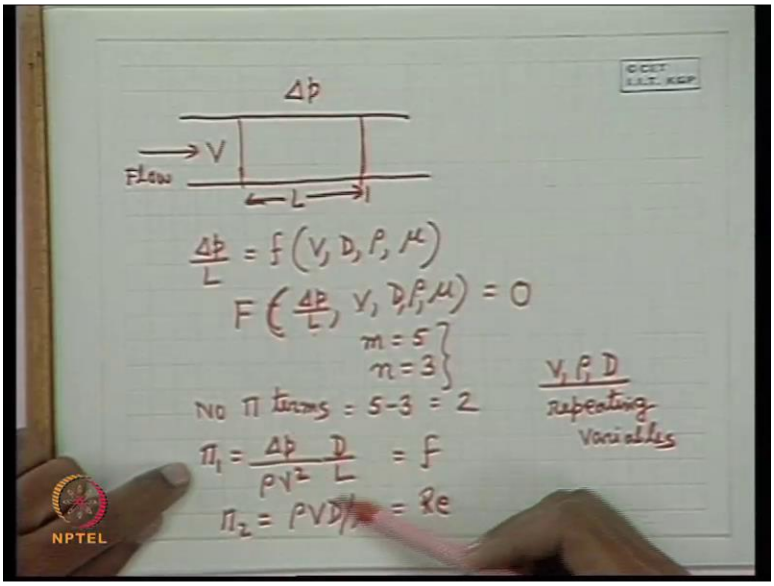

Now, this can be well explained, if we consider a pipe flow problem. Let us consider a pipe flow problem. The flow takes place a pipe flow problem. The flow takes place now we know that delta p the pressure difference over a length l, let the pressure difference over a length l delta p by l in this type of problem is a function of the flow velocity v, let the v is the flow velocity. Well is the function of the diameter of the pipe, function of the density of the liquid viscosity of the liquid. So, therefore, we see that the pressure drop per unit length in case of a pipe flow problem, when the flow is govern by the pressure force and the viscous force depends on the flow velocity, diameter of the pipe, density of the pipe and the viscosity of the pipe.

This is a very simple problem as we have already started earlier. So, that we can write that the problem can be described by like this; that means, a problem of pipe flow can be described by five dimensional variables that is pressure drop per unit length, the flow velocity, the diameter of the pipe, the density of the liquid, and the viscosity of liquid. So, now if we apply the dimensional analysis to find out the pi terms at the criteria of similarity, so first we see that one two three four five; that means, the number of variables m is equal to five. Now these variables can be expressed by three fundamental dimensions; that means, the number of fundamental dimensions is equal to there.

So, therefore, the number of pi terms is equal to five minus three is equal to two. What are those pi terms, if we find out by any method of dimensional analysis that as you know there are two methods; one is the Buckingham pi theorem, another is the Rayleigh initial method. So, if we take v rho and d as repeating variables; pi one comes like this, delta p by rho v square into d by l, and pi two comes as rho v d by mu. As you know this pi one is defined as the friction factor, and pi two is defined as Reynolds number, but what you observe is that this number is a dimensionless number, this number is also a dimensionless number.

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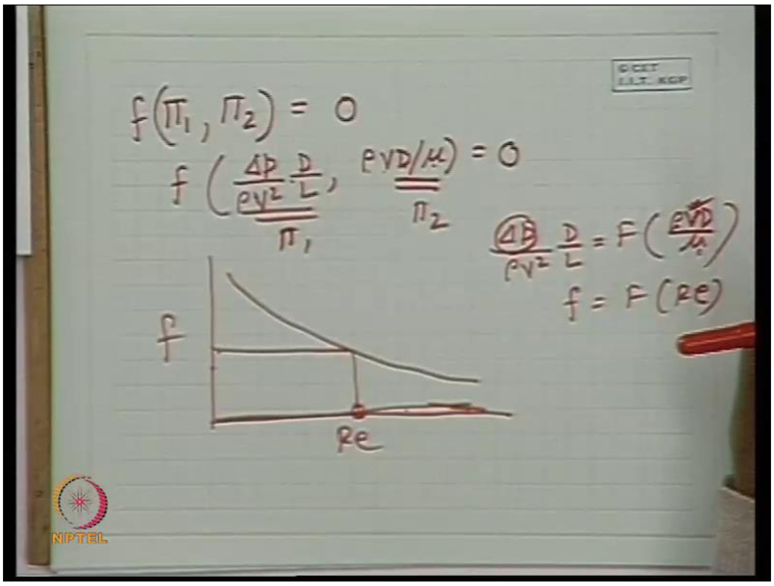

So, therefore, the pipe flow problem, you see is now express in terms of two dimensionless number like this function of delta p by rho v square into d by l and rho v d by mu. So, instead of five variables now these two variables pi one and pi two describe the problem. So, the one thing is very clear now. If we make the experiments of pipe flow problem in laboratory or if you consider the two systems of pipe flow problem, the pressure drop, the velocity, the diameter, the length, the density, and viscosity of the liquid may vary. To maintain the similarity, what we have to do, the pi terms; that means, pi one and pi two terms there is typical combinations of these two, the typical combinations of the variables in this fashion have to be maintain same.

Let me revisit the variations are there. In the two sets of experiments, in respect of the dependent variant independent dimensional variables, but the ranges of the non-dimensional term should be made fixed to maintain the similarity. That means, if you perform the experiments in pipe flow problem in laboratory, we will have to choose our density of the fluid viscosity, of the fluid velocity, of flow diameter of the pipe. In such a way that the typical combinations denoting the pi terms must be same or within the same range as it happens in actual practice.

And another very important theory is that we can immediately show that the relationship can be expressed as for example, this functional relationship can be expressed as d by l that is it is a function of rho v d by mu; that means, friction factor is a function of

Reynolds number. That means, we can express by single curve for a particular problem that is laminar flow for example, or both laminar and turbulent flow combined with the flow in a smooth pipe. So, relationship between friction factor and Reynolds number now this relationships shows that the variation of friction factor with Reynolds number. Now the pressure drop in a pipe flow problem may vary with different input parameters like velocity of flow diameter of pipe density of the liquid viscosity of the liquid.

But we may vary any one of them into show the influence of others. That means, for example, in laboratory, if we vary the velocity v, we can change the Reynolds number and we can find out the corresponding friction factor. For example, if this be the Reynolds number this is the friction factor and we can choose the velocity of flow to be to vary to represent the variation of rho d mu, because in a laboratory it is very difficult to vary the diameter, we have to go for different pipes. If you want to vary the density to show its influence then we have to take different liquids.

Similar is the case for viscosity, but if we simply vary the velocity of flow which is done very easily by controlling a valve be a particular pipe using a fixed liquid then we can vary the Reynolds number. And we can show its influence on f and through that we can tell also the influence of rho and d. For example, if velocity is doubled means that rho may be doubled, d may be doubled and mu may be half; that means, change in a Reynolds number may be brought by a change in any of the parameters. So, with a change in one parameter we can show the influence of other parameter.

Now it is clear how this similarity between the two systems of the same class of problem can be made, what are the criteria of the similarity, these are the pi terms, they are found by an dimensional analysis. And then we can predict the influence of all the operating dimensional variables on the performance of the system by performing a lesser number of experiments by varying a lesser number of independent operating parameter. Now, the same physical principle now if we apply to a fluid machines, let us find out if we apply to a fluid machines.

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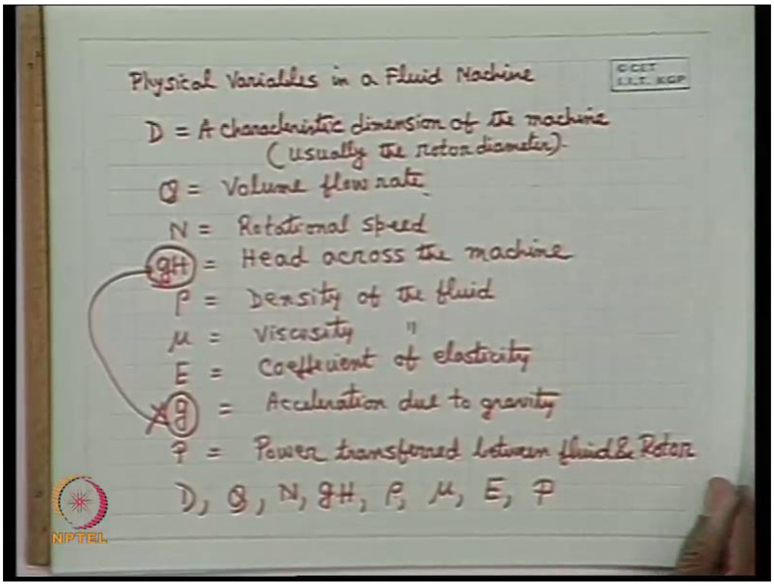

Now, therefore, we will have to recognize first the variables, so physical variables physical variables describing physical variables in fluid machines; that means, describing the problem of fluid machine. So, we have to first find out a physical variables in a fluid machines what are those this is d let us first d which is equal to a characteristic dimension of the machine, a characteristic dimension of fluid machine of the machine a characteristic dimension of the machine, which is usually the rotor diameter, well the characteristic dimension.

Then comes q volume flow rate through the machine, volume flow rate, n - rotational speed, then h the head across the machine, then density the fluid property let density of the fluid. These are the real logical properties of the fluid then mu viscosity of the fluid then e the coefficients of elasticity. Then g comes always - acceleration due to gravity, and then p power transferred, power transferred between fluid and rotator. Now we see that these are the physical variables in general in a fluid machine handling compressible fluid.

Let us first talk about compressible fluid d, a characteristic dimension of the machine which is usually the rotor diameter, q is the volume flow rate through the machine, n is rotational speed, h is the head across the machine; that means, this is the head in case of a turbine. This is the head given up by the fluid or in case of a pump or compressor, this is the head developed by the fluid; that means, the head across the machine. Then these are the liquid property or fluid properties that density, the viscosity and coefficient of elasticity which comes when the compressibility of fluid comes into consideration which is not beyond the scope of this class.

Of course, we are a discussing the fluid machines handling in compressible fluid, but general coefficient of elasticity comes when the compressibility is taken care of the fluid, handles compressible fluid machine. Handles the compressible fluid, g is the acceleration due to gravity and p is the power transferred between fluid and rotor; that means, the difference between p and h is taken care of by the hydraulic efficiency. This is the power transferred between fluid and rotor, and this is the head across the machine you must understand the difference between these two. That means, in case of turbine, it is the head that is being giving that is being giving by the fluid to the rotor and this is the power, which is being obtained by the rotor, so difference is there in terms of hydraulic efficiency.

Similarly, in case of pump, this is the power which is being received by the rotor, and h is the head developed by the fluid machines. Now in almost all cases in fluid machines the free surface does not exist. So, the variation of the influence of g is neglected because influence of g is in any problem comes when there is a free surface. So, when there is no free surface, the problems without a free surface, the variation of g, we do not take care. And this g is rather coupled here with g h is instead of g, we take g h as the variable rather than g. So, therefore, we arrive at this number of variables, this variables d, q, n, g h; that means, energy per unit mass at the machine is not the head g h then rho then mu then e then p. Then how many variables we do have, one, two, three, four, five, six, seven, eight, so therefore, we see that eight variables describe the problems in fluid machines.

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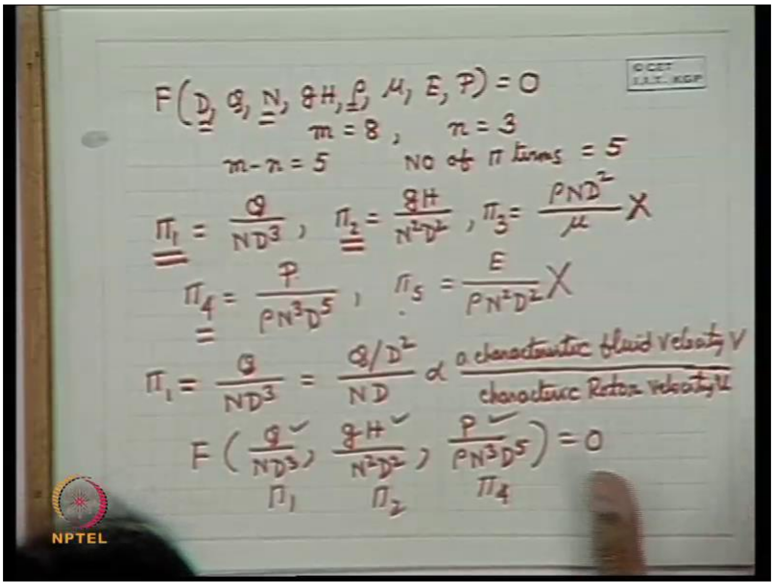

That means we can write this the functions of these eight variables; that means, D, Q, N, G, H, rho, mu, E, P is zero. Now to stick for the pi terms as the criteria of similarity what we have to do, we have to apply the dimension analysis. We have the number of variables is eight, and number of fundamental dimensions to express this variables are three mass, length and time. So, therefore, the number of pi terms m minus n is equal to five; that means, number of pi terms is equal to five. Well if we take d n and rho as the limiting variables then we get the pi terms like that pi 1 as Q by N D cube, pi 2 g h by N square D square, pi 2 rho N D square by mu. By the typical analysis, Buckingham pi theory, we get those terms pi four is equal to p by rho N cube D five and pi five as e by rho N square D square.

So, we get this five distinct pi terms q by n d cube pi two is g h by n square d square pi three is rho n d square by mu pi four is p by rho n cube d five and pi five is e by rho n square d square. Now let us see let us try to recognize the physical interpretations or significance of these pi terms. What are the physical significances of this pi terms. Let us first consider the first pi term, because all this pi terms represent now the principle of similarity or the similarity criteria; that means, if we make a test on fluid machines under altered state of conditions, we will have to make these pi terms same with the actual cases.

So, let us see physical significances of the pi terms. Let us consider the first pi term pi one, it is Q by N D cube, which can be written in this fashion Q by D square divided by N D. Now this Q by D square can be written as this that Q is the volumetric flow rate and D square is the area. So, the Q by D square represents a characteristic fluid velocity. So, we can write it is a characteristic fluid velocity. And N D is the characteristic rotor velocity. So, therefore, the pi term represents, let u the ratio is proportional to the ratio of the characteristic fluid velocity to rotor velocity.

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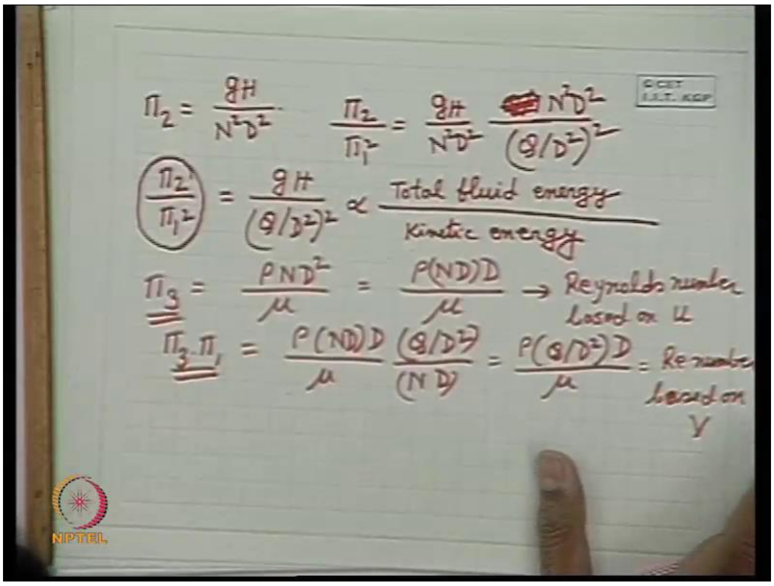

So, what is then pi two terms pi two is equal to g H by N square D square, what is this pi two term g H by N square D square. Now if we see, if numerator is the energy, total energy of the fluid either gained or giving; denominator represents the square of the rotor velocity to get a more clear idea, if we multiply or divided by pi two by pi one square what we get it is g H by N square D square. And what is that pi one square is pi one is Q by D square by N D; that means, it is divided by; that means, N square D square divided by D by D square whole square. That means, we can tell that pi two by pi one square, this is equal to g H divided by Q by D square whole square, which is proportional to total fluid energy divided by kinetic energy.

Now here you see that this pi two terms alone represents the total fluid energy divided by some energy represented in the rotor velocity. So, there we have to make a manipulations with the pi one terms. So, any combinations of the pi terms also represents the another pi term that is a corollary of the pi theorem as a similarity parameters. So, pi two by p one square is also a similarity parameter, which represents that total fluid energy to the kinetic energy of the fluid.

Now we come to the third pi term third pi term represents rho n d square by mu what is the physical significance very simple it is rho n d we take separately d by mu. So, you see that n d is the characteristic rotor velocity. So, this is a short of Reynolds number based on rotor velocity based on u - the rotor velocity. We can make the Reynolds number based on fluid velocity, if you multiply with pi one; that means, rho N D D divided by mu if you just multiply with pi one, what is pi one Q by D square by N D; that means, q by n d k. So, in that case, it becomes rho q by d square d by v; that means, it becomes the Reynolds number based on fluid velocity; that means, pi three is the representation of the Reynolds number based on rotor velocity or you make a combination with pi one like this, we get the Reynolds number based on fluid velocity.

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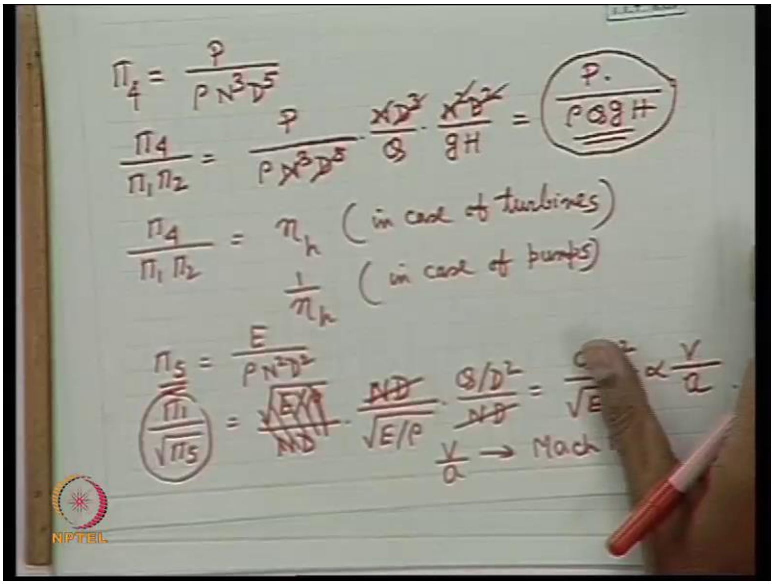

Then we come to pi four, what is the physical significance of pi four, this combination p by rho N cube D five; p is very straightforward thing there is a power transferred between the fluid and the rotor. But what is rho N cube D five as such it does not come out to be a physical concept rho N cube D five, but what we can make. If we make this combination pi four divided by pi one pi two then what we get P by rho N cube D five what is pi one cube by N cube D cube. So, N D cube by Q and what is pi two g H by N square D square; that means, N square D square by g H So, therefore, we see this n and d almost cancels. So, we get P by rho Q g H. So, therefore, now you see that f we make a manipulations with pi one pi two then we get this is equal to one P by rho Q g H; that means, power transmission between fluid and the rotor divided by the total head available by the fluid or gain by the fluid.

That means, in case of turbine it is nothing, but eta h, in case of turbines. Obviously, because P is the power available by the rotor or transferred to the rotor and this is head available by the fluid; that means, head given up by the fluid. Or 1 by eta h in case of pumps, because in case of pumps, it reads that p is the input power to the rotor, and rho q g h is the head developed by the fluid. So, therefore, pi four by pi one pi two represents the hydraulic efficiency or one by hydraulic efficiency depending upon whether it is a turbine or a pump.

Now we come to the last one - pi five. What is pi five as we have sit for obtained is E by rho N square D square. Now, if we make this pi one by root over pi five; that means, we can write this way E by rho square root with little adjustment and then N D. And what is pi one by root pi that means it will be n d denominator will be root over E by rho and what is pi Q by D square and N D, that means, n d as a whole cancels this is equal to q by d square by root over E by rho. Now Q by D square is proportional to the characteristic fluid velocity and what is root over E by rho, can you tell what is root over e by rho e by rho is the velocity of sound in that media. That means, velocity of sound in that fluid media which we can write as a - acoustic velocity in the fluid media. And this ratio V by a is known as mach number. So, therefore, we see the implication of pi five is the ratio of fluid velocity to the sound velocity which can be directly obtained if we make a manipulation of pi one by root over pi five.

Now let us come back again to the basic pi term. So, therefore, now we see the physical implications of each and every pi terms as the similarity parameter; that means, if this terms or the combinations of the basic variables in this fashions are maintained same into two kinds of investigation the entire physical similarity is obtained. And we have found the physical significances of these parameters.

I a fluid machine handling in compressible fluid, we can get rid of this parameters, because there the compressibility is not coming into picture. And moreover, it has been found out that the influence of the viscosity liquid viscosity in a fluid machine is very much negligible as compare to the influences of other parameter. So, therefore, the variation of this third pi term pi three is also discarded. So, therefore, we only take or only we are let only with the pi terms pi one, pi two, pi four, pi five; pi one, pi two, pi four only this three pi terms.

So, therefore, the problem of fluid machine handling in compressible fluids are defined in terms of the dimensionless pi terms as this three pi terms only pi one, pi two, and this is pi one, this is pi two ,and this I give the nomenclature pi four as it came after this pi three. So, pi one, pi two, pi four. So, these three dimensionless pi terms or dimensionless terms of the pi terms defined the problem of fluid machines handling incompressible fluid; that means, handling liquid.

So, therefore, we see that if we express the functional relationship in terms of this three parameters the similarity in the fluid machines are maintained. That means, a particular fluid machines of a particular geometrical shape will behave in a similar conditions provided this pi one, pi two, pi four are maintained same in all the machines even if the variables give Q, N, D, P, H varying. This is the principle of similarity, I will discuss it further in the next class.

Thank you.