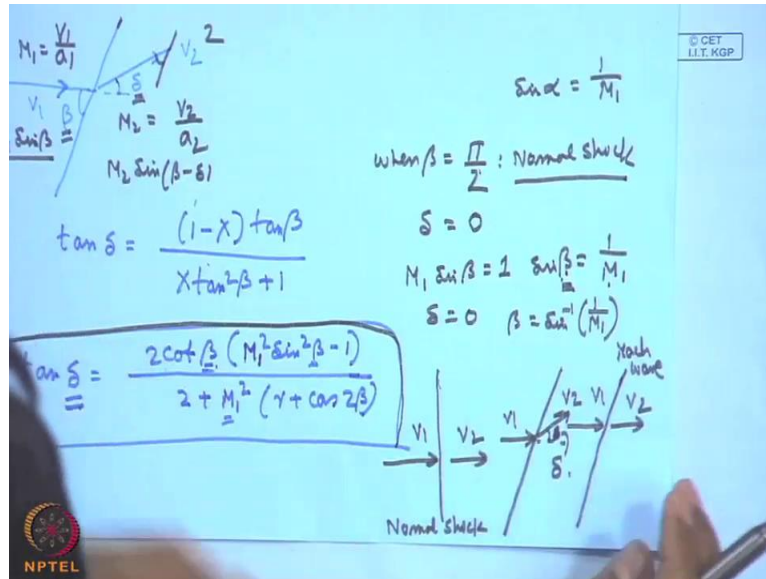


**Introduction to Fluid Machines and Compressible Flow**  
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**Lecture - 39**  
**Oblique Shock (Part – II)**

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Good morning, and welcome you all to this session of the course. Now last class, we were discussing about the oblique shock wave, and where we ended, just we have to look the last class. We were discussing again we just make a recapitulation that a oblique shock wave is like this, where the inlet velocity or the incoming velocity  $V_1$  is being deflected. And this angle is the angle of the deflection and this is beta. And what we discussed last class is that this oblique shock wave can be treated as a normal shock provided the mach number mach 1 which is  $V_1$  by  $a_1$  is modified the  $M_1 \sin \beta$  and the mach two actual mach number is  $V_2$  by  $a_2$  the condition here 2. Section 1, so it is  $M_2$  that is the normal component  $\sin \beta$  minus  $\delta$ , that is this angle made by this velocity  $V_2$  with the line parallel to the shock wave, and similarly this is beta. So, this we discussed and finally, we underappreciated that a relationship between beta and delta and the mach number initially  $M_1 \sin \beta$  or  $M_1$  whatever you tell is very much important and then from a trigonometry relationship, we arrive that thus relation and delta is  $1 - x$  where  $x$  was some quantity define.

So, finally, this is the expression this is the expression we derive where  $\delta$  is this deflection angle,  $\beta$  is this shock wave angle well  $M_1$  is the approach mach number  $M_1$ . So, this is the very very important relationship. Now two characteristic feature of the relationship is that you see when  $\beta$  is  $\pi/2$ , this already we discussed earlier,  $\beta$   $\pi/2$  means that where in the velocity of approach to the shock becomes ninety degree makes ninety degree angle to the shock; that means, this becomes a normal shock. This is the normal shock condition, then what happens what is the value of  $\delta$  when  $\beta$   $\pi/2$   $\cot \beta$  will be 0 when  $\beta$  is  $\pi/2$   $\cot \beta$  will be 0. So, therefore,  $\delta$  is 0.

Now another case, you see when  $M_1 \sin \beta = 1$  then also  $\delta$  is 0. This is because this becomes 0 and this  $M_1 \sin \beta = 1$  means  $\sin \beta = 1/M_1$ . And this is the mach angle; that means,  $\beta$  becomes equal to the mach angle. Because we know the for a mach way the angle which is makes with the flow stream direction is  $1/M_1 \sin$  of that; that means,  $\sin^{-1} 1/M_1$ ; that means, this  $\beta$  becomes a mach angle; that means, the shock wave becomes a mach wave.

So, therefore, two extreme conditions where normal shock we already appreciated that the value of  $\beta$  that lies between two extreme values  $\pi/2$  and  $\sin^{-1} 1/M_1$ ; that means, we you can write  $\beta$  is equal to  $\sin^{-1} 1/M_1$ . So, within this two limits, there is no deviation. This is because for a normal shock the velocity before the shock and after the shock at the same direction. Similarly for a mach wave, the velocity before and after the shock we send and this again can be represented by this like this that a oblique shock is that oblique shock is between this to two limit 1 is normal shock normal shock.

And another is the mach this is the mach wave mach wave here also there is no deflection  $V_1 \sin \beta = V_2$  and the oblique shock to remain somewhere here where there is a deflection of the velocity that is the  $\delta$  sorry you cannot understand for a  $\delta$ . So, therefore, the oblique shock wave lies between these two limit. Now you see that this particular formula this can be little bit we can do with this since  $\beta$  is  $\delta = 0$  between  $\pi/2$  and  $\sin^{-1} 1/M_1$  which means that  $\beta$  may have ideal maximum or minimum in between this two value because it is 0 zero to 0. So, that can be found out by making a derivative of this that if we can make this with respective  $\beta$   $d\delta/d\beta$  making 0. And checking for the second derivative, whether it is less than 0 or greater than 0 we can find out the condition maximum and minimum. And if you perform this

then this is your task I am not doing this the entire algebra in the class, but I tell you this thing.

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$$\frac{ds}{dB} = 0, \text{ when.}$$

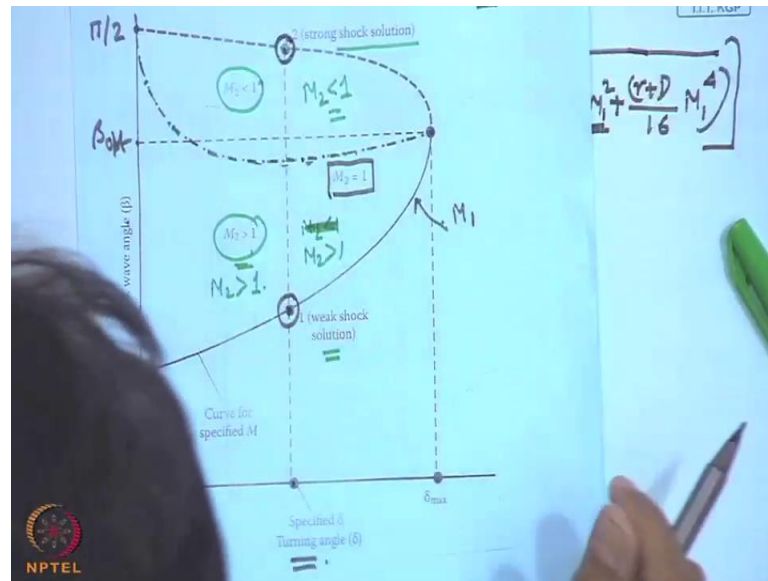
$$\sin^2 \beta_{opt} = \frac{\gamma+1}{4\gamma} \left[ 1 - \sqrt{(\gamma+1) \left( 1 + \frac{\gamma-1}{2} M_1^2 + \frac{\gamma+1}{16} M_1^4 \right)} \right]$$

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If you do it then you get the delta has a maximum that  $d\delta/d\beta$  is 0, when beta is given by this expression  $\sin^2 \beta$ . Now this beta well in book the sometime write the max, but I will write the optimum value of the beta, where it is maximum delta that is gamma plus 1 by four gamma this is relatively compression 1 by gamma M 1 square into 1 minus. You can do it at your home this is a task for you 1 plus gamma minus 1 by two M 1 square plus gamma plus 1 divided by sixteen M 1 four really a little complicated 1.

So, this is the expression for the optimum value of beta, where delta is maximum. And if you put this value and you get the maximum value of delta that expression you can find out and more important thing is that how does it look now. If you make that you will can find out the value of delta maximum and that all depends upon the mach number. So, therefore, the optimum value of beta where delta is maximum and the maximum value of delta which will be found out by replacing this in this equation for the value of beta 1 can get this is not very simple, but straightforward thing is little complicated. So, 1 can get a curve like this that I will explain.

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Now, what is this curve the see very carefully the curve is like this now we can find out we see that from here that now we what we are doing we are drawing the deflection angle with beta or beta with delta for different values of  $M_1$ . So, a familiar curve of delta beta with different values of  $M_1$  now for a given value of a  $M_1$  let us understand the curve as we have seen that the beta is 0 here where sorry delta is 0 here where beta is  $\sin^{-1} 1/M_1$ . So, this is for a particular value of the  $M_1$  this is for a given adverse  $M_1$  clear and  $\delta_{max}$  is a maximum that is the maximum delta that is the delta maximum clear.

Now, this as a value like that this what is this this is the  $\pi/2$ ; that means, this is the normal shock limit this is the mach. So, this is the shock wave angle for a given value of  $M_1$  or all right and this clearly has a maximum value that is  $\delta_{max}$  here, another 1 curve is drawn this 1, this is redrawn because this gives the value of the beta optimum. Now what is this this is the locus  $M_2$ . Now if you find out the locus of  $M_2$  in the beta and delta plane just like that this is the locus of a given  $M_1$  that how beta and delta is changes for a given  $M_1$ .

We know that the  $M_1$  is related to  $M_2$  this  $M_1$  is related with  $M_2$  this  $M_1$  is related with  $M_2$  because  $M_1 \sin \beta$  earlier we did that  $M_2 \sin(\beta - \delta)$ . They work as the mach number approach or inlet or at the upstream for a normal shock and that is after the normal shock the way the normal shock relates the downstream mach number to

the upstream mach number with the same relation we have shown that is valid for this case. So, therefore, if we do that mathematics then we can plot in the figure the locus of the constant  $M_2$  that mean this is 1 and this is done for  $M_2$  is equal to 1 I will explain it. Afterward this is done for  $n$  two is equal to 1  $M_2$  is 1 means  $M_2$  is the mach number after the oblique shock. So, that locus is giving. So, this is the picture.

Now, another interesting picture which is found from this figure is that for a given value of delta that is the deflection angle, we get two values of beta that means, there are two values of beta solution is obtained. So, now, why exercise is not coming exercise not coming given value sorry given value of delta there are two values now it is ok.

Yes.

It is focus all ready it is. So, for a given values of delta we have two values of the shock angel now this delta is specified by the geometry of the flow this delta is determining angle that we will understand solving problems that delta depends upon the geometry of the problem because delta is determining. So, shock wave take place depending upon the tangent of the flow. So, the flow is turn by the geometry of the flow for example, flow passes through wage. So, therefore, this deflection angle infect is the input to the real problem. So, therefore, for a given delta depending upon geometry of the problem we get two solutions from this as you see that two beta value.

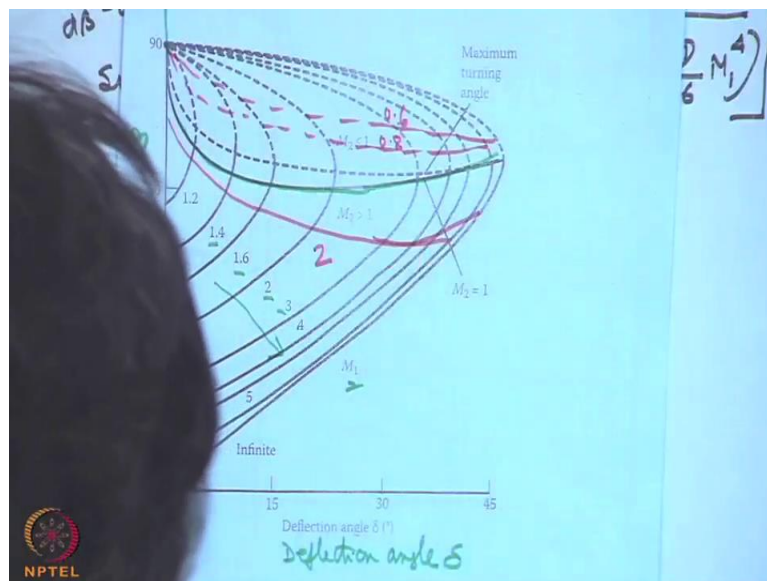
Now, if this two solutions, if we see, we will find out that you see this that is why a mach number 1 curve the locus of mach number 1 is given here  $M_2 = 1$ , mach number means  $M_2 = 1$  and the value of  $M_2$  below this curve. In this plane is into less than 1 and the value of the sorry  $M_2$  less than 1 I am sorry it is  $M_2$  greater than 1. It is already given  $M_2$  greater than 1 a above this we can get the locus of all constant  $m$  which is  $M_2$  less than 1. That means, this solution of this shock wave angle gives a shock where after the oblique shock the velocity that is again  $M_2$  is greater than on the flow remains in the supersonic that all ready we discuss that after the oblique shock the flow may be supersonic may be sonic.

Because there restriction is on the  $M_2 \sin \beta - \delta$  which has to be less than 1. So,  $M_2$  may be greater than 1, whereas this solution that this value of shock angle that higher value gives raise to  $M_2$  less than 1 that mean is if we read the  $M_2$  value here; that means, this is a strong shock because it reduces the velocity to subsonic label and

this is the weak shock. So, therefore, two solutions of beta out of which when the lower value corresponds to a weak shock solution and in the higher value corresponds to a strong strong shock solution. So, usually it is very difficult it depends upon the circumstances which 1 will prevailed.

But it has been found experimentally that flow pass external bodies or the external flow the solution or the shock wave the which shock wave is found there means after the shock the flow become supersonic, but it may be subsonic depending upon the circumstances. So, if nothing is told for our purpose for solving problems, specifically we usually take the solution of weak shock that we will do when will solving the problem. But this is the total physical picture that the shock wave angle dependence with the turning angle for difference values M 1 and at the same time what will be the corresponding value is of M 2 that is the mach number after the shock.

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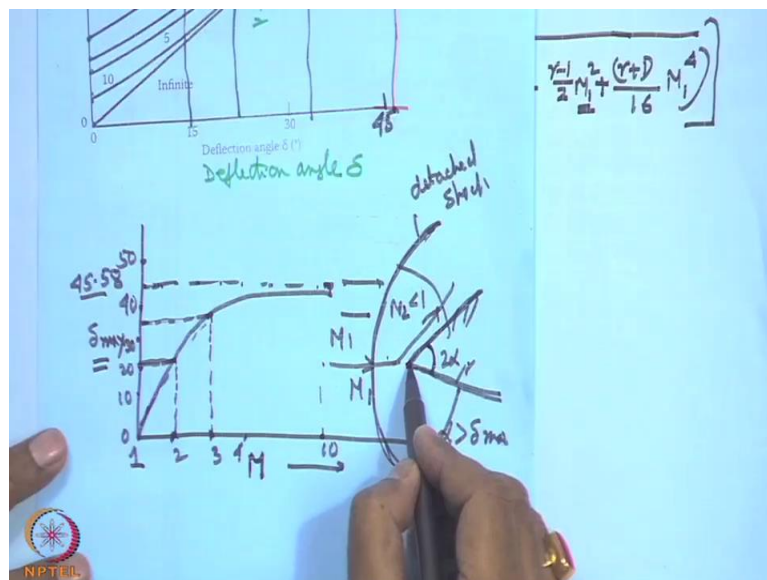


Now, this thing can be shown here in a more compact form like this just you see this 1 is your this curve is your M 2 greater than 1. Now you see this is a curve you can see probably the this is deflection angle deflection angle delta this is deflection angle delta this is shock angle this is beta now you see this are the families of curves for different m 1. So, this is what now 1 thing is that this is 1 by M 1 for given m given 1 this is the for a given M 1 this are all corresponding mach wave limit, but the maximum limit is pi by two for all initial mach number.

So, therefore, all these curve merges ninety degree. So,  $M_1$  values goes on this is  $M_1$  values goes on increasing in this direction goes on increasing in this direction this obvious because if you see here the  $\sin^{-1} 1/M_1$  is small when a  $M_1$  is high. So, therefore,  $M_1$  increasing in this direction you see this is this one point four one point six two three. Now  $M_2$  is equal to 1 this demarcates two series of two categories of mach number 1 category of mach number is here. I will draw these are the mach numbers these are the mach number greater than 1  $M_2$  this less than 1 point eight point six like that where as we get mach number here which are  $M_2$  is greater than 1 that means.

Let for example 2. So, therefore, this curve this curve is  $M_2 = 1$  demarcates, the region of strong shock and the region of weak shock clear and this one, this curve is the locus of the maximum deflection angle this is the locus of the this. Again coincides at the maximum value of the delta that I will come afterwards that delta max. Now if what is this value, why I stop why the curve is like this this is because if delta max is found out with  $M_1$  then what is the value delta max by  $M_1$ . If you plot you will see that there is a limit; that means, if we now find just a minute I will show you that if we plot the delta max which calculate delta max.

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And we plot delta max verses m then interesting result will be there from 1 delta max for example, delta max 0 ten this scale. I am just doing otherwise it will be easier thirty forty degree fifty then somewhere here for air. It is calculated the values this values represent

for air a limiting value that forty five point five; that means, with increasing  $M_1$  like that somewhere it is near ten or something some value ultimately asymptotic; that means, the maximum angle forty five. That means, the turning angle cannot be more than forty five point five value whatever may be the value of  $m$  whatever may be the value of  $m$  with  $m$  the turning angle changes that we have already seen the turning angle changes with  $m$ .

For a given shock wave angle you see we can have the turning angle changes with  $m$  with  $m$  changing the turning angle changes. But if we this is the curve if we now plot the  $\delta_{max}$  the maximum turning angle; that means, this is the maximum turning. For example,  $m = 1$  will choose this  $m$  this  $m$  is equal to two this  $m$  curve what is the maximum turning angle this is the maximum turning angle. If you choose this  $m$  this is the maximum turn turning  $m$  is reduced it if  $m$  increases this turning angle increases a  $m$  is increasing in this direction I told you. So, as we increase the  $m$  the turning angle increase, but this becomes very closed to each other. And finally, this is totally become compact and finally, it reaches a value of forty five point five this is forty five point five eight for yeah this is a representative figure for that mean.

If we increase the mach number very high also this will not reach go more than that. What is this; that means, this physically implies one thing that if the angle is more than that just I am telling you two things come into picture. For example, the mach number is this  $M_1$  then I find out this is the maximum  $\delta_{max}$  mach number is this this is the maximum turning angle. For example, if mach number is two, for example, I know this is the maximum turning angle if my turning angle is more than that. What will happen do you know that if turning angle is more than the maximum; that means, a flow through an angle what is this turning angle turning angle is the flow is turned like this.

So, if the wedge angle for example, this  $2\alpha$ ; that means, the  $\alpha$  is more than  $\delta_{max}$  for a given  $M_1$  this which gives you the turning angle actually for flow, then what happens for a given mach number the shock wave is not attach to this. There becomes there occurs a detach shock, this is known as detached shock. The flow like this the detach shock is like this the flow takes place and this is a typical the detach shock this is  $M_2$  is less than 1 this is  $M_1$ . And this becomes curve this is not at the this is not attach to the wedge this is detach of this becomes curve and this type of shock is not amenable to be mathematical analysis we have done for a straight oblique shock.



So, this is out of this scope of our discussion, but you must know that this is there, but another interesting fact that if a body of this shape. For example, of any shape, which gives a turning angle given turning angle is accelerating within the air; that means, air flow is velocity is changing if it accelerated that this may. So, happen and when the lower mach number is turning angle may be more than that, but air mach number if you increase the mach number the turning angle increase a when it accelerate and achieves a higher mach number then it may be such that the turning angle may be lower than delta max.

So, therefore, when a body accelerated sometime it is observed that the shock is detached that lower mach number again attached higher mach number we will do we will solve some problem. So, that we can get some filed of this let us solve some problem then only you will let me see that I have some figure let me have a check that I have some figure which I will show like to show you hah this is the oblique shock is that shock curves shock detach shock ok

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nd a temperature of  $30^{\circ}\text{C}$  passes over a component of an aircraft that can be modeled as a wedge with an included angle of  $8^{\circ}$  that is aligned with the flow, i.e. the flow is turned by both upper and lower surfaces of the wedge through an angle  $4^{\circ}$  leading to the generation of a oblique shock wave. Find the pressure acting on the wedge surface.

$M_1 = 2$     $\delta = 4^{\circ}$     $\beta = \dots$   
 $\beta = 33.4^{\circ}$  (Wentz shock solution)  
 $M_{N1} = M_1 \sin \beta = 2 \sin 33.4^{\circ} = 1.10$   
 $p_2/p_1 = 1.245$   
 $p_2 = 1.245 \times 80 = 99.6 \text{ kPa}$   
 $M_2$  (from Normal shock)  $= M_{N2} = M_2$

Let me solve a problem and then we will discuss the next part let us solve a problem first this is a first problem example 1 air flowing at mach number two with a pressure of eight kilo pascal's. And a temperature of thirty degree celsius passes over component of an aircraft that can be modeled as a wedge with an in included angle of eight degree that is aligned with the flow. That is the flow is turned by both upper and lower surfaces of the

wedge through an angle of four degree leading to the generation of a oblique shock wave. Find the pressure acting on the wedge surface. Just now I told you that this is an example this type now what is this there is a wedge shape; that means, this is the only the simple example the total angle included angle is eighty. That means, this angle is four degree this side four degree this side now this is the incoming air  $V_1$ . Let the pressure  $p_1$ , the temperature  $T_1$ , mach number is  $M_1$ .

Now, what happens the flow will be turn to four degree both this sides; that means, 1 side will just; that means, this is the flow which will be deflected because the stream line what will happen the stream line will be going like this. So, because of this two edge placed only the stream line is deflected and if the approach mach number is greater than 1 this defection of stream line the because of the presence of the wedge creates the oblique shock that is the concept that is the basic concept. So, this is the forty this is the basic concept and this is the basic idea of the problem  $V_2$  that is all. So, what we have to find out this is given that that is aligned with the flow, that is the flow is turned by both upper and lower surfaces. This is upper and lower surfaces of the wedge through an angle of four degree leading to the generation of a oblique shock wave find the pressure acting on the wedge surface; that means, this surface  $p_2$ .

So, this is same in both the side and just showing it here mach number will be a  $M_2$  temperature will  $t_2$ . So, how to solve the problem problem is very simple now  $m$  is 2. So, first of all what we have to do with this  $M_2$  the first job with this turning angle; that means, a  $M_1$  here is a 2 turning angle here is four degree. So, with this  $M_1$ , with this turning angle, we have to calculate the beta either from the equation, you can tell sir I will calculate those who are very much the interested. And using the analytical equation that analytical expression they can solve analytical expression or you can see that chart because it will be easier that we  $M_1$  to and delta four if we see this chart  $M_1$  two we have to go for the different values this  $M_1$  2.

And delta four degree 1 can find out what is the value of beta here beta also get two values 1 is for weak another is for strong and we take the strong solutions and from this figure we get the value of  $m_1$ . So, from this, what is the value of  $M_1$  delta 4  $M_1$  sorry beta the value of beta is thirty three point four degree. That means, we take the weak shock solution weak shock when nothing is stored this is just a methodology we

steep we can take strong of solution and find the result and which 1 will be correct that will be validated by the experiment.

So, it is difficult to take for the time being I am taking the weak shock solutions. So, therefore, the weak solution lower value of beta from the graph we are taking that the beta thirty where beta beta is known things are done. That means, what is the procedure then we know  $M_1 \sin \beta$  we find out  $M_1$  is two into sin thirty three point four degree, and what is that value  $m_n$  that is  $m_n$  that is stored at  $m_n$  is  $M_1 \sin \beta$  and that become equal to one point one zero. Now we see the normal shock wave.

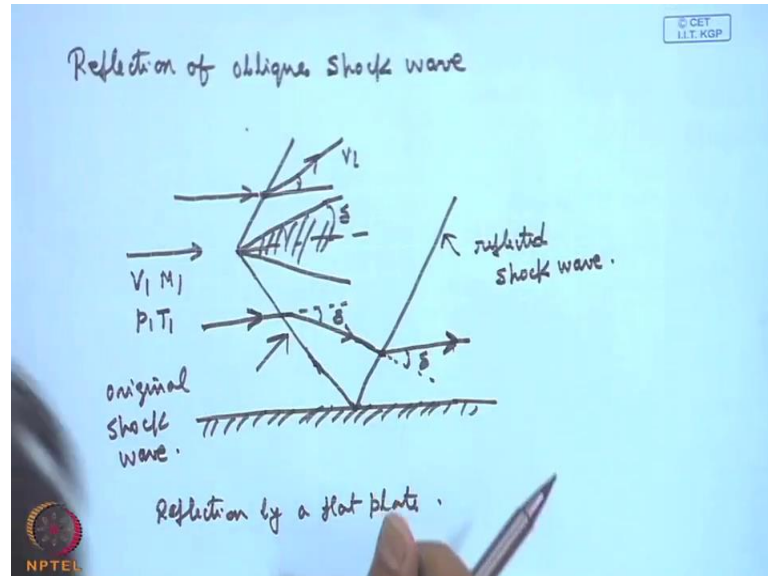
So, from the normal shock wave with this as the  $M_1$  plus plus as I showed you that 1 the normal shock wave there are will at  $M_1$  the column for a given  $M_1$  we can find out the value of  $p_2$  by  $p_1$ . So, for this  $M_1$  this as the approach  $m$  to the shock for the inlet  $M_1$  to the shock we you can find out the value of  $p_2$  by  $p_1$  and that values is 1 point two four five that value is 1 point two four five. So, this value if you just multiply what is the date  $t$ . So,  $p_2$  is 1 point two four five into eighty eighty kilo pascal's. So, the value will be I am telling you the value I have done it is ninety nine point six kilo pascal's ninety nine now here this is the problem that pressure acting on the wedge.

If it is store find out the velocity or the mach number here, what is the mach number here. I will find out the mach number after the flow. How for this  $M_1$   $m_n$  from the shock table to find out  $M_2$  that is not ask for, but I am adding this thing the book from where I have taken the problem which is not ask for now. If I have to find out the  $M_2$  here, what you have to do you have to find out  $M_2$  from shock table that  $M_2$  is what from shock table which we get from normal shock table is actually what is, the present case is  $m_n$  two is, actually  $m_n$  two that is equal to  $M_2$ , that is  $M_2$  from normal shock table is  $m_n$  two and that is equal to  $M_2 \sin \beta$ . That means, we know that for a oblique shock it is the sin from component of the  $n$  act as the inlet and outlet  $n$  for a normal shock.

So, when we solve the problem as equivalent to a normal shock always we have to read  $M_1$   $M_2$  with respect to the normal shock which is nothing, but  $m_n$  1 that is  $M_1 \sin \beta$  and for  $M_2$  it is  $\sin \beta$  minus dell. So, when we know both beta and delta we can from this we can find out the actual into for this problem this this is not is not told we have to find out. So, it is. So, this is 1 problem that we can discuss that a for the shock

then we will solve another problem oh then before that I think we have to discuss the reflection of shock wave reflection of shock wave we will discuss

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Now, the reflection of oblique shock wave what is that reflection of oblique shock wave oblique shock wave reflection of oblique shock wave what is meant by reflection of oblique shock wave. Now, it is very important that how the shock wave is reflected now let us consider a case like this physically first of all that there is in wedge there is an wedge there is a wedge it is the. So, there will be shock flow is approaching which some  $V_1 M_1$  b 1 t 1 now here if I flow the velocity.

So, it will go like this and this will be the  $V_2$  turning this is the half angle here also what happen the flow will turned like this now if there is a plate here flat plate for example,. So, first I consider reflection by a flat plate simple case reflection reflection by a flat plate then what will happen physically. You see that this thing will ultimately become parallel to the plate afterward the stream line will become parallel to the plate this means that the shock comes here this is the shock wave shock wave trays the plate and is being reflecting such way that this when comes to this reflected shock wave.

So, this is the deflection for example, this value is delta. So, this is the deflected this is again this is this direction deflected in the same value delta to make it paragraph; that means, initially this was the flow direction which was parallel to the plate. For example, it is deflected by the delta by the original shock wave this is original original shock wave

well and this is the reflected shock wave reflected shock wave reflected shock wave well. So, therefore, you see the original shock wave deviates this 1 by a angle delta that is the turning angle of this that is the angle impose by this wedge shape body.

So, then what happen this side is a plate then again the flow stream will get reflected by the reflected shock wave of a same angle of deflection to make it parallel. This is a typical picture very simple case that how does a oblique shock wave gets reflected by the flat plate let us solve a problem based on that then only this thing I think deflection of shock wave will be better clear from a problem.

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Reflection of oblique shock wave

$M_1 = 2.5$   
 $p_1 = 60 \text{ kPa}$   
 $T_1 = -20^\circ\text{C}$   
 $\delta = 4^\circ$   
 $\beta = 2^\circ$   
 reflected shock wave at  $\delta = 4^\circ$   
 $M_2 = 2.5$   
 $\beta = 26.6^\circ$   
 $M_{N2} = M_2 \sin(\beta - \delta) = 1.12$   
 $\frac{p_2}{p_1} = 1.336; \frac{T_2}{T_1} = 1.087$   
 $M_{N2} = 0.897 = M_2 \sin(\beta - \delta)$

original shock wave

Plane wall

reflection by a flat plate.

ms the oblique on flat and is shock wave

Let us see this problem example two when the air flowing the m is equal to two point five mach two point five with a pressure of sixty kilo pascal. And the temperature of minus twenty degree celsius passes over a wedge, which turns the flow through an angle of four degree simple case. That means, there is oblique shock takes place attach oblique shock to the wedge this is again told in the problem that leading to the generation of oblique shock waves. This is true 1 of the oblique shock wave as I adjust just kind of same physics is being repeated in the problem with numerical data impinges on a flat wall which is parallel to the flow upstream of the wedge and is reflected from it find the pressure and velocity behind the reflected shock wave.

That means this is the problem find the pressure and the; that means, precisely this is the problem let this is 1 let this two let this is 3. So, we have to find out what is p 3 now what

is giving  $p_1 / M_1$  is two point five well  $p_1$  is sixty kilo pascal and  $t_1$  is minus twenty degree Celsius. These are the given data and  $\Delta$  is  $\Delta$  in both the cases this four degree it becomes again parallel to its original degree which is parallel to the wall this is the wall plane wall plane wall problem is very very simple. First we find out the condition at two, now  $M_1$  is to first first due to is to find out beta; that means, at  $\Delta$  is equal to four degree and  $M_1$  is to two point five by find beta the weak shock solution beta very lower value of beta and from the figure.

And this becomes is equal to twenty six point six degree; that means, I just search for this  $M_1$  is to two point five I go somewhere here that is the  $M_1$  is two this is 3. And with the beta four degree, we find out this find out this  $M_1$  is to point somewhere it is thirty. So, it may be twenty two point six; however, I am not doing it here, but this was found as the this is the. So, whenever I know beta. So, this side  $m_n / M_1$  is  $M_1 \sin \beta$   $m_n / M_1$  is  $M_1 \sin \beta$ , whose value is one point one two now. I can rewrite from normal shock table that this one point one two.

What is the value of  $p_2$  I can find out  $p_2$  by  $p_1$   $p_2$  by  $p_1$ . And that  $p_2$   $p_2$  by  $p_1$  that is found out is equal to one point three three six. And  $t_2$  by  $t_1$  is found out this is all from normal shock table because now it is equivalent to a normal shock I am reading  $M_1 \sin \beta$  that 1 point 1 two as  $M_1$  in the normal shock table to find out  $p_2$  by  $p_1$  and  $t_2$  by  $t_1$ . Therefore, I can find out  $p_2$   $t_2$  here because  $p_1$  and  $t_1$  is are giving now what will happen I have to find out the  $M_2$  here. How to find out  $M_2$  here corresponding to this  $m_n / M_1$  I know an  $m_n / M_2$  from the table that is the  $M_2$  from the sock table.

And that value is 0 point eight nine seven; that means, this is the value of  $M_2$  in the shock table; that means, the downstream mach number corresponding to an upstream mach number of 1 point 1 two minus ten corresponding to an upstream mach number of one point one two. I find the downstream mach number  $m_n / M_2$  which downstream mach number  $M_2$  which is are  $m_n / M_2$  which is actually  $M_2 \sin \beta$  minus  $\Delta$  here beta I know twenty six point six. This is value I know the twenty six point six this is this is the twenty six point six twenty six point sorry six this is twenty six point six this is the beta we know.

So, delta is four degree. So, we can find out m 2. So, what is M 2 m two find out as point eight nine seven I write it sin of twenty six point twenty six minus four degree and that become equal to two point three three. Now is another problem that second reflected shock wave that M 2 is two point that is approaching with a M 2 - two point three three four which is being deflected this way by delta. So, therefore, now again with delta is equal to four degree and this M 2 as m 1; that means, approach mach; that means, this M 2 that is here it will act as M 1 for this table we find out what is beta that means.

For this table when we will use to find out this table this table this mach number will be this 1 this two point 3 3 four. So, for this the shock wave angle will be twenty eight point five degree; that means, for the reflected shock the shock wave angle is different. So, shock wave angle is twenty eight point five degree

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through an angle of  $4^\circ$  leading to the generation of oblique shock waves. One of the oblique shock waves impinges on a flat wall, which is parallel to the flow upstream of the wedge and is reflected from it. Find the pressure and velocity behind reflected shock wave

$$M_{N2} = 2.334 \sin 28.5^\circ = 1.113 \quad \frac{p_3}{p_2} = 1.297 \quad \frac{T_3}{T_2} = 1.072$$

$$p_3 = 104.0 \text{ kPa} \quad T_3 = 296 \text{ K} \quad a_3 = \sqrt{\gamma R T_3} \quad M_{N3} = 0.9$$

$$V_3 = a_3 M_3 \quad M_3 = \frac{M_{N2}}{\sin(28.5^\circ - 4^\circ)} = 749 \text{ m/s}$$

This is now this is twenty eight point five degree. Now turned again by delta is four degree with this twenty eight point five degree I decode again m n two this m n two is what this is. With this approach a three point three three four, which is our M 2, which I am using as if M 1 in this shock table two point 3 3 from approach or to find out the value of beta with the delta M 1 that figure delta beta for different the M 1 that figure.

So, m n two is two point three three four into sin of this eight point five two I think you will understand this can clearly; that means, this becomes a sin sin component the normal component of the mach number though mach number is though. You should use that

component mach number is a scalar quantity it is the ratio of the velocity, but this is the we can understand that is sin component of the mach number which act as the normal shock mach number. Now with this this  $m_n$  two this becomes a equals to I tell you sin this is sin twenty eight point five and this becomes a is equal to 1 point 1 one 1 3 something like that that you see 1 point 1 as I found find the solution.

Now, again this mach number in the shock table now this is the approach mach number for the reflected shock I can find out the pressure ratio; that means, I can find out the pressure ratio. That means, I can find out the  $p_3$  by  $p_2$  and this  $p_3$  by  $p_2$  I can find out 1 point two nine seven and I can find out  $t_3$  by  $t_2$  which will be read as  $t_2$  by  $t_1$ . They are in the shock table; that means, I will use this approach mach number in the shock table as  $M_1$  and find out  $p_3$  by  $t_3$  earlier we find out  $p_2$   $t_2$  because we knew that  $p_2$  by  $p_1$  and  $t_2$  by  $t_1$ .

So, therefore, we find out  $p_2$   $t_2$  multiplying with  $p_1$  and  $t_1$  then  $p_3$  by  $p_2$  and  $t_3$  by  $t_2$  is known. So, we can find out  $p_3$   $t_3$ . So, final result is that  $p_3$  is equal to hundred four point 0 kilo pascal and  $t_3$  two ninety six k now velocity has to be found out. So, find the pressure and velocity behind the reflected shock, how to find out velocity velocity is very simple you find out and 3 at root over gamma  $r$   $t_3$  well root over gamma  $r$   $t_3$ . And when you get the velocity a 3 you get the  $V_3$  as a 3 into  $m_3$  what is  $m_3$   $m_3$  is the this 1  $m_n$  two you get here.

And you find out the corresponding value of  $m_3$ ; that means, you find out here a value of  $m_n$  3 from this. With this value of the approach mach number and that  $m_n$  3 value equals to I tell you zero point nine one zero eight zero nine zero zero eight zero point nine 1 zero eight. Then you find out  $m_3$  decoded that is  $m_n$  3 is  $m_3$  divided by that is  $m_n$  3  $m_3$  into sin beta minus delta. So, therefore, sin beta this beta is the beta for the reflected shock wave; that means, this beta twenty eight point five delta remains the same in both the cases. So, therefore, you get a value of  $m_3$  two point 1 seven fine. So, this  $m_3$  if you put a 3 if you put you get a value of  $b_3$  seven forty nine meter per second.

So, therefore, we find out the pressure temperature and everything. So, see that a mach of two point five is ultimately reduce to mach of two point one seven. Because we have chosen always the weak solution always the weak solution I am also not sure which 1



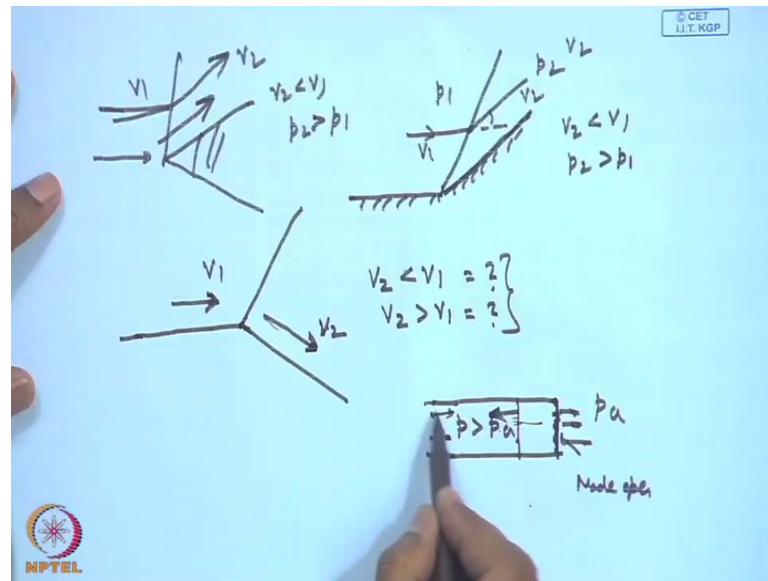
will prevail, but we can have this strong solution is just only a change in the value and we will get the same we will follow the same procedure, but finally, experiment will tell which 1 is correct. So, this is the way we have to solve the problem. So, therefore, it is clear now again I am telling you for closing the lecture on oblique shock that the oblique shock has to be treated always as a normal shock by considering the fact that the this the velocity component parallel to the oblique shock does not change.

So, therefore, the normal component of the velocity acts as the same thing that is the normal that is the velocity of a approach for a normal shock. So, we consider this mach number  $M_1 \sin \beta$  if  $\beta$  is the shock wave angle and  $M_2 \sin(\beta - \delta)$  if  $\delta$  is the deflection angle as the mach number before and after the shock. And we can treat this as a normal shock that is the main thing and  $\delta$  the turning angle is impose by the problem geometry for which 1 has to find out the  $\beta$  depending upon the inlet mach number  $M_1$ . This reduce mach number is for using the normal shock table; that means, to convert the problem to a normal shock problem. So, this is the total thing for the oblique shock.

So, therefore, today I will stop for the oblique shock discussion and now I will start the another chapter I do not know whether the time is not or is there or not rather let me start at least the now the next question is that. So, far we are discuss about a discontinuity of flow field where the pressure changes in a way that pressure is decreased. And the supersonic flow is change to either the supersonic flow or a subsonic flow in the form of the deceleration. Either a supersonic flow or a subsonic flow in a form of deceleration for a normal shock it has to be subsonic for a oblique shock. It may be subsonic, it may be supersonic, but it has to be decelerated where pressure temperature will increased this type of wave is known as compression wave where the velocity is decreased then pressure is increased.

And this type of wave may be stationary with respect to a static frame of a frame of reference or with respect to a coordinate frame of reference. You can say and this can be moving also and in that case moving with a constant velocity we can attach the coordinate frame with the shock wave and we can analyze, but there are certain cases where this is not this compressor wave does not occur. For example, I tell you just consider that

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So, far we have considering a wedge shape for example, a wedge shape where the the direction is change like that let us consider a we have consider shape like this. So, this is the case where we have seen that there is a this we have not consider there will be a shock wave like this oblique shock wave where this will be there will be change like this the deflection ok.

So,  $p_2$  this is  $p_1$  this is  $V_2$  this is  $V_1$  and it is true that  $V_2$  is less than  $V_1$  and; obviously,  $p_2$  this as is improve  $t_2$  is  $t_1$  here also the oblique shock wave is there. So, that sorry that is the and  $V_2 < V_1$  and  $V_2$  is less than  $V_1$  this is the wedge and  $p_2$  is greater than  $p_1$ , but what happens is if a supersonic flow approaches a convex corner it may be smooth it may be sharp what happens. That means, this what happens how what will happen that if this is the thing what will happen will there be shock wave and the velocity will be turned like this and  $V_2$  will be less than  $V_1$  or  $V_2$  will be greater than  $V_1$  to which 1 will prevail which 1 will prevail. So, this is here you can show from the geometry that  $V_2$  will be greater than  $V_1$ .

Another example, I am telling in that if there is a pipe this pressure is an atmosphere pressure this pressure is  $p$  which is much greater than atmospheric pressure. There is a bulk and diameter from whatever you tell it is suddenly captured it is made open made open the fluid is suddenly come out. Then what will happen the side the pressure will be

released and velocity will suddenly slowly slowly in a compressible flow depending upon the extend of compressibility the velocity in the upstream will slowly the reduced.

And in the pressure the upstream will slowly reduce. So, what will happen. So, this means and expansion wave will move in this direction here also this will be a static expansion wave means this  $V_2$  will increase, whereas the pressure will fall. Here also the fluid will start; that means, there will be a; that means, what will happen in the upstream velocity will be generate, because the flow has static suddenly opened will take in an incompressible. For the same distance will be instantaneously felt here, but in a compressible fluid is not felt here or the pressure felt here is  $p_a$  then immediately the pressure will not be felt here as  $p_a$ .

So, therefore, this will take time; that means, there will be a propagation of which is an expansion wave why which makes the pressure low as it passes through and makes the velocity be high, because it was zero. Similar thing happens this is a stationary wave where this  $V_2$  will be higher than  $V_1$ . I will explain this in this case if it is a convex wedge or a convex corner rather sharp or smooth corner these type of oblique shock this is oblique shock will not occur this is oblique shock. So, this is oblique shock what happened this will be expansion here we will get expansion wave which I will discuss in the next class I think time is up. So, today will finish it here.

Thank you.