Introduction to Fluid Machines, and Compressible Flow
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Lecture - 38
Oblique Shock Part I

Good morning, and welcome you all to this session of the course. Now last class we mentioned about the reflection of normal shock wave from the close end of the act, and just now we solving a problem to understand how the fluid properties are changed, because of this reflected shock wave well.

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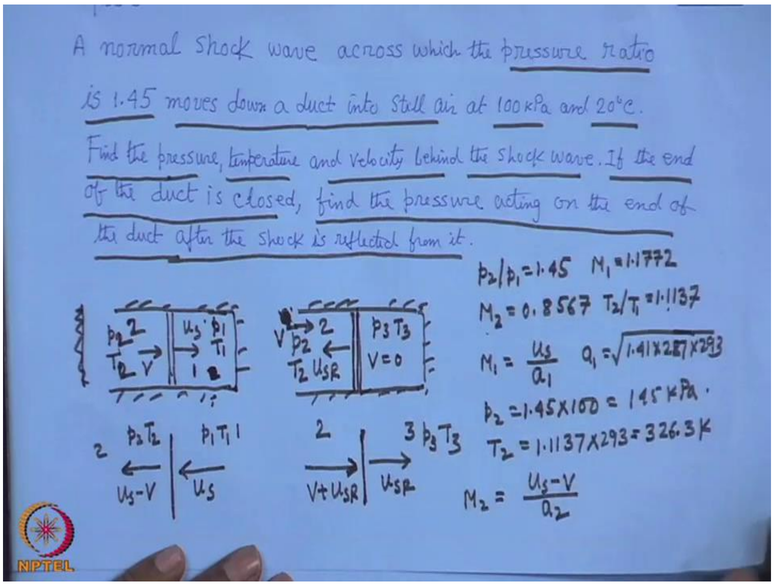

So, let us then concentrate on a problem the problem like this a normal shock wave across which the pressure ratio is 1.45 pressure ratio is moves down a duct into still air at 100 kilo pascal, and 20 degree celsius this is again a repetition of the earlier work problems that day moving pressure wave in still air find the pressure temperature, and velocity feel behind the shock wave that is the induced by the shock wave velocity induced pressure, and temperature rise these all.

If the end this is the new part of the duct is closed find the pressure acting on the end of the duct after the shock is reflected from it. Now let us make the physical model like this let us make this way know let us make this physical problem like this, this is the first 1 that this is the duct close end, and let this is the physical problem it is first moving with a velocity u s. Let this section is 1 this section is 2 here the pressure is p 1 temperature is T 1 this is this is actually sorry this is actually 1 2 is wrong here I write 1 here, because with respect to the shock this will be the other way I am writing one. So, this will be p 2 T two; that means, this is the pressure, and temperature after the shock as pass through it, and induces velocity V as we did earlier, and here this section is 1 sorry, pressure p 1 T one.

So, this is the initial before reflection of the shock, then after that what happens shock is reflected shock is reflected shock is reflected let us consider this theme shock theme shock it is reflected with the velocity u s r now what is the picture here that if we translate these in terms of the physical model with respect to the coordinate system attached to the moving shock, then what will happen that the similar 1 it is coming with the velocity u s at a pressure p 1, and temperature T 1 that is the static pressure, and the temperature of the still air which was initially at rest. So, therefore, with respect to the shock as usual u s, and then it goes out with a velocity this is the section 2 p 2 T 2 two one.

And these goes out with a velocity u s minus V as we did earlier, and when we make this 1 this is coming, then sorry what happens when reaches, sorry when this reaches the end of the duct the entire duct is at 2 state 2 that is p 2 T 2 alright. So, therefore, this side when it is moving again after the reflection this side is the undisturbed side is 2 p 2 T 2 try to understand, and this side led it is p 3 T 3 where the velocity V is made zero finally, this is the problem that when it is reflected back from this end with the velocity is zero; that means, if we make the picture with respect to the shock wave, then what will be this picture here the velocity is zero here it is coming with a velocity u s r.

But earlier these velocity there is a velocity V 2 here there is a velocity I am sorry V 2 here, because when these moves here situation of the duct is T 2 p 2, and the velocity V v 2 rather we have denoted it by V simple V v. So, therefore, with respect to the shock the velocity will be what; that means, this is u s r coming in this direction. We will give by opposite direction; that means, this will be the section 2 from where it is the upstream of the shock relative to the shock this is the upstream where the velocity is V plus u s r clear, and the downstream velocity which was zero in actual case when the shock was moving V 2 will be u s r, and this is 3 this is p 3 T 3. Now, if we can draw this diagram, and can treat this thing properly upstream, and downstream of a stationary normal shock.

Then the problem can be done very easily by the ahh application by by using the normal shock table. So, therefore, what we do first we find out the pressure temperature now this problem p 2 by p 1 is giving p 2 by p 1 is pressure ratio is 1 point four five. So, p 2 by p 1 is 1 point four five it is giving at this p 2 by p 1 1.45. What happen? this p 2 by p 1 at 1 point four five we have a mach number m 1 which is equal to 1 point 1 seven seven 2, and a mach number from the shock table as I told in the last class zero point eight five six seven, and corresponding T 2 by T 1 equals to 1 point 1 1 3 seven. Now what is this mach number 1 with respect to this, now this has to be translated to hear the mach number 1 is m 1 is u s this is the velocity u s by a one. So, what is a 1 a 1 is root over gamma r T 1 1 point four 1 into 2 eighty seven into what is T 1 T 1 is giving twenty; that means, 2 ninety three. So, this is m one.

What we have to find out first up all we have to find out the pressure very simple pressure p 2 is 1 point four five into hundred that is 1 forty five kilo pascal this is the pressure pressure is 1 forty five kilo pascal, then T 2 as we know T 2 by T 1 is 1 point 1 1 3 seven into T 1 is 2 ninety 3, and that becomes equals to that becomes equal to 3 twenty six point 3 k now a 1 is known to us again the same formula that to find out the first we find out V these V induced by how it will be found out it is very very simple that m 2 is known m 2 what is m 2 m 2 is just you see here m 2 here m 2 is u s this picture u s minus V divided by a 2 well.

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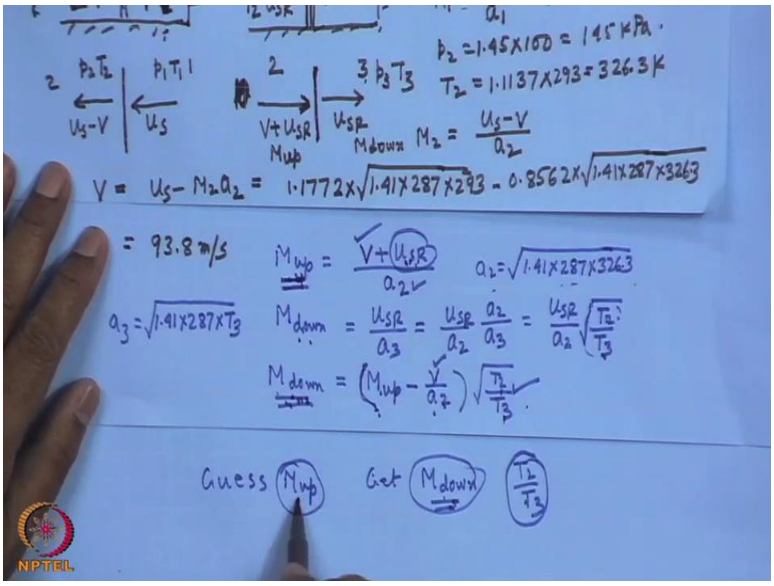

So, therefore, we can write now little bit I check it, then m 2 is this. So, therefore, V is equal to what V is equal to u s minus m 2 a 2 what is u s u s we have got it is clearly seen u s now u s is what it is it is seen no u s is this 1 m 1 into this one. So, this is m 1 m 1 is 1 point 1 seven seven 2 this is found from the table into root over these velocity this sounds 1 point four 1 into 2 eighty seven into 2 ninety 3 this is u s minus m 2 m 2 is zero point eight five six 2 into root over gamma are T 2 1 point four 1 into 2 eighty seven into T 2 T 2 is we are already find out 3 twenty six point 3 well.

So, therefore, we get the value of V 2 or V here the value of v. So, therefore, the value of V as I have calculated I tell you ninety 3 point eight meter ninety 3 point eight per second now you have to see this 1 this is reflected wave now here if you are write here 1 thing the mach number if you write first of all here what you will write in this case the first of all you will write the mach number now the mach number 3, and 2 here actually these 2, and these 2 will be difference. So, that sometimes ahh well this this actually 3, and this is the stage now I will not use this m 2.

Now rather this will be confusing already m 2 I have used here. So, therefore, in this situation let us consider this as coming m m upstream sorry here this is the upstream this is the upstream called as m upstream, and here m downstream m upstream, and m downstream I think I should use a in the a upstream, because this 2 3 will be unnecessarily confusing now with respect to this shock wave what is m upstream m upstream is now V plus u s r let us see nothing is known now pressure ratio only this pressure p 2 is known V plus u s r divided by a 2 a 2 is of course, known where a 2 is root over 1 point four 1 gamma into, because temperature is already determined 3 twenty six point 3.

Now what is m downstream m downstream; that means, here is equal to u s r by a 3 where a 3 is here I write is root over gamma r T root over 1 point four 1 2 eighty seven into T three, but T 3 I do not know T 3 I do not know let it be like that. So, this can be retain as u s by a 2 a 2 is known into a 2 by a 3 u s r by a 2 into a 2 by a 3 now this can be retain as u s r by a 2 into root over this is in terms of temperature T 2 by T 3, and this can be little arranged, because u s r by a 2 is m upstream minus V by a two. So, m downstream can be retain while I am writing I will tell you afterword this is some algebraic rearrangement m upstream this is u s r by a two; that means, m upstream minus V by a 2 this is u s r by a 2 into root over T two; that means, the upstream, and downstream mach number is related to each other this is algebraic rearrangement, but you see here things are not known, but this can be solved algebraically.

This is because we know T 2 we do not know T 3 we do not know u s r we know a 2 we know T two, but at the same time we have the relationship from the shock tables between m downstream, and m upstream, and also we know the relationship between T 2, and T 3 in terms of the m upstream. So, all these relation together if you write number of unknowns, and the number of equation will be same, and you can solving by a program by writing a program, and you can solve in a computer, but I can tell you a very simple way of solving it by iterative technique using the shock normal shock table now is the most simple way of solving you first make a guess of upstream mach number make a guess any guess. So, as long as soon as you make guess of upstream mach number try to understand here.

U s r a 2 a 2 you know. So, you get the value of here V by a 2 you are knowing. So, since you know this V by a 2 you unknowing. So, u s r by a two. So, you know the value of whenever upstream mach number your assuming you know the value of T 2 by T 3 as soon as you know the value of T 2 by T three. So, you can find out T 2 already you know. So, u s r, and you find out m downstream as you can calculate m downstream from here, because V value you know again I am telling you guess a up streaming a 2 you know you find out u s r.

So, as soon as you find out u s r what you do now let it be there now as soon as you guess in upstream you get T 2 by T 3 from the shock table. So, you this one. So, you find out the m downstream by this equation corresponding to the m upstream there is 1 downstream guess m upstream; that means, you get guess m upstream you get what you get get m down first trial T 2 by T 3 now if you know T 2 by T 3 without this m down that you can keep it you can, then find out, because V you know a 2 you know, because a 2 is this root over gamma r t. So, you can calculate by this formula m downstream you compare with this m downstream if this m downstream is not equal to the m downstream obtain from the chart that we guess value of m upstream, then you modify the m upstream by using the calculated m upstream, and repeat the calculation.

And make a convergence. So, that every think you can get; that means, how you can formulate the problem the solution proceeded, because you solve if you write all the shock equations relation between m upstream, and m downstream of a normal shock wave all the unknowns will be evaluated from the number, because there were simultaneously algebraic equations, but simple solution when you have this shock table by guess m upstream find out m downstream u s r is known. So, you can find out the m ups downstream. So, m downstream this way you can find out. So, when m downstream is your finding out, then these m downstream will be compared with corresponding m downstream from m upstream, because guess upstream will be T 2 by T three. So, T 2 by T 3 is known V by a square is known for a guess you get m upstream you get m downstream.

Again for the guess m upstream you get a m downstream from the shock table shock table in downstream, and these m downstream compared, and then you change the m upstream by the computed m downstream, and again repeat the calculation this clearly understood by hope. So, this way 1 can solve a problem of reflection; that means, here 1 thing is made clear I tell you a value. If you make this type of iteration, then you get a value of ultimately p 3 I write here the value of p 3 equals to 2 zero seven find the pressure acting at the end after the shock is reflected you can find out p 3 also only p 3 is calculated; that means, p 3 becomes equal to here 2 zero seven kilo pascal; that means, when the shock passes here, it was that hundred kilo pascal shock induces a velocity, and changes the pressure that pressure equal to 1 forty five kilo pascal when it is reflected back, then again velocity is created velocity is coming to zero, and then again the pressure changes in this side close to the ultimately this pressure will be there throughout when this will come in this end, then the pressure will be 2 zero seven kilo pascal. So, we have to set the problem with respect to the stationary with respect to the shock by imposing a velocity in the opposite direction.

Things becomes steady, and a stationary normal shock wave, and then you write the equation upstream downstream mach number solved it with help of the shock table or with the algebraic equation.

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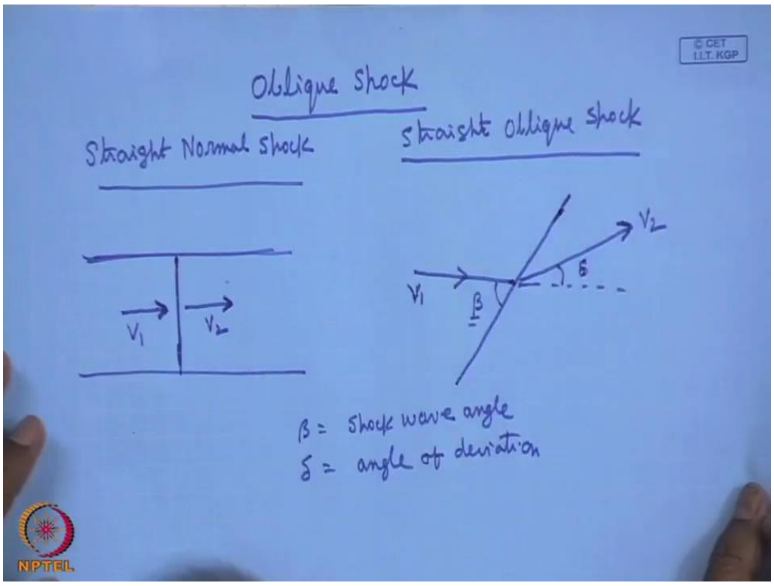

So, with this I will well I will finish this normal shock chapter now I will go to another topic which is oblique shock, which is oblique oblique shock oblique what is oblique shock most of the cases this shock which we get is an oblique shock first of all we define the 2 first of all straight normal shock what is straight normal shock. So, far we are discussed that the shock with respect to a duck it may be flow or an external flow also the shock will be there normal shock is the shock straight means this is there is no curvature in the shock, and this is very thin the di dimension is the order of the molecular dimensions thin straight, and normal shock means the velocity of approach V 1, and the velocity behind the shock after the shock both are normal to the shock wave; that means, these angle included angle is ninety degree.

And since they are both of them normal to the shock wave they have in the same direction; that means, both are in the same direction which is normal to the shock wave now if I show the same thing we do not consider a straight shock wave straight, but not normal oblique shock the basic difference from the nor oblique shock from the normal shock is that. Now I am not drawing any duct I am drawing that be shock wave is not per perpendicular to the incoming velocities first of all this is the first V 1 that is the incoming velocity is not perpendicular to the shock will makes an angle to the shock wave, let this angle be beta, and moreover when it passes across that shock through the shock, then what happens this change its direction V 2 this is also not normal to the shock.

This deviation in its path is given by delta this angle beta is known as shock wave angle shock wave angle that is the angle that incoming velocity makes with the thin straight shock which is oblique in nature it is not ninety degree, and the velocity which is go out; that means, the velocity after the shock is not in the same direction of its of that before the shock there is the change in the direction in case of oblique shock, and that is known as delta. So, delta is the angle of deviation that is angle by which this velocity derives the magnitude; obviously, this is in this direction angle of deviation this is an oblique shock well now I will we will analyze the oblique shock. So, now, this concept in the background as the definition of the straight oblique shock now what I will do is see.

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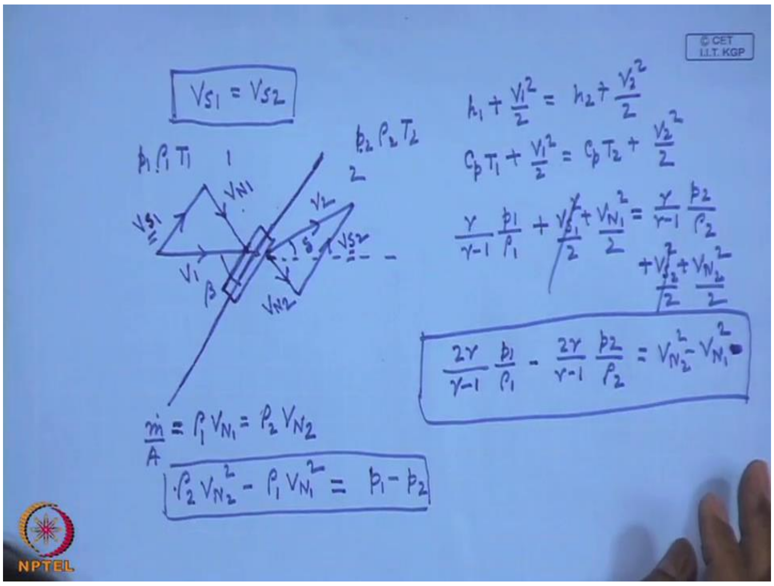

Let us consider a oblique shock like this, and let us consider the oblique shock a control volume very small control volume as we did for normal shock for derivation for all equation remember fundamentally by the application of the conservation equation of mask momentum, and energy is the same thing now I will do. So, I take a control volume this is a control volume c V this is a control volume now this is my incoming velocity V 1, and this V 1 is having an angle this angle beta shock wave angle, but this can be resolved into 2 component 1 is along the shock wave that is V s 1, and another is perpendicular to the shock wave V n 1 at the n is the normal s is along the distance that is usually presented as now the outlet velocity here it has got a deflection. So, this is the deflection delta it has also can be it is also can be resolved into to component V n 2, and V s 2 this angle is delta this is the angle of deviation this is the beta well.

Now, you see that what we do first consideration is that before applying any conservation equations we will tell you that since we know along the shock the fluid property do not be changing change is only across the shock. So, therefore, there is no force acting along the shock there is no change in momentum which means if we take in very finite control volume this V s 1, and V s 2 is same this is very very important assumptions of the oblique shock this is the key assumption you will see after words the entire analysis depends upon that; that means, they have a velocity V 1 they have a velocity V 2 outlet outlet means after the shock, but they are such a way that the tangential component to the component along the shock wave, but equal. So, only inequality is that V n 1, and V n 2 they are not equal. So, this is first thing we have to remember.

Now, if you write the therefore, there is no change in momentum in this direction the force acting in this direction is zero now if we write the continuity equation mass flax row since this 1 is V s 2 they cannot carry any extra mass additional mass in the control volume. So, control volume mass can be retain this is the cross sectional area, and for the same cross sectional area the continuity equation row 1 V n 1 this is 1 this is two. So, the properties are like that row 1 T 1, and p 2 row 2 this this concept will remain same that p 2 will be more than p 1 row 2 will be more than row 1, because it is a compression process across the shock T 2 will be more than the T one, but now you just see how in a very simple way we did this is the conservation of mass or mass continuity now let us write the momentum a theory; that means, the equation of motion that is the momentum theorem that is the momentum theorem with respect to the control volume now what is the rate of rate of momentum reflects from the control volume in the normal direction, because in the axial direction it is zero we have already discussed.

So, this will be the mass flow rate is this this is the mass flow rate. So, mass flow rate into V n one; that means, it will be net momentum a flux will be row 2 V n 2 into V n two. So, therefore, V n 2 square minus here I will write the mass as row 1 V n, because I write the mass the outlet velocity or the velocity after the shock I write this 1, and when at the inlet or at the approach I write this 1 this is the standard practice. So, this is the net rate of momentum reflects, that is what momentum momentum normal direction momentum reflects from the control volume, and under steady state this will be the net force acting on the control volume the normal direction, and that is nothing but the pressure force; that means, it becomes p 1 minus p 2 the, because this is mass flow rate per unit area.

I am sorry this is the mass flow rate per unit area. So, therefore, area cancels from both this sides. So, this is your equations in the this is the momentum equation this is the momentum equation. Now, if we write the energy equation what is the energy equation you write now energy equation you write that h 1 plus V 1 square by 2 neglecting the kinetic, and potential energy as you know the the similar treatment the shock across this shock process is adiabatic. So, therefore, we can write the energy quotient without any work, and heat interaction h 1 plus V 1 square divided by 2 is h 2 plus V 2 square by 2 h 1 is what c p T 1 plus V 1 square divided by 2 equal to c p T 2 plus V 2 square by 2 now this c p can be expressed as gamma by gamma minus one.

R T 1 is sometimes this I did earlier also if you remember the normal shock wave deductions this is this was done this way the equation in compressible expressed in terms of only p, and row here now this V 1 again can be retained at V s 1 square plus V n 1 square, and this must be equal to gamma by gamma minus 1 p 2 by row 2 plus V s 2 square plus V n 2 square now as our assumption by 2 very good by 2 by two. So, this, and this cancels. So, therefore, here also we can write the equation this 2 if you write like this 2 gamma by gamma minus 1 p 1 by row 1 minus 2 gamma by gamma minus 1 p 2 by row 2 is nothing, but V n 2 square minus V n, sorry sorry V n 1 square.

So, this is the extract from this now you see V s 1 equals to V 1 V s 2 V s 1 equals to V s 2, then you see that automatically they have no contribution if you now compare these with the normal shock equation you see state of V 1, and V 2 V 2, and V 1 here also V 2, and V 1 we have just substituted the normal component of the velocity; that means, all the relation of normal shock will hold good if we just change the actual velocity by its normal component. So, therefore, this is clear, and the other relationship that which we get in rankine hugoniot relations your relationship the which we het that we do by p 2 by p 1 row 2 by row 1 rather row 2 by I am sorry row 2 by row 1.

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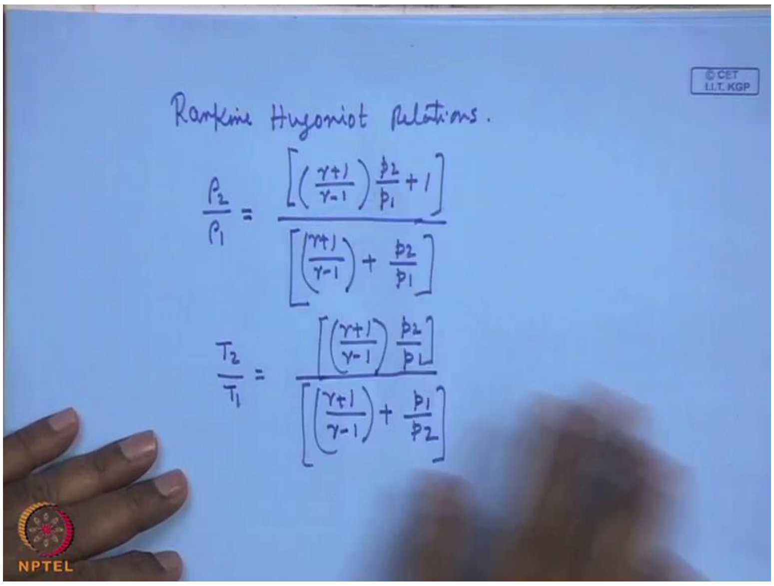

Let me write that again the rankine hugoniot relations that we already did used for this normal shock rankine hugoniot relationsrankine hugoniot relations is the same; that means, rankine relation is row 2 by row 1 in terms of p 2 by p 1 this is usually expressed in terms of p 2 by p 1 I write it for your recapitulation here only just recapitulation I write plus 1 divided by gamma plus 1 by gamma minus 1 plus p 2 by p one. So, this is 1 relation. So, this is relation is not all, because this is the relationship between the scalar quantity T 2 by T 1 is gamma plus 1 by gamma minus 1 into p 2 by p one divided by gamma plus 1 gamma minus 1 plus p 1 by p two. So, this is the thing. So, this remains as it is this is just for your recapitulation. Now I come again to this same thing that now; obviously, the intuition tells when you compare this if you substitute this normal velocity you feel that all the conservation equation for normal shock, and oblique shock remains same since V s 1 equal to V s 2. Now if you want to do with the mach number, because what we want we want the relationship with the mach number, then what we will see that V n 1 now before that away we are just to write 1 thing V n 1 here itself that V n 1 what is V n 1 V n 1 we can write V n 1, we can write this 1 side this is beta; that means, this is beta.

The V n 1 is V 1 sin beta what is V n 2 V n 2 is this one. So, this will be V 2 this is beta minus delta V 2 sin this angle is beta minus delta why, because this is beta, and this is delta. So, therefore, this angle is beta minus delta, because this angle this angle this angle my drawing is not in scale, these are 2 parallel line this angle is what this is beta with this, this is the shock, and this is the original line, and this is the deviation delta. So, this is beta minus delta you can make it out by yourself V 2 sin beta minus delta; that means, if I substitute V 1 V 1 sine theta V 2 V 2 sine beta minus delta.

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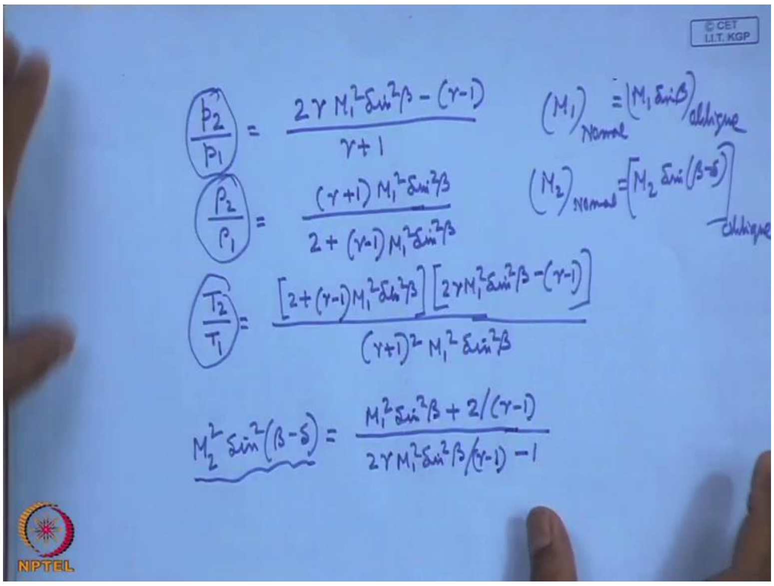

This exactly matches with the relationships of the normal shock, and then most important relationship which we want that will come out that is what p 2 by p 1 you remember that was in terms of the approach mach number upstream mach number what is that, it was m one. So, now since this is with the normal component. So, automatically m 1 will be m 1 sine beta actually I want V 1 by. So, m 2 is equal to m 2 sine beta minus delta what is this m 1 m 1 sine beta; that means, sine beta is equal to one; that means, m upstream this m 1 is m upstream actually m 1 sine beta; that means, what we will do; that means, this is m 1 sine beta that you can forget this it will give confusion that if you use m 1 sin beta as the approach m 1, and m 2 sine beta minus delta is the mach number as the mach number after the shock wave you can use this same relationship of the normal shock.

hat means, the normal shock relationship the m 1 should be replaced by m 1 sine beta, and m 2 should be replaced by m 2 sine beta where m 1 m 2 is the actual mach number here for the oblique shock Oblique shock mach number is V 1 by a 1, and a 1 is, and this mach number is V 2 by a 2 there is no doubt this is the definition of the mach number, and a 2 is root over gamma r t. So, if we just normalize this mach number by sine beta, and normalize this mach number by sin beta minus delta, because of this logic that we have seen this is V s 1 equal to V s 2, then we can use this m 1 sine beta in the place of m 1 in the normal shock relation, and replace m 2 sine beta minus delta is the m 2 in normal shock relation that is the same relation that is the same relation which you will get oblique shock it was 2 gamma m 1 square. So, this was m 1 square sine square minus gamma minus 1 divided by. So, in case of normal shock p 2 by p 1 was twice gamma m 1 square minus gamma minus 1 divided by gamma plus 1 similarly similarly if you write row 2 by row 1 that will be gamma plus 1 into m 1 square.

But here it will be gamma plus 1 into m 1 square sine square beta ; that means, m 1 for normal shock wave for normal shock wave m 1 sine beta for oblique shock similarly m 2 for normal shock wave will be replaced by m 2 sine beta minus delta for oblique shock wave for oblique oblique shock wave for oblique shock wave; that means, we are replacing this, and if we do that we will get the required relationship for the oblique shock 2 gamma minus 1 m 1 square; that means, again this is m 1 square sine square beta similarly if you recollect last class I did T 2 by T 1 equals to 2 plus gamma minus 1 into m 1 square. So, this will be m 1 square sine square beta, and 2 gamma m 1 square sin square beta minus gamma minus 1, and denominator was gamma plus 1 whole square gamma plus 1 whole square m 1 square; that means, m 1 square sine square beta another interesting thing is that there was a relationship between m 2, and m 1 when m 1 is given I can get all parameters p 2 by p 1 the change in the property along with the m two.

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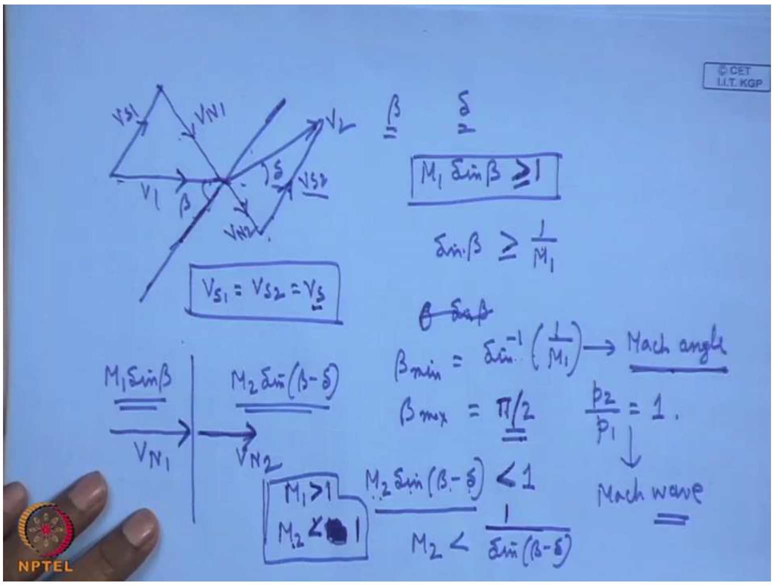

That is the mach number after the shock here it to that relationship m 2 has to be again replace by m 2 sine beta. So, actual relation here was m 2 square equals to m 2 square sine square beta minus delta equals to what parallel, you see that if you remember that equation for the normal shock 2 by gamma minus 1 divided by 2 gamma m 1 square sine square beta divided by gamma minus 1 minus 1 this is.

If you see this expression compared with the normal shock only m 1 is replaced by m 1 sine beta m 2 is replaced by m 2 sine beta. So, this is the theme; that means, now I can tell you that an oblique shock that an oblique shock again this is an very interesting thing at an oblique shock which has got 2 component which is striking the velocity V 1 with the shock angle beta which is striking with the V 1 well as got a normal component, and the tangential component, and this is being deflected with an angle delta V 2 which also has normal, and V s 2 this parallel to this, and V n 2 this is V s 2 with the premise that V s 1 is V s 2 can be considered equivalent to; that means, if we now axis which is moving V s 1, and V s 2 along the wave shock wave; that means, V s 1 equal to V s 2 V s if we attach an axis.

If we make this representation with respect to axis which is moving with V s velocity; that means, magnitude in this direction this is nothing, but a shock wave normal shock wave; that means, this is like this; that means, this thing is with this premise is resolved to this this can be reduced to this with respect to this can be reduced to with respect to coordinate system which is moving with V s with respect to the coordinate system this is the relative velocity normal component, and similarly with respect to the coordinate system which is moving with V s the velocity V 2 relative to V s will be V n 2 clear. So, therefore, this will be resolved to a normal shock wave V n 1, and V n 2, and all normal shock wave relation seems will be valid provided the actual mach number is normalized with sine beta, and here m 2 sin beta minus delta beta, and delta are very important parameter shock wave angle, and the angle of deviation.

So, this is clearly deduced to a normal shock wave with the normal component of the velocity, and with a modified mach number now with similar treatment of the normal shock wave from consideration. First we can tell m 1 sine beta has to be greater than 1, because this has to be greater than 1 the flow approaching the shock is to be supersonic one. So, therefore, this is 1 very important criteria, which means that from here 1 can write it can be a case equals to one; that means, sine beta will be greater than 1 by m 1 now u see this is very important information that is the minimum value of beta I write the sine beta beta minimum is equal to sine inverse 1 by m what is this this is match angle which was told earlier this is the match angle; that means, minimum shock match angle 1 by m 1 what is the maximum shock match angle that is the maximum that is equal to pi by two; that means, the maximum value is the normal shock, and the minimum value is the match angle; that means, the oblique shock limit is between the match angle, and the normal shock 1 by m 1 is the match angle you just, now substitute this match angle sin beta is sin inverse 1 by m 1 you get p 2 by p 1 is equal 1, if you put m square 1 2 gamma minus 1 that is by gamma plus 1 1 that is in that case p 2 by exactly the relationship you you got for the match wave; that means, the this the match wave; that means, the oblique shock wave this case becomes the shock wave 1 limiting case, and the another limiting case with the maximum value of the shock wave angles it becomes totally it the normal shock wave like this this is 1 think.

Next is that again from the same thermo dynamic consideration we showed earlier, because it is reduced to the normal shock sin beta minus delta has to be less than 1 as we know that the this is very interesting will show m 2 is less, then m 1, and m 1 has to be greater than one; that means, therefore, sorry m 2 is less than 1, then m 2 is definitely less than 1 m 2 less than 1, and m 1 is greater than 1, and m 2 is less than 1, sorry m 2 is less the 1 or. So, this 2 think we proved we proved this 2 thinks that a shock wave is such that supersonic fluid is changed to sub sonic fluid. So, therefore, match number is less than one, but here you see in the oblique shock wave m 2 sin beta minus delta will be less than one, but the same consideration.

If you go back to the normal shock relation, and their analysis which means that m 2 which is less than 1 by sin beta minus delta since the sin beta minus delta is less than 1 always this is not necessarily that always m 2 will be less than one; that means, m 2 sin beta minus delta less than one; that means, in that case m 2 may be greater 1 depending on the value of sin beta minus delta. So, this is very important consideration now m 2 is actual match number sometime it is asked if your ask that if there is a oblique shock wave is the supersonic flow after the yes, because the limitation is m 2 sin beta minus delta is depends upon the shock wave angle, and the deviation angle of deviation this is the restriction that reduce the match number has to be less than one. So, the actual match number may or may not be less than 1 it may be less than 1 or it may not be less than 1, and in some circumstances it may it may be greater than 1 depending upon the value of the sin beta minus delta, this is one very important consideration. Now we will develop a hone time is there know we will be develop some expression.

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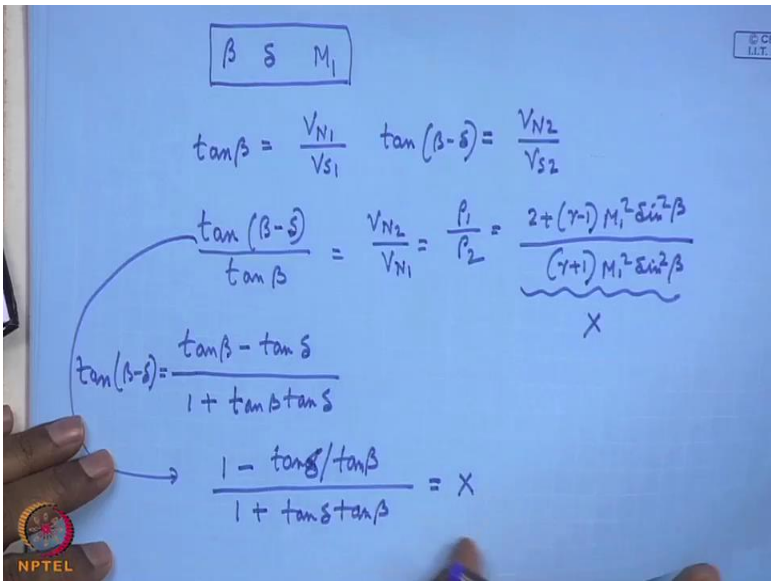

Now, we are interested to develop 1 expression between beta delta, and m 1 why you will understand afterwards let us do some algebraic calculations to do it now you see that what is beta now beta is this one. So, beta 1 beta 1 is V n 1, and V s 1 tan beta 1 sorry similarly this is beta minus delta, because this is delta, and this is beta this is shown in another diagram beta minus delta.

So, tan beta minus delta is V n 2 by V s two. So, let us write that that tan beta is V n 1 by V s 1, and tan beta minus delta is V n 2 by V s 2 now sorry if you make a ratio, then tan beta minus delta divided by tan beta is equal to V n 2 by V n 1, and from continuative what is this V n 2 by V n 1 is row 1 by row 2 now this row 1 by row 2 is again can be return as this row 1 by row 2 can be again return in terms of 2 plus gamma minus. So, on as I have done earlier 2 plus gamma minus 1 m 1 square sin square beta divided by that just now I have down divided by gamma plus 1 sin square square beta at the present moment let I consider this as x, then tan beta minus delta is tan beta minus tan delta divided by what 1 plus tan beta tan delta this is tan beta minus delta this is tan beta minus delta that divided by tan beta therefore, this thing becomes what 1 minus tan beta tan alpha by tan beta tan alpha by tan delta sorry tan delta by tan beta divided by 1 plus tan delta tan beta equals to x. Now if you put value of x, and make some adjustment, then first of all put the value of if you you clear this thing this thing will come as tan if you just make a rearrangement.

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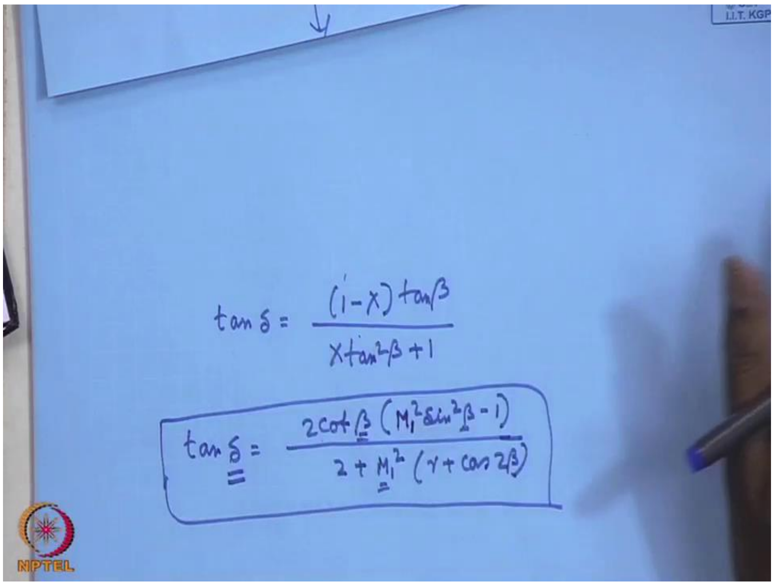

You will get tan delta is 1 minus x x is what x is this 1 minus x into tan beta divided by x tan square beta plus 1 if you put the value of x finally, you get some expression that with some rearrangement tan delta is that is important 1 2 cot beta this will not come very first next will, but you have to make first 2 3 simplification, but this will come very straight forward no other relationship required only we have to know the trigonometry identity; that means, tan delta this coming clearly from here 1 line next step it comes here substitute x. We replace this as x here this will be little complicated in a sense it will b a big 1, and if you make a rearrangement, and clear it ultimately you will get 2 cot beta m 1 square sin square beta minus 1 2 plus m 1 square gamma cos 2 beta this 1 very important relation which expresses the angle of deviation in terms of the shock wave angle, and approach match number. So, I will continue on the next class since the time is up out, and I think we do not have time.

Thank you for today.