

Introduction to Fluid Machines and Compressible Flow
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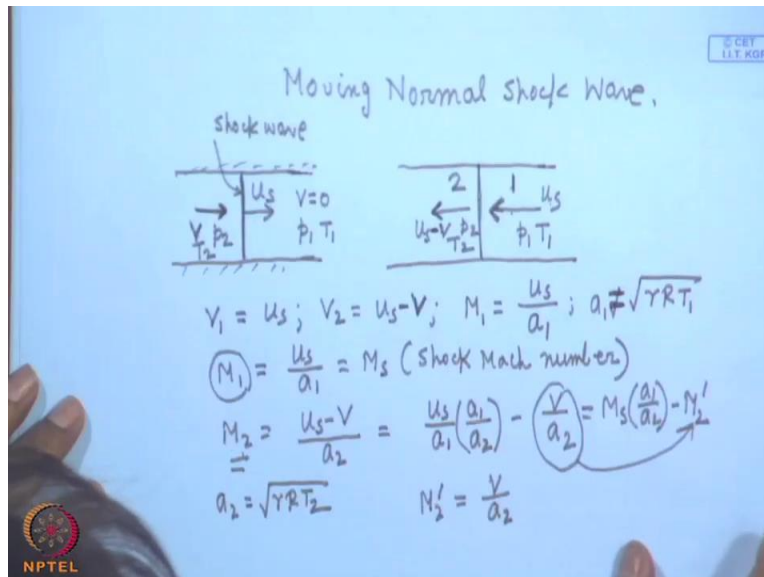
Lecture - 37
Normal shock part V

Good morning and welcome you all to this session of this course. So far we were discussing the shock, normal shock and in all those discussions, we have made the shocks stationary at a locations while the fluid flowing and acquires the shock there are certain discontinuities in the fluid properties that we have seen.

Now, there are situations, where the shock is moving sometimes in a flow, relative to a flow, flow may be raised at raised flow also may be moving, which are observed in certain practical cases like in air compresses, which we discussed in turbo machines. Sometimes in the exhaust and inlet manifold of internal combustion engines, which may occur due to explosion a shock wave propagates through a medium, which may occur in case of fluid flowing in a pipe line, we just close or open the valve at the downstream. So, a shock wave is generated, which is propagated or moving to the in the fluid medium.

In those cases it is easy to analyze the mathematical relations or analyze the problem with respect to a coordinate system attached to the shock moving with the uniform velocity then the entire thing appears to be a stationary shock wave acquires, which there is a change of fluid properties, but how these analysis are being made and results are being obtained we have to see. Most important thing here in these context to find out the velocity imposed by the motion of the shock wave to the fluid medium. So, to understand this let us consider a case like this.

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Let us consider a case like this, rather I will write here moving normal shock wave, let us consider a case like this, where in a duct a shock wave, this is the shock wave, let us consider a situation like this that a shock wave is moving in a duct with a velocity of u_s constant velocity of u_s and the gas behind the shock that means, downstream part of the shock it is at rest there is no velocity v is zero ok. And pressure is p_1 and temperature is T_1 , whereas these shock has created a velocity in the fluid through, which the shock has passed let these velocity is create a velocity v , let us consider a velocity v ok. Let us consider a velocity v , which is be and it has created a pressure p_2 and temperature T_2 this is the velocity of flow v , that means, due to the shock passing over this fluid a velocity is imposed or induced by the shock.

Now, this problem, if you analyze in a way that this is a picture with respect to a coordinate frame attach to the wall, that means; they coordinate frame static frame of reference, which is addressed. Now, if you attach the coordinate frame on the shock that means, if we see these thing relative to the shock it is just like a stationary shock wave, normal shock wave that means, shock is addressed here that means, then it will look like that the fluid with a velocity u_s and a pressure p_1 and temperature T_1 approaches the shock and while it goes this side its velocity is reduced $u_s - v$, usually this u_s is higher than v . And therefore there is a deceleration in the same direction and this is $u_s - v$ and the pressure is p_2 and T_2 . So, there more with respect to the coordinate axis attached to the shock wave this is the picture.

Now, if this you make an analogy with the stationary shock then what you see? You see that in the stationary shock the approaching velocity is given as v_1 , which equals to u_s and this is v_2 , which equals to $u_s - v_{sh}$ then $p_1 T_2$, that mean, section one and section two ok alright. Then v_2 is $u_s - v_{sh}$ so ok. Now, if this is the thing now, we if we define the Mach number at the inlet that is upstream of the shock ok then what is the Mach number? Now, here I can write the Mach number M_1 as u_s / a_1 , where a_1 is these acoustic speeds sorry, a_1 is root over $\gamma R T_1$ well a_1 is equal to root over γ this is known as this M_1 is u_s again I am writing a_1 is known as M_s this is known as this is just a definition shock Mach number M_s ok.

Now, what is M_2 now? That means with respect to a stationary waves, so this is the picture so M_2 will be the velocity after the shock wave that means; $u_s - v_{sh}$ by a_2 , where a_2 is what? a_2 is root over $\gamma R T_2$ clear. This can be written as u_s / a_2 , which can be written as a_1 / a_2 this is a style of writing a_1 / a_2 ok minus v_{sh} / a_2 . So, these be is the v_{sh} generated due to the passing of the shock wave, but in a steady condition with reference to a coordinate frame attached to the shock wave that means, if you make the shock wave stationary by imposing velocity u_s in the opposite direction to the entire system this is the situation. So I am writing in that case the M_2 is the velocity that means, the Mach stream a Mach number after the shock that is the stream velocity $u_s - v_{sh}$.

So, these become this one that means this equal to M_s into a_1 / a_2 minus this is v_{sh} / a_2 is written as M_2 dash that means, M_2 dash is nothing but v_{sh} / a_2 that means this v_{sh} / a_2 is written as M_2 dash this is an important parameter. So, therefore, these relationship you have to first of all remember or you have to write that relations that means, it happens that a fluid with a Mach number M_1 given by u_s / a_1 and with a velocity u_s and $p_1 T_1$ is passing through a stationary shock wave and its velocity is reduced that is mach number rather is reduced to M_2 , which is defined as $u_s - v_{sh} / a_2$. Now, these quantities are with respect to this problem this is unsteady problem where the shock is moving and these velocities are defined with respect to the coordinate frame static coordinate frame, which is attached to the wall coordinate frame which is attached to the wall.

Now, if we accept this now next part is that if we recall the analysis if we now recall the shock relationships just I am writing this in earlier classes in few earlier classes.

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Normal Shock relations

$$\frac{p_2}{p_1} = \frac{2\gamma M_1^2 - (\gamma - 1)}{(\gamma + 1)}; \frac{p_2}{p_1} = \frac{(\gamma + 1) M_1^2}{2 + (\gamma - 1) M_1^2}$$

$$\frac{T_2}{T_1} = \frac{[2 + (\gamma - 1) M_1^2][2\gamma M_1^2 - (\gamma - 1)]}{(\gamma + 1)^2 M_1^2} = \left(\frac{a_2}{a_1}\right)^2$$

$$M_2^2 = \frac{(\gamma - 1) M_1^2 + 2}{2\gamma M_1^2 - (\gamma - 1)} \quad \left. \begin{array}{l} M_1 > 1 \\ M_2 < 1 \end{array} \right\}$$

$$\frac{p_2}{p_1} = \frac{2\gamma M_2^2 - (\gamma - 1)}{(\gamma + 1)}; \frac{p_2}{p_1} = \frac{(\gamma + 1) M_2^2}{2 + (\gamma - 1) M_2^2}$$

$$\frac{T_2}{T_1} = \frac{[2 + (\gamma - 1) M_2^2][2\gamma M_2^2 - (\gamma - 1)]}{(\gamma + 1)^2 M_2^2} = \left(\frac{a_2}{a_1}\right)^2$$

We have discussed this thing that the suffix 2 is always the shock, now I am writing the shock relations, normal shock relation, normal shock just recapitulation normal shock relations recapitulation of normal shock relation this equal to this p_2 by p_1 that means, the suffix 2 as you know is a usual symbol is after the shock and this is before the shock.

Usually in shock more useful relationships are the ratios of pressure, temperature, density in terms of the inlet Mach number, inlet Mach number is the input parameter. So p_2 is usually specified by inlet Mach number so, I am recollecting these formulas that we already deduce $\gamma + 1$. Similarly ρ_2 by ρ_1 has an expression, which is $\gamma + 1 M_1^2$ square divided by $2 + \gamma - 1 M_1^2$ square. Similarly we can write T_2 by T_1 , which was all deduced earlier this is this, $2 + \gamma - 1 M_1^2$ square, if you remember this relationship $2\gamma M_1^2 - \gamma - 1$ ok and divided by $\gamma + 1$ whole square M_1^2 square.

Now, this T_2 by T_1 can be written also equals to what is T_2 by T_1 ? a_2 by a_1 whole square, why? Because the acoustics speed a_2 at the section 2 is root over γT_2 and acoustics speed a_1 is root over γT_1 . So, ratio of temperature is the ratio of the acoustics speed at that state corresponding states a_2 by a_1 square.

Another very important relation is there that is the Mach number after the shock, which is the subsonic Mach number $\gamma - 1 M_1^2 + 2$ divided by $2\gamma M_1^2 - \gamma - 1$ square

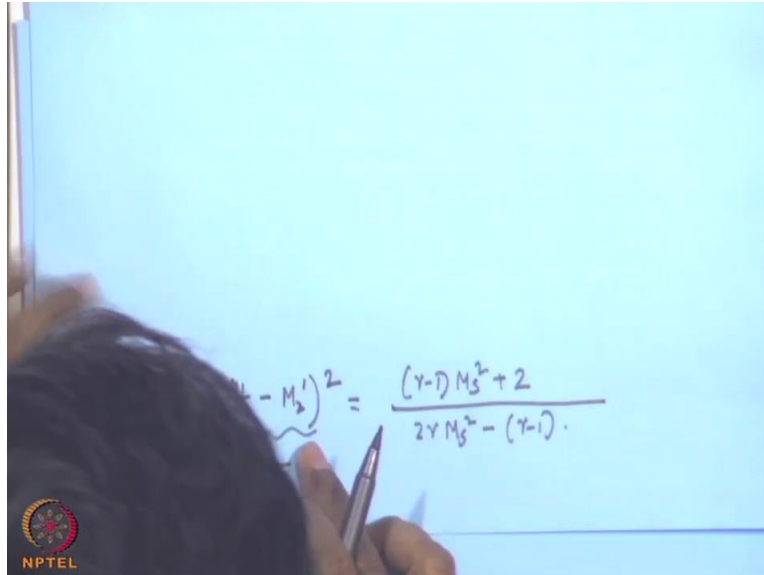
minus 1. So, at present I recall only these relationships, again I tell you, what are those relationships this is the change in the fluid property in terms of the ratio pressure ratio the temperature T_2 by T_1 rho 2 by rho 1 the density and the Mach number after the shock in terms of the initial Mach number.

Now, this relationship are exploited to find out a chart, where against a Mach number one that is M_1 against M_1 against a Mach number M_1 we can found we can find all these properties. Another thing you have to remember with this or the recapitulation, which has been already discussed earlier from the consideration of entropy change, as a corollary of the second law we always tell that M_1 has to be greater than 1 for a shock to occur and M_2 will always be less than 1 that means; a shock always decelerates the fluid approaching the shock from a supersonic one to a subsonic one after the shock. So, this if you recapitulate or you remember now here what happens now this problem this is similar to a stationary shock, where M_1 is this one M_s and M_2 is this one $M_s a_1$ by a_2 minus M_2 dash or you can write u_s minus v by a_2 so, this is our M_2 .

Now, what you do, if you write the same these replace this you get this that means instead of M_1 you replace M_s that means in this case for this moving shock wave I can found p_2 by I can find p_2 by p_1 just again I am writing just replacing this in terms of M_s that means, rho 2 by rho 1, I am writing little fast, because it is just for your taking note. M_1 is because M_1 in this case is M_s u_s by a_1 M_s square so according to the present nomenclature, I write this one and T_2 by T_1 will be again same that M_1 is replaced by M_s that means, $2 + \gamma - 1$ M_s square into $2 \gamma M_s$ square minus $\gamma - 1$ so this is also equals to a_2 by a_1 whole square.

So, this a_2 a_1 related to T_2 T_1 this T_2 T_1 p_2 p_1 they are scalar quantity they are not dependent on the coordinate transformations. Now, the M_2 similarly has to be replace, if you write the substitute equation for this then what will be the equation? This equation will be instead of M_2 this equation we write here M_2 is M_s that in our earlier thing that M_2 is $M_s a_1 a_2$ minus M_2 dash square.

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$$(M_2 - M_2')^2 = \frac{(\gamma-1)M_3^2 + 2}{2\gamma M_3^2 - (\gamma-1)}$$

So, therefore M_2 square will be therefore M_3 a 1 a 2 minus M_2 dash this is the square and that equal to same thing just we are replacing the corresponding $M_1 M_2$, where 1 2 is the upstream and downstream section of the M_1 square minus gamma minus 1. So, this is our M_2 at the present moment M_3 a 1 by a 2 minus M_2 dash ok.

Now, what happens? If we take this equation and if we use the a 1 by a 2 then we get a very important relationship of M_2 dash, let us do that. Now, let us again write this equation, let us write this equation again now, let this equation is, you just remember these equation, this equation is written ok, this equation again I am writing M_3 a 1 by a 2 minus M_2 dash I take a square root and I write like this. I think you can see that thing I write that gamma minus 1 M_3 square plus 2 to the power 0.5 and here $2\gamma M_3$ square minus gamma minus 1 to the power 0.5.

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The image shows a handwritten derivation on a whiteboard. At the top, there is a partial equation: $M_2' = M_s \frac{(\gamma+1) M_s}{[2 + (\gamma-1) M_s^2]^{0.5} [2\gamma M_s^2 - (\gamma-1)]^{0.5}} - \frac{[(\gamma-1) M_s^2 + 2]^{0.5}}{[2\gamma M_s^2 - (\gamma-1)]^{0.5}}$. Below this, the expression is simplified to $M_2' = \frac{M_s^2(\gamma+1) - [2 + (\gamma-1) M_s^2]}{[2 + (\gamma-1) M_s^2]^{0.5} [2\gamma M_s^2 - (\gamma-1)]^{0.5}}$. To the right of this step, there is a small calculation: $\gamma M_s^2 + M_s^2 - 2 + \gamma M_s^2 + M_s^2$. The final result is $M_2' = \frac{2(M_s^2 - 1)}{[2 + (\gamma-1) M_s^2]^{0.5} [2\gamma M_s^2 - (\gamma-1)]^{0.5}}$. An NPTEL logo is visible in the bottom left corner of the whiteboard image.

Now, a 1 by a 2 is what? a 1 by a 2 is the under root of this thing so you just see that from your note that as we have taken here I am writing then M_2' is equal to M_s into a 1 by a 2 what is a 1 by a 2 is under root of that, that means, it will be M_s this is a 2 by a 1 that means a 1 by a 2 is the under root of reciprocal of that. That means, denominator will be 2 plus gamma minus 1 M_s^2 to the power 0.5 into 2 gamma M_s^2 minus gamma minus 1 to the power 0.5 that means, numerator will be denominator it is a 2 by a 1 I am writing a 1 by a 2 and under root, because I am its a square and now the denominator will be under root gamma plus 1 whole square M_s^2 . That means this gamma plus 1 ok, whole square that means gamma plus 1 into M_s ok and denominator it will be sorry, M_s very good, denominator it is M_s very good denominator it will be M_s very correct so denominator it will be M_s alright.

So, M_2' is M_s into this minus if you do that you will get gamma minus 1 M_s^2 plus 2 to the power 0.5 divided by 2 gamma M_s^2 minus gamma minus 1 to the power 0.5 you can see very directly ok. Then this becomes equal to what? Then this becomes is equal to M_s^2 gamma plus 1 so this will be the denominator 2 plus gamma minus 1 M_s^2 to the power 0.5 let me write this 2 gamma M_s^2 minus gamma minus 1 to the power 0.5 then minus so this one is 2 gamma that means this one 2 plus gamma minus 1 M_s^2 plus that means; this will be simply 2 plus gamma minus 1 M_s^2 that means the same thing 0.5, 0.5.

So, if you now clear this thing then the numerator will be what? denominator will be this big thing $2 + \gamma - 1 M^2$ square it is little tedious I know, but you just follow that whether I am doing any mistake or not $M^2 - \gamma - 1$ to the power 0.5. And now, $M^2 - \gamma + 1 - 2\gamma - 1$ so $\gamma M^2 - \gamma M^2$ is canceled, so this is M^2 and then γ now, if you write this γ here, I make a calculation $\gamma M^2 + M^2 - \gamma - 1$ what is that? $2 + \gamma - 1 - \gamma M^2 + M^2$ that means these cancels that means $2 + \gamma - 1$ this become $2 + \gamma - 1$. So, this is one very important deductions that is here I can write M^2 dash, I think you can see equals to which is very important M^2 dash why I am writing it is v by a , it is not M^2 , M^2 is this one M^2 dash this is M^2 , but I am finding out an expression of M^2 dash in terms of M . Now, you see what interesting results we have achieving.

Now, you see here what it is given? That it is given not the downstream Mach number that means Mach number after the shock with respect to the shock the coordinate with respect to the shock, but it is given the ratio of the velocity induced, because of the shock in actual case divided by $a^2 M^2$ dash in terms of the inlet of shock Mach number here M you remember is u by a that is equal to actual M in this case.

Now, is a very interesting result it gives. Now, what interesting result? That if this if we increase the Mach number, let us consider the inlet Mach number is very very high, inlet Mach number is very very high that means; the flow approaches with a very high supersonic velocity. And as you know the strength of a shock as I told you earlier is expressed by the pressure ratio that means; the ratio of pressure after the shock to that before the shock and it is a direct function of Mach number, I tell you is a direct function of inlet Mach number, more is the Mach number more is this ratio, that means; the strength of the shock is usually expressed by this ratio, which is a function of inlet Mach number or approaching Mach number, more is the Mach number more is the strength of shock. So, therefore, a Mach number approaches a shock a fluid approaching with high Mach number to a shock wave is told as the shock is of very high strength. So, therefore, when the shock Mach number is very high we usually call that this is a shock of high strength.

Now, whatever high strength shock comes there is a limiting value for M^2 dash physically. Now, look it mathematically is there any limiting value M tends to 0, if I ask this question that,

when M_s tends to infinity what is the value of what is the limit rather M_s tends to infinity what is the limit?

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Handwritten mathematical derivation on a whiteboard:

$$M_2 = \frac{2 + (\gamma - 1)M_1^2}{[2\gamma M_1^2 - (\gamma - 1)]^{0.5}}$$

$$\lim_{M_1 \rightarrow \infty} M_2 = \lim_{M_1 \rightarrow \infty} \left(\frac{2}{2\gamma} \right)^{0.5} = \sqrt{\frac{2}{\gamma(\gamma - 1)}}$$

For air $\gamma = 1.4$, $M_2 = 1.89$

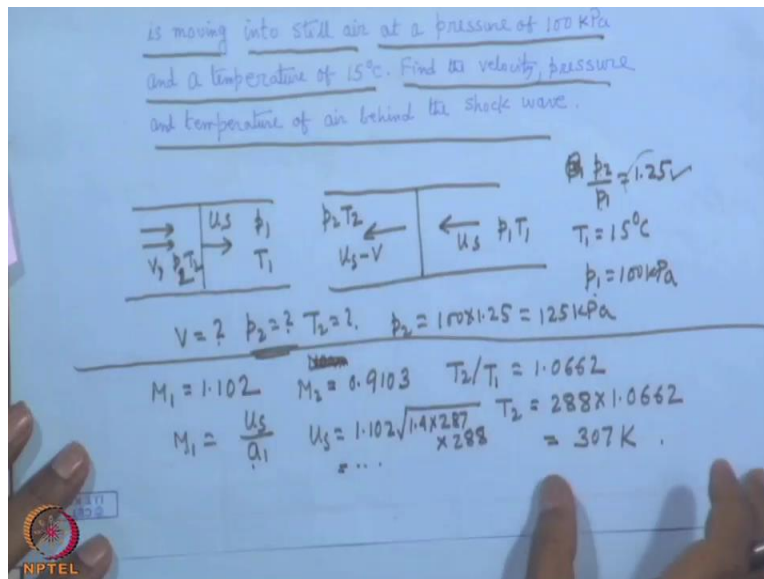
That limit I will now show you. Now, if M_s tends to infinity M_2 can be written as so these type of thing as you know elementary mathematics that in school level you know M_s tends to infinity let me express in terms of $1/M_s$ that means both numerator and denominator should be divided by M_s^2 this is the standard technique, see that whether the limit is there or not so M_s^2 means one M_s I can take here and one M_s I can take here. And when it goes within the bracket again M_s^2 that means; this will be $2/M_s^2$ that means this gives directly and indicates and that there is a finite limiting value that term is becoming free of M_s . M_s by this M_s , where dividing numerator and denominator by M_s^2 this M_s goes inside.

So, therefore M_s^2 here also $2\gamma - \gamma - 1$ so beautiful that means this give us an indication what you want that there is a finite limiting value so an M_s tends to infinity it is 0, it is 0 and it is 0. So, the limiting value there is a limiting value is $\sqrt{2\gamma / (\gamma - 1)}$ this is $\sqrt{2\gamma / (\gamma - 1)}$ and 2 under root and this is 2 . So, therefore, 2 under root will go up. So, therefore, you see the limiting value that I was telling that limit M_2 dash or equal to M_s tends to infinity limit v by 2 as M_s that means the shock Mach number tends to infinity is not infinity it is square root of $2\gamma / (\gamma - 1)$. This is a beautiful result and you see that for air, if we take $\gamma = 1.4$ this M_2 dash is 1.8 that means;

you can say that in air whatever strength may be of the shock even in infinite strength shock very high strength shock can only induce a finite velocity, which will be always less than this corresponding Mach number 1.89. The actual velocity will depend upon the temperature we have to multiply this with a 2, which is root over gamma or T 2 that means velocity is usually specified in compressible flow by the mach number so you can tell for example in air whatever strong Mach number may be at the upstream of a moving shock this can or whatever rather this way you tell whatever strong shock may be a moving shock whatever strong the shock may be whatever velocity the shock may be moving that the M s u s by a it cannot induce a flow whose Mach number will be more than 1.89 in case of air, so this is the limiting Mach number in case of air ok.

Now, next is next we will rather now after this I think we will solve some problem.

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So, that your all this things will be made more clear by solving the problem before I go to a this refraction of the shock, let us ah solve some problems ok. Now, problem one, example one, a this is the first problem example say shock wave across, which just you see this thing problem which the pressure ratio is 1.25 this is not a this is a moving shock, a shock wave across which pressure 1.25 is moving yes it is written is moving into still air at a pressure of 100 kilo Pascal and a temperature of 15 degree Celsius this is the still air. Find the velocity pressure and temperature of air behind the shock wave that means it is simply the application of that, how to do it?

Now, let us again solve the problem, the shock is moving with a velocity u_s moving into still air and we do not know its velocity let u_s similarly that u_s and this p_1, T_1 the same problem and then it induces velocity v and p_2, T_2 . So, with respect to the shock the problem is with respect to the shock the problem becomes steady with a velocity u_s and it goes with velocity $u_s - v$ and p_2, T_2 , what are given in the problem? p_1, T_1 is not given rather p_2/p_1 is 1.25, what are the given, T_1 is given 15 degree Celsius, p_1 is given 100 kilo Pascal, what we have to find out velocity, pressure, air behind a shock that mean, we have to find out v , we have to find out pressure p_2 and temperature T_2 the first one is the school level thing p_2/p_1 because the pressure ratio is given 1.25, I know the pressure p_1 so p_2 will be 100 into 1.25 is 125 kilo Pascal straight way, but we do not know T_2 , how to calculate?

Now, I have told you earlier that in shock all those relationship, we have derived that means for example p_2/p_1 is known I can find out M_1 , because p_2/p_1 is expressed in terms of M_1 . Again I tell you that just also at the beginning of this class I told you that all these relationships, which was derived earlier for example this important relationship here one can find out algebraically p_2/p_1 I know I can find out M_1 ok. So, if I know M_1 I can find out $\rho_2/\rho_1, T_2/T_1$ over these algebraic calculations sometimes are tedious. So, therefore, a table is drawn based on this calculation as usually drawn for different cases that table as I now tell you earlier also I told which is known as normal shock tables. So, this is their just you see how to see a normal shock tables a normal. Now, I not I will do here, I will rather do it here a normal shock table we just how does it look like you can see if in any book normal shock table just I show you probably, I told you earlier also.

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Normal Shock Table

M_1	M_2	P_2/P_1	T_2/T_1	ρ_2/ρ_1	P_{02}/P_{01}
1.00					
...					
5.00					

Example 1. A shock

Normal shock table looks like that in the left hand column these are the different column M_1 the corresponding nomenclature you know that is the Mach number approaching the shock with respect to a stationary shock this is the aftershock Mach number then this is the ratio of pressure. 2 is the section after the shock and this is all the ratios of the static pressure, temperature, the density and there is a ratio of stagnation pressure also because of the losses shocks is an across the shock the process is universally, so therefore p_{o2} by p_{o1} ok.

Now, these there are a number of Mach numbers different values starts from 1.00, why? Because shock does not occur when the approach Mach number is subsonic, so therefore, it goes some for example 5.00 so corresponding Mach number all these things and there. So, then if I know the p_2 by p_1 now, we have to see that what are the quantities given in these problem, we know p_2 by p_1 , we know p_2 by p_1 is 1.25. So, if we know p_2 by p_1 I can find out everything M_2 , T_2 by T_1 that means, any one of these is known other things are known through the equations. So, therefore, it is not that always M_1 has to be known anything has to be known, we can find out from the chart ok. So, from these chart, if you read with p_1 p_2 by p_1 1.25, you will get a Mach number of M_1 , which from normal shock table you will always M_1 and M_2 , M_1 means inlet Mach number M_2 means outlet that means the downstream after the shock Mach number 9 and T_2 by T_1 you get 1.0662.

Now, when you get T_2 by T_1 straight T_2 is T_1 is given T_1 is 15 that means; 288 into 1.0662. So, you get T_2 , your T_2 is 307 k in this case. Now, in your case M_1 is what? M_1 is u_s by a 1 now, what is a 1? a 1 is root over $\gamma R T_1$. So, therefore, u_s is the Mach number, I have got from the table into root over γ that is 1.4 into R is 287 into the T_1 , T_1 is 15 degree means, 288. So, therefore, you straight forward get the value of u_s , if you know M_1 already you know a 1 root over $\gamma R T_1$ and you get the value of u_s ok.

Now, you have to find out the velocity behind the shock that means this velocity this is very simple to do this what you have to do, you have to write the expression for M_2 in your case.

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Handwritten notes on a blue background showing calculations for Mach number and velocity behind a shock. The top part shows:

$$V = ? \quad p_2 = ? \quad T_2 = ? \quad p_2 = 1.07 \times 1.25 = 125.14 \text{ kPa}$$

$$M_1 = 1.102 \quad M_2 = 0.9103 \quad T_2/T_1 = 1.0662$$

$$M_1 = \frac{u_s}{a_1} \quad u_s = 1.102 \sqrt{1.4 \times 287 \times 288} = 307 \text{ K}$$

The bottom part shows the derivation of velocity V behind the shock:

$$M_2 = \frac{u_s - V}{a_2} \quad V = u_s - M_2 a_2$$

$$= 1.102 \sqrt{1.4 \times 287 \times 288} - 0.9103 \sqrt{1.4 \times 287 \times 307}$$

$$= 55.2 \text{ m/s}$$

In your case M_2 is, what is M_2 u_s minus v , which you have to find out by a 2. So, therefore, you can write v is equal to $a_2 a_2 u_s$ minus $M_2 a_2$, u_s is always I know that this is found 1.102 into root over 1.4 into 287 is the value of R into 288 minus M_2 , which I have found out 0.9103 into again a 2, a 2 is root over 1.4 into sorry, 287 into T_2 , T_2 I have already calculated, because I know T_2 by T_1 from the table, so T_2 is calculated 307. So, most important parameter is this we want to know this, this comes out to be 55.2 as per as this calculation, I know the answer, this is the answer. So, this is a straight use of this formula, which we derive the analysis, which we have made for a moving shock.

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Example 2
 A normal shock wave across which the pressure ratio is 1.17, moves down a duct into still air at a pressure of 105 kPa and a temperature of 30°C. Find the temperature, pressure and velocity of air behind the shock wave. This shock wave passes over a small cylinder (calculator) as shown in Figure. Assume that the shock is unaffected by the small cylinder, find pressure at the stagnation point on the cylinder after the shock has passed over it.

$\frac{p_2}{p_1} = 1.17$
 $M_1 = 1.07; M_2 = 0.936$
 $\frac{T_2}{T_1} = 1.046$
 $T_2 = 1.046 \times 303$

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Now, another interesting problem you see this is a relatively more interesting problem. This problem what does it say, what does it say, a normal shock wave across, which the pressure ratio is 1.17 moves down a duct into still air at a pressure of this is almost the same and a temperature of 30 degrees rest part is little interesting. Find the temperature, pressure and velocity of air behind the shock wave, this part is routine that means; you can say that said this is the total repetition of the earlier problem we probably difference in the data.

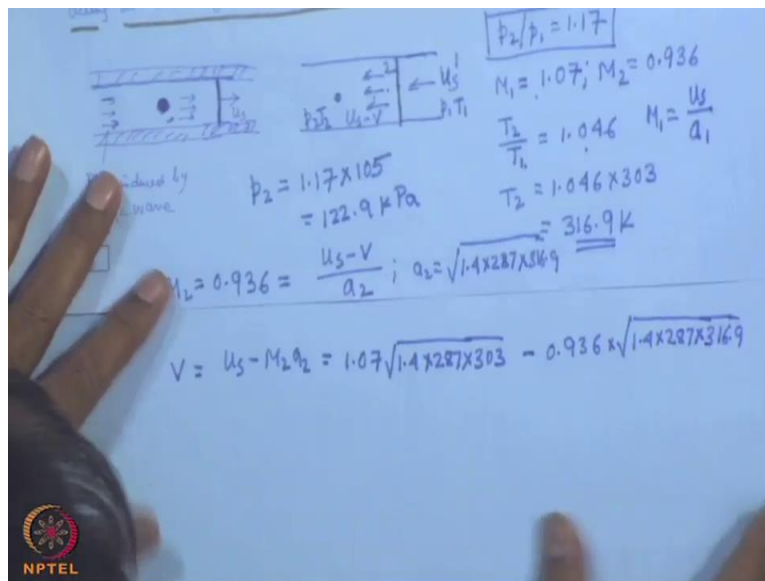
Now, next one this shock wave passes over a small circular cylinder, it is actually it will be here circular cylinder as shown in figure that I will show you in the figure. Assume that the shock is unaffected by the small cylinder, find pressure acting at the stagnation point on the cylinder after the shock has passed over it. Now, this is a problem here, now this problem if you solve then how you, what is the problem? Problem is this one that this is the shock, shock and this is the small cylinder. So, shock has started from somewhere else in the upstream it has passed different sections so at any instant as the problem tells when the shock has passed over the cylinder and shock remains unaffected, what is the condition, that means; in the entire upstream part of the shock through, which the shock has already passed the fluid these are the attain some induced velocity v ok.

Now, let us find these induced velocity v . So, therefore, our case is similar to this then our case is similar to this that let this is be cylinder our case is similar to this so u_s same thing u_s minus v

this is one and this is two. And let this is $p_2 T_2$ so $p_1 T_1$ so I can find everything, but first of all I have to see with respect to the stationary shock, what is the thing in pressure ratio same thing that means; I am having pressure ratio that means I can immediately go to the shock table normal shock table and see against this pressure ratio what is my value $M_1 M_2$. So, this pressure against this pressure ratio the M_1 has found from the shock table is 1.07 M_2 has found from the shock is 0.936 ok and T_2 by T_1 has found is 1.046.

So, therefore, since T_2 by T_1 is known I can immediately calculate the temperature behind the shock that means this temperature behind the shock in this case there that means this temperature T_2 I can find out T_1 is what T_1 is 30 degree that means T_2 is 1.046 into 273 plus 30 that means 303. So I can find out this equals to 316.9 k.

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So, I know this and p_2 I can find out how? p_2 is equal to this is the school level thing 1.17, it is already given 1.17 so 1.17 into 105 kilo Pascal so I can find out and this becomes is equal to 122.9 kilo Pascal.

Now, what I have to find out next, assume that the shock is unaffected ρ , before that I have to find out the velocity of air behind the shock this is again the same thing that M_2 I know that means M_2 is 0.936 is equal to what? u_5 minus v divided by a_2 . Now, a_2 is what? a_2 is equal to root over $\gamma R T_2$ that means 1.4 gamma R 287 into T_2 , because T_2 is 316.9 so I get the value of a_2 . I will just substitute to get the value of $a_2 M_2$ that means I can write better v is

equal to $u_s \text{ minus } M_2 a_2$ that means u_s what is u_s ? u_s is again M_1 is what is M_1 here, M_1 is u_s by a_1 and a_1 is root over $\gamma R T$ so u_s is M_1 times a_1 root over $\gamma R T$, T is 303 minus M_2 , M_2 is 0.936 times root over γR and T_2 that means 316.9 so that is very simple 316.9 you understand 316.9.

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Assume that the flow is isentropic at the stagnation point on the cylinder after the shock has passed over it.

Flow reduced by shock wave

$p_2/p_1 = 1.17$

$M_1 = 1.07; M_2 = 0.936$

$T_2/T_1 = 1.046$

$T_2 = 1.046 \times 303 = 316.9 \text{ K}$

$M_2 = 0.936 = \frac{u_s - v}{a_2}; a_2 = \sqrt{1.4 \times 287 \times 316.9} = 316.9 \text{ K}$

$= 39.38 \text{ m/s}$

$M = \frac{39.38}{\sqrt{1.4 \times 287 \times 316.9}} = 0.11$

$p_0/p = 1.0085$

$p_0 = 1.0085 \times 122.9 \text{ kPa} = 123.9 \text{ kPa}$

So therefore I get the value of v , that this v becomes equals to some 33 ok, this becomes equals to v becomes equals to 39.38 meter per second I think it is absolutely alright so it is just the repetition of the earlier problem.

Now, the question is this that we have to find out the stagnation pressure. Now, what happens, if you see this picture now this cylinder actually here this is the model physically translated but actual problem is this. This is the problem as we look with reference to a coordinate frame attached to the moving shock and flow become steady and this becomes stationary. So, actually this is the problem this v , we are getting which is 39.8 that means, now it is a problem that a cylinder circular cylinder is exposed here another thing is told stagnation point. Assuming that the shock is un effected by the small cylinder find pressure acting on the stagnation point on the cylinder after the shock has passed over it and there is another thing, which has to be told that consider this flow to be isentropic consider the flow the pressure acting at the stagnation point on the cylinder assuming that the flow is isentropic.

Now, if we assume the flow to be isentropic then what happens, there is an isentropic flow a cylinder whose velocity is 39.38 meter per second. So, we can find out the stagnation pressure. Before that I have to tell you that, if you recall the this isentropic flow relation, the isentropic table that isentropic flow relations are unnecessary without going to all algebraic calculations through the algebraic equations of isentropic flow relations rather isentropic flow isentropic flow table that is normal shock table it is given in a tabular form. Similar to normal shock table that one column the extreme left column is M then these are the stagnation, because here most important part of calculation is the ratio of stagnation temperature to static temperature.

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Isentropic Flow Table

M	T_0/T	p_0/p	ρ_0/ρ	a_0/a	A/A^*	θ
0.1						
0.2						
0.3						
0.4						
0.5						
1.0	1	↓			1	90°

Stagnation to static, all stagnation property with respect of density to the corresponding static is the function of the Mach number of flow. So, therefore, if you would know the Mach number of flow at that particular location we can find out all this thing.

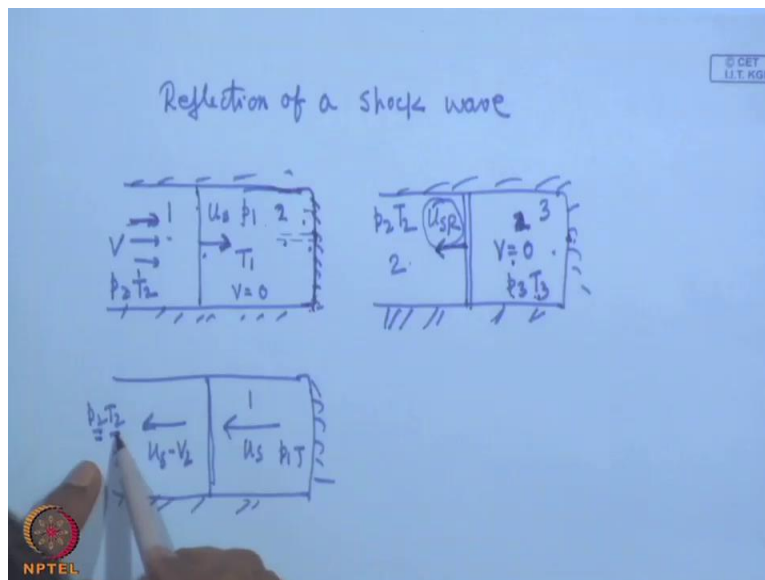
Similarly, a 0 by a what is that acoustic speed at the stagnation condition to the local acoustic speed A by A^* probably you recall this thing A^* is the area, where the sonic velocity is reached that is the minimum area known as throat area corresponding to $M = 1$ and this A is the area local area at which we are concerned with. And another parameter is θ I told probably earlier that does not come into picture here that will come when you discuss the expansion wave this is the angle related to the expansion wave this is not coming and that there also the problems are treated as an isentropic flow that is why in the isentropic flow table itself this that is given this is not required now.

So, now, if we know the Mach number at a particular location in a isentropic flow this here Mach number start 0.01 0.04 like that very small, because all subsonic supersonic all labels are there. So, there will be somewhere one when the value of A by A^* will be one this thing probably I have discussed earlier. So, now, if we know the local Mach number I can find out the p_0 by p_0 by T , I can read that everything so here I have to find out the local Mach number. Now, how to find out, because I know, now, the actual problem the cylinder is exposed to this uniform velocity 39.38.

So, local Mach number is M for example here is 39.38 by the local acoustic speed, which is root over 1.4 gamma into R into this temperature, that means; this is 316 that means, it is nothing but a 2, it is nothing but in our this problem this is a 2. So, you find out this M and this M , if you calculate this M will become equal to I tell you the value, which is already calculated by me, this M has got a value this M , if you find out this M is I tell you just wait, I will tell you the value of M this is a problem, where the M^2 . So, therefore, we take a value of M this M is equal to this is a v^2 and a pressure the Mach number is 0.11 just I see the calculation only so that you can check 0. it has been calculated by me earlier so 0 see that. So, therefore, in the table, if you see this Mach number you can find out the value of p_0 by p and that p_0 by p , if you see from the table this also I tell you for this problem is this.

So, therefore, you can very well find out p_0 is 1.0085 into the static pressure there, which is $p_2 = 122.9$, because the condition prevailing is this p_2 . So, this becomes is equal to 123.9 let me see the value 123.9 as I have calculated already kilo Pascal. So, this is the value, so it is clear that you can find out the value of the Mach number you can find out the value of the stagnation point pressure ok. So, this is one very interesting problem.

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Now, I will tell you something regarding the reflection of a shock wave, regarding the reflection of a shock wave ok. What is reflection of a shock wave? So, what is reflection of a shock wave, let me tell you what is reflection of a shock wave, this happens when the shock wave is moving so moving shock wave analysis we have done reflection of a shock wave. Now, reflection of a shock wave if we have to understand then just how does it happen, I tell you the physical picture. Now, if there is a cylinder, we have so far consider that it is moving with u_s ok and it is creating a velocity v and a pressure p_2 T_2 and this side the pressure is p_1 and initial velocity v is 0 and temperature T_1 that was our thing that it is moving in this direction so with respect to these things are happening now this is the actual problem physical problem as we have discussed.

Now, what happens if this thing is closed at the end? That means, there is a closed end or the pipe was open suddenly the valve is closed that means, the shock wave is moving and the pipe is closed at the end or is being closed by the closer of a valve at the end, what will happen? Physically you try to understand what will happen, this velocity will be induced so long the

shock moves now try to understand what happens in reality. Now, when the shock comes here the velocity will be imposed v but after that what happens this is the closed end so at this solid end the velocity will be zero there has cannot be any velocity the velocity cannot penetrate into the solid like that so there will be normal velocity so therefore this velocity will be zero.

And slowly in a compressible fluid they entered fluid will attain a velocity zero for an ideal incompressible fluid this will be instantaneously zero but for a compressible fluid the velocity will first fall here zero and its it will take some time it may be infinitely small depending upon the compressibility of the fluid that the entered velocity will again be zero. So, this thing is perceived in a way as if the shock wave after reaching there is being reflected back in a way that it creates a zero velocity here is reflected back in a way that usually the velocity it created in this direction v again it created a velocity v in this direction in the opposite direction so that the final velocity is zero, that means; the strength of the shock will be such for the reflected wave so that it can create a zero velocity, while passing through it, so that is the philosophy of the shock, reflection of the shock.

Now, let us do that reflection of the shock let us do that the reflection now if reflection of the shock is to be understood let us consider not the initial one the shock is being reflected, the shock is being reflected now and we can just consider a case that when the shock is reflected ok. And this is the reflection, that a reflected shock velocity and here let one and two, if we see that one sorry, this is why one and two I am telling that this can be expressed as we did earlier in this way that this is coming with $u_s p_1 T_1$, so this one and this is two this is $p_2 T_2$ and this is going with u_s minus v_2 .

So, now, what happened so this two will now prevail here that is here two that is $p_2 T_2$ so now this is $p_2 T_2$ I am sorry, now this will be sorry, sorry, this when you will come here so entire thing was then $p_2 T_2$ that means two when it reaches when it reached here that the entire fluid was at two. So, when it is coming here this side is two that is $p_2 T_2$ I think you have understood and this side is three, that is $p_3 T_3$ and velocity is zero. So, this is the reflected shock that means the reflected shock, if we analyze you will analyze this way.

Now, with the help of the shock tables we can calculate things that means; shock will again be reflected back here. So, this is a typical reflection and under certain circumstances shock will go back again. So, there will be a repeated movement of the shock, so one such reflection of shock

problem, we will now solve that how a shock moving ultimately moving in this direction and a particular direction ultimately reaches the closed end and backs, when it reaches the close end automatically the velocity become zero this is the reality that means; this is conceive as a movement of a reflected shock wave with velocity u_{sr} such that this velocity become zero. And the condition which is being created is p_3, t_3 again it is changed that means from p_1, t_1 one this is p_2, t_2 the shock this is with respect to the shock then the entire thing will become p_2, t_2 here also the when it reaches here the entire thing becomes p_2, t_2 pressure p_2 temperature then again it will be changed to p_3, t_3 when the reflected shock will pass through that.

So this is the scenario physical scenario, I think that this will be more ah easily understood if you solve a problem and this time we cannot I cannot continue because the time is up. So in the next class i will solve a problem in relation to the reflection of a shock wave.

Thank you, close.