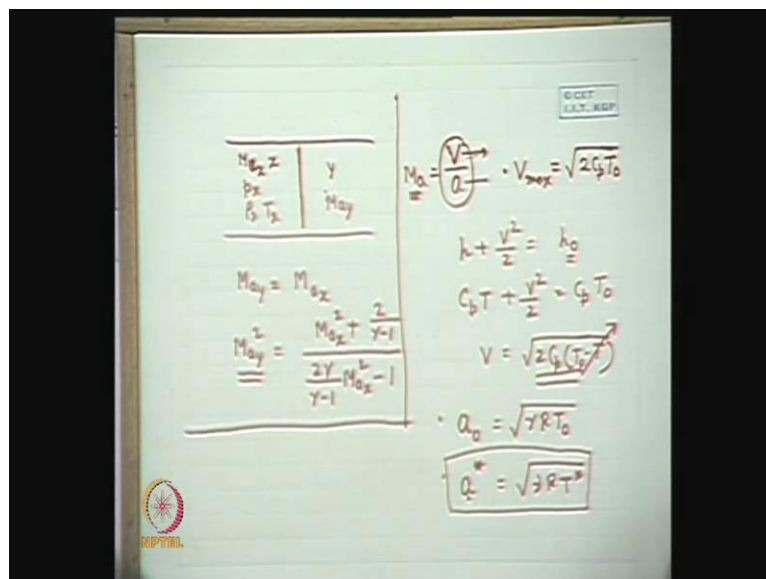


**Introduction to Fluid Machines, and Compressible Flow**  
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**Lecture - 35**  
**Normal Shock Part – III**

Good morning I welcome you to this session. We will continue the discussion on normal shocks. So, last class actually, we were discussing that for a shock, if we know the properties at the upstream of the shock, we are interested to know the properties at the downstream section; that means, for a given low properties at the upstream of the shock what are the flow properties after the shock waves; that means, across the shock waves there is a change in flow properties, and we are interested to find that. So, let us continue the discussions in this way that let this is the duct, and let this there is a shock here a shock wave

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And if the upstream section is designated by x with all given properties that mach sorry m a x p x rho x t x we are interested in all those properties across the shock. So, last class we derived 1 very important relation relating to the first the mach number after this shock downstream of the shock, and we probably, if you recall we get 2 solutions from a quadratic equation connecting the mach number at this section y after this shock, and the before this shock; that means, m a y, and m a x, and we found 1 trivial solution that is m

a y is m a x that is the equality another 1 is the which is the possible solutions that m a y square in this way can be written as, and it is very simple from that equation 2 by gamma minus 1 by 2 by gamma by gamma minus 1 m a x square minus one.

So, this is the relationship; that means, if we know a value of m a x; that means, the mach number at the upstream of this shock, then the downstream of the shock we get the mach number which is given by this after that what we was discussing is the non dimensional of just for the moment we come to a different aspect that non dimensionalization of velocity dimensionalization of velocity in compressible flow now the velocity in compressible flow is non dimensionalized by three reference velocity now here 1 very pertinent question comes that this v by a the mach number at any section is itself a dimensionless parameter.

But this cannot be used as a dimensionless velocity this is, because a change in mach number implies both in change in v, and a this is, because when the flow changes from section to section the flow velocity along with that the velocity of the sound also changes, because this state properties change. So, therefore, mach number cannot be used in as a dimensionless or non dimensional flow velocity, because the reference value a which is used in the denominator here by its definition also change with the change in v. So, therefore, three reference velocities actually this was discussed in last class also, but I feel that I was little fast. So, there were some confusions.

So, that is why I want to repeat it again. So, there are three reference velocities used in this connection 1 is the maximum velocity 1 is the maximum velocity maximum velocity is given by root over 2 c p into t 0 you know relating to stagnation condition in an isentropic flow if we write the energy equation h plus v square by 2 is equal to h 0 stagnation step, and any other step given by velocity v, and for a perfect gas c p by c p t that h is c p t is equal to c p t 0 you know that. So, in general v is equal to root over 2 c p or any adiabatic flow it is the equation like that.

So, the maximum velocity by theory comes when t is equal to 0 this hypothetical case. So, this a theoretical velocity can be used as the maximum velocity another 1 is the this is number 1 number 2 is the acoustic velocity a at the stagnation condition this is given by gamma r t 0 well another 1 is the acoustic velocity at the critical condition a star which given as gamma r t star. Now usually in compressible flow the velocity is

dimensional non dimensionalized or is made dimensionless by use of this velocity as the reference velocity, it is the velocity of sound at the critical condition  $\gamma r t$  dash t star.

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The whiteboard contains the following handwritten derivations:

$$\frac{V}{a^*} = Ma^*$$

$$a^* = \sqrt{\gamma R T^*}$$

$$Ma^* = 1$$

$$h + \frac{V^2}{2} = h^* + \frac{V^{*2}}{2}$$

$$c_p T + \frac{V^2}{2} = c_p T^* + \frac{V^{*2}}{2}$$

$$\frac{a^2}{\gamma - 1} + \frac{V^2}{2} = \frac{a^{*2}}{\gamma - 1} + \frac{V^{*2}}{2}$$

$$\frac{V^2}{2} + \frac{a^2}{\gamma - 1} = \frac{(\gamma + 1)}{2(\gamma - 1)} a^{*2}$$

$$\frac{Ma^2}{2} + \frac{1}{\gamma - 1} \frac{a^2}{a^{*2}} = \frac{\gamma + 1}{2(\gamma - 1)}$$

Other notes on the board include:

- $c_p T = \frac{\gamma R T}{\gamma - 1}$
- $\frac{a^2}{\gamma - 1}$
- $Ma = \frac{V}{a}$
- $Ma^* = \frac{V}{a^*}$
- $\frac{a}{a^*} = \frac{Ma^*}{Ma}$

Now, if we use this  $v$  with a star the ratio of the flow velocity to the  $a$  star. So,  $a$  star is the unique value in a particular flow condition it does not change from section to section it has an unique value that is the value of velocity of sound at the critical condition this is defined as the  $m a$  star. So, confusion here was there the last class the asterisk or the star is used for all properties at the critical condition for example, by  $p$  star we mean the pressure at the critical condition by  $t$  star, we mean the temperature at the critical condition by  $\rho$  star we mean the density at the critical condition.

Similarly, a star used here mean means the sound velocity of the critical condition; that means, it is nothing, but root over  $\gamma r t$  star in case o perfect gas, but this critical condition is defined when the velocity is equal to the velocity the sound flow velocity; that means,  $v$  star the flow velocity at the critical condition is a star. So, therefore,  $m a$  at the critical condition is one, but we do not use the symbol  $m a$  star to denote the mach number at the critical condition rather  $m a$  star there may be a confusion this symbol this is the convention  $m a$  star is used to denote the dimensionless velocity  $v$  by a star.

So,  $m a$  star is not the mach number at the critical condition, because this cannot be used as  $m a$  star, because this is unique, and this is the value of this is 1 at the critical

condition. So, we take the critical condition as defined by  $M_a$  is equal to 1 when the flow velocity is equal to velocity of sound, but  $M_a^*$  is not the critical mach number at the critical section it is  $v$  by  $a^*$  now from a simple energy equation if I write the energy equation with at any section given by the velocity  $v$ , and the critical section we can write  $h^* + \frac{v^{*2}}{2}$  all right now  $h$  can be written again  $c_p t + \frac{v^2}{2}$  is equal to  $c_p t^* + \frac{v^{*2}}{2}$ .

Now, what is  $c_p c_p t c_p t$  here we can write  $c_p t$  is equal to  $\frac{\gamma}{\gamma - 1} r t$ . So, this is nothing, but a square. So, a square by  $\gamma - 1$ . So, if I substitute this here we get a square by  $\gamma - 1 + \frac{v^2}{2}$  is equal to similarly this will be  $a^{*2} + \frac{v^{*2}}{2}$  now I write this first. So,  $\frac{v^2}{2} + a^2$  by  $\gamma - 1$ . Now  $v^*$  is a star that is true at the critical condition. So, this can be written as if I take common. So,  $2 \gamma - 1$   $a^*$  into a star square.

Now, if I divide by  $a^{*2}$  this equation let hand side, and right hand side we get  $\frac{a^{*2}}{2} + \frac{v^{*2}}{2}$  by definition is  $M_a^* + 1$  by  $\gamma - 1$  a square by  $a^{*2}$  is equal to  $\frac{\gamma + 1}{2 \gamma - 1}$  all right now  $a$  by  $a^*$  can be replaced like that by definition what is  $a$   $M_a$  is  $v$  by  $a$ ; that means,  $a$  is  $v$  by  $M_a$ , and  $M_a^*$  is that is the dimensionless velocity mach number is also a dimensionless quantity containing velocity, but the flow velocity, and the sound velocity, but here it is the flow velocity, and a reference velocity which is a sound velocity at a particular condition.

So, that is the difference here both the quantities change with the flow, but here only the flow velocity change, and this is being normalized with a reference quantity at the denominator this is the difference. So, therefore, I can write  $\frac{a}{a^*}$  by  $\frac{a}{a^*}$  is equal to  $\frac{M_a^*}{M_a}$ ; that means, we divide this by this  $\frac{a}{a^*}$ . So, if I substitute this  $\frac{a}{a^*}$  in terms of  $\frac{M_a^*}{M_a}$  we get an equation this is this we get an equation connecting  $M_a$ , and  $M_a^*$  all right  $M_a$ , and  $M_a^*$  let me write this if we write this we get an equation can you see that ok.

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$$\frac{Ma_x^2}{2} + \frac{1}{\gamma-1} \frac{Ma_x^2}{Ma_x^2} = \frac{\gamma+1}{2(\gamma-1)}$$

$$Ma_x = f(Ma_y)$$

$$Ma_y = f(Ma_x)$$

$$Ma_y^2 = \frac{\frac{2}{\gamma+1} Ma_x^2}{1 - \frac{\gamma-1}{\gamma+1} Ma_x^2}$$

So, we can write an equation in a star square by 2 plus 1 by gamma minus 1 a by a star means in a star square by m a square is gamma plus 1 by 2 gamma minus 1 m a star by m a is a by a star. So, a by a star square is m a star square now with this equation now can be expressed in 2 fashion 1 is that m a as a function of m a star very simple 1 can do or m a star as a function of m a either of these 2 if it is written you will get most important 1 is m a star square m a square m a as a function of m a star 2 by gamma plus 1 m a star square divided by 1 minus gamma minus 1 by gamma plus 1 m a star square.

You do not have to remember all this formulae you have to know the logic, and the steps through which it is there being reduced similarly 1 can express m a star in terms of m a as gamma plus 1 by 2 m a square divided by 1 plus gamma minus 1 by 2 m a square now out of these 2 this 1 is used this is very important that mach number is mach number is expressed in terms of the the dimensionless velocity m a star now if we use this in the earlier 1 now we stopped here now if I express m a y in terms of m a y star, and m a x in terms of m a x star what we get.

Now, before that you see that this is the expression well now this is not the thing where I did it sorry I think we should concentrate here otherwise you will be in trouble. So, m a y these are the 2 solutions we get we got m a y into terms of m a x now here 1 thing is sure that if m a x greater than 1 we already recognized that the shock takes place when the upstream condition is supersonic; that means, in a supersonic flow, then from this

equation 1 can prove that when simple mathematics can prove this that when  $M_a x$  is greater than 1  $M_a y$  will be less than 1 with feasible values of gamma between 1.67. As you know the gamma for compressible fluids that gases varies between 1 is the absolute minimum, and 1 point six seven is the maximum 1 for all polyatomic cases. So, taking any representative value between that it can be proved that when  $M_a x$  is greater than 1  $M_a y$ ; that means, the shock changes the supersonic flow to subsonic flow this will be more clear if we express this mach number at the 2 section in terms of the  $M^*$ ; that means, dimensionless velocity; that means, by the use of this equation the equation which we justify by the use of this equation.

That means  $M_a y$   $M_a x$ ; that means, the  $M_a y$  or  $M_a x$  whatever you tell; that means, the mach number in terms of the corresponding  $M^*$  if we do that.

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Handwritten notes on a whiteboard showing the derivation of the temperature ratio across a normal shock wave. The notes define Mach number  $M^* = 1$  at the shock, distinguish between supersonic ( $M^* > 1$ ) and subsonic ( $M^* < 1$ ) flow, and derive the relationship  $\frac{T_1}{T_2} = \frac{1 + \frac{\gamma-1}{2} M_1^2}{1 + \frac{\gamma-1}{2} M_2^2}$ .

Then we will get The relationship like that  $M_a y$  star. So, this is a very simple relationship is equal to one; that means, here it is obvious that the gamma factor is not there. So, when  $M_a$  star  $M_a x$  star is greater than one; that means, supersonic  $M_a y$  star is less than one; that means, supersonic to subsonic now there are certain routine calculations or the pressure values now we are interested with temperature, and pressure.

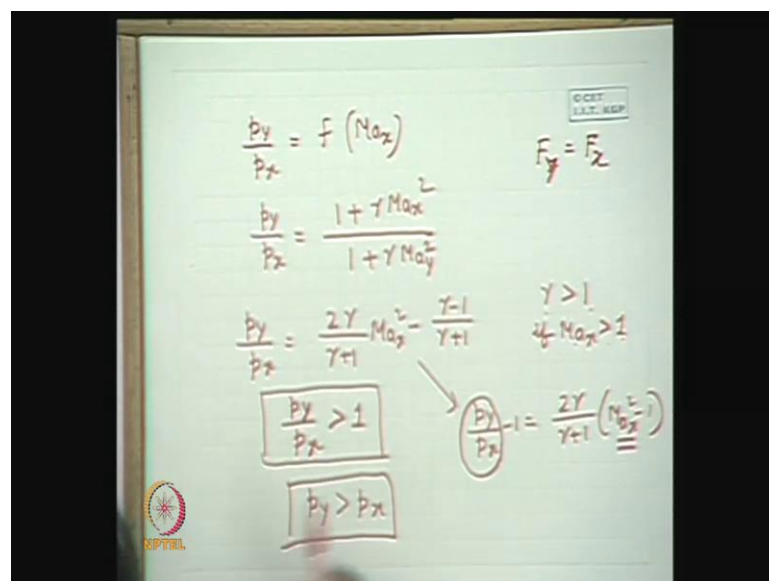
That means our basic motto is again to find out all the quantities after the shock quantities at temperature pressure density rho y these are the main flow properties or the state points. So, these we will calculate how we will calculate if you remember the

temperature ratio now it is very simple when we have already derived the relationship between  $m a_y$ , and  $m a_x$  now if we can write or we recall the relationship between  $t_y$ , and  $t_x$  in an isentropic flow for a perfect gas it will be  $1 + \gamma - 1$  by  $2 m a_x$  square plus divided by  $1 + \gamma - 1$  by  $2$ .

We deduce it with the help of the relationship between the stagnation properties, and the local properties that is  $1 + \gamma - 1$  by  $2 m a$  square. So, logic is like that, and this was derived from starting from the basic energy equation for an adiabatic flow, and considering the flow is in visit, and using the perfect gas as the ideal gases as the working fluid well. So, here if we use  $m a_y$  if we just here sorry  $m a_y$  in terms of  $m a_x$ , then we get a relationship like this which is a big 1 you do not have to remember again I am telling just the logical step you have to know.

Let me write  $1 + \gamma - 1$  by  $2 m a_x$  is square into  $2 \gamma$  by  $\gamma - 1$   $m a_x$  is square minus  $1$  divided by  $\gamma + 1$  whole square divided by  $2 \gamma - 1$  minus one. So, objective is that when we know the property at the upstream of this shock by the  $m a_x$ , then we can find out the temperature ratio; that means, knowing  $t_x$ , and  $m a_x$  we can find out the  $t_y$ ; that means, the temperature after the shock; that means, the temperature ratio  $t_y / t_x$  is expressed in terms  $m a_x$  with same philosophy the pressure ratios are expressed in terms of the mach number as a function of  $m a_x$ .

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The procedure is like that if we recall the most simple ratios of  $p_y$  by  $p_x$  which was derived by the exploitation of the impulse function equality of the impulse function which sorry  $f_y$  better to write  $f_y$  is  $f_x$  which was derived with the use of momentum equation with the use of momentum equation equation of motion, then probably if you recall this was  $\gamma m a_x^2 + 1$  plus this deductions are very simple. So, only thing is that you will have to know the logic that this is from this.

So, simply again the substituting  $m a_y$  in terms of  $m a_x$ ; that means, the same equation  $m a_y$  in terms of  $m a_x$  we can derive the  $p_y$  to  $p_x$   $p_y$  by  $p_x$  is equal to  $2\gamma$  by  $\gamma + 1$   $m a_x^2$  minus  $\gamma - 1$  by  $\gamma + 1$ . Now here 1 interesting thing you can immediately prove you can immediately prove that for any value of  $\gamma$  greater than 1 if  $m a_x$  is greater than 1  $p_y$  by  $p_x$  is also greater than 1 this can be proved immediately; that means, this quantity is greater than 1 provided  $m a_x$  is greater than 1, and  $\gamma$  is greater than one.

This can be proved easily, because if you subtract minus 1 from here, then you will see that  $2\gamma$  by  $\gamma + 1$  will be thing; that means, if you subtract minus 1 it will; that means,  $p_y$  by  $p_x$  minus 1 is a simple thing that you can do  $2\gamma$  by  $\gamma + 1$  into  $m a_x^2$  minus one; that means, for any value of  $m a_x^2$  greater than one. So, this is positive; that means,  $p_y$  by  $p_x$  is greater than one. So, this can be proved this is a very important conclusion; that means, after the shock the flow reaches subsonic; that means, the flow is decelerated, and at the same time the pressure is increased; that means,  $p_y$  is greater than  $p_x$  ok.



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Handwritten mathematical derivations on a whiteboard:

$$\frac{\rho_y}{\rho_x} = f(M_{ax}) \quad \frac{p_y}{p_x} = F(M_{ax})$$

$$\frac{p_y}{p_x} = \left(\frac{\rho_y}{\rho_x}\right) \left(\frac{T_x}{T_y}\right) = f(M_{ax}) \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} p = \rho R T$$

$$\frac{V_y}{V_x} = \frac{\rho_x}{\rho_y} = \frac{1}{f(M_{ax})}$$

$p, T, \rho, V$        $M_{ax}$

So, this is the ratio, then again it is at the routine affair that rho y by rho x as a function of m a x very simple thing. So, rho y by rho x we have already found out the function that p y by p x as a function of m a x is known similarly we have found out t y by t x. So, if we can express rho y by rho x by these 2 ratios from the equation of state we can find it we know that p is equal to rho r t. So, rho y by rho x is p y by p x into t x by t y. So, t x by t y as a function we know t y by t x similarly this as a function of m a x we know. So, we can find out rho y by rho x. So, similar way we can find out v y by v x.

V y by v x from the continuity is rho x by rho y; that means, rho y by rho x means the reverse v x by v y. So, this way we can find out the functional relationship m a x let this is some function or this 1 by function m a x. So, another function of m a x. So, this way we can find out the ratio of the properties any properties p t rho v after the shock to that before the shock in terms of the mach number m a x

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$$\frac{p_{0y}}{p_{0x}} = \frac{p_y}{p_x} \frac{\rho_x}{\rho_x} = \frac{p_y}{p_x} \left(1 + \frac{\gamma-1}{2} Ma_x^2\right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{p_{0y}}{p_{0x}} = \frac{\left[\frac{\gamma+1}{2} Ma_x^2\right]^{\frac{\gamma}{\gamma-1}}}{\left[1 + \frac{\gamma-1}{2} Ma_x^2\right]^{\frac{\gamma}{\gamma-1}}} \cdot \left[\frac{2\gamma Ma_x^2 - (\gamma-1)}{\gamma+1}\right]^{\frac{-1}{\gamma-1}}$$

Measure of irreversibility

$p_{0y} < p_{0x}$   $Ma_x > 1$

Now, 1 important thing is the stagnation pressure  $p_{0y}$ , and  $p_{0x}$  now see in a shock  $p_{0y}$ , and  $p_{0x}$  are not maintained same this is, because shock is an irreversible process friction is there. So, stagnation pressures are not same later.

So, this is the measure of the irreversibility measure of irreversibility irreversibility now before going for a routine evaluation of this you must know this thing that in a flow the stagnation temperature remains constant when the flow is adiabatic, because stagnation temperature is the index of this stagnation enthalpy all right, because in a perfect gas when there is no heat transfer in adiabatic flow this stagnation enthalpy remains constant total energy enthalpy plus the kinetic energy, and in case of an idea gas the enthalpy can be expressed as  $c_p t$ . So, therefore,  $c_p t + \frac{v^2}{2}$  is known as the stagnation enthalpy.

That means  $c_p t_0$  that is  $h_0$ . So, stagnation temperature is fixed provided there is no energy added or energy taken out, but stagnation pressure will not be same stagnation pressure by definition is the pressure which could be reached if the flow is decelerated or comes to the stagnation condition isentropically without friction; that means, the entire kinetic energy is converted only in the pressure energy understand only in the pressure energy not in the enthalpy through the internal energy that is a very useful concept I think in fluid mechanics also you know that is why in energy conservation whether friction is there or not there is no loss total energy remains same.

But difference is that when friction is there some part is converted intermolecular energy, but in in visit flow or isentropic flow without heat transfer the stagnation pressure remains same; that means, there is no degradation in the intermolecular energy; that means, the entire kinetic energy becomes 0 when it comes to stagnation test the entire kinetic energy is converted only to the pressured energy, and that pressure is known as this stagnation pressure. So, stagnation pressure will be equal when the flow will be frictionless.

But fir any natural flow when the flow is brought to race that is the stagnation condition the pressure there is not exactly this stagnation pressure by its definition. So, therefore, you see the ratio of this stagnation pressure in any flow is the measure of the irreversibility; that means, its departure from 1 is the measure of the irreversibility. Let us find out  $p_o y$  by  $p_o x$  the routine procedure is very simple we can express in terms of their local values, and  $p_x$  by  $p_o x$ , and we know  $p_o y$  by  $p_y$   $p_x$  by  $p_o x$ ; that means, the ratio of the stagnation properties to the local properties in case of pressure will be  $1 + \frac{\gamma - 1}{2} M^2$  by  $\gamma - 1$ .

And  $p_o y$  by  $p_x$  also this ration also we have deduced in terms of the mach number; that means, we know this  $p_y$  by  $p_x$ ; that means,  $p_y$  by  $p_x$  just now I derived; that means,  $p_y$  by  $p_x$  is this. So, we can replace  $p_y$  by  $p_x$  in terms of the  $M^2$   $p_o y$  by  $p_y$  in term of the  $M^2$ , and  $p_x$  by  $p_o x$  in terms of the  $M^2$ , and this  $M^2$  is again substituted in terms of  $M^2$ . So, finally, we get an expression only in terms of  $M^2$  like this this is a big expression again I am telling you do not have to remember this only thing is that you'll have to remember the way it is being deduced  $1 + \frac{\gamma - 1}{2} M^2$  to the power  $\gamma$  by  $\gamma - 1$ .

Well I think is equal to here, then divided by  $2 + \frac{\gamma - 1}{\gamma + 1} M^2$  square minus  $\frac{\gamma - 1}{\gamma + 1} M^2$  whole to the power  $\frac{1}{\gamma - 1}$ . So, this is the relation between  $p_o y$ , and  $p_o x$ , and we can see the value of  $p_o y$  by  $p_x$  is not unity; that means, it is different from one. So, stagnation pressure, and we can prove that  $p_o y$  is less than  $p_o x$  when  $M^2$  is greater than one; that means, this is, because of the friction when  $M^2$  is greater than one; that means, supersonic flow is changed to subsonic flow through a process shock where  $p_o y$  is less than  $p_o x$ .

That means it is the effect of friction that this stagnation pressure is lower than the initial stagnation pressure.

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Handwritten derivation on a slide:

$$s_y - s_x = ?$$

$$T ds = dh - v dp$$

$$T ds = c_p dT - \frac{R T}{p} dp$$

$$ds = c_p \frac{dT}{T} - R \frac{dp}{p}$$

$$\int_x^y ds = \int_x^y c_p \frac{dT}{T} - R \int_x^y \frac{dp}{p}$$

$$s_y - s_x = c_p \ln \frac{T_y}{T_x} - R \ln \frac{p_y}{p_x}$$

$$= c_p \ln \left\{ \frac{T_y/T_x}{(p_y/p_x)^{\frac{R}{c_p}}} \right\}$$

Additional notes on the slide:  $\gamma = \frac{c_p}{c_p - R}$ ,  $R = \frac{\gamma - 1}{\gamma} c_p$ ,  $p v = R T$ .

Now, after this we are interested to find out what is the change of entropy. Let a specific entropy small  $s$  we are interested in that what is the change of entropy earlier in  $h-s$  plane we have recognized that for a shock to occur the entropy has to increase according to second law of thermodynamics. So, what is the value of this. So, routine calculation starts from the thermodynamic property relation  $T ds = dh - v dp$  for a perfect gas we can write  $T ds = c_p dT$ , and  $v$  is what is  $v = R T / p$  or well  $v = R T / p$  is equal to  $R T / p$  by  $p dp$ .

So, therefore,  $ds$  is equal to very simple that you have already learned in your basic thermodynamics  $R dp / p$ . So, now, the job is to integrate on  $ds$  from  $x$  to  $y$  this upstream to downstream section. So,  $c_p dT / T$  by  $T_x$  to  $T_y$  minus integral we can take  $R$  outside the characteristic gas constant which is constant. So, we can take outside the integral. Now, therefore, we can write  $s_y - s_x$  is simply  $c_p \ln(T_y/T_x) - R \ln(p_y/p_x)$  now this  $R$  can be written with this formula  $c_p$  you know is equal to  $\gamma R / (\gamma - 1)$ ; that means,  $R$  is equal to  $(\gamma - 1) c_p / \gamma$  all right into  $c_p$ .

So, if I replace this value of  $R$  here, and take  $c_p$  common that is  $\ln$ , then it becomes  $\ln(T_y/T_x) - (\gamma - 1) \ln(p_y/p_x)$  and then this coefficient can go as a power here; that means,  $p_y / p_x$  it is a

minus. So,  $\ln x$  minus  $\ln y$  as  $\ln x$  by  $y$  using this formula to the power  $\gamma$  minus 1 by  $\gamma$ . So, I can write this  $\gamma$  minus 1 by  $\gamma$   $c_p c_p$  is coming common. So,  $\ln t_y$  by  $t_x$  minus  $\ln p_y$  by  $p_x$  raised  $\gamma$  minus 1 by  $\gamma$  this comes. So, this is the  $s_y$  minus  $s_x$  now this can also be written in terms of this stagnation properties, and it is very simple that we can use this stagnation properties.

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The whiteboard shows the following derivation:

$$s_y - s_x = c_p \ln \left\{ \frac{T_y/T_x}{(p_y/p_x)^{\frac{\gamma}{\gamma-1}}} \right\}$$

$$= c_p \ln \left\{ \frac{T_y/T_x}{(p_y/p_x)^{\frac{\gamma}{\gamma-1}}} \right\}$$

$$= -c_p \ln (p_y/p_x)^{\frac{\gamma}{\gamma-1}}$$

$$= -R \ln (p_y/p_x)$$

$$\frac{s_y - s_x}{R} = -\ln (p_y/p_x)$$

Additional notes on the right side of the whiteboard:

$$\frac{T_y}{T_x} = 1 + \frac{\gamma-1}{2} M_y^2$$

$$\frac{T_x}{T_x} = 1 + \frac{\gamma-1}{2} M_x^2$$

$$\frac{p_y}{p_x} = \left( \frac{1 + \frac{\gamma-1}{2} M_x^2}{1 + \frac{\gamma-1}{2} M_y^2} \right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{p_x}{p_x} = \left( \frac{1 + \frac{\gamma-1}{2} M_x^2}{1 + \frac{\gamma-1}{2} M_x^2} \right)^{\frac{\gamma}{\gamma-1}}$$

At the bottom, it is noted that  $p_{0y} < p_{0x}$ .

This can be straightaway written as let me write it again otherwise there will be problem  $s_y$  minus  $s_x$  is equal to  $c_p \ln t_y$  by  $t_x$  well divided by  $p_y$  by  $p_x$  to the power  $\gamma$  minus 1 this can be straightaway written  $c_p \ln t_o y$  by  $t_o x$  divided by  $p_o y$  by  $p_o x$  to the power now you may say sir why are you writing like this as if it appears the ratio of the properties is equal to the ratio of their stagnation properties it is not, so but by the relationships it appears so; that means, if you write  $t_o y$  by  $t_o y$  that is is equal to  $1 + \gamma$  minus 1 by 2  $m_a y$  square.

Similarly, if you write  $t_o x$  by  $t_o x$  is equal to  $1 + \gamma$  minus 1 by 2  $m_a x$  square; that means, the ratio  $t_y$  by  $t_x$  is no equal to the ratio  $t_o x$  by  $t_o y$  by  $t_o x$ , but they will carry the ratio between these two, but at the same time the  $p_o y$  by  $p_o y$  if you do this calculation it will be obvious immediate  $m_a y$  square to the power just the reciprocal of it  $\gamma$  by  $\gamma$  minus 1, and similarly  $p_o x$  by  $p_o x$  will be the same quantity with mach number  $x$  square to the power  $\gamma$  by  $\gamma$  minus one.

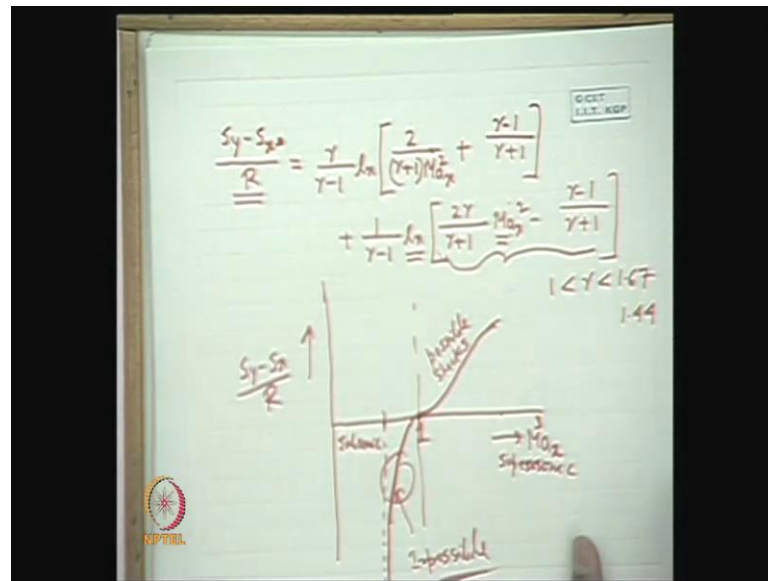
So, with the help of these four equations, if you express  $t_y$  by  $t_x$  the ratio of the temperatures in terms of their stagnation temperature, and the ratio of the pressures in terms of their stagnation pressures you will see  $\gamma - 1$   $\gamma$  by  $\gamma$ . So, these ratio of these 2 quantities will cancel from the numerator, and denominator. So, that ultimately it is very interesting that the same ratio; that means, the variables in terms of the local properties can be just changed or substituted in terms of the stagnation properties. So, that we can express this...

Now, here 1 thing is that  $t_o y$  is equal to  $t_o x$ . So, this is 1 in case of shock, because it is an adiabatic condition there's no energy is added or extracted. So, therefore, the stagnation temperature as I said that is one; that means,  $\ln p_o y$  by  $p_o x$  to the power  $\gamma - 1$  now if I take this thing here in the coefficient, then it can be written, and again replaced it by  $r \ln p_o y$  by  $p_o x$ . So, 1 simple expression that  $s_y - s_x$  by  $r$  it is a non dimensional quantity this side  $\ln$  now since  $p_o y$  is already we have proved that  $p_o y$  is less than  $p_o x$ .

So, therefore, this is always greater than 0 that entropy is always increased all right now a very routine, and very tedious calculations I can make again to have the same conventional things that the entropy change in terms of the mach number that as a function of upstream mach number what he has to do he has to substitute this big expression of the  $y$  by  $p_o x$  in terms of the mach number somewhere earlier we have deduced it that  $p_o y$  by  $p_o x$  in terms of the well in terms of the mach number  $M_a x$ .

If we do that we ultimately get the expression  $s_y$ ; that means, we get the expression  $s_y - s_x$ . I think you've understood that already we have deduced this. So, this is very interesting equation in very simplifies form.

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But if we want to express in terms of mach number at the upstream section  $m a$ , then we have to substitute this ratio as a function of  $m a$  which we have deduced earlier, and final result is that  $s y$  minus  $s x$  by  $r$  is equal to is a very big  $1$  gamma by gamma minus  $1$  there is no reason to remember it, but you must know this gamma plus  $1$   $m a$  x square plus gamma minus  $1$  by gamma plus one.

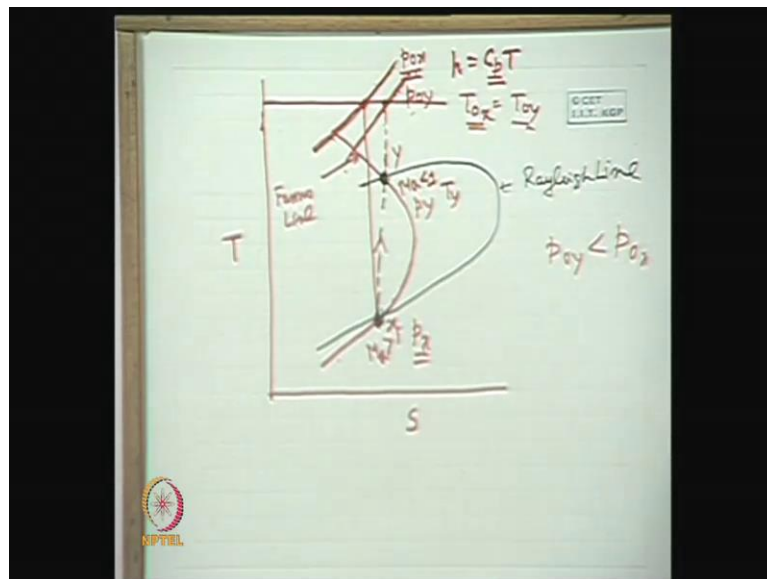
Plus  $1$  by gamma minus  $1$   $1$  n well  $2$  gamma by gamma plus  $1$   $m a$  x square minus gamma minus  $1$  by gamma plus  $1$  now usual convention is to draw now we can show in figure the variation of entropy change  $s y$  minus  $s x$  by  $r$  with mach number  $m a$  x now it has been proved already that  $p o y$  is less than  $p o x$ . So,  $s y$  minus  $s x$  is positive greater than well greater than  $0$ , I am sorry greater than zero. So, greater than zero; that means, the entropy increases greater than  $0$  it is not greater than  $1$  greater than zero. So, this can be expressed in terms of  $m a$  x now we can plot.

So, if we take a value of gamma between  $1$  to  $1$  point six seven let us consider a value any value  $1$  point four four for the diatomic gases, then we can plot this, and we will see that well. So, this is the  $1$  mach number let this increases three like that in a supersonic region this is supersonic super supersonic, and this is subsonic well sub sonic. So, we will see the mach number if I this entropy is going like this . So, therefore, this is an impossible this is the possible shock waves where the entropy increases possible shocks possible shocks.

And this is the impossible region; that means, where the  $s_y$  minus  $s_x$  are negative; that means, the possible shocks corresponding to an upstream mach number which is greater than one, but for an upstream mach number which is less than 1 a process will reduce the entropy in an adiabatic flow which is violating the second law of thermodynamics mathematically we can see that there is an asymptotic approach to the minus infinity when the mach number reaches a particular value in the subsonic region.

This can be found out from this expression here you see that this argument becomes 0 implying an infinite value for this in the negative access, then we can find that a particular value of  $Ma^2$  which becomes equal to  $\frac{\gamma - 1}{2\gamma + 1}$ . So, that value of  $Ma^2$  makes this argument zero. So, this argument 0 this is the 1 n. So, therefore, this becomes. So, a particular value of  $Ma$  this becomes that this part of the curve is impossible impossible this part of the curve is impossible all right.

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So, this is the change in entropy, now we just show at the end the come to this to have an idea of stagnation pressure temperature let us draw the fanoe line in t s diagram we know that we drew fanoe line earlier in h s diagram in a perfect gas  $h$  is equal to  $c_p t$  for a calorically perfect gas  $c_p$  is constant. So,  $h$ , and  $t$  are same; that means, they are just changed with a scale factor. So, the same graph we can draw in t s diagram with the



same qualitative picture like this. So, this is fanoe line all right. So, this is fanoe line this is fanoe line fanoe line.

So, if you recall the Rayleigh line which is like this, this is the Rayleigh line this is the Rayleigh line, and the intersection between the fanoe line, and the rayleigh line is the shock; that means, this is the direction of the shock; that means, the shock takes place in such a way that there is a change in entropy. So, this is the region where mach is greater than 1 m a; that means, supersonic this is the region where mach is less than one. So, this is Rayleigh line I must draw that this is Rayleigh line I must write r a y l e I g h Rayleigh line all right.

So, therefore, we see that these are the intersection now here you 1 thing is very clear that this is the upstream side, and this is the downstream side; that means, this is the x, and this is the y we have already discussed the shock wave the upstream, and downstream points must match both the Rayleigh line conditions, and fanoe line conditions. So, this is the direction of the shock. So, shock takes place in this directions from x to y. Now, therefore, we see that in a shock this stagnation temperature is fixed that is t o x, because this is adiabatic is equal to t o y; that means, if I draw an isentropic line from here vertical line.

So, this cuts here the constant pressure lines in t s diagram like this these are the constant pressure line for a gas for example, the perfect here. So, this is the p o x, but now this step if I draw an isentropic line i; that means, if I draw an isentropic line from x this cuts the isentropic temperature corresponding to this step, and this point corresponds to a particular constant pressure line which physically signifies this stagnation pressure corresponding to this step; that means, this stagnation pressure corresponding to the pressure at this step similarly this is valid for any step that step y it has got a pressure p y, and it has got a temperature t y, and it has got a stagnation temperature t o y.

If x and y are the points in an adiabatic flow; that means, between these 2 points no energy is added or extracted; that means, the isentropic a stagnation temperature is same; that means, if I draw the isentropic line, and I we just come to the same stagnation temperature, because t o x is t o y. So, it come up to this point, then the constant pressure line crossing through this point will indicate this stagnation pressure corresponding to this point since this point is right of this it is obvious geometrically also; that means, the

entropy has increased for this point from this point, then a isentropic the vertical line will cut the same horizontal line to a point where the constant pressure line  $p_o y$  which is less than  $p_o x$ .

That means this is the difference in the stagnation process stagnation is decreased where the stagnation temperature remains same. So, this is precisely the graphical representation of the shock this is the fanoe line low, this is the rayleigh line flow again I am telling this curve is the fanoe line flow; that means, the flow without heat transfer, but with friction, and the steady state condition that is same mass flow, and this is the rayleigh line where the flow takes place without friction, but in general heat transfer is there, and this steady state mass flow condition is maintained that the same mass flow.

But the shock occurs in such a way that upstream, and downstream both satisfies the conditions of no heat transfer conditions of no friction, and also the steady state condition it is very important there the lies the concept that the shock is such that interior details of the shock friction is there shock is a frictional process a natural process which increases the entropy, but upstream, and downstream of the shock refers to the conditions that now energy is added, and ultimately the friction is taken to be negligible in a sense that when we take the momentum equation that across the shock wave the control volume is. So, thin that we have neglected the momentum neglected the frictional effect.

So, this is an approximation, but it is a very good approximation. So, that equality of impulse functions are valid. So, therefore, the both the upstream, and downstream sections corresponds to the corresponds to both the fanoe line, and rayleigh line conditions. So, therefore, they correspond to the intersections of rayleigh, and fanoe line points. So, therefore, again coming to this picture you see. So, this at the 2 points. So, this has to be upstream this has to be downstream, because of the second law, and this figure shows you the corresponding stagnation pressure where the stagnation temperature remains same all right I think any question, question.

Thank you.