Introduction to Fluid Machines, and Compressible Flow

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Lecture - 34
Normal Shock Part – II

Good morning, I welcome you to this session. Today we will continue the discussion on normal shock which we have already started the last session. So, therefore, if we continue our discussion on normal shock, and recall what we have done in the last class that for a friction

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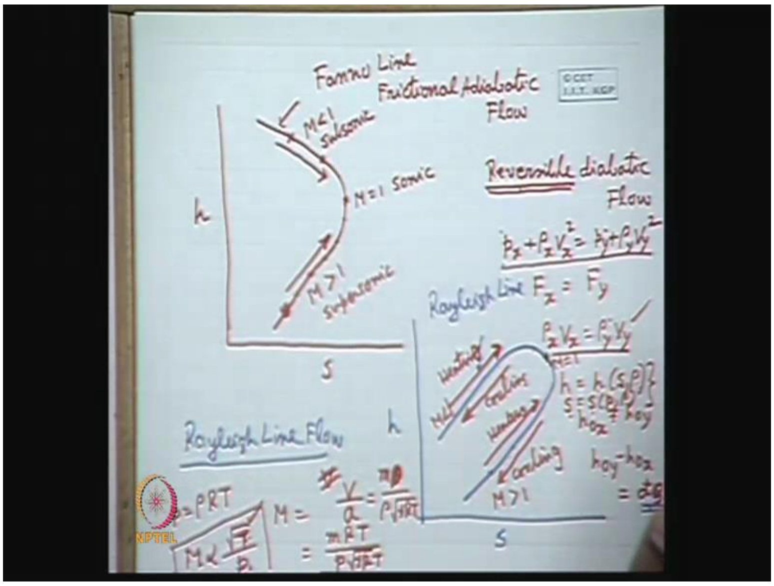

Flow without heat transfer; that means, an adiabatic flow the locus in h s plane is known as fanno line was drawn like this this was known as fanno line, and the flow were referred to fanno line flow, and this is for a frictional frictional adiabatic flow adiabatic flow.

If we recall that adiabatic flow; that means, the points on this curve represents the step points of a flow with constant stagnation enthalpy, because of the condition of this adiabaticness that is adiabatic flow, but they are this step points in a flow which in which friction will be there; that means, they satisfy the adiabatic condition of the constant stagnation enthalpy as well as the steady state continuity equation; that means, the same mass flow, and also we have recognized that this part that is the positive slope in h s diagram this part represents the supersonic m greater than one supersonic flow supersonic flow, and this part m less than one, and represents subsonic flow.

And we have also recognized the point where the entropy becomes maximum this represents m is equal to one or sonic flow, and also we have recognized that according to second law thermodynamics if a upstream point is there in any part of the curve either in supersonic or subsonic the downstream point will always be along the curve in the right direction. So, that the flow takes place in such a way that the entropy of the system which is flowing increases. So, therefore, we always proceed in the right direction which concluded one interesting thing is that the effect of friction; that means, if we see the effect of friction in supersonic flow will be to make the flow towards the sonic, and at the same time the effect of friction in subsonic flow was also to make the flow towards the sonic.

So, this will be the direction of the flow with the effect of friction in case of an adiabatic flow of course,. So, that the entropy increases in the flow according to the second law of thermodynamics now we consider another class of flow which is reversible without friction reversible; that means, without friction, but diabatic flow; that means, diabatic flow; that means, where the heat flow is there; that means, in this case we satisfy the reversible that is zero friction case the momentum equations which ultimately tells that p x plus rho section x the suffix x represents one upstream section is p y already we have recognized that in absence of friction the equation of motion or momentum theory on applied to a control volume where the inlet of the control volume refers to the section x, and the outlet of the control volume refers to the section y can be written like that where this is known as the impulse function p plus rho v square.

So, this is the impulse function corresponding to state x this is the equality of impulse function along with the mass conservation for a steady flow that is the equation of continuity, and the equation of state thermodynamic equation of state h is a function of s, and rho, then we can draw the locus of points in similarly in h s plane for this type of flow like this. If we draw this how can we draw this let us follow a routine procedure as I told in case of fanno line flow that you consider first of all the initial state suffix for a given initial state corresponding to p x rho x, and v x first of all we assume some arbitrary values of v y.

And we can calculate the values of rho y from here, then we can find out the values of first of all values of rho y from here I am sorry the values of rho y from here, then substituting the rho y we can find out the values of p y, and then we can find out the values of enthalpy from is this functional relationship h as a function of s, and rho. So, therefore, we can calculate this curve like this this very important a curve like this. So, here one thing is that in this case. If we try to find out the h o x that probably I discussed in the last class will be not equal to h o y. So, if I find the difference h o y minus h o x I will be getting a value which is not zero, and this will represents the value of the heat transfer during the process.

If it is negative the heat has been taken out of this system, and if it is positive; that means, heat has been added to the system; that means, well; that means, if we find out the equations from the locus of h s from these sets of equations of course, here you will have to consider another set that another equations probably the thermodynamic equation of state s as a function of p, and rho. So, knowing the p, and rho you'll can find out s,and then only you can find out h. So, that you can draw this curve; that means, we are satisfying the momentum equation for in visit flow satisfying the continuity equations for steady flow, and using the thermodynamic property relations we can draw this again I am telling in a routine matter.

First you assume v y, then for a given step points at upstream section x we find out rho y similar to that of fanno line as we did, then by knowing rho y from the momentum equations I find p y, then I can use this property relation to find entropy,and then we can find out enthalpy from the this property relation enthalpy as a function of entropy, and density. So, this way we can find out this curve, and this curve is known as Rayleigh line this curve is known as Rayleigh line this is known as Rayleigh rayleigh line, and this flow is referred as Rayleigh line flow Rayleigh line flow; that means, this is precisely the reversible diabatic flow.

Now, you see in the Rayleigh line flow; that means, the flows which are reversible, and diabatic; that means, there is an heat transfer here you see where I have not used the condition for equality of this stagnation enthalpy. So, therefore, this stagnation enthalpy in general will not be equal it may or may not be equal, but the difference in stagnation enthalpy for Rayleigh line flow indicates the heat transfer; that means, if h o y minus h o x y is the downstream section is greater than zero heat is being added otherwise it is being rejected taken out.

So, therefore, in this case since the heat transfer is there from the working system which is flowing that is the medium the entropy will may increase may decrease depending upon the situation whether heat is added or heat is taken out. So, therefore, in this case both the directions we can move in the curve; that means, entropy can increase entropy can decrease this is, because there is an heat transfer. So, therefore, when entropy is increasing this direction represents the heating; that means, in this direction this is the heating; that means, when heating is done. So, we will move in this direction along the curve while when the cooling is made we will move in this direction this is cooling; that means, we can move towards right along the curve when heating is there; that means, which will effect, and increase in entropy.

Similarly, when cooling is made we will move along the left of the curve; that means, when heat is taken out. So, we can move in both the directions again it can be shown that this part of the curve the lower part of the curve is associated with m greater that one, and this upper part of the curve is associated with m less than one; that means, this is for supersonic flow, and this is for subsonic flow where this point is for the sonic condition m is equal to one. So, if we split this curve we can say in supersonic flow if we make heating; that means, heating in a supersonic reversible diabatic flow will cause the flow to decelerate, and going towards the sonic condition while the cooling in the supersonic region; that means, for a supersonic flow maintaining the reversible condition will make the flow accelerating, and going more towards more supersonic region the reverse is happening for subsonic flow; that means, if the flow is subsonic the heating in a reversible manner will make the flow accelerating, and going towards sonic while in a subsonic flow if cooling is made, then the flow is decelerated, and if the flow is made reversible, then the flow is decelerated, and going towards more subsonic region.

So, therefore, from this curve we can infer the effect of heating, and cooling in both supersonic, and subsonic region for reversible diabatic flow all right now this of course, is very difficult to conclude mathematically from these two equations you see that if you write the velocity expression that velocity well the mach number for example, not the velocity the mach number is precisely v by a. Now if we write the v expression v v is given by for a given mass flow the v, and v is equal to rho sorry v is equal to mass flow rate divided by rho for a given area, and a is root over gamma r t, and if we just substitute rho in terms of pressure, and temperature rho is p by r t, because we know that p is equal to rho r t. So, rho is p by r t; that means, m r t by p root over gamma r t.

So, therefore, we see m is proportional to root over t by p. So, therefore, the effect of heat addition will change the m by the combination of the change in t, and p. So, both the temperature, and pressure changes. So, whether m will increase or not when heat is added or heat is subtracted depends upon the relative change in root t, and p. So, both the temperature, and pressure changes simultaneously. So, it is very difficult to be conclusive from this equation of this type until, and unless we go for a detailed calculations or a particular type of flow for example, reversible diabatic flow.

And this particular calculation helps us in doing this curve or doing this figure where we can be conclusive that the effect of heating in supersonic flow is to increase the mach number towards the sonic, and similarly the effect of cooling is to decrease the mach number sorry to decade the mach number towards the sonic, and to increase the mach number towards more supersonic region now the physical now the question is graph this Rayleigh line graph is or the figure is refers to the flow which is reversible diabatic flow, and fanno line refers to the flow; that means, this fanno line refers to the flow frictional adiabatic flow.

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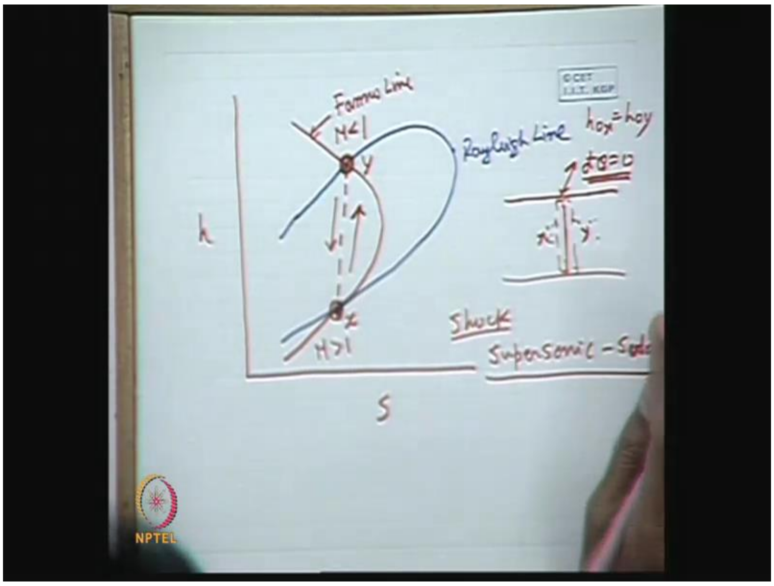

Now, a shock wave a shock refers to both conditions of fanno line flow, and the Rayleigh line flow; that means, if I now just draw the fanno line, and Rayleigh line in a single plane; that means, h s diagram if I draw this fanno line, and this is the Rayleigh line let this is fanno line, and let this Rayleigh line, and let this is Rayleigh line now you see that since the shock represents both the fanno line, and Rayleigh line flow we have already discussed that the shock in case of shock these equations are valid this equation is valid this equation is valid these are the thermodynamic equations of state or the thermodynamic property relations, and along with that the constancy in stagnation enthalpy h o x is h o y is also valid.

So, therefore, for a shock o occur both the upstream, and downstream points will satisfy all these conditions; that means, they will lie on both the rayleigh, and fanno line; that means, this intersection of rayleigh, and fanno lines will represents the upstream, and downstream points of a shock. So, therefore, in h s plane if I have a fanno line for a particular stagnation enthalpy, and if I have a Rayleigh line, then we can find out this intersection of the rayleigh, and fanno lines are the two points on the shock; that means, if I join these two points we can get the direction of shock we can sorry we can get the two points; that means, the line; that means, the line line indicating the process of shock.

Now, question comes whether it is in this direction or it is in this direction; that means, whether our upstream if upstream point is this, then downstream point will be that, because they are the two extreme points or if the upstream point is here the downstream point will be here, but as we have already as we have already recognized that the process of shock will occur because in case of a shock you see it is. So, thin, and if we take a control volume. So, this is the upstream x, and y. So, d q in either direction is consider to be zero.

Therefore in the process of shock; that means, across the shock the process occurs which is precisely adiabatic that is why we have made this equation to be valid. So, in this case the entropy change of the system is the entropy change of the universe; that means, the entropy change will be greater than zero or the change of entropy will be positive. So, therefore, he process will take place in such a way from upstream to downstream which will cause the entropy to increase. So, therefore, there is no other go than this point to be the upstream point x, and this point to be the downstream point, and these two points intersecting the two regions one is the m greater than one, and another is the m less than one.

So, this prove that the shock takes place from supersonic shock takes place from supersonic to sonic subsonic supersonic to subsonic. So, take place from supersonic to subsonic well. So, therefore, it proves that shock takes place from a supersonic region. So, that it takes place from supersonic region to subsonic region; that means, due to the shock the flow changes from supersonic to subsonic which finds the direction of the shock.

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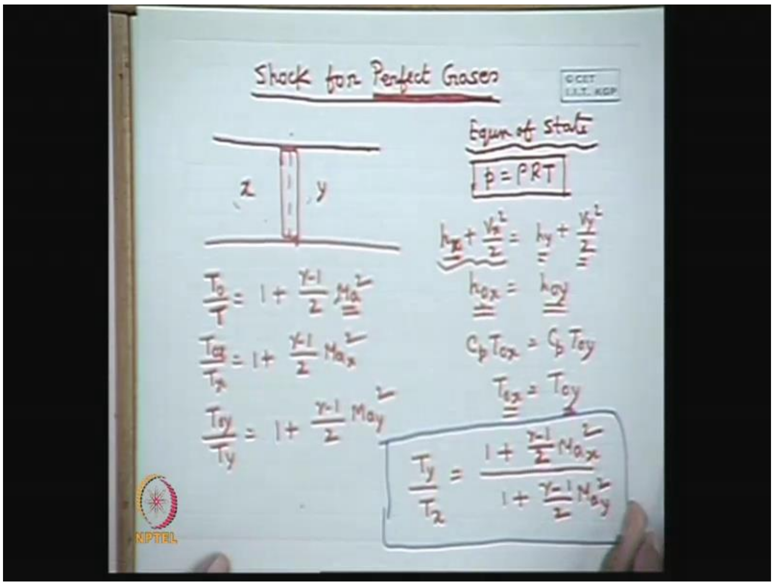

Now, after this I will just discuss the shock in normal shock for perfect gases perfect gases well shock for perfect gases now we have now we are interested as I have told you earlier not in the not in the interior details of the shock, but let this is the control volume the upstream, and downstream flow properties across the shock; that means, if we know the upstream properties of the shock what are the corresponding flow properties downstream the shock now to know this explicitly we have to find out the we have to know rather explicit algebraic relationships of these thermodynamic property relations or the relationship between pressure temperature, and volume; that means, precisely we have to know the equation of state.

So, therefore, equation of state which is very important equation of state has to be known explicitly; that means, explicit algebraic form of the equation of state should be known for which you have to specify a particular system as the working fluid or the flowing fluid. So, we consider the flowing fluid to be perfect gases or to be a perfect gas in that case we can write the equation of state as p is equal to rho r t, and all other hypotheses leading to other thermodynamic property relations for perfect gases can be used.

Now, let us find out, and exploit all those property relations to find out the relationship between flow properties across a shock wave. So, if you recall we first started with the energy equation h x plus v x square by two is equal to h y plus v y square by two; that means, x refers to the upstream section that is the enthalpy plus the velocity head that is kinetic energy per unit mass this is the specific enthalpy is equal to h y plus v y square by two or precisely we told that h o x is equal to h o y, because this represent the stagnation enthalpy referred to this x condition, and this is the stagnation enthalpy referred to this y condition.

And since there is no heat transfer in this control volume that is from section x to y this stagnation enthalpy will be equal. Now since we have now since we have considered the perfect gas as the working fluid, now I can write this as c p t o x is equal to c p t o y. So, therefore, for a perfect gas we tell that since this stagnation enthalpy is same this stagnation temperature will be same the stagnation temperatures are the index of stagnation enthalpy, because for a perfect gas we can write that enthalpy is c p times the temperature.

Now, if we recall he relationship between the local temperature to this stagnation temperature, because earlier we deduce these ratios taking this stagnation properties as the reference properties if you recall we deduce this equation in terms of the mach number; that means, if the mach number is m a the local temperature; that means, the temperature corresponding to the mach number at any location is expressed in terms of this stagnation temperature by this relation. So, if we utilize this equation for both t x, and t y; that means, if I write t o by t x; that means, with this stagnation temperature m a x square. So, this t o is t o x, and t o y by t y is equal to one plus gamma minus one by two all right m a y square, then we can write from this two that t x t y rather I write by t x t y by t x is equal to one plus t y by t x that is this divided by this; that means, it will be one plus gamma minus one by two which one will be there t y by t x m a x.

Yes sir.

One plus gamma minus one by two m a y square all right. So, this is one very important relationship; that means, the ratio of temperatures can be expressed in terms of the mach numbers at x, and y. Now if now there are certain algebraic steps there is no such concept of fluid mechanics now at present. So, it is only algebra with the equations.

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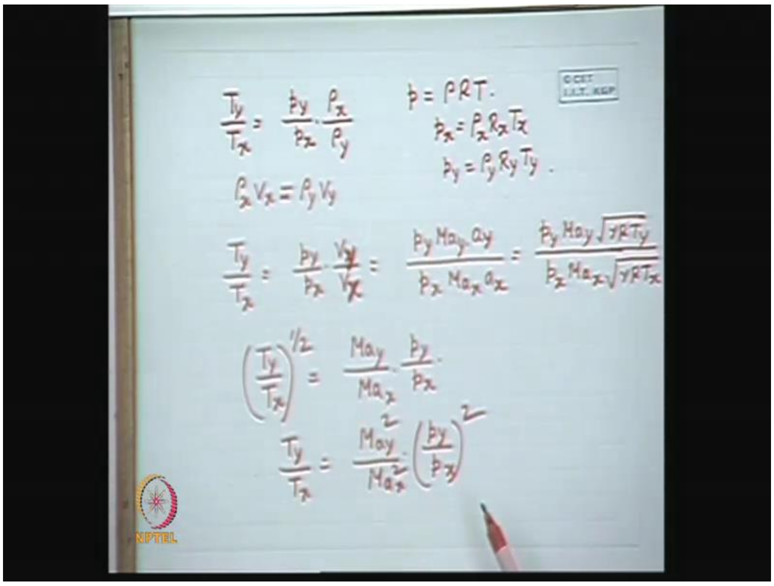

Now, we can write also t y by t x from the equation of state; that means, p is equal to rho r t see that t y by t x can be written as p y by p x into rho x by rho y this I write from the equation of state p is equal to rho r t; that means, p x is rho x; that means, you can write p x is rho x r x t x well, and p y is rho y r y t y.

So, therefore, we can write t y by t x is p y by p x into rho x by rho again from the continuity we can write rho x v x is rho y v y this is well known continuity at steady state to accommodate the same mass flow rate well. So, therefore, rho x by rho y can be substituted from these equation here. So, we get t y by t x is equal to p y by p x into v x by v y again I can write v x by v y in terms of the mach number.

P x sir.

O, well p y by p x into v y by v x I am sorry v y by v x I am sorry v y by v x well now this can be written in terms of the mach number; that means, p y v y can be written as m a y into a y similarly it can be written as p x m a x into a x a y a x are the sound speeds at the condition y at the condition x; that means, this can be written for perfect gases p y m a y p x m a x a y is root over gamma r t y, and this is root over gamma r t x this is for perfect gases we can replace this sound speed or the acoustic speed in terms of the temperature as root over gamma r t y root over gamma.

So, if we take this t y t x here we get t y by t x to the power half is equal to what we get m a y all right by m a x into p y p x all right, now I can well p y by p x. So, I can make this t y by this is square. So, we can use we can make t y by t x is equal to m a y by m a x into p y by p x whole square all right p y by p x m y m a y by m a x whole square, and m a y by m a x also whole square I am sorry m a y whole square m a x whole square p y by p x whole square all right m a y by m a x p y by p x this is clear all right t y by t x.

Now, if I substitute this what we will get; that means, if I substitute now here just you see that if I substitute you can see this thing no, if I can substitute this t y by t x here.

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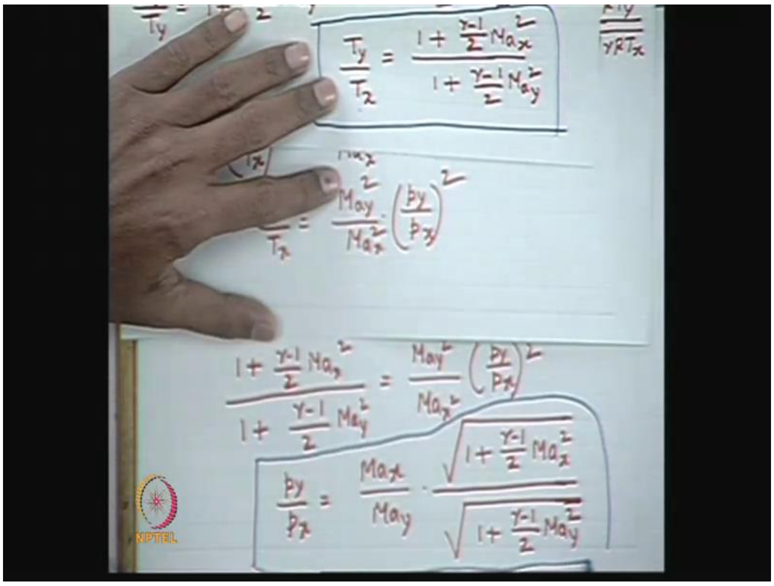

What I get I get one plus gamma minus one by two m a x square divided by one plus gamma minus one by two m a y square is equal to m a y square divided by m a x square into p y p x whole square this is all right. That means, t y by t x I am substituting here this is p y by p x whole square now I can write p y by p x is equal to m a x; that means, taking p y by p x n one side by m a y into under root one plus gamma minus one by two m a x square divided by under root one plus gamma minus one by two m a y, please check it.

So, this is a relationship which is similar to that one t y by t x; that means, the ratio of the pressure at the downstream to the upstream is m a x by m a y under root of one plus gamma minus one by two m a x square one plus all right well now. So, far I have deduced the relationship of t y by t x where I have used only the energy equation for an adiabatic flow; that means, the constancy of stagnation enthalpy, and with the use of the equation of state for a perfect gas; that means, with this condition the constancy of stagnation temperature, and using the continuity equation, and of course, the continuity equation I have used here.

So, continuity equation, and energy equation continuity equation for steady flow, and energy equation for a adiabatic flow; that means, we have developed these relationship; that means, you should be very careful that these relationships are valid or adiabatic flow, and the steady flow; that means, the friction the absence of friction has not been taken so far

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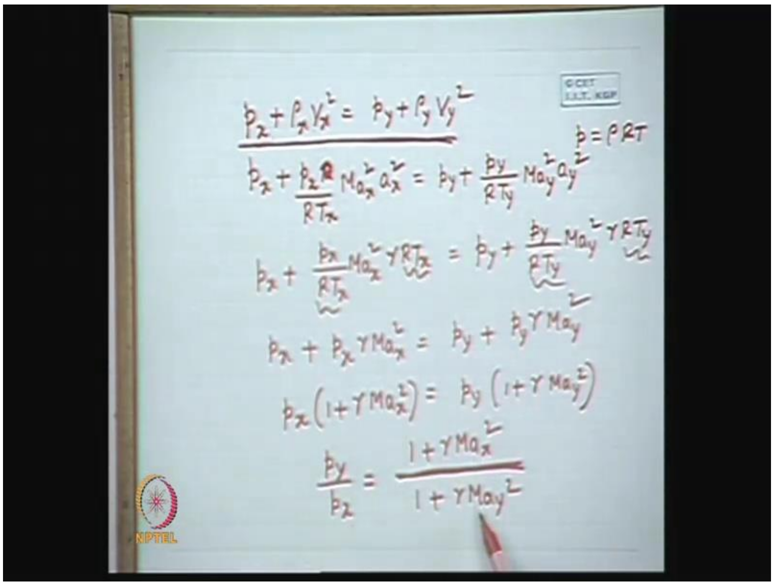

Now, if I consider the momentum equation in absence of friction, then I can write p x plus rho x v x square is equal to p y plus rho y v y square all right this is the momentum equations without friction. Now if you have got any question you can ask me yes, but I can also stop in between if you have got any query do you have any query in this relationships t y by t x, and p y by p x its clear.

Yes sir.

And these relationship have been derived by using the energy equation for adiabatic flow, and the continuity equation; that means, this equation continuity equation for steady flow. So, therefore, these equations are valid; that means, p y by p x, and well t y by t x is valid for an adiabatic flow adiabatic, and steady flow well now if I consider the frictionless flow, then p x plus rho x v x square is p y plus rho y v y square all right. So, now, we can write p x plus what is rho x rho x I can write p x for again using the perfect gas as the working fluid p x r p is equal to rho r t. So, p by r t very good p by r t x, and v x I can write m a x square into a x square a x square well is equal to again I can write plus p y by r t y I am going probably little fast I understand that m a y square, but these are very simple algebraic arrangements a y square.

Now, I can write p x plus p x by r t x m a x square what is a x square again gamma r t x. So, similarly here p y plus p y r t y m a y square gamma r t y; that means, we get p x; that means, r t x r t x cancels r t y r t y cancels p x plus p x into gamma m a x square is equal to p y plus p y into gamma m a y square; that means, I can write p x into one plus gamma m a x square is equal to p y into one plus gamma m a y square; that means, from here I get a ratio of p y p x p y by p x is one plus gamma m a x square divided by one plus gamma m a y square; that means, using the reversible condition that is frictionless condition; that means, exploiting the momentum equation or the equation of motion in case of no friction; that means, the equality of the impulse function I can straight away deduce p y by p x in terms of the corresponding mach numbers m a x, and m a y.

Now, if I equate these two, because for the shock two points two extreme points in a shock; that means, the upstream point, and the downstream point of the shocks satisfy all these conditions; that means, the adiabatic conditions; that means, the same stagnation enthalpy, and the frictionless conditions; that means, the same impulse function this is, because the shock is. So, thin that across that shock the friction if you take a control volume enveloping that shock the friction is zero. So, therefore, therefore, the equality of impulse functions along with the continuity equation for steady state has to be satisfied for both these two points; that means, the upstream, and the downstream points across of the shock the process of shock.

Then we can make these two equal; that means, the p y p x developed earlier without considering the equality of the impulse function with the help of the equation of motion, and this p y p x here is the concept that why we will make these two equal, then we can write, then expression that.

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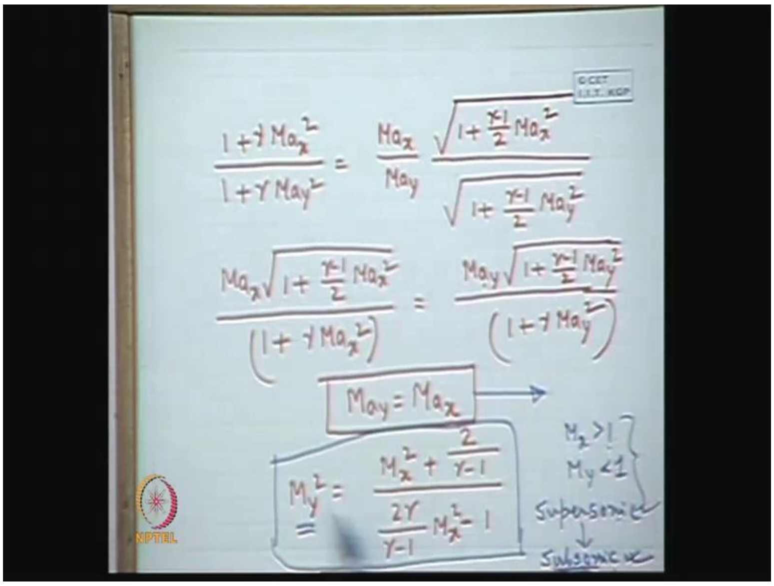

If we make these equal, then just you can see from your notes that I can write one plus gamma m just simply equal x square divided by one plus gamma m a y square is equal to m a x you check it whether it is correct I am I am writing correctly that root over all right one plus gamma minus one by two m a x square by root over one plus gamma minus one by two m a y square.

Or I can write in this fashion that well I can write in this fashion also that m a x into root over one plus gamma minus one by two m a x square divided by one plus gamma you check m a x square; that means, m a x this one divided by this is equal to this go here that is m a y well root over one plus gamma minus one by two m a y square divided by; that means, taking the expression for m a y at one place, and m a x at one place now from this equations m a y if you take m a y this in the left hand side, and this is in the right hand side, because our main objective is to solve for the flow properties at y that is the downstream of the shock if the upstream conditions are known.

So, now we can say that from these equations one can straight forward find a solution of m a y in terms of m a x, and it is not very complicated it is very simple it looks like that if you make if square it, if you square both the sides, and equate it you will get a very simple solution like that m a y is equal to m a x this is one trivial solution you get, and another solution which is very interesting m y square which is the non trivial solution it is m x square plus two by gamma minus one two by gamma by gamma minus one m x square minus one; that means, from these equation if we solve m a y in terms of m a x which is very simple, and straight forward comes like this if you make the square both sides, and then equate, then you will see a trivial solution m a y is equal to m a x, and another non trivial solution is like this m y square is this.

Now, this trivial solution has got no importance, because it physically implies that both the conditions are same; that means, they refer to the same section, but this is the most important relation where you see that if you put m x is greater than one this equation shows that if m x is greater than one that already we have proved that the shock wave or shock process or shock takes place rather you can tell shock takes place in supersonic flow the shock takes place at supersonic flow when the flow is supersonic this shock takes place.

And the result is deceleration; that means, the reduction in the mach number the flow comes from supersonic or goes from supersonic to subsonic that already we have recognized in the h s diagram with fanno line, and Rayleigh line, and here also we see from this expression that if m x greater than one we get m y less than one this means the shock occurs in supersonic flow; that means, upstream point of the shock wave or the shock is the supersonic where it makes the flow after this shock subsonic; that means, across their shock the flow is supersonic at this upstream, and subsonic at this downstream that is due to the shock a supersonic flow becomes subsonic if you take m x greater than one, then m y becomes less than one.

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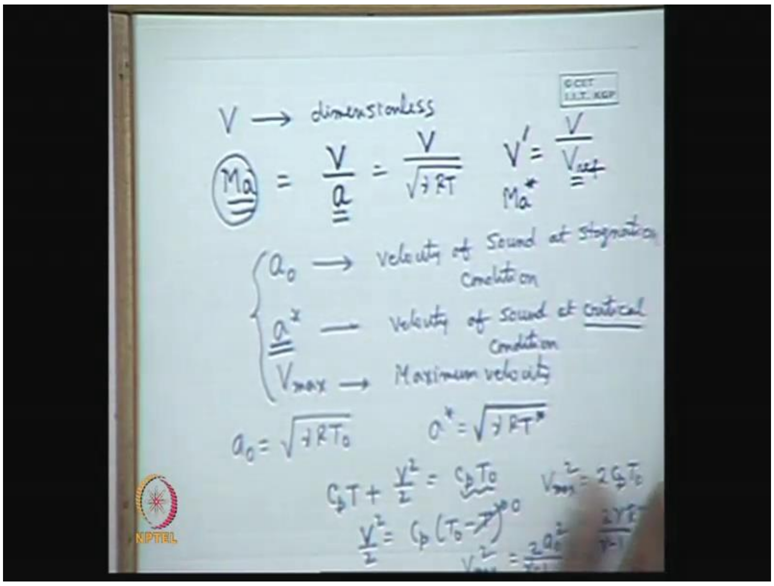

Now, this expression is sometimes expressed in terms of a dimensionless velocity now I tell you before that well that sometimes in velocity in a compressible flow is made dimensionless in with v made dimensionless v is made dimensionless v is made dimensionless with respect to three reference velocities in this context you can ask me sir why mach number is not used as a dimensionless velocity mach number is a dimensionless term which contains the velocity, but difficulty is that mach number can never be used as a dimensionless velocity, because this value a also changes, because when the velocity changes the value of a is also changes, because value of a is the sound velocity root over gamma r t; that means, with the flow when the velocity is changing.

So, sound velocity is changing velocity of sound is not a fixed one. So, this cannot be taken as a reference velocity for normalizing the velocity. So, therefore, usually the three reference velocities are taken one is the sound velocity corresponding to stagnation condition that is the velocity it is the velocity of sound at stagnation condition velocity of sound at stagnation condition at stagnation condition velocity of sound at stagnation condition another is a star that is velocity of sound at the sonic condition velocity of sound when the flow reaches sonic velocity of sound at critical condition; that means, when the flow reaches at critical condition sonic condition means when the flow velocity is sonic rather it is critical condition another is the maximum velocity maximum velocity.

Now, you see the definition of a zero as you know for a perfect gas is root over gamma r t zero well the definition of a star is root over gamma r t star we have already recognized this star or asterisk; that means, this is the section where the flow velocity has reached sonic that is the critical section that is t star is the critical temperatures what is v max v max concept is like that. If we write the energy equation in terms of c p t for an adiabatic flow; that means, you can write c p t already you have recognized this is v square by two is equal to c p t zero; that means, this is the stagnation enthalpy; that means, taking the section at stagnation point; that means, that the reservoir an at any local point with velocity v.

So, here I we can write v square by two is c p t zero minus t. So, v will be maximum when t will be zero. So, a reference maximum velocity is defined as v maximum square is two c p t zero c p can be replaced as two gamma by gamma minus one into r t zero; that means, it is precisely gamma r t zero is again what a zero square; that means, two by gamma minus one a zero square. So, therefore, the v max is the maximum velocity is related to the sound speed velocity of sound at stagnation condition by this equation.

So, three of these are used as the reference velocity; that means, if I denote a non dimensional velocity v let us denote it by v prime that the actual velocity by the reference velocity. So, any one of these three is taken as the reference velocity, but most usual convention, and very convenient is to use a star that is the velocity of sound at critical condition as the reference velocity to express the dimensionless veolocity v dash, and this is expressed as m a star.

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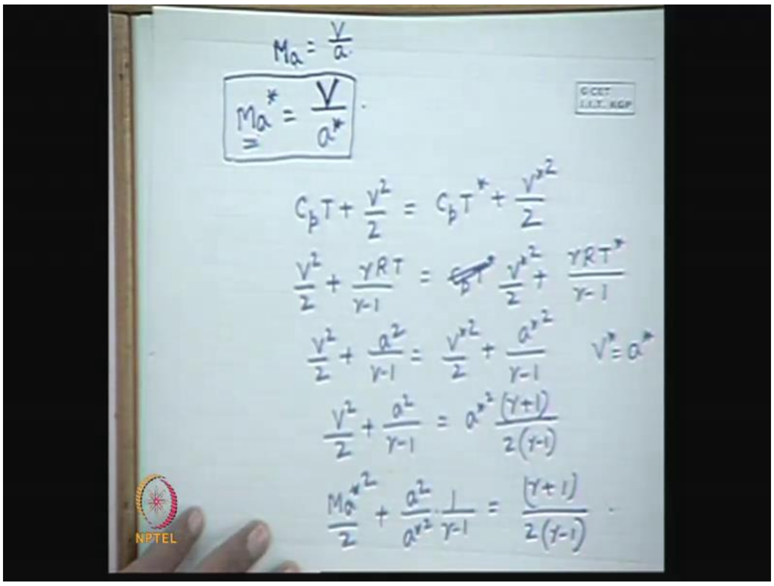

So, therefore, one can use or write one can use the a star as the reference velocity, and write as m star which is nothing, but v by a star. So, m star represents a non dimensional counter part of a velocity v; that means, if mach number is m a; that means, this m a mach number refers to a flow velocity v, and the corresponding sound velocity a both v, and a changes where m a star represents the non dimensional counter part of the flow velocity v by a star. So, this is the definition of the reference non dimensional sorry non dimensional velocity.

Now, one can make a relationship between m a star, and m a through the use of this equation, if I write this equation again c p now you see that c p t plus v square by two if I write this at the two sections that at critical section v star square by two whether I write v square by two first v square by two plus c p t what is c p t c p t is we can write gamma by gamma minus one r t is equal to c p t star sorry I am sore v star square by two plus gamma r t star by gamma minus one. So, simply v square by two plus a square this is a square by gamma minus one I can write v star square by two plus gamma r t star is a star square by gamma minus one.

Now, I can write this v square by two plus a square by gamma minus one now v star, and a star is same, because at the critical condition v star is equal to a star. So, I can write therefore, a star square into half plus gamma minus one which becomes what I can write two gamma minus one what I can write gamma minus one plus two that is gamma plus one a star a star a a star square by two gamma plus one gamma minus one now if we divide both the side by a star we get m a star square by two plus a square by a star square into one by these are very simple that is why I am going little fast gamma plus one by two gamma minus one all right.

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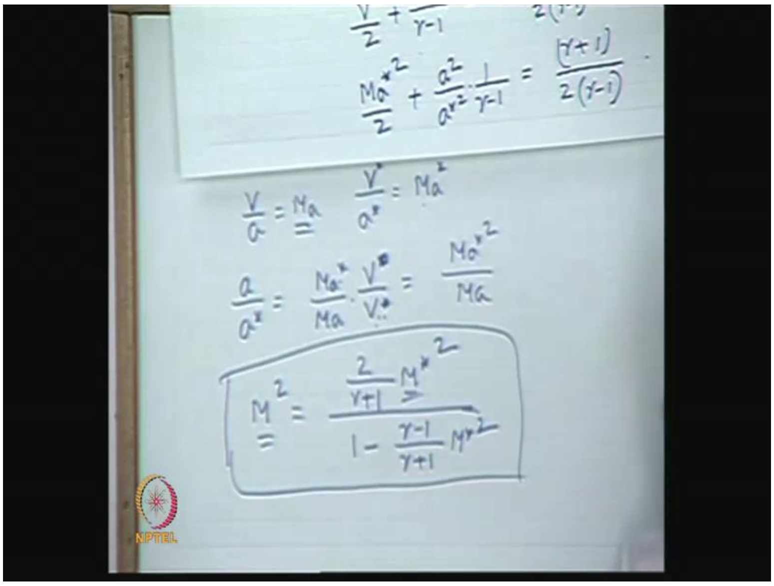

Now, if I express this a by a star in terms of the mach number like this that a is equal to what root over gamma r t, and a star is root over gamma r t star or I can write v by a is equal to m a, and v by a star v star by a star is equal to m a star. So, what is a by a star. So, a by a star please v star by a star that is v by v star that is v by v star. So, you can substitute a by a star in terms of m a by m a star. So, a by what is a by a star is equal to m a star by m a into v by. So, please a by a star; that means, a a is there; that means, m a star that is v star by v; that means, it becomes simply v by v star is again m m star that means.

That will be v by v star.

That will be v by.

V star.

V star.

A no.

Yes sir.

V by v star.

So, what is v by v star that is m a star. So, m a star square by m a ok.

Sir m a star is such that v by a star (( )).

A star is v star. So, v star is a star. So, v by a star is m m a star v by a star is m a star. So, you can have it. So, if you substitute here finally, you get an expression like that m in terms of m square in terms of these are simple algebraic steps m star square. So, this is the relationship between the mach number, and the reference dimensionless velocity.

Thank you.