Introduction to Fluid Machines, and Compressible Flow
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Lecture - 33
Normal Shock

Good morning, I welcome you to the session. Today we will be discussing normal shocks now is a very important phenomena the shock is very important phenomena in compressible flow. Now earlier we have seen that in case of a convergent divergent flow through a convergent divergent duct well convergent divergent duct, it is not possible to have the solution for an isentropic flow under all possible conditions of pressures inlet, and outlet pressures that we are seen that for a given inlet pressure in case of convergent divergent duct that is a throat in between there are has to be design pressure at the outlet to have a solution for the isentropic flow; that means, to have an undisturbed flow.

But what happens if the pressure at the downstream section is kept in between we have seen that there occurs a certain discontinuity in the flow field; that means, there is a certain change in the pressure, and velocity in the flow field within the duct sometimes outside the duct if the pressure at the outlet is below that of the design pressure these are met by the phenomena known as shocks. So, usually if we define the phenomena shock you can define this way that in any fluid flow actually in certain circumstances we will see the after wards that it happens in case of supersonic flow.

That is sudden changes a very rapid change is in pressure, and velocity takes place, because of certain circumstances in the flow, and this changes takes place within a very short distances these are affect to their very short distances in the flow field which is in the order of molecular distances that is order of mean free path few times of mean free path it is. So, small; that means, we can simulate it for its analysis by a sharp discontinuity in the flow field, and in our analysis we will be more interested in knowing the flow properties before this discontinuity or before this shock, and after this shock without going into the details of what happens within this shock which is in the order of a mean free path whose.

And thickness is in order of mean free path. So, without going to the inlet details of the shock wave we are interested in the flow properties before, and after the shock wave. So, well this can be done with a simulation type with a simulation like this we can consider a discontinuity in the flow field; that means, a section upstream of which the flow properties are steady, and downstream of which flow properties are also steady, but there is a change in the flow property properties usually we will find later on that this shock are generated or take place in case of supersonic flow where after this discontinuity the flow becomes subsonic; that means, the velocity is reduced that flow is decelerated, and subsequently the pressure is increased.

It is true that is increase of pressure, and decrease of velocity is substantial; that means, there may be an increases of pressure by five times sometimes five times it becomes the decrease in velocity becomes even in three to four times, but still the thickness of the shock ways are very, very small. So, now, let us try to make a formulation in a simplified form for a one dimensional shock before that I tell you that there are two types of shocks sometime when the shock found is normal to the direction of the flow; that means, this discontinuity takes place in a direction perpendicular to the direction of the flow.

Then the shocks are called normal shocks in cases when the shock found is oblique to the direction of the flow at the discontinuity takes place in an oblique direction with respect to the direction of flow the shocks are called oblique shocks just I give you some examples of normal, and oblique shocks just you see that example in case of a convergent divergent duct

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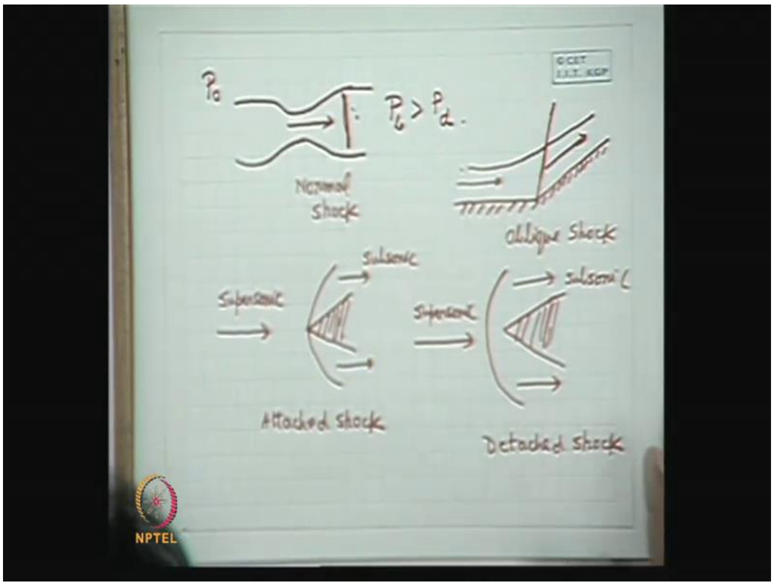

You see in case of a convergent divergent duct this is a shock if the design pressure that is a back pressure is not is equal to the dead line design pressure; that means, if it greater than the design pressure as you know corresponding to an inlet stagnation pressure. Then a shock occurs in the divergent part when the flow is supersonic. So, this shock found is normal to the direction of the flow, and this region upstream of shock the flow is supersonic downstream it becomes subsonic, and thick thickness of this shock is extremely small, and there is a sub discontinuity in the pressure pressured increases sharply, and the velocity decreases this is an example of normal shock normal shock similarly a oblique shock can be seen in case of flow pass shock scan occur, in case of both flow through a duct flow passed a body first surface like that where the flow takes place in this direction let us consider this stream lines like this the flow takes place.

Then a oblique shock may occur like that. So, the direction of flow is the shock found is of oblique. So, it is oblique shock, these are some examples of shocks in case of supersonic flow passed a body for example, let us consider a wretch type of thing sometimes the depending upon the flow situation a supersonic flow when it approaches the body this is supersonic supersonic this we will prove that always it the shock takes place from supersonic to subsonic flow this is subsonic flow this is definitely an oblique shock.

And this is defined as attached shock attached attached shock which is attached to this body sometimes depending upon the situations the shock found may not be attached this is known as detached shock this is the shock found, where this is the flow supersonic this is a wretch shape structure the flow is supersonic if the supersonic flow faces this type of abstract flow, then a shock occurs; that means, in the flow field they are occurs is sudden discontinuity sudden jump from supersonic to subsonic flow with an increases in pressure.

And decrease in velocity. So, this is subsonic, and this type of detached detached shock this type of shock is known as detached shock. So, these are certain pictures how the shock takes place creating a sharp discontinuity in the flow field now for a mathematical analysis

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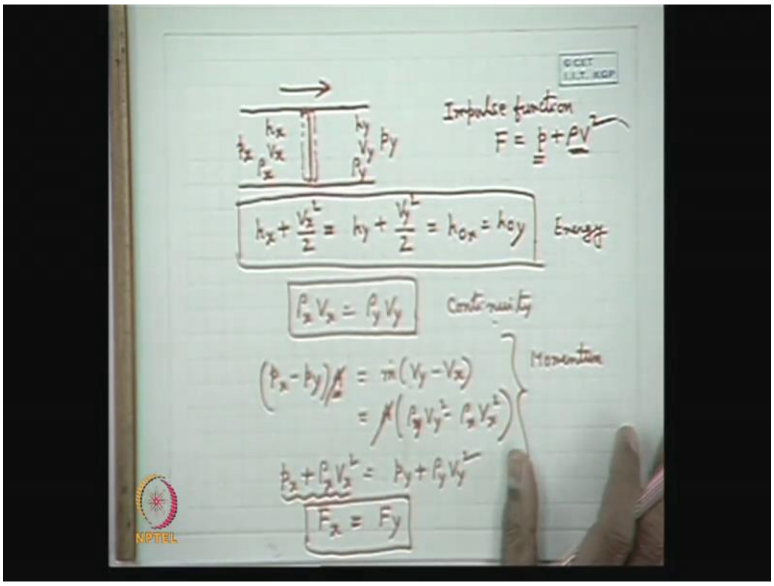

Let us consider things in this way that let us consider a duct certain part of the duct where the shock takes place a let us consider this is a thin shock way. And let us specify the properties upstream the shock way by a suffix x let the enthalpy is defined as h x the velocity is v x the density is rho x the pressure is p x, and all this quantity at the downstream of the shock is given by a suffix y, that is the enthalpy velocity density pressure now well if we write the energy equation for control volume enveloping the shock we can write that h x class v x square by two is equal to h y plus v y square by two you know writing the energy equation in this form assumes that within the control volume there is no heat transfer actually what happens this takes place very rapidly.

And we can consider this process; that means, which through the property changes across a very thin shock wave is adiabatic; that means, if we impose the conditions of this (( )); that means, no heat transfer condition, then we can write this enthalpy plus the velocity heat that is the kinetic energy per unit mass basis is equal to h y plus v y square, and this equals to this stagnation enthalpy h o x or h o y which means the stagnation enthalpy corresponding to the situation at upstream x is equal to that corresponding to the situation y.

Because stagnation enthalpy will change only when there will be an energy addition on energy depletion energy either energy is added or energy is extracted, otherwise this will remain same this is the very important equation this comes this is the energy equations.

Now if I write the continuity equation continuity equation, if I write the continuity equation we see that the cross sectional area remains same across the shock, because of the thinness.

So, we can write this rho x into v x is equal to rho y into v y since the cross sectional area remains same that is another important conservation equations which comes from the continuity equation. Now if I write the momentum equation. So, this is energy this is continuity well, now if I write momentum equation or momentum theorem for this control volume, then what I can write I can write that net force acting the direction of flow p x minus p y into a now see that since the shock way is very, very thin I have told just now that it is in the order of molecular distances the main (( )) path, we can neglect the frictional force in this small control volume.

So, therefore, without friction the only force says acting at the pressured force size, and that mass b is equal to the mass flow rate m dot times in v mass flow rate remains the same under steady conditions, this is the momentum may flux in the direction of flow now mass flow x mass flow rate can be expressed as rho x v x times the area cross sectional area a. So, accordingly we can write rho a into rho x v sorry, sorry rho y v y square minus rho x v x square this area is getting cancelled.

So, we can write p x plus rho x v x square is equal to p y plus rho y y square now this term can be written as the impulse now in compressible flow. We defined a function as the impulse function. So, this is the outcome of momentum now we define, and function known as impulse function impulse function in case of compressible flow we define the impulse function f as the sum of pressure plus the product of density in square of velocity; that means, v plus rho v square this is the find as an impulse function which is also a flow properties the combination of flow properties like this pressure plus rho v square.

So, therefore, we this definition we can write the outcome of the momentum equation is that the impulse force impulse functions sorry impulse function of the upstream is equal to impulse function of the downstream. So, this three equations are obtained from the conservation equations conservation of energy conservation of mass that is continuity.

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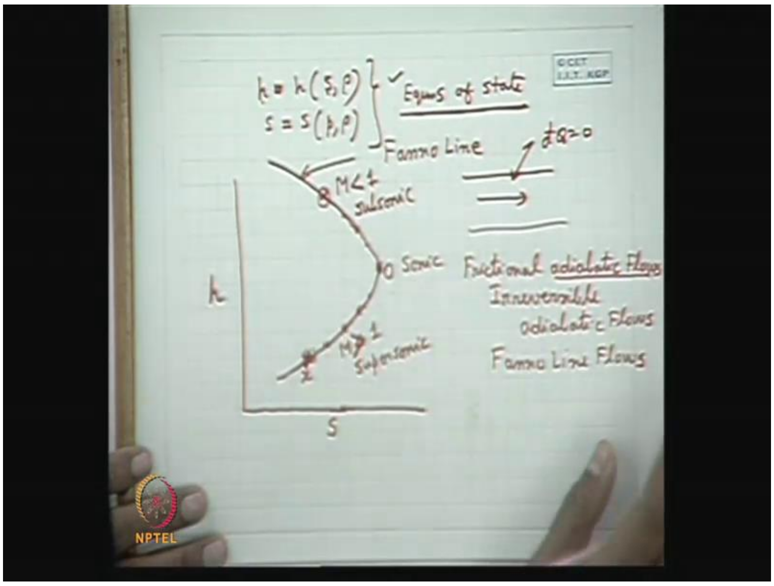

And the conservation of momentum that is the equation of motion for this control volume along with that along with this, we have just see along with this we have the equation of state defining h is we can write in implicitly as a function of s, and rho or s as a function of p n rho this at the implicit functional relationship as you know for a oh one component one single phase flow that thermodynamic properties any of the property can be expressed as any two independent properties; that means, two properties are independent.

So, therefore, h can be expressed as a function of a entropy density entropy can expressed as a function of pressure, and density. So, these are expressed in an implicit form an implicit relationship, because the implicit relationship depends upon the type or the nature of the system. So, for any system we can write the implicit relationship or thermodynamic properties which are nothing, but the equations of state these are define known as the equations of state; that means, thermodynamic equations of state can be written like that now you see that if I am interested with the help of this energy equation continuity equation momentum equation, and the equation of state to find out the locus of points in h s plane which satisfy try to understand which satisfy these equations which satisfy this energy equation continuity equation, and the equation of states.

So, physical implication will be made clear after sometime. now just you follow in routine manner that, if I try to draw the locus or the points having the same stagnation enthalpy satisfying the continuity; that means, for the same mass flow rate, and equation of state not the momentum equation not the momentum equation, then we can draw the locus like this show can draw this first we fix a particular point x; that means, to do this what we can do. So, we can choose particular conditions now before that let me tell you my basic intension of doing that you must follow it very carefully and seriously.

Here lies physical concept the basic motivation is that, if I have got certain fixed state here. So, whether it is possible by shock to attain several states here or not or other way we can put the questions for a given properties here, are all state points which different properties are accessible through shock or not the answer to this is no there is an unique state mathematically there is an unique state corresponding to a given state at upstream which can be achieved through shock.

But do this we left to go through this let us find out the locus of points all points in h s plane which corresponds to this states; that means, which of the same stagnation enthalpy with this state; that means, points of constant stagnation enthalpy, and satisfying the continuity, and the equation of states. So, routine process of doing this mathematically that we consider first fix this h x v x rho x p x everything. So, that a stagnation enthalpy is defined, then we choose a v y here any arbitrary value we chose a v y we can find out h y.

So, when we find h y v y is which select v y we find h y now when we know v y from the continuity we can know rho y since we know rho y from a thermodynamic equation of state which can be expressed enthalpy as a function of s, and rho we can know this. So, therefore, we can know the value of h, and x by doing. So, with different values of v y arbitrarily chosen v y we can find out the corresponding h y for a given stagnation enthalpy, and the corresponding rho y, and finally, the corresponding h, and we can construct a curve like this, this curve known as fanno line is very important fanno line this curve is known as fanno line.

This has got a point here where this curve shows a maximum or minimum as you tell that is maximum in s. So, this curve is known as fanno line physically this curve implies the locus of the points which mathematically first which satisfy the energy equations, and continuity equations, and the equation of state. Now you see these energy equation does not put any restriction to the friction friction may or may not be there where the stagnation enthalpy remains equal; that means, these energy equation this put constant only on the heat transfer; that means, the flows the adiabatic that mean this is an adiabatic flow which satisfies the steady state condition that is the same mass flow rate across the sections.

And also the equation of state for the particular system used as the working fluid. So, therefore, this type of curve represents the points having the same mass flow, but flowing with adiabatic boundary condition; that means, without heat transfer, but friction may or may not have this is, because we have not put any restriction of friction. So, long we have not used this equation that the equality of impulse functions which comes from the equation of motion or the momentum theorem or momentum equation where definitely the constant of zero friction has been put has not been taken in deriving this locus.

So, therefore, this refers to a flow where the flow takes place without heat transfer; that means, d q is zero, but they are may be friction; that means, we can tell the flows this refers to flows which are frictional frictional adiabatic flow, these are not isentropic flows frictional adiabatic flows or irreversible adiabatic flows; that means, adiabatic flows irreversible adiabatic flows this flows are irreversible with frictions, but no heat transfer this flows are known as fanno line flows.

That means this step points in these flows are determined are on this line; that means, frictional adiabatic flows or a fanno line flow that is irreversible adiabatic flows are flows with constant stagnation enthalpy. So, there enthalpy, and entropy changes along this line well now it can be shown just I will show you that this region of this curve that is the fanno line represents the supersonic region m less than one supersonic this part represents the subsonic region m less than one sorry, sorry subsonic region.

And this is the point let this point is o this is sonic region. So, before showing that let us again see that we can have a an idea that in a supersonic flow you see in a the effect of friction is to reduce; that means, if I have got a point here now try to understand that if I got a point at any point that let we consider this is the supersonic part; that means, that any point here in the supersonic region along this line we have to move in this direction. So, starting from any point that point cannot come; that means, if the flow takes place if the upstream point is this one.

So, downstream point cannot come here it has to go there why can you tell that for any point let example in a supersonic flow the upstream point. If we fix at particular point x as the upstream point why that downstream point will be always along the curve in the right direction.

Entropic cannot.

Entropic cannot very good very good this is, because this is the flow is adiabatic. So, therefore, entropy of the system will increase, because this is second law of thermodynamic tells that the entropy of an isolated system is always greater than zero entropy change of an isolated system is always greater than zero since this system is an isolated system there is no heat transfer it this surrounding. So, entropy change of the system will be greater than zero similarly if I have a point in this region upstream. So, downstream point will be towards this direction; that means, process should take place in such a way; that means, the flow should take place in such a way that it must in increase the entropy of the system since the entropy change of the surrounding is zero, because of no heat interaction the entropy change of the system will correspond to the entropy change of the universe.

So, therefore, the entropy change of the universe to make entropy change of the universe greater than zero we must make the entropy change of the system greater than zero. So, entropy will always increase. So, flow will take place along the right. So, from this we can also conclude one thing that in a supersonic flow therefore, the effect of friction is that it will decrease the velocity it will move towards the sonic flow; that means, you can visualize this that in a supersonic flow if you increase the friction usually this is manifested in terms of increasing the length of the duct we will go on decreasing the supersonic flow to the sonic region up to the point to after which a further increasing friction will not change the flow to subsonic region until, and unless the inlet condition is altered.

Similarly, in a subsonic flow the influence of friction; that means, this friction can be visualized in terms of the increasing a dart length it will change the for a given mass flow rate to accommodate, it will change the subsonic flow towards the sonic region where you see that if you go on increasing duct length, and to have maintained the mass flow rate you will change the flow more towards a sonic region. So, when sonic flow will reached, then for the same mass flow an increase in dart length will not change the flow from sonic to supersonic region until.

And unless the inlet conditions are change. So, that is the physical explanation of this fanno line. Now let us mathematically prove that at o the sonic condition is reached. So, let us prove the condition at o let us consider a infinite small process in any part of the subsonic line infinite small process.

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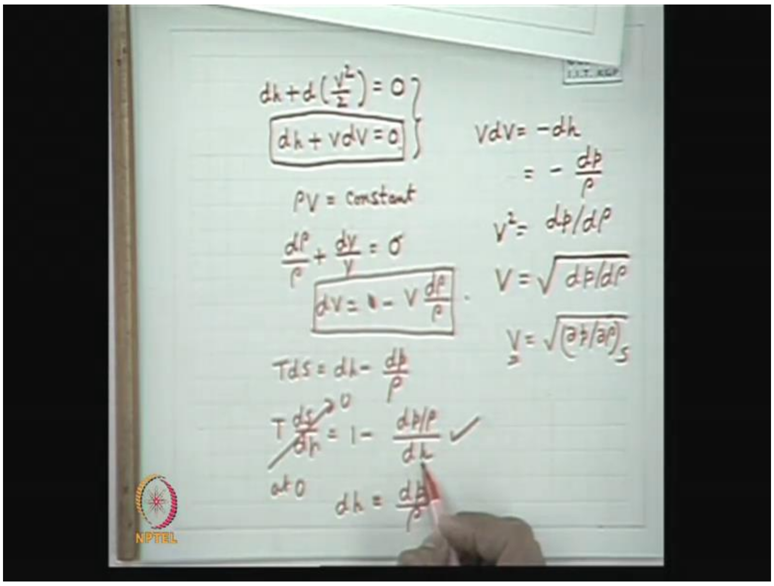

Now, you see that the energy equation could be written in this way d h plus d of v square by two is zero in case of an adiabatic flow the integrated form of which is h plus v square by two is constant which already we have used.

Let me see that which already we have used that h x plus v x square by two h y plus v y square by two; that means, it differential form it is d h plus d v square by two is equal to zero or d h plus v d v is equal to zero all right. So, this is the energy equation for adiabatic flow the continuity equation tells rho v is equal to constant since the cross sectional area remains constant across the shock.

So, this can be written as d rho by rho plus d v by v logarithmic differentiation equation is zero all right from which we can write d v is equal to minus is equal to sorry minus v into d rho by rho now thermodynamic property relations can be written as t d s is equal to d h minus d p by; that means, t d s is d h minus v d p by rho v d p v is the specific volume which I which we use in case of thermodynamics we use in case of fluid mechanics as one by rho. So, you know these relations we have already discussed earlier t d s is d h minus d p by rho now if I write this equation t d s d h this form is equal to one minus d p by rho divided by d h.

Now, you see therefore, this is the relationship developed from the thermodynamic property relations this is the outcome of the continuity equation, and this is the outcome of the energy equations now for fanno lines we have used these energy equations for adiabatic flow this is the equations for continuity, and this is the thermodynamic property relations now at the point s we see that d s d h is zero d s d h is zero true d s d h is zero true. So, at the point o d s d h is zero; that means, this is zero at o. So, therefore, what we get d h is equal to what we get d h is equal to d p by rho. So, now we if I write v d v is equal to minus d h is equal to minus d p rho.

So, what is d v d v is minus v d rho by rho. So, if you put that you will get v square is d p by rho if you put that v is minus d rho by rho than we get v square is d p by d rho. So, therefore, simply we get very simple expression d p by d rho now since the process is considered adiabatic, and if we consider the shock as an isentropic process in case of shock we can write v I is equal to del p del rho at constant s. Therefore, in case of shock this is at there is o attains the sonic condition that is the sonic velocity similar way we can write that this part of the curve is supersonic.

And this part of the curve is subsonic how is a very simple intuition in that case we will not put d s d h zero here in case of the lower portion of the curve; that means, you see this part of the curve d s d h is positive, and this part of the curve d s d h is negative very good. So, this part of the curve if you consider that d s d h which is positive, then d p by rho divided by d h has to be less than one; that means, d p by rho has to be greater than d h. So, if you d h has to be less. So, if you put like that.

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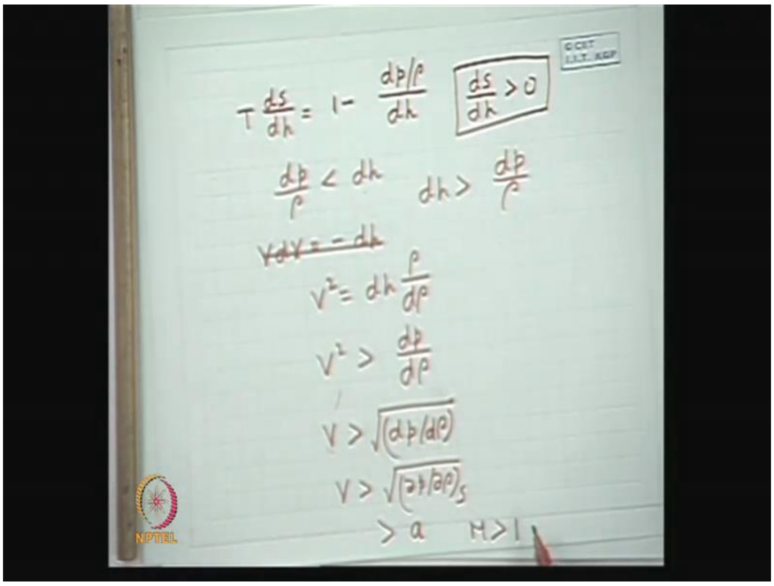

Let us see this thing we have got t t s d h is one minus d p by rho by d h all right. So, we can show that in this part d s d h is positive; that means, d s d h is also positive; that means… So, in this part what will happen? So, this will be less than one; that means, d p by rho will be less than d h d p by rho will be less than d h otherwise this will not be less than sorry it will be d s by d h is positive; that means, positive means this will less than one; that means, in this region d s by d h is greater than zero in the lower part of the curve which means this has to be less than one.

So, d p by d rho is less than d h all right, then we can use this curve use this equation v d v is equal to minus d h. So, using this we can prove that v d v is minus d h by, but here what will prove that d p by rho. So, v d v. So, let us. So, put d v d v is what let us see here d v is. So, v d v is equal to minus d h. So, d v is minus v d rho by rho. So, minus cancels. So, v square d rho by rho is equal to d h. So, v square this is the general expression is d h rho by d rho. So, we write this expression that v square this expression is equal to d h rho by d rho in earlier case we had d h is d p by d rho when d s d h is zero at the point o.

So, therefore, it becomes d p by rho, but in this case d h is greater than d p by rho, because d p by rho is less than d h. So, v square is greater than d p by d rho; that means, v is greater than root over d p by d rho. So, putting the additional constant in case of shock that the flow is isentropic without friction that we can write v is greater than del p del rho at s; that means, greater than the acoustic speed where earlier we proved that v is equal to acoustic speed; that means; obviously, by exploiting this relationship; that means, it is m greater than one; that means, exploiting the relationship that the slope of the curve h s is positive here slope of the curve is negative here, and the point o it is zero.

That means d s d h is zero or otherwise d s d h is infinite we can use the equations like this the property relations thermodynamic property relations the continuity equations, and the energy equation to finally, express v in terms of a d h v square d h rho by d rho, and using this expression when d s d h is equal to zero d p by rho is d h, then v square is d o by d rho when d s d h is less than one, then we can prove that v is greater than this. And similarly when d s d h is greater than one; that means, in upper part of the curve this will be less than this that you can prove when this corresponds to mach number less than one.

So, therefore, this represents a fanno line, but you must understand one thing very clearly that is fanno line represents the locus of the points which follow the adiabatic conditions, and this steady state continuity equation; that means, this at the locus of points having the same mass flow rates without any heat transfer, and following the particular equation of states we will deriving after wards these equations for perfect gases; that means, this is precisely referring or this precisely refer to frictional adiabatic flow.

Because the frictional that is the zero frictional or absence of friction is not a constant foot here. So, this is simply the locus of all these state points in a situations of flow there may be friction, but no heat transfer no heat transfer, and also following the steady state condition; that means, which mathematically satisfies the continuity equation the energy equation h plus v square by two is constant; that means, the energy equation for a adiabatic flow, and at the same time the thermodynamic equation of states those flows are called fanno line flows now you may ask questions sir you started with the shock.

And immediately you refer to derive the equation locus of points in h s plane which corresponding to certain flow with where friction may be incurred really this has got directly no reference to the shock, but ultimately you will find that through this we can explain certain restriction in shock, because our final motto is to prove that shock will take place only in supersonic flow at it will decelerate the supersonic flow to subsonic flow not that supersonic flow will again go to a more supersonic flow the shock is a phenomena where the fluid velocity is decelerated.

And it jumps from a supersonic to subsonic flow it is just like your hydraulic jump have you read the hydraulic jump that trunk will rapid flow or rapid flow to hydraulic jump takes place at of rapid flow becomes trunk wheel flow similarly to that is supersonic flow becomes subsonic flow with can increase in pressure, and velocity or decreases in velocity. So, for giving supersonic flow there is a unique subsonic state where the flow will reach, because of the shock to prove that we have to go through this; that means, step by step first we deduce the locus or draw the locus of the points in a h s plane for a type of flow where friction may be there.

But there is no heat transfer, and the fall there the flow obeys the steady state condition; that means, the mass flow rate across each section. So, this is called fanno line flow well next we can consider. So, therefore, you understand this is subsonic, and supersonic flow, and in this case again I repeat that at any point if you start either in subsonic or in supersonic region to know the downstream point that it in which direction the flow occurs or flow take place we will have to move along this curve definitely for fanno line flows towards the right. So, therefore, the influence of friction by making in a more more duct length of the duct.

Because all the points in the supersonic or subsonic region towards the right represents the length of a duct physically; that means, if you increase the length of a duct in a supersonic flow the flow tends to becomes sonic ultimately when it will reach on o the point o the flow will be choked.

So, a further increase in length will not change the flow until, and unless the this condition; that means, this inlet condition will is changed similarly the influence of increasing duct length or friction in a subsonic flow without heat transfer will change the subsonic flow towards the sonic flow the point moves towards the sonic flow. We will have to proceed to along the curve to the right say following the second law of thermodynamics that is the increases in entropy for the system without heat transfer will be greater than zero now well the time will have to see…

So, next we see that we consider a flow where the heat transfer is there. Now if we try to well, now if we try to draw the locus of points which satisfy this equation that is the continuity equation well which satisfy. Now we take this equation the impulse function; that means, these momentum equations for zero friction, and the thermodynamic equation of states; that means, the implicit functional relationship like this depends upon the particular working system.

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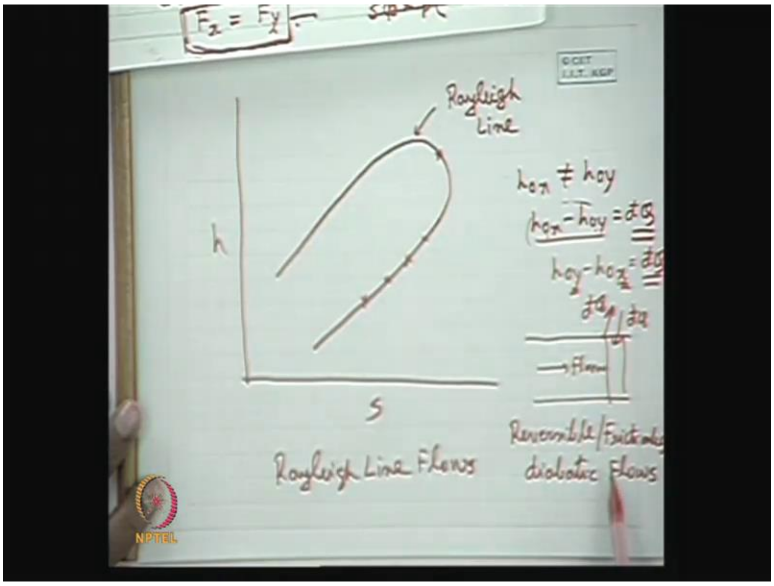

And if we try to draw locus in the h s plane the locus will be like this if we draw that in h s plane the locus will be like this. So, there will be a point like this. So, what does this curve represents this curve is known as rayleigh line r a y l e I g h, now again I tell first look mathematically the rayleigh line is drawn by joining the points or the locus of the points which satisfy the continuity equation the momentum equal where the equality of impulse functions are made.

And the equation of states; that means, a h is a function now implicit form s rho, and p is function of or s is sorry s is a function of anyway you can write s is a function of e n rho; that means, thermodynamic equation of state equation of motion, and continuity it refers to a situation of flow where friction is not there, because you are satisfied these equations the flow is steady, but this condition we have not satisfied; that means, in case in this case if we find h o x at any point x, and any point y we will see that that may not be equal to this; that means, h o x minus h o y will have some value let us consider these values as d q.

Because we know this difference in stagnation enthalpy is be heat transfer; that means, is h ox minus h o y if we find or rather if we write h o y minus h o x x considering any upstream, and y considering any downstream section, and let the quantity denoted as d q which is of physical significance as like this if this positive, then we will consider there is an increase in stagnation enthalpy which represents an amount of heat transfer heat is added to the flow or if this is negative heat is taken away from the flow; that means, it represents a flow where friction is absent.

But there is an heat transfer; that means, heat is either added d q with cart I use, because to differential it from perfect differential or heat is taken as here. So, this refers to flow which are reversible or frictionless reversible or frictionless you write frictionless diabetic sorry frictionless diabatic flow that mean flow with heat transfer frictionless or diabatic flows flows with heat transfer, but without friction that are known as rayleigh line flow rayleigh line flows; that means, here the points represents the locus or of this take points in a flow without friction, but with heat transfer. So, I think time is up today.

So, next class we will be discussing again this rayleigh line flows. So, this is rayleigh line flow; that means, these are the locus which satisfy the equation on motion for an frictionless flow the continuity equation, and the thermodynamic equation of states. So, therefore, the locus in a h s plane represents the state points in a flow without friction with heat transfer, and obeying the steady state conditions that is the same mass flow rate these flows are known as rayleigh line flows they are reversible or frictionless diabatic flow diabatic flow means there is heat transfer either heat is added or heat is taken out; that means, the stagnation enthalpy will change; that means, all the points in this locus in this curve the different points represent the different in general the different stagnation enthalpy.

Thank you.