Introduction to Fluid Machines and Compressible Flow

Prof. S.K. Som
Department of Mechanical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 40

Introduction to Expansion Wave and Prandtl Meyer Flow

Ok, good morning to all of you and welcome to this session of course. Now, last class we were discussing, that what will happen if the fluid flows through a convex corner or convex ((Refer Time: 00:31)) surface. Let us see that what happens.

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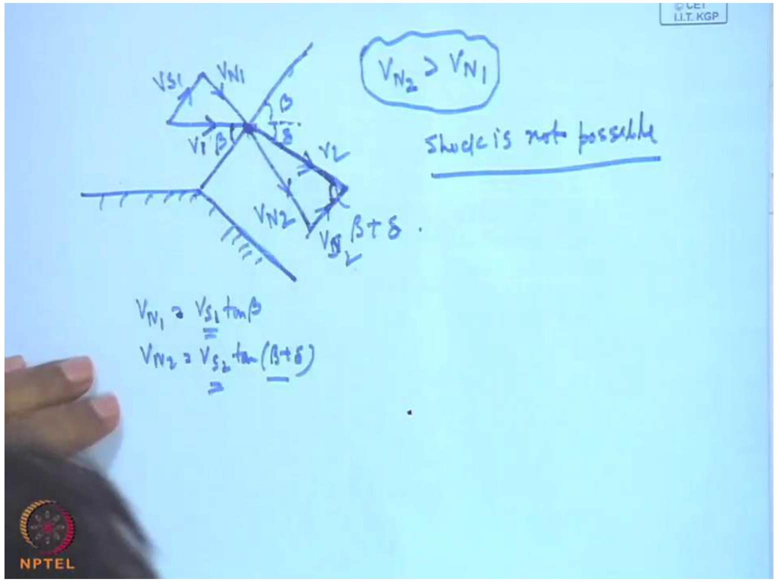

Now, if we considered similarly a type of oblique shock and you just draw the diagram, that V 1 is there, now it will be deflected. As for the geometry, the surface is a convex surface, this will be V 2, this line is parallel to this and these are the normal and the tangential velocities, normal and the tangential velocities.

Now, according to our earlier considerations, along the shock, the property does not change, no need force is acting along this direction, so V s 1 is V s 2. Now, for the same V s 1 and V s 2, the geometry source, clearly V N 2 is more than V N 1, see that. Mathematically, also you can see, that if you consider this angle as beta, which was oblique to, which was the oblique shock wave of angle, then the delta is this one. So, this angle is beta plus delta. So, this is the angle here, beta plus delta, because this line is the parallel to the shock, so this angle is beta plus delta. So, V N 1 is, then what will be that, V S 1 tan beta, ok, where V N 2 is V S 2 tan beta plus delta and V S 2 and V S 1 are same. So, therefore from geometry and from trigonometric relations we see, that V N 2, that means, V N 2 is greater than V N 1.

So, this cannot be in a shock, so shock is not possible, so therefore shock is not possible; shock is not possible. And therefore, this case, there will be an expansion wave or expansion flow with expansion waves. This we will discuss. So, before coming to the discussion on expansion waves, what we will do first? We will make some mathematical analysis.

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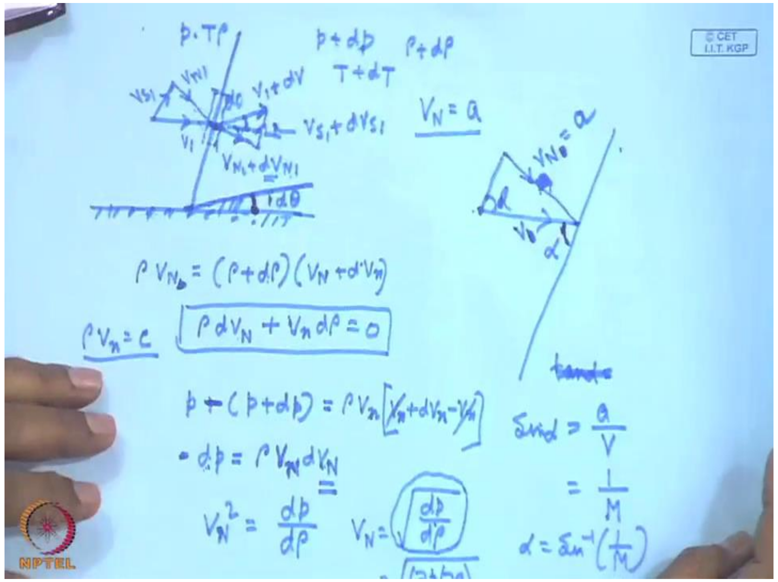

Let us first do a mathematical analysis like this. It is that a, like this a plate type of, let us make a slight change in the orientation theta d theta. What happens to the flow condition? Let us consider first oblique, this shock may be oblique shock or a Mach wave oblique shock wave. Just we are now making a generalized thing, that there will be a wave, which may be Mach wave, which may be shock wave.

For that now, what we are doing? We are making a finite small turning of the flow, that means, flow is turned by an infinite, that means, if flow is V 1, which has the component V S 1 and V N 1 is turned through an infinite small amount d theta. And this value is V 1 plus, let us consider d V a small change, ok, and this is the, this is the normal component, which is V N 1 plus d V N 1 and this is, is the V S 1 plus d V S 1.

Now, let us first find out with this small change in this direction what will be the changes in the flow properties. Now, for that we make a representation with infinite small changes in the quantities, that velocity quantities. Similarly, the pressure p 1, let the pressure p, just to represent by p, p plus dp. So, temperature, let us consider T here, so this will be T plus dt, and the density is rho, which will be rho plus dp. That means, all the infinite small check.

Now, let us find out whether this change are positive or negative and here, this change on theta in this direction will consider as a positive change in theta, which I will explain earlier how it is known as positive. And all these things, at first we consider this type of infinite small change in this, initially it was flat, so this make a change in the shape. That means, this is a corner type of thing. So, where there is a wave and this wave changes the flow, turns the flow according to this by an angle d theta, that means, if you just, this is make, this is the original direction, this is, this angle is d theta, this angle with the original direction of the velocity.

Now, if this be the thing, let us find out first the continuity equation. So, continuity equation we can write, that rho V N 1 is equal to rho plus d rho into V N 1. Now, we will not write any one, we will make V N because here we are already making with the infinite small change. So, therefore we will consider all as V N, V N plus d V N. So, 1 I will not use though I have written it in the diagram, please change it V N d V N. And neglecting the product of the, for example, d rho d V N, we get this equation, rho d V N is plus N, that is, V dV N, that is, V N d rho is 0. This we can take by simply telling rho V N is constant making a logarithmic differentiation or simply differentiation rho d V N plus V N d rho is 0. So, this is one equation from the differential form, from the ((Refer Time: 06:52)), very simple.

Then, if you write the momentum equation with respect to an infinite small control volume here, then the momentum equation was a p, momentum equation means, the theorem, momentum theorem in the direction this along this normal direction. That means, the net rate of momentum aflux, normal momentum aflux is equal to the net fore in the normal direction. That means, here area cancels out for both the terms, both the sides. Here also, it will do like, that p minus, sorry, p plus d p. This side is p plus d p, this side, this p net 4 is equal to rho V N, well, into V N plus d V N minus V N. So, therefore this cancels, so therefore we get d p is equal to the rho V N d V N. d p, what we get here from here, minus d p is equal to rho V N rho V N d V N, sorry, V N, small, d V N, rho V N d V N rho V N dv N.

Now, if I, here what we do? If I substitute the d V N from here, that means, this dV N, if I write in terms of minus d p, that d V N is minus d p rho by V N, then what we get? If we substitute d V N here, d V N is minus d p by rho V N minus d p by rho V N, so V N V N square. So, we will get a very good relationship, that V N square is d p by d rho d p by d rho or V N is equal to d p by d rho, clear. Just d V N is substituted here, minus d p by rho V N.

Now, since this change is infinite small, we can calculate the process to be free from dissipation and adiabatic, so that this can be considered as an isentropic condition. That means, this can be written as and it can be shown that this way the process is isentropic. Also, del p del rho at constant ((Refer Time: 09:35))

So, therefore this expression is the expression for sound velocity at this state. So, therefore in this case, the normal velocity is sound, which means, that an infinite small turning due to this infinite small disturbance type of thing, infinite small turning, infinite small angle of turning d theta makes a wave, which is nothing but the Mach wave, V N is equal to a, that means, this can be shown this way now. That this is simply a Mach wave.

That means, if this be the V 1, then this is the V 1, then the normal component of velocity. This is the, sorry, this is the, I am sorry, this be the V 1. This will look nice, this normal component of velocity V N 1, simply you write V not N V N is equal to a and therefore, this angle is the Mach angle. This angle, this angle, that means, this angle means, which angle, that means, this angle, well, this angle, this is alpha with, so alpha is what? Tan alpha is, tan alpha is, tan is this divided by this, ok. So, sine alpha, not tan alpha, I am sorry, sine alpha is, I am sorry, sine alpha is this divided by this, that means, a by V, that is, this equal to 1 by Mach, Mach M.

So, therefore this is the, this angle alpha is 1 by M sine inwards, that means, alpha is sine inverse 1 by M, that means, this small divergent will call, we will find, we will ultimately make a Mach wave. We will create Mach wave and this will be inclined with this wave by an angle, which is the Mach angle.

Now, next is, we do it, well after that we write the equality of this, equality of this. Let me do it here, otherwise we will be in little trouble, that this V S 1 is equal to V S 2 and V S 1 is what? Now, let us consider is V and this is V plus d V, ok. So, therefore if we write this, we can write V S 1 is V S 2. So, that means, V cos alpha. Now, I define this as alpha, cos alpha, V cos alpha is equal to, V cos alpha is equal to V plus d V, that means, V plus d V and this angle then changes, cos alpha plus cos of, not plus, minus alpha, sorry, cos alpha minus d theta. This angle changes because this is alpha that means, this is, this is the alpha and this has been changed by d theta. So, therefore this angle is alpha minus d theta, earlier we did it.

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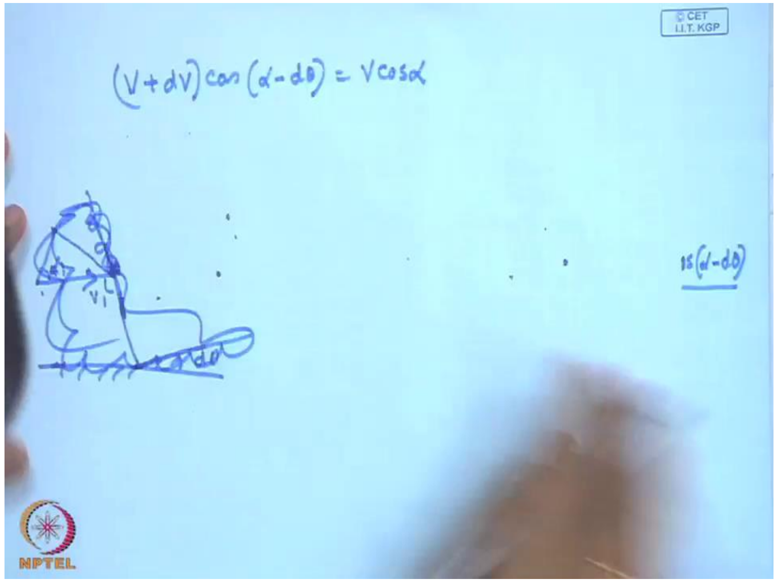

So, therefore we can write, that equality of this V plus d V cos alpha minus d theta is equal to V cos alpha. Well, again I draw that figure for you because I understand you are in trouble to understand, sorry, this is small angle theta, I am just, d theta, I have just exaggerated it. Let us consider this, that this is the, sorry, this is the V 1, V 1 and this is the, ok. This is the V 1 and this is the alpha 1. So, therefore this is the alpha 1, ok.

So, sorry, I have done some mistake, sorry, then I have done some mistake, that is why there is a problem, I am sorry.

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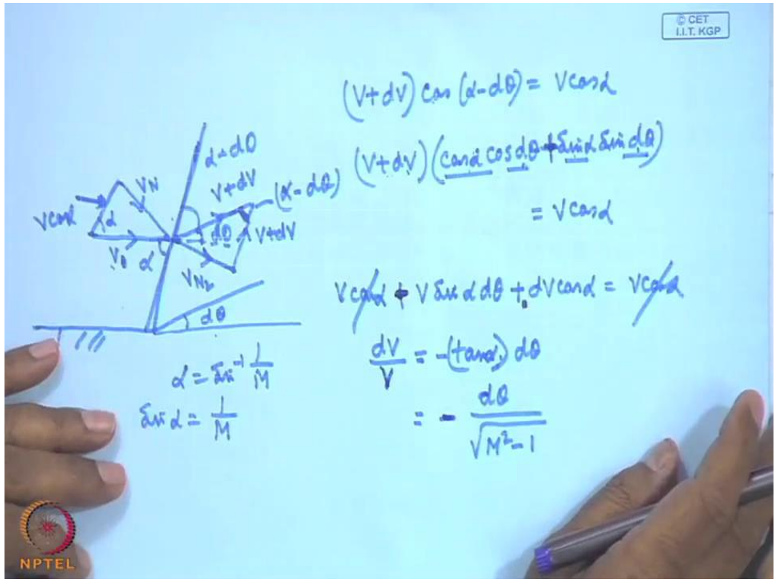

Let me have this thing, ok. This is d theta, so let us, we, because this is a positive change, so therefore we make a thing like this, ok, as I did earlier. So, this is this. So, now this is very clear. So, this angle is, ok, this angle is alpha, this angle I made a alpha, this is the wave and this angle is this angle alpha. So, therefore this will be this, V 1. This is a V N 1 so this this is V N 1 and simply V. So, this will be V cos alpha.

Then, what will happen here? This will change here, so this will be going to this direction, this is V plus d V. Well, so this has also a, this is V N 2. Again, I am writing, so this is, what is that? This is V plus d V, Now, this angle, this angle, this angle is, all this is alpha minus d theta, clear. This angle, because this angle, this is alpha and this is d theta. So, therefore this angle is alpha minus d theta, which is equal to this angle, the same thing.

Now, I have to be little fast, that now if I write V plus dV, equality of the tangential component or parallel component, cos of alpha minus d theta is V cos alpha. Now, we have to be little bit fast now, let us do it. V plus dV now cos alpha, sorry, cos alpha minus d theta is cos alpha cause d theta minus sine alpha sine d theta is equal to V cos alpha.

Now, make the left hand side. Now, V cos alpha cos b theta, now for small b theta cos b theta 1, so V cos alpha. Now, if I multiply with this V sine alpha, sine d theta is d theta, V sine alpha d theta plus dV cos alpha because cos d theta is 1 minus d V sine alpha d theta. So, dV d theta, product of two small term, sine d theta is very small, sine d theta means d theta. dV, this is neglected, so neglecting the higher order term, the product of this two we get what? V sine alpha, minus V sine alpha d theta plus d V cause alpha is equal to 0. This is a very interesting result we get from which we can write dV by V is equal to minus tan alpha into d theta minus tan alpha into d theta ((Refer Time 17:40)).

Now, tan alpha can be written as minus d theta root over M square minus 1, why? Because we know, that alpha is sine inverse 1 by M for such small turning, small angle d theta, so that sine alpha is, simple trigonometry, tan alpha is 1 minus root over M square minus 1, ok. And dV by V is sine alpha by cos alpha, that is, tan alpha dV by V is tan alpha d theta.

Student: (( ))

Professor: Which one? That will be, this is minus, this is minus, this is plus.

Student: minus will not come?

Professor: This is minus.

Student: dV by V is equal to minus tan alpha.

Professor: dV by this, oh, then that will be plus cos A cos B is plus, oh sorry, I am sorry, you are very correct. Cos A cos B will vary. This is the final results that I know. Yes, Cos A minus B is Cos A Cos B, sorry, plus sine A sine B, the minus will come if it is sine A minus B or cos A plus B. Correct, very correct. So, there will be minus d theta by root over, I am happy, it is very good, dV by V is equal to minus d theta by M square. Now, we go to the, well, energy equation.

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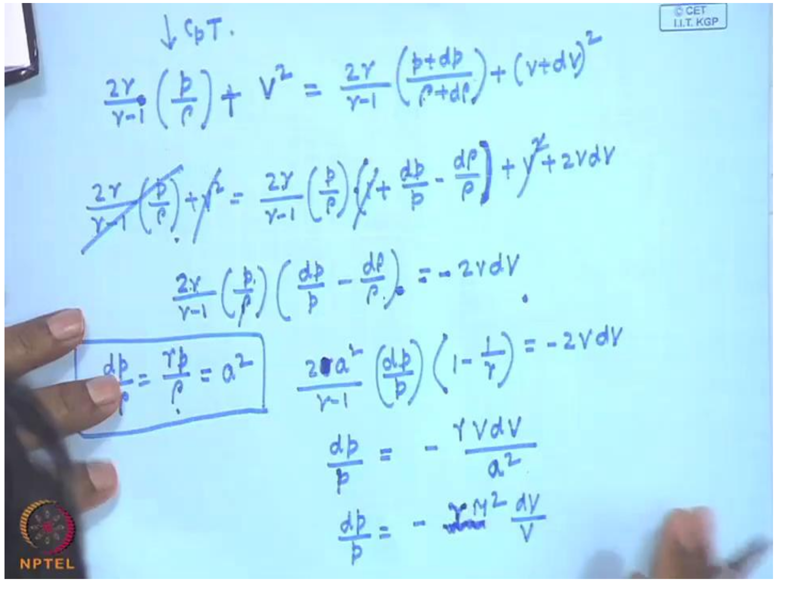

Now, we go to the energy equation. I am in little, you know, that energy equation, we wrote, that 2 gamma, that is, h 1 C p t, that is written as gamma by gamma minus 1 into t by rho plus V square. Why 2? Because V square by 2, I am just, I am writing this thing, it is V square by 2 gamma by, gamma minus 1 is what? C p into r. C p is r gamma, gamma, gamma minus 1, that means, it is C p by r and p is equal to rho r t.

So, therefore it comes from what? It comes from C p into T, but 2 C p T, this, this is C p into T gamma r by gamma minus and r t is p by rho and V square by 2. So, I am multiplying 2 on both the side, that means, this is the static ((Refer Time: 20:00)) plus the kinetic energy. We discussed this earlier, that is why, I am writing straight because here I do not want to repeat it again.

Now, for an infinite small change the quantities are shown like this. This is a rho plus d rho, yes, plus p plus dV whole square. Now, if you do it now, you again, you will see, that 2 gamma by gamma minus 1 into p by rho plus V square is equal to this side 2 by gamma minus 1. Now, you take p by rho common that means, you take p from the denominator and rho from the numerator. If you take p from the denominator, this will be p plus d p to the p 1 plus d p by p to the power 1, that is, 1 plus d p by p, that means, 1 plus d p by p. So, therefore 1 plus d p by p, I am writing in one shot, and rho if you take, 1 plus rho by rho 2, the power of minus 1.

And if you make the binomial theorem and second order terms you neglect, then it becomes equal to 1 minus d rho by rho only. Here, this will be multiplied, so that finally you can write 1 plus d p by p, which is there already, p common d p. So, denominator contribution is, that 1 plus d rho by rho to the power minus 1 will be multiplied for which only one term will come, 1 plus d p by p minus 0 by rho, alright. Sorry, this is in the first bracket, I get plus V square plus 2 V dV. I just again neglect the higher order term, so V square V square will be cancelled, so this thing will be cancelled, 2 gamma by gamma minus 1 p by rho 1 will not be there.

So, therefore 2 gamma by gamma minus 1 p by rho d p by p minus d rho by rho will become equals to 0. So, ultimately will become equals to 2 V dV. So, ultimately we can make this one like this, 2 gamma by, I can write it p by rho into d p by p minus d rho by rho, ok, d rho by rho is minus 2 V dV. If there is any mistake you tell me. This is ok. Now, what we can do? We use a formula, that d p by d rho, you know, that d p by d rho is equal to for an isentropic process because already we have taken the process, because the entropy is constant and that becomes equal to a square. So, we will use that.

If you use this, that and what do you do? This gamma p by rho already you use as a square gamma p by rho so therefore you can write 2 gamma gamma p by rho a square so just see that what I am doing this I am using and d p by p I am taking common

then what is there so d p by p that is p by rho d rho by d p and if we use this equation this will be ultimately 1 by 1 minus gamma is equal to minus 2 V d V what I am doing gamma p by rho is a square d p by p I am taking common that means this will be d rho by d p into p by rho and d rho by d p into p by rho is 1 by 1 minus gamma. So, if you equal this two, then you will find this. Then, gamma minus 1 in gamma minus 1 will cancel.

So, therefore from here we get straightaway dM, d p by p we get straightaway, that d p by p, d p by p is what, now gamma minus 1 and gamma minus 1 we will change. So, therefore gamma, only gamma, that means, 2 2 cancels minus V dV by gamma, that gamma will not be there. So, gamma minus 1, so that gamma, so only gamma V dV by, no gamma will be there, 1 minus 1 by gamma 2 a square. There gamma will not come gamma p by a rho is a square. So, this gamma, this gamma will not come, so therefore gamma V dV divided by a square, that means, d p by p is equal to minus gamma V square. If I multiplied gamma by a square V square gamma m square, sorry, this one, d V by V gamma minus, sorry, this is not gamma M square because I multiplied with V, V square by a square is M square.

Now, we know the value of dV by V, so dV by V we have already found out, just I see dV by V is equal to minus d theta by root over M square minus 1. So, therefore we get d p by p is equal to dV by V M square. dV by V is, again I see, minus d theta root over m square. So, minus minus cancel, so it will be gamma M square root over m square minus 1 into d theta. So, so now I just make it like this, dv by V, dV by V well. So, this is, d V by V is this one and I get d p by p is this one from the energy equation ok

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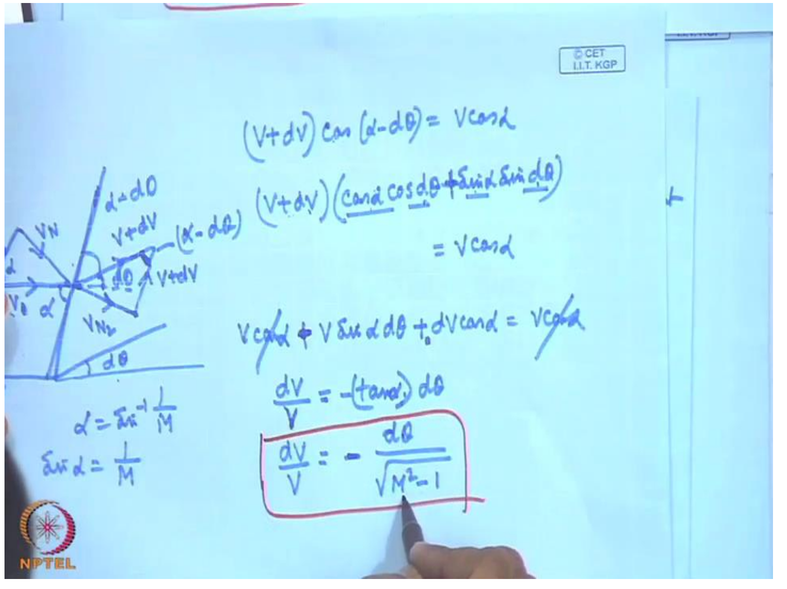

Now, after that I will do for another same thing is the Mach number. Now, Mach number, you can define in terms of square. Mach number square is V square by a square because a, unnecessarily contains under root of that, that is why, we are now, if we considered a is gamma of the, a square is gamma p by rho, then this will be gamma p because a square, I am writing, gamma p by rho, that we know, already we wrote.

So, therefore if we make a differentiation, logarithmic differentiation, then 2 M dM by M is equal to 2 V, 2M, no, 2 dM by M. This will be 2 V, 2 dV by V, 2 dV by V, I am taking logarithmic 2 dM by M, ok. 2M, that means, 2 l N M dM by M is equal to 2 dV by V plus d rho by rho minus d p by p. This is ok. 2 dM by M is equal to 2 dV by V. Now, you just substitute these values, but before that you have to know, d rho by rho, d rho by rho is another one, which you have to find out. Now, before that we will find out what is d rho by rho.

How you will calculate d rho by rho? It is very simple. We have considered the isentropic flow, p rho to the power gamma is constant, so take a logarithmic differentiation. So, d p by p is E plus or there we can take that side minus gamma d rho by rho minus d p by p plus gamma. d rho by rho is 0 minus, so d rho by rho is minus 1 by gamma d p by p and that it is alright, d p by p plus gamma d rho by rho is 0, ok.

Student: p rho to the power gamma

Professor: p rho to the power gamma is constant for the isentropic flow; for the isentropic flow p rho to the power gamma is equal to constant. We are considering the flow where the entropy does not change, that is, the isentropic flow. So, therefore the d p by p plus gamma d rho by rho is 0 minus gamma d rho by rho. So, minus 1 by gamma.

Student: gamma (Refer Time: 29:06))

Professor: no, no. p p into the rho to bar gamma is constant is a p rho to the bar minus gamma, exactly, you are correct, p rho to the power minus gamma p by rho to the power gamma is constant. I am sorry, so therefore, it will be then plus, plus p rho to the power minus gamma, I am sorry, p by rho to the power gamma. Sorry, sorry, p by rho to the power gamma is constant, very correct, p by rho to the power gamma is constant. So, d p by p is equal to gamma d rho by rho. So, d p by d rho by, there is 1 by gamma d p by p.

What is d p by p? d p by p 1 by gamma M square, that means, that gamma will not be there. So, therefore, very good, d rho by rho will be M square by root over M square minus 1, very good, very good. I am sorry, p V to the power gamma is constant, but it is p by rho to the power gamma is constant. I am extremely sorry, this will be p into rho to the power minus gamma is constant, very correct. p by rho to the power gamma is constant. So, therefore there will be a minus sign M square root over M square.

So, this is d rho by rho. Now, if you substitute, now dV by V, dV by V dM by M. Now, dV by V where we have got, now again I am telling, that dV by V in term, that minus d theta by root over M square minus 1. So, d p by p is gamma M square by root over M square minus 1 d theta and d rho by rho if you substitute all those things. Finally, you can write, dM by M become equals to, if you do it you will see, dM by 1, you can write like this one line, minus 2 plus M square for d rho by all M square minus 1 will get common minus gamma M square because d theta by root over M square minus 1 is common to all. So, if you clear this thing you will get, this is a very standard expression, 1 plus gamma minus 1 by 2 M square. This is a very standard expression, minus d theta by, this is minus d theta, I am writing minus here, M square minus 1. So, this will come like that. Finally, you will get this.

Now, if you compare all these things, again I am showing it, dV by V is minus d theta by root over M square minus 1 d p by p is gamma M square by root over M square. Rather, I write it again, that will be better for you.

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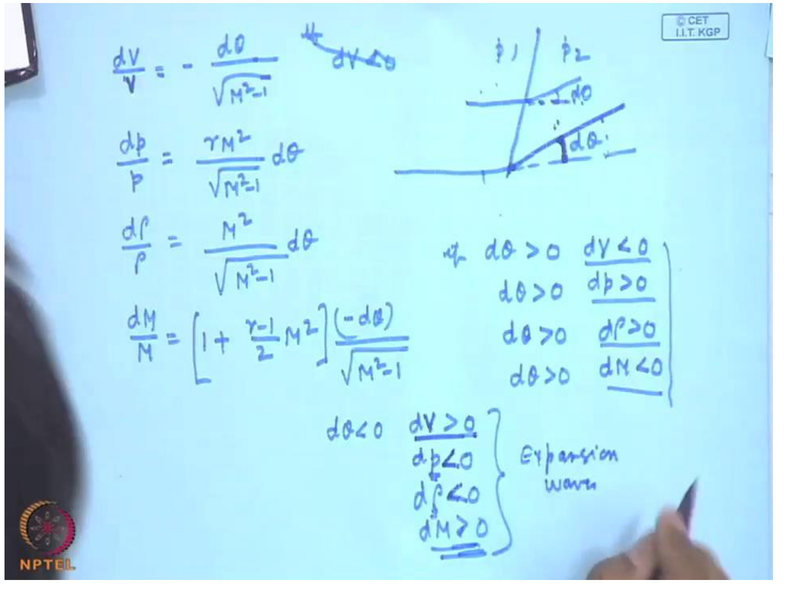

So, now I write everything, dV by V is equal to minus d theta by root over M square minus 1. Now, d p by p I write, which we deduced, d p by p is gamma M square by root over M square minus 1 d theta, ok. d rho by rho is 1 by gamma, so M square by root over M square minus 1 d theta, good, and d V by V, dM by M, sorry, then dM by M is equal to 1 plus gamma minus 1 by 2 M square, this, into minus d theta.

So, this is plus d theta, this is plus d theta, so minus d theta, I write here, which I write earlier, root over M square minus 1. Now, you see, the beauty is, that we made a change like this. This is the concept now, where the wave was like this, we change, change the thing like this, this is d theta. Now, this d theta is positive and this is considered to be positive in this direction. This change is positive. This is because this makes a change in a way, that it makes this velocity, that there is a reduction in the tangential within a normal component of velocity. Or we find out, that p 2 is more than p 1, that here we will see, that if we consider this d theta as a positive one, then we see, that the positive d theta. That means, in this direction it change, we will make the velocity dV less than 0.

If here, if d theta is greater than 0, then what happens? dV less than 0, d theta greater than 0, d p, d p greater than 0, d theta greater than 0, d p d theta greater than 0, d rho greater than 0 and d theta greater than 0, d M less than 0. This is typically the shock wave. That means, where with a positive change in the angle here, the velocity decreases, pressure increases, rho increases and Mach increases. But the other way, if d theta is less than 0, then dV will be greater than 0 and d p will be less than 0, d rho will be less than 0 and dM will be greater than 0. That means, the flow will accelerate with a change in, reduction in pressure and density and with a flow will accelerate, that means, with an increase in velocity, with the increase in Mach number also, with a reduction in pressure and the reduction in density and this is known as expansion waves or expansion flow, that I will explain now, I will show you. Is there anything, is ok?

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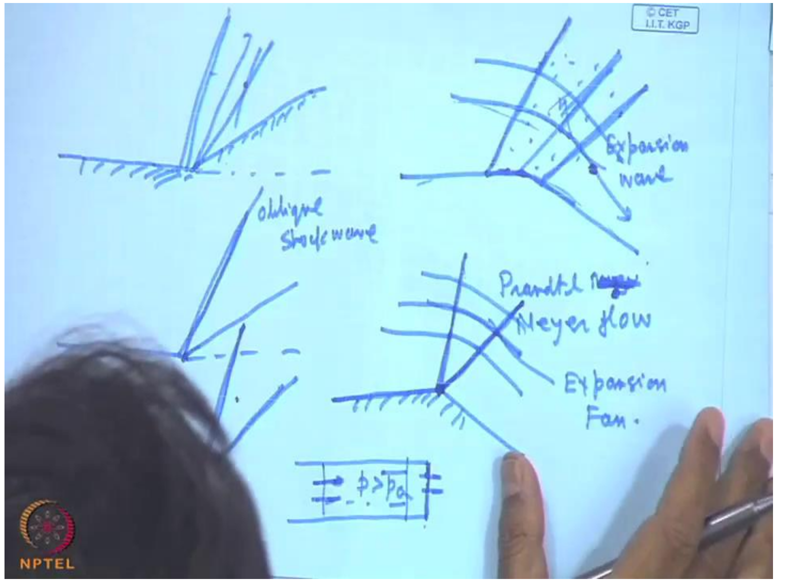

Now what happens, I will just tell you. Now, if you consider a change like this, now any finite change a, now you think a change, that is a finite change in the angle, that means, a concave corner or concave type of thing, concave surface like this, that is a concave change, then this can be considered as some of many infinite small changes. And what happens? The number of waves will be generated where the Mach will be changes and this wave will be ultimately converging and will give you a finally converging wave, that is, the converging wave, that is, the shock wave, that is, the oblique shock wave, oblique shock wave.

This nature will be like this, but if we have a thing like this, then what will happen? This will start somewhere here that may be for example, here not it may not be very sharp, then what will happen? This type of thing will give a diverging type of wave and this wave is known as, these are known as, this is, this is just for drawing, I am showing this is the diverging wave, this is known as expansion wave, expansion wave. This is expansion wave, this will never converge, this will never converge expansion wave.

And within this, the flow takes place like this, ultimately goes on diverging like this and throughout this expansion wave region. The flow remains isentropic, the entropy change will be very less and this flow is known as Prandtl Meyer flow, Prandtl Meyer flow. This flow is known as M E, Prandtl Meyer flow, M E Y R, Prandtl Meyer flow, this flow is known as Prandtl Meyer flow. This is, and now for example, if this is also like this, but if this is a sharp one and this is a sharp one, this I have shown also, then it will give you a converged oblique shock, but this will give a this type of fan type of thing. So, that is why, this is known as, this is known as expansion fan, expansion fan, so this is the thing.

That means, if this is a sharp corner, like this is a convex, all these oblique shock wave merge one another and gives rise to a single oblique shock wave across which there will be increase in pressure, increase in density and a decrease in velocity and Mach number. And just the opposite will be there in the case of a concave surface like this, in the flow takes place, there will be a series of expansion waves and which will be diverging in nature and create a fan like thing. And if we think of a sharp corner, then they will make a fan like that. This will be attached here and throughout there and the same thing will happen. I will explain afterwards.

In case of a flow fluid at some pressure p, higher than p atmospheric pressure, if this valve is opened or the diaphragm is ruptured, then the flow will commence, then the flow will commence and the expansion will go like that. And you will see, that there will be a zone of expansion wave that will be created here. So, that I will explain afterwards. So, this is the picture by which an expansion wave is created.

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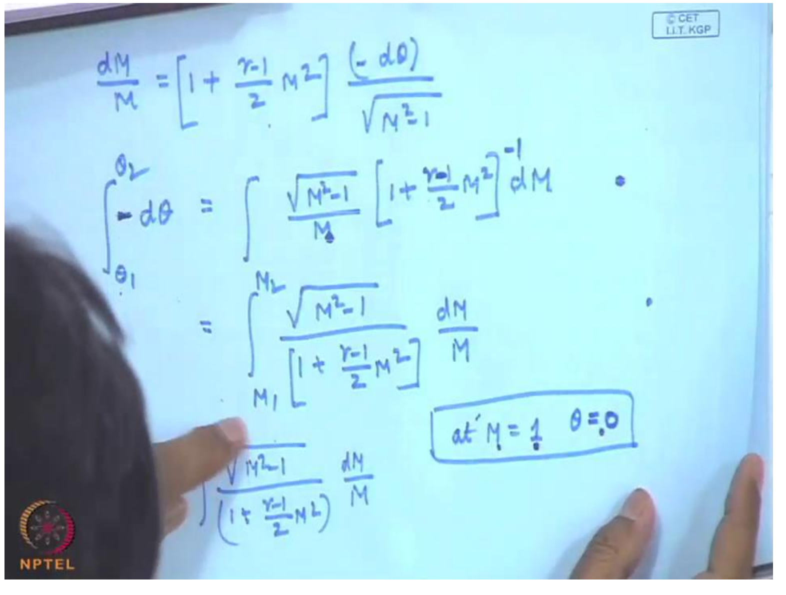

Now, what we will do? We will now create, calculate d theta Mach number relationship. What is that now, what is most important is, that dM by M d theta, ok. Let me write that dM by M is 1 plus gamma minus 1 by 2 M square into minus d theta divided by root over M square minus 1. Now, for an expansion wave I now do not give this minus d theta sign, rather, rather first of all, let me give this minus d theta sign.

Then, I will tell you, now if we will integrate this with respect to M, that means, that is, M square root over M square minus 1 by M root over M square minus 1 by M into 1 plus gamma minus 1 by 2 M square dM. If we integrate this and this is a 1 plus gamma minus 1 m square whole to the power minus 1, yes, whole to the power minus 1 dM whole to the power minus 1 dM. Rather, I must write in a different fashion, root integration, root over M square minus 1 divided by, you are correct, gamma minus 1 by 2 M square into dM by M. I think, this is ok. Now, if I calculate from 1, theta 1 to theta 2 and we can calculate from M 1, we can integrate from M 1 to M 2. So, we can get a relationship that what should for a given change in the angle what should be the change in the Mach number. So, we can make an integration.

Now, better giving any limits because we do not know, that the limits we better integrate it simply by considering, that an indefinite in integral. We do not take, first of all, this minus sign because it is the case where we only consider the concave surface. So, we take theta is equal to integration of this root over, simple integration of this root over 1 plus gamma minus 1 by 2M square dM by M.

Now, if you make this differ indefinite integral, you have to impose some constant, some constant because the integration constant will be there, you have to impose some condition, rather constant is, should impose some condition to element the constant of integration. Then, we consider, at M is equal to 1, theta is equal to o, arbitrarily. That means, we consider, that when the flow is, that M is equal to 1, then theta is equal to 0.

But this thing I will explain afterward, that when M is equal to 1, what could have been the value of theta, that I will tell afterwards. That means, now we put a condition, that at M is equal to 1, when the M is equal to 1, then theta is equal to 0, is we put a condition and integrate this. Then, you will get a relationship, that I will now write to you, that integration I am not doing, that can be done by making integration by parts, and just I am giving you the result now.

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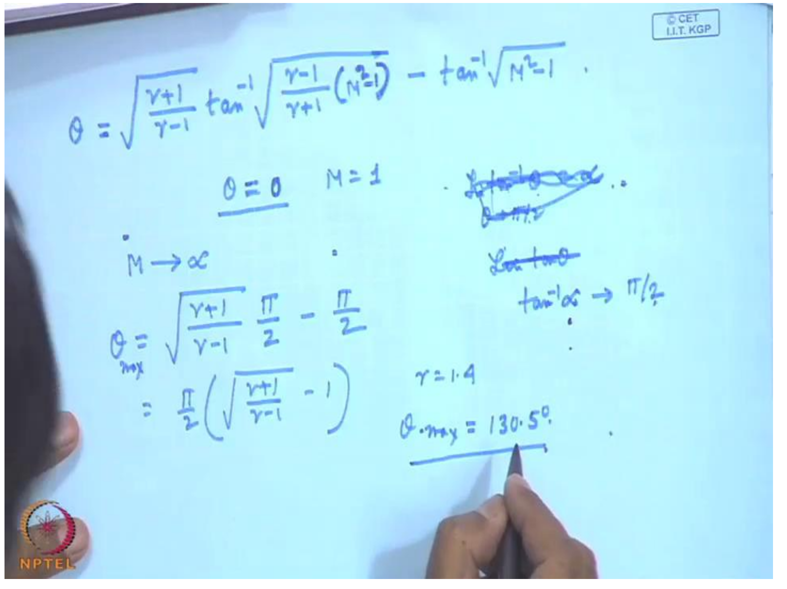

Now, theta is root over gamma plus 1 by gamma minus 1 tan inverse root over gamma minus 1 by gamma plus 1 M square minus 1 minus tan inverse root over M square minus 1. Now, this is the expression where theta is 0 arbitrarily when M is equal to 1. This I will be discussing afterward when, especially I will solve the problem. This arbitrary means, will be understood, that how it is taken care of. Let us first, for the sake of mathematics, consider some arbitrary condition, that theta, that the turning is 0 in M is equal to 1, just mathematically I get.

Now, here another very interesting observation is, that when M tends to infinity, then what is the value of this turning? This is not infinity because tan inverse infinity pi by 2 tan inverse theta limit as theta tends to pi by 2 is infinity, ok. No, not, sorry, sorry, sorry, limit tan theta, a limit tan, enough tan, tan inverse infinity is equal to pi by 2, I am sorry, tan pi by 2 is infinity. So, therefore tan inverse infinity is pi by 2, that means, limiting value, that is, the limiting value I wanted to write, that when theta tends to pi by 2, I wrote correctly, limit tan inverse theta theta limit tan inverse theta, theta tends to pi by 2, it is alright. So, this is tan inverse infinity is pi by 2.

So, therefore theta is, this is the trignometric relationship, because tan pi by 2 is what? That sine pi by 2 cos pi by 2 cos pi by 2 is sine pi by 2 1 cos pi by, it is ok, means, tan pi by 2 is infinity, ok, because sine pi by 2 1 cos pi by 2 is 0. So, therefore it is infinity. So, therefore this will be root over gamma plus 1 divided by gamma minus 1 into pi by 2 minus pi by 2, that means, is equal to pi by 2 into root over gamma plus 1 by gamma minus 1 minus 1. And this value for gamma is equal to 1.4 for a year, this theta Machs or the limiting value. You can take Machs because M tends to infinity is 130.5 degree, ok.

Now, what happens is, that 130.5 degree, that means, if the turning angle is more than that, what happens I will just show you.

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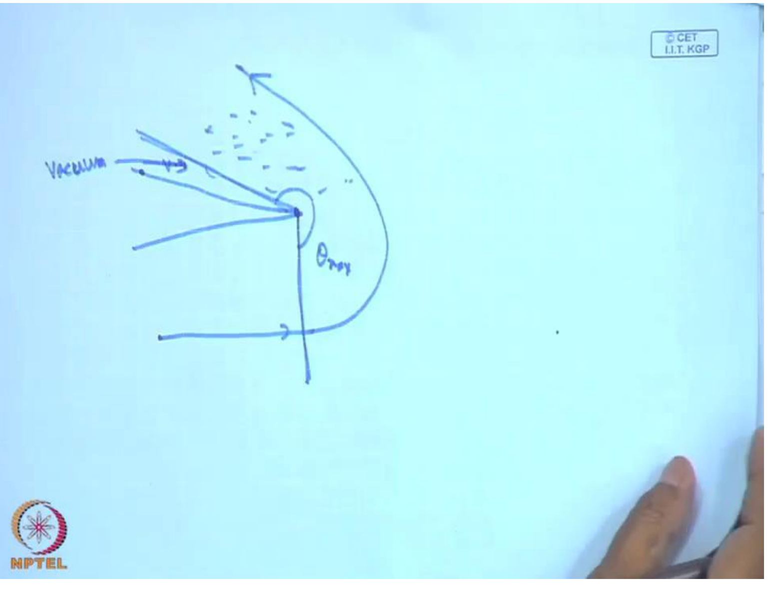

If the turning angle is more than that, let us consider a turning like this that at this is the turning, which is equal to theta max. Let this is theta max, so a flow, which is taking place this way is turned like this and they may go to this. The pressure may fall to 0 and if there is a turning, which is like this, then there may be a pressure zone which, here is in this zone. The pressure is vacuum pressure, vacuum pressure. So, this zone is actually the vacuum pressure zone or the 0 pressure zone. That means, if the turning angle is more than this maximum angle, but we are not interested in this type of situation now. So, only this turning angle, this concept is there.

Now, I will tell you, we will solve some problem, then only this things will be make more clear, otherwise it will be difficult. Just let me do this things. So, ((Refer Time: 47:37)) problem, yes, let us make, let us solve some problems.

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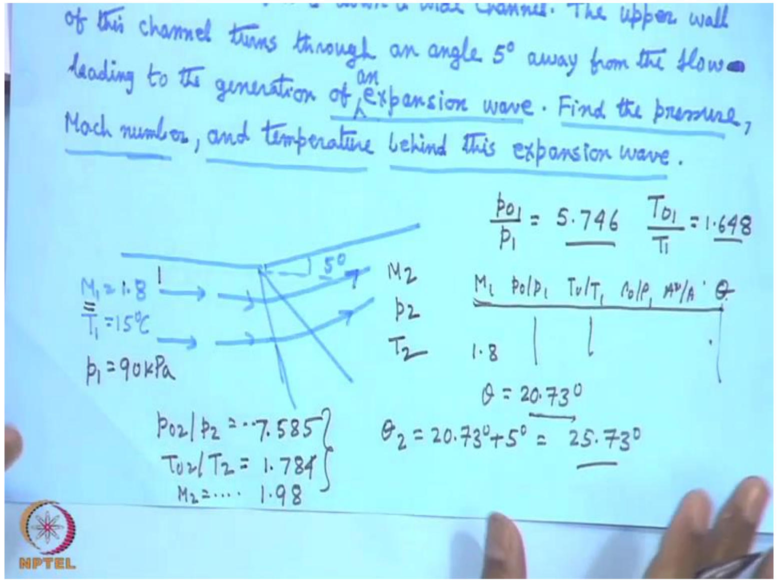

Now, example 1, air flows at M 1 is equal to 1.8 with a pressure of 90 KiloPascals and a temperature of 15 degree Celsius down a wide channel. The upper wall of this channel terms through an angle 5 degree away from the flow leading to the generation of an expansion wave, leading to be generation of an expansion wave. Find the pressure, Mach number and temperature behind this expansion wave; find the pressure, Mach number and temperature behind this expansion wave.

Now, what is this? Let us see the problem, problem is like this. Air flows at Mach number 0.8 with a pressure of 90 KiloPascals and wide, wide channel. The upper hill of the channel, that means, this, the upper one of this channel turns through an angle of 5 degree. So, therefore what happens, there is a this type of flow takes place, that means, the flow is taking place, flow changes like this, the flow changes its direction like this, ok. Now, here the Mach number is 1.8. Well, this pen is not working well. Mach number is 8, T 1 is 15 degree Celsius, I think this pen works better. p 1 is 90 kiloPascals, ok.

Then, what happens, that this one, now this one, M 2 p 2 and T 2. Now, you see, this one will make you clear about that arbitrary constant, that from the flow leading to the general expansion, generation of an expansion. So, first of all what you have to do? We have to find out from the isentropic table. First of all, this flow is an isentropic flow with this Mach number 1.8. What is value of p 0 by p 1 from isentropic table? The value of p 0 by p 1 from the isentropic table is 5 point, that is already done, 5.746 and the value of T 0. Let this section is 1, this section is 2. So, p 0 1 T 0 1 by T 1 and that is 1.648. Try to understand that.

Now, in isentropic table, if you remember earlier I told you, that isentropic table shows p 0 by p 1 T 0 by T 1 rho 0 by rho 1 A star by A. All these things I told and there 1 theta value was there at the end. For all isentropic table there is 1 theta value, that theta value is the theta value for the Prandtl Meyer expansion or expansion wave, which is also isentropic. They follow these isentropic table values. So, therefore corresponding to this Mach number 1.8 I d p 0 by p 1, this one, I d T 0 by T 1 I d is the value of theta and that theta value is equal to the value 20, theta becomes equal to 20.73 degree.

Now, you understand what does it mean. That means, this Mach number is generated by turning an angle 20.73 degree Celsius where the Mach number was 1, you understand. That means, from a 1 Mach number these turning will be then the final turning angle theta. This was the theta, so theta for this thing theta 2, that the final theta is 20.73 degree plus 5 degree is 25.73 degree against this theta. You will find out the p 0 2 by p 2, we will find out T 0 2 by T 2 from the isentropic table. And what we will find out? We will find out the value of M 2, the Mach number, everything you will find out if we know this thing. So, therefore you will find out the Mach number p 0 2, now p 0 2 by p 2. Now, this value will be 25.73. So, p 0 2 by p 2 in these case is 7.585 T 0 2 by T 2 is 1.784 and M 2 is 1.98.

Well, that means, try to understand what is the thing that we are finding out from this Mach number? That to get this Mach number through an expansion because this theta is kept against a Mach number from, now your concept will be clear from this expression. What is that expression I just wrote? After integrating this thing, immediately I wrote an expression wave is, that well, what, where is that expression, just now, theta, yes I wrote this expression, this expression is valid provided theta is equal to 0 M. This condition is satisfied, that means, when I compute this isentropic table at a given Mach number, this theta means, from Mach number 1, this Mach number is obtained with this angle of time. So, therefore for this Mach number if we have an angle of turn 20.73 through an expansion wave, we can generate this Mach number from Mach number 1.

So, therefore the, if we have to find out the property here by turning with respect when expansion wave, we have to add 5 degree with this value as if from Mach number 1 this turning is being made. So, then we relate this to be Mach number 1 condition through the calculation of theta for this Mach number 1.8, that is the beauty of this type of problem. So, we find out this against this turning angle, we get 0 2 by p 2 T 0 2 by T 1. Now, p 0 1 and p 0 2 is same because this is an isentropic flow. So, therefore p 0 2 by p 2 is this, p 2 is p 0 2 by this one and p 0 2 is p 0 1, which is p 1 into this one.

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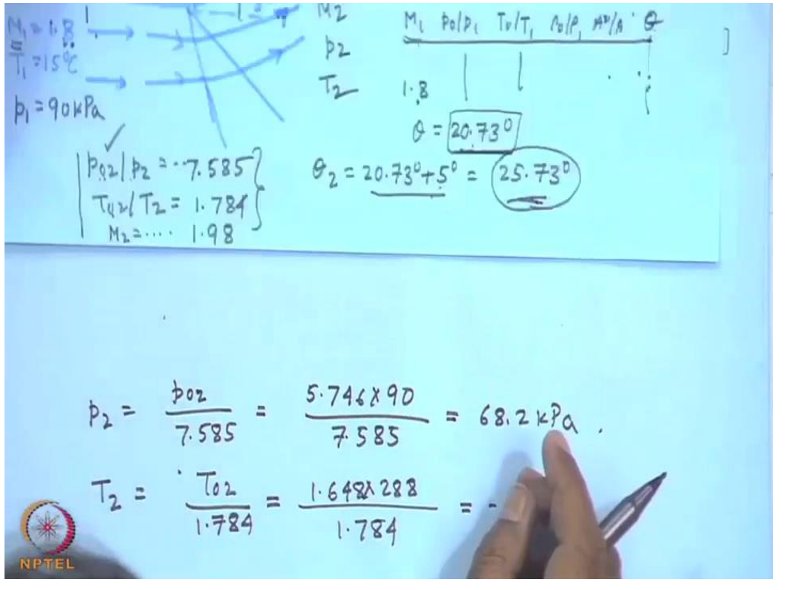

So, therefore we can calculate, that p 2 is p 0 2 divided by 7.585 and p 0 2 is p 0 1 and that is equal to 5.746 into p 1 is 90 KiloPascals divided by 7.585 and that becomes equals to 1 that becomes equal to, well p 2, I am giving the value 68.2 KiloPascals.

And similar way if you calculate T 2, that is, we can calculate T 2 in the similar way because we know, T 0 2 by T 2, so T 2 is T 0 2 divided by 1.784 and that becomes equal to T 0 2 is T 0 1. And T 0 1 we have calculated, 1.648 into T 1 that means, 1.648 into T 1. T 1 is 288 divided by 1.784 and that gives you a value, which is already calculated and that value is what? T 2 value is minus 7 degree Celsius, so this we, we can calculate, ok.

We can solve another problem, but I do not think, time is less, so we cannot solve any problem, any more time is over. Well, another problem, just I give you for your exercise.

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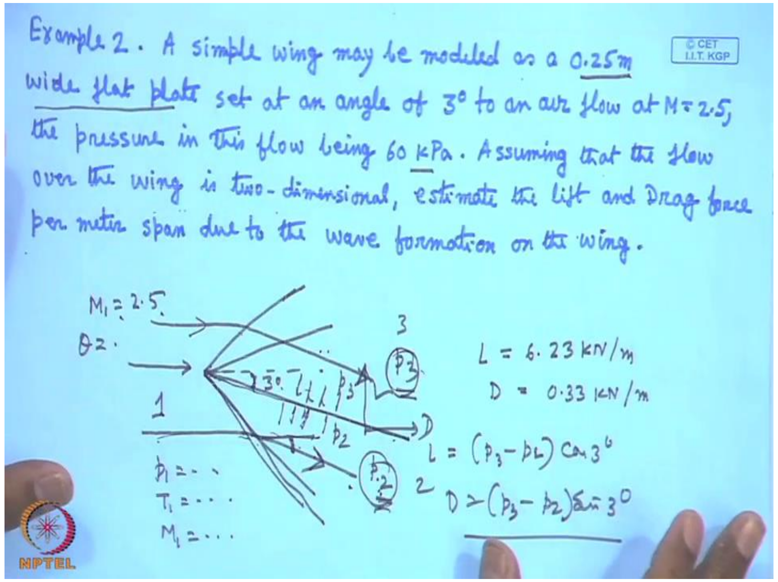

A simple wing, just you write this problem, may be modeled as a 0.25 meter, I will give you hint, wide flat plate set at an angle of 3 degree to an air flow at Mach number 0.25, ok. And the pressure in the ((Refer Time: 56:49)), the pressure in this flow being 60 KiloPascal. Assuming, that the flow over the wing is two-dimensional, estimate the lift and drag force per meter span due to the wave formation on the wing. So, this is a very interesting problem.

So, this problem, actually you can solve the problem is like this. There is a flat plate, it is modeled by a flat plate, which is 3 degree, like this. Now, when the flow takes place like this, what happens? There is a, this is shock, an oblique shock and there takes place this expansion wave. So, flow goes by expansion wave in the upper part and the shock wave. This is the shock wave, shock wave by lower part, so there will be an increase in pressure. This is 1, this is 2, this is 3, you can find out the pressure here in this upper part. From the expansion wave theory you can find out the pressure p 2, this is p 1 is given, T 1 is given, M 1 is given. T 1 is not given, p 1 is given, however. So, you can find out the p 2 and p 3.

When you can find out p 2 and p 3, then you can make the vertical component as the lift force and the horizontal component as the lift force. This width is given per meter span in this direction. I can tell you the, well, this result that the lift force will be, lift force will be 6.23 kilo Newton per meter of span length and drag force will be very small, 0.33. This is only because of the pressure that means, you calculate the oblique shock concept as I did with this Mach number, finding out the shock wave angle. The turning angle is 3 degree and find out this p 3. Similarly, sorry, find out the p 2 here, sorry, find out the p 2 and find out the p 3 by using the expansion wave theory for this Mach number.

Mach number is what? Mach number is 2.5, you find out what is the value of theta, with that add 3 degree and for that turning angle you find out the ratio of pressure, make the stagnation to static, stagnation pace is same as that, so you can find out this static pressure. Here, you can find out this pressure from the oblique shock angle for this M 1 and this delta. Here, the deviation is delta, then you find out the shock wave angle beta and then modify the Mach M 1 sine beta. Find out the shock table, normal shock table, what are the pressure ratio, you find out p 2.

The problem is simple but very interesting, that you can find out the ((Refer Time: 59:58)). It happens, that one part there is an expansion wave, another part there is a shock wave takes place. So, because of the shock wave this part pressure is more, this pressure will be more, p 2 will be more and here, the p 3 will be less. So, finally if this is the 3 degree, so it will be p 3, lift will be p 3 minus p 2 into, there is a vertical direction lift and horizontal direction drag is cos of 3 degree, and drag will be p 3 minus p 2 sine of 3 degree. So, that I give you the hint, that this way you can solve this problem, ok.

Today, up to this, so therefore, we, this I close the lecture on this introduction to compressible flow and I will close this course here. So, I think you just go through all the courses, all the material that have been taught and I wish you all the best to learn the subject, ok.

Thank you.