Introduction to Fluid Machines, and Compressible Flow
Prof. S. K. Som
Department of Mechanical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 32
Isentropic flow through Convergent Divergent Duct

Good afternoon I welcome you to this session. Last class we discussed the concept of choking in relation to isentropic flow of a compressible fluid or isentropic compressible flow in continuation to that. Today we will be discussing the isentropic flow through a convergent divergent duct at first we again recall the relationship between the stagnation, and local properties again. So, let us concentrate that the relationships are like this

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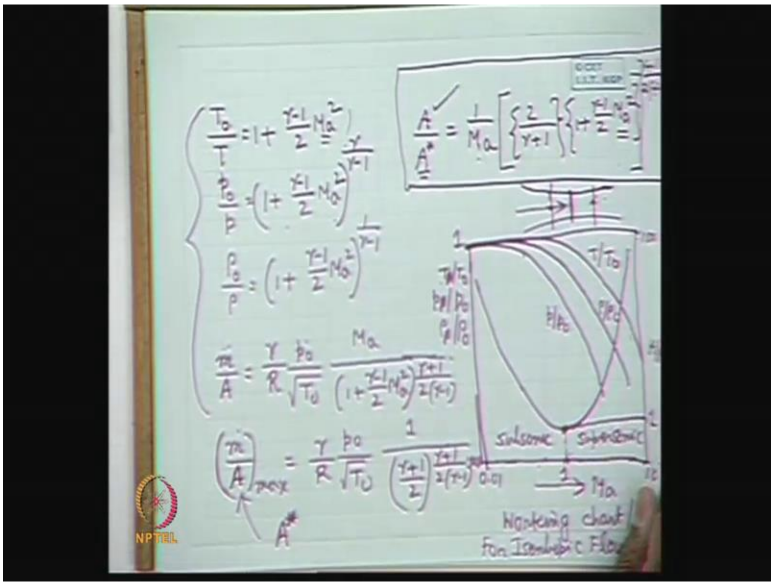

If we denote the properties with a o suffix a zero suffix as the stagnation one, and without any suffix is the local one we know that, because these are very important things gamma minus one by two m a square or m a is the mach number. Similarly, p zero by p the corresponding pressure is equal to one plus a gamma minus one by two the same thing using the relationship between pressure, and temperature in isentropic flow gamma by gamma minus one this gamma is the ratio of specific heats these are all valid for an isentropic flow of a perfect gas similarly rho zero by rho is one plus gamma minus one by two m a square into one by gamma minus one. So, the properties with suffix zero at the stagnations, and without any suffix that t p, and rho are the local properties local temperature pressure, and density well at the same time if we recall the mass flow rate per unit area expression.

That mass flow rate per unit area was expressed for a perfect gas as the fluid p zero by root over t zero into m a divided by one plus gamma minus one by two if you recall this m a square to the power gamma plus one by two gamma minus one if you recall this last class we deduced this, and at the same time by differentiating we saw that for a given stagnation pressure, and temperature, and for a given perfect gas where gamma, and r are constant. So, this function this m dot by a is a function of mach number m a, and this shows a maximum when mach number equal to one.

That means the maximum value of this quantity corresponds to a mach number one, and if mach number one is substituted, then we get gamma by r p zero by root over t zero into one divided by this becomes gamma plus one by two raised gamma plus raised to the power gamma minus one all right now at the same time we know that this area the m dot by a maximum occurs when this area is given as when the area where this occurs mach number is equal to one; that means, the sonic condition that the flow velocity becomes the velocity of sound.

Those properties at those conditions are denoted with an asterisk as the superscript or star we tell that a star; that means, in that case a becomes a star all right. So, if we put a is equal to a star, and divide this with this value m dot by a we get a area ratio a by a star which is very important a by a star if we divide this with this divide this; that means, we will get one by m a, this m a is there the mach number one by m a into this be two by gamma plus one you see that two by gamma plus one one by this. So, this becomes two, because this comes one the top one plus gamma minus one by two m a square, and everything to the raise to the power raised to the power gamma plus one by two gamma minus one.

So, this is the a very important relation; that means, this is the relationship between a their area at any section of a duct to a star where a star represents the area where the sonic conditions is reached m a is equal to one see if you put m a is equal to one this a by a star becomes one all right. So, a by a star a star is here used as a reference area to make this area at any section normalize. So, a by a star is a function of mach number area. So, one interesting thing is that there are two interesting things not one.

That you see that for any value of m a m a is always greater than zero it may be either less than one or greater than one depending upon whether the fluids are sonic or supersonic. So, for any value of m a greater than zero either less than one or greater than a by a star is always greater than one; that means, a star is the minimum area where the mach number one occurs, and another interesting fact that we will discuss afterwards also in relation to convergent flow through convergent divergent duct that for a given value of a by a star m a has got two values this is a quadratic equation m a square.

So, for any given value of a by a star; that means, for any given area corresponding to a particular a star they are a two values of m a one less than one another greater than one the physical reasoning like that if we have a convergent divergent duct for example, the flow takes place like that if the flow is subsonic in the upstream region downstream supersonic. So, a same area we may get in both these sides from the throat area this is the throat where the m are different here m is less than one here m is greater than one. So, therefore, there may be two areas there will always be two areas when there is a throat which are equal two equal areas, but the mach numbers are different.

That means for a given area ratio we may have two mach numbers now all these formulae analytical formulae a by a star t zero by t p zero by p rho zero by rho can be expressed graphically like this plot can be plotted like that with m a as the independent variables you see the ratio the local, and stagnation properties, and the area ratio with m a, if we plot, then this will give the result like this if we plot t zero by t let this is very small point any value point zero one let here ten it starts with point zero zero one for all this quantity let this is maximum one t zero by t p zero by p rho zero by rho this axis the maximum value is on.

And this here we write a by a star the value of a by a star now we will see for this the graph will be like this the figure will be like this it is like this the qualitative trend is like that for example, this t by t zero, I am sorry it is not t zero by t p by p zero rho by rho zero. Now there it is t by t zero p by p zero rho by rho zero; that means, this to the power minus one this to the power minus the reciprocal of this t by t zero p by; that means, this is t by t zero, let this is rho by rho zero, and this is p by the qualitative trend I am showing; that means, when mach number zero this t by t zero p by p zero rho by rho zero all asymptotically reach one, because that is the stagnation values.

That in local pressure temperature, and density reach their respective stagnation values, and they monotonically decrease as the mach number is increased now these are the three curves for these ratios now if we plot this curve it is interesting that this curve will show a minima this curve is like that there is the minimum this a by a star one will reach when the mach number is equal to one, and it will again the a by a star is like that at one it is the minimum. So, if we decrease the mach number, then a by a star is increasing, and as mach number reaches zero. So, a by a star reaching infinity.

You see with zero mach number a by a star reaching infinity similarly mach number reaches infinity also a by a star reaches infinity this is the peculiar characteristics of this curve; that means, it reaches a minima when mach number is one this zone is supersonic zone supersonic, and this zone is subsonic; that means, a star reaches the minimum here; that means, in this subsonic zone as we increase the velocity or mach number same thing we see there is a decrease in the area, and the supersonic zone as we increase the mach number there is an increase in the area.

So, the a by a star increases in both the direction here in the subsonic with a decrease in m a, and in the supersonic with an increase in m a showing a minimum one. So, this is one here, and this is a very high value here if may be ten hundred like that. So, that it goes on infinity asymptotically as the mach number tends to infinity, and here it tends to zero. So, this is the trend of the curve. So, all these curves together forms a figure or chart this is known as working chart working chart or figure for isentropic flow for isentropic flow.

That means if we know a value of p by p zero we can find the value of m a, and the corresponding value of the area ratio. So, this will be useful for solving problems without using these equations. We can straight forward use the equations or we can find from the graphs, but these graphs will be important in illustrating some physical phenomena.

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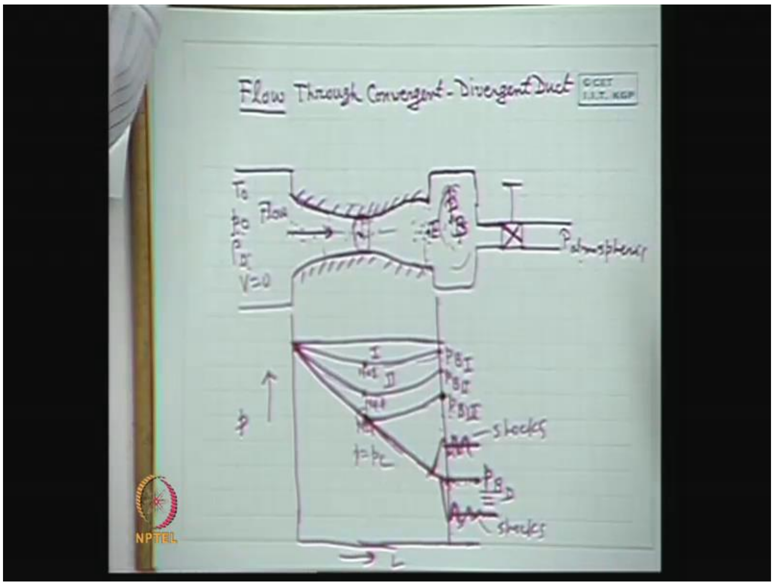

So, let us come to that what is that flow though convergent divergent duct flow though convergent divergent duct flow through convergent divergent duct that flow is isentropic flow flow through convergent divergent duct.

Let us consider if convergent divergent duct like this let us consider a situation like this well let us consider a situation like this. So, this is a reservoir the stagnation conditions that is the temperature t zero pressure p zero rho zero where v is equal to zero the flow starts from here from a reservoir from a stagnation condition through a convergent divergent duct which is the insulated, because the isentropic flow should be essentially adiabatic flow. So, this flows in that direction, and this chamber is main like that with a valve arrangement this pressure is the atmospheric pressure atmospheric pressure. So, that by operating this valve this pressure p b that is the back pressure of this convergent divergent duct; that means, the surrounding ambience at which this duct discharges that is known as back pressure can be varied by operation of this valve. So, when the valve is closed it is equal to the value of p zero. So, p b from a maximum value of p zero can be reduced by gradually opening the valve up to a pressure of p atmospheric now if we do that what do we see physically let us see. So, this is the exit plane let this is the exit plane of the nozzle this is the exit plane.

Not nozzle, sorry this is the exit plane of the duct sorry exit plane of the duct no initially when the valve is closed throughout the pressure is p zero; that means, the graph is like that there is no flow, and the pressure is constant; that means, I am drawing the figure for p versus this length along this nozzle length along the duct sorry along the duct the length pressure variation. So, pressure is zero throughout now if we slightly open the valve. So, if flow starts which is a very low which is the flow velocity is very low very small flow starts. So, therefore, it starts from an incompressible region now when the flow starts with a very small velocity mach number below point three three, it may be an incompressible flow.

So, in that case for flow of any incompressible fluid; that means, that a mach number lower than point three three such a convergent divergent duct it is convergent in that direction in the direction of flow fast in the upstream part, then the downstream part is divergent. So, it will behave like this the pressure decreases first, and reaches its minimum at the minimum cross section that is the throat, and again it increases like this you see. So, this will be the curve first part of the curve when the valve is slightly open that p b is slightly less than p o. So, this is condition where p b is given by p b one this is the curve one.

So, it will act as an venturi meter this type of flow is known as venturi flow, and the duct will act as a venturi meter; that means, the convergent part will act as a nozzle where, then pressure decreases, and velocity increases. So, at the throat the minimum pressure is reached, and the velocity is maximum, then in the divergent part the pressure increases, and velocity decreases. So, pressure meets up here if the two areas are equal, and the flow is reversible; that means, in, then the pressure will assume this valve of course, pressure may not assume this value if there is a velocity.

So, both the areas are equal. So, velocity will be same, and pressure will be same, but in that case we have considered the velocity is zero it is of infinite area this is a infinite reservoir. So, this is a stagnation condition. So, pressure will end up here with a velocity here discharging with some velocity. So, this will be the pressure distribution if we still lower the pressure by opening the valve. So, we will see that the mach number increase, because still flow rate will be increased, and the mach number may be beyond the quinn compressible flow region more than point three three, but will be in subsonic region. So, that the what happen still it will expand only up to the throat, and then it will go on t.

This is the two. So, and this pressure corresponds to p b two. So, if we further reduce the pressure for example, here if we go on further reduce the go on further reducing the pressure you see that while reducing the pressure the flow rate is increased, and if we notice the velocity at any cross section that will be increasing at any cross section the velocity is increasing. So, the see that if we consider the velocity at the throat which is the maximum velocity in the duct. So, a pressure will be reached when the throat will reach to a velocity equal to the sonic velocity.

And, then also at a this is a limiting case at some pressure it may act as a diffuser also that it will though sonic velocity has reached I told earlier class that it depends upon the design pressure. So, at some pressure if we go on slowly varying gradually varying the pressure we may reach at a pressure here where it will act as a venturi meter; that means, the convergent part there will be expansion that is a nozzle action expansion that is a reduction of pressure, and increase in velocity, and this maximum velocity will reach one; that means, m is equal to one here, but till, but still there will be a reduction that diffusion reduction in the velocity or increase in the pressure in the divergent duct.

So, this is the condition reached when the back pressure reaches a particular value p b three corresponding to a fixed values of p zero now what will happen if we gradually vary the pressure, and come to this condition when it will act as a venturi flow initially the reduction in pressure as I said it will increase in velocity, and finally, a increase in pressure or deduction in velocity the divergent part with the attainment of sonic velocity; that means, the flow velocity equal to the sonic velocity; that means, with the attainment of m is equal to one at the throat.

So, in that case the maximum velocity or the maximum mach number here was less than one. So, far; that means, the flow of incompressible flow it will be in it was in incompressible region it was in the subsonic compressible region now the sonic condition has been achieved at the nozzle, then what will happen if we reduce the back pressure further what will happen this is of very importance now it has been found that if we just reduce the back pressure.

So, what we will expect we will expect that it will if you reduce the back pressure further what will happen whether the flow will be decelerated or accelerated. So, when we reduce the back pressure it is obvious if the flow is accelerated, then means it has to go beyond mach number one all right. So, what; that means, that if we reduce the back pressure the expansion the fluid will still go on accelerating provided there is a particular back pressure maintained here; that means, just we consider at the time being there is a back pressure known back pressures somebody tells you where known back pressure I am just give these value with p b d.

There is a particular back pressure corresponding to the stagnation condition, and this duct if we maintain that we will be seeing that a continuous expansion will take place now first thing we will have to understand that when sonic velocity is reached here a further reduction in the back pressure will not be able to generate any increase in the flow, and this section attainment of this section the properties at this section will remain unaltered which I already discussed to you earlier. So, that the mach number one conditions, and the pressure distributions in this portion will remain unchanged.

So, this part will not be changed, and here the flow velocity will attain the mach number a mach number one that is a sound velocity. So, therefore, further expansion continuous expansion or further continuous reduction in pressure associated with an increase in velocity occur provided; that means, we can reach from subsonic to supersonic velocity or continuous expansion or continuous acceleration in the supersonic region provided we at a particular back pressure maintained here, and that particular back pressure depends up on the stagnation pressure, and the particular area ratio that I will come afterwards this the particular design or particular dimensions of the duct.

Now, what happens let us understand that if this pressure is kept somewhere here if I know there is a design pressure like that if this information we have accepts some pressure above these; that means, in between this or some pressure below this; that means, if we know there is a design pressure if our ambient pressure is first something hard than this pressure, but something lower than this pressure where the mach number one is was attained, then what will happen the expansion will take place in the nozzle, and what will be done that a fluid will be over expanded.

That means fluid will go on expanding to a pressure which is lower that the pressure back pressure maintained here, and it will catch up the back pressure just before the discharge plane through a discontinuity series of shock waves like that; that means, the fluid will immediately jump to the back pressure little upstream from the exit plane through it a discontinuity through a pressure wave this known as series of shocks shock, and this shocks at this discontinuity in catching up abruptly the high pressure cartels the energy of the fluid total energy of the fluid, and this is an irreversible conditions. So, the undisturbed expansion throughout the divergent part will not take place if the pressure lies the back pressure lies between these two; that means, one is the design pressure defined as the design pressure.

And the pressure where the mach number one was reached fast. So, this is the limiting conditions for the venturi flow with mach number one at the throat now if we have the back pressure or if we maintain the back pressure below the design pressure the lower pressure, then what will happen, then it will go in expanding like that, but it has to catch this pressure. So,, then what will happen it is having an under expansion always. So, just before the discharge. So, to meet up these expansion it will again go through like this through a series of shock wave to finally, adjust the back pressure.

And this occurs little before little upstream the discharge plane; that means, this discontinuity this is also shock through series of shocks it will occur. So, that we see that if we cannot keep a particular pressure here which is known as the design pressure the undisturbed expansion throughout the divergent duct that the divergent duct will act as a nozzle throughout is not possible otherwise what will happen if we keep pressure in between these or here or any other pressure altered from this value. So, the undisturbed expansion will not be there; that means, it will expand up to some point, then it will catch up either a higher pressure than the design pressure or the lower pressure than the design pressure either it will be suddenly expanded or suddenly comprised through a through some discontinuities known as shocks which will incur losses in the fluid. So, to have a continuous expansion without any such irreversibility's we will have to fix a particular pressure known as the design pressure. So, this is most important thing now this can be explained from this you see that y it is. So, if you see here is the that for a given value of p zero for example, here that given value of p zero. For example, this p b this is the design pressure p b let for example, we say that we have any back pressure p b we have got any back pressure p b we do not know what is the back pressure.

Any pressure we have we have we consider that this should be the back pressure the back pressure is not in our hands to vary. So, we have any pressure back pressure p b. So, we can have a point here p b by p o. So, corresponding to that point we have a particular area ratio it is a by a star. So, a particular area ratio is fixed for a given value of p b by p zero; that means, for a given stagnation pressure for a given back pressure there is a particular area ratio which corresponds to an continuous undisturbed flow if the area ratio is different from that, then the undisturbed flow will not be possible at that design pressure you understand.

In a other way we can say that if the area ratio is fixed; that means, this for a given a star this a is fixed; that means, when the area ratio is fixed the design pressure is fixed; that means, if the design pressure is different if this pressure is here. So, if we have a back pressure here first of all with respect to p zero we will see these back pressured is lower than the critical pressure of not; that means, this is the critical pressure; that means, if it is lower than the critical pressure there may be number of back pressures, but all these back pressure gives the unique value of a by a star in supersonic region.

That means for a particular back pressure we have a unique area ratio you understand a by a star; that means, a particular area corresponding to a fixed a star will corresponding to this back pressure to give a undisturbed expansion this is very important this is very important that if we have a back pressure we can find out for a undisturbed expansion if it is supersonic region for a given stagnation pressure that if we see that this back pressure is lower than the critical pressure; obviously, this will give a undisturbed expansion provided we have a particular area ratio.

So, this is precisely the concept of convergent divergent flow flow through convergent divergent duct; that means, if the back pressure is above this p b three this pressures, then the duct will act as a venturi meter. So, p b three is the back pressure corresponding to which you see that mach number one is reached; that means, here the pressure is the critical pressure here the pressure is the critical pressure. So, this is the back pressure where this back pressure this critical pressure the critical condition is reached; that means, if back pressure is in between these two the after that for a particular area ratio there is the design pressure to give an undisturbed expansion through the enter convergent divergent duct ok.

So, therefore, in short what we can tell that we have got a stagnation pressure for example, we have got a back pressure all right we want the expansion continuous just we are discussing one day that what happens in a air craft air craft for example, a jet engine is moving at a certain altitude all right. So, at that altitude we know the back pressure, because at that altitude you know the pressure we know the back pressure for example, now we want to continue as expansion in a propelling nozzles what do you want we want a continuous expansion to get more velocity or for any purpose we want to continue as expansion.

So, what we will first see that whether this back pressure corresponds to a critical pressure is not than or less than the critical pressure. So, if it is more than the critical pressure corresponding to the stagnation pressure what is the stagnation peruse means the inlet pressure that we consider as the stagnation pressure we neglect the velocity is the approximately, because fluid approaching the nozzle with a very small velocity. So, that pressure we consider as a stagnation pressure. So, that pressure considering the stagnation pressure we can find out what is the critical pressure.

So, that pressure considering the stagnation pressure we can find out what is the critical pressure. So, if it is more than the critical pressure; that means, we can tell that a convergent duct will give the acceleration all of you understand. So, that critical pressure more than that we will design for a convergent duct, and if you go for a smaller area. So, it will give to a higher velocity, but for an expansion or reduction of pressure, and increase in velocity for an accelerating flow a throat is not required a throat is not required.

That means it is only a convergent duct, but if we see that this pressure is lower than the critical pressure; that means, if we want to expand up to that pressure continuously; that means, that back pressure which is lower than the critical pressure we will have to provide a convergent divergent duct; that means, if we want to expand the gas or accelerate the gas from that high pressure we have to go into the supersonic region the acceleration will give to a flow velocity which is more than the velocity of the sound at that state.

So, therefore, we will have to use a convergent divergent duct it is the number one number two whenever the supersonic expansion is there, then we will have to be very careful; that means, to have a continuous acceleration undisturbed expansion in the supersonic region without any irreversible shocks we will have to design the supersonic duct in that; that means, any divergent duct will not do, then we will have to design the divergent part to give a convergent divergent nozzle in such a way that if should give an undisturbed expansion.

Then what we have to do we have to find out the correct area ratio corresponding to that back pressure that area ratio is known as the design area ratio either way we have to know the design area ratio or if for example, a duct is fixed for a given area ratio the back pressure at which it will give the undisturbed expansion or acceleration up to the supersonic label is known as the design pressure corresponding to that duct or if the back pressure is fixed which is lower than the critical pressure the area ratio.

That means the ratio of the area at the discharge for the divergent part to that at the throat for that corresponding to that back pressure where you will get undisturbed expansion continuous acceleration is known as the design area ratio; that means, for that particular back pressure which is lower than the critical pressure to have a continuous acceleration in the supersonic region undisturbed avoiding the reversible shocks we will have to search for that design areas ratio where you will get you will get from the isentropic charts; that means, precisely from this formula a by a star as a function of mach number.

So, with that mach number value what is value of a by a star or from the isentropic chart we will find out this design area ratio. So, we will have to give that design area ratio for that convergent divergent duct, but sometimes in jet engine even if that probably you know you have already read in your gas turbine even if back pressure at that altitude is lower than the critical pressure sometimes deliberately the divergent part is not added; that means, the supersonic region is avoided; that means, a convergent duct is only used.

What happens at the end of the convergent duct as you know the pressure will reach the critical pressure, it will not be expanded up to the back pressure, because the back pressure is lower than the critical pressure a convergent duct can only make the expansion up to the critical pressure, and accordingly the acceleration will be there up to the point when the mach number one will be reached we will not gain a momentum thrust more corresponding to we will get the momentum thrust corresponding to mach number one.

So, we will sacrifice some momentum thrust more corresponding to we will get a momentum thrust corresponding to mach number one. So, we will sacrifice some momentum thrust which we could have obtained if the acceleration could have been made up to a value of m greater than one that supersonic acceleration, but you deliberately avoid it why because of the fact that when the pressure is high there, then the fluid immediately comes to the back pressure, there is a pressure loss we get an extra pressure thrust if you take the control volume of the jet engine you see since the pressure is high at the nozzle exit plane.

And throughout the atmospheric pressure in duct the pressure is atmospheric pressure. So, in the direction of motion, we get an additional pressure thrust you know we get an additional pressure thrust. If the expansion is not there up to the back pressure at the outlet end of the nozzle you get an additional pressure thrust. So, the total thrust in the propelling nozzle is the momentum thrust plus the additional pressure thrust. So, we have to decide that whether this additional pressure thrust will be more or less than the additional momentum thrust that could have been obtained, because of supersonic acceleration in the supersonic region by obtaining a supersonic velocity.

So, up to certain mach number ranges this is beneficial. So, therefore, it tradeoff I discussed earlier is made whether we will; that means, tradeoff between the pressure thrust, and the momentum thrust is made that whether it will be given or not. So, it is not always that when the pressure goes below in case of jet engine is a practical application I am telling that pressure goes below the critical pressure the back pressure always you will be using a convergent divergent duct.

But it is true that if you do not use the convergent divergent duct, then you will not be able to expand the gas up to the back pressure. So, therefore, again I am telling what is the final conclusion that. So, long the back pressure is above the critical pressure corresponding to a stagnation pressure; that means, the inlet pressure to a duct which is the pressure where the velocity is approximately zero, then if we provide a convergent duct the complete expansion up to the back pressure is possible the fluid will ultimately attain the back pressure at the exit plane of the nozzle.

And accordingly we will find from the energy equation the velocity of the fluid, and this will be always less than sound velocity that is mach number less than one, but when this pressure back pressure will attain the critical pressure there is a critical pressure; that means, corresponding to the stagnation pressure there is a pressure when the mach number one will be attained when the fluid accelerates through a convergent duct that is known as critical pressure.

But if the back pressure is lower than the critical pressure, then it is not possible for a convergent duct to expand the gases from the stagnation pressure to the back pressure that is below the critical pressure. So, the condition will remain same as that of the critical pressure which was reached when the back pressure was exactly equal to the critical pressure that is the condition in mach number one; that means, when the outlet in mach number one will be reached nothing will change in the nozzle flow. So, further expansion will take place if you put a divergent duct.

And next point is that when you put a divergent duct further expansion or acceleration will take place, and the flow will go to the supersonic regime, but to have a undisturbed expansion in the supersonic regime up to the discharged plane of the divergent duct with some mach number more than one we have to have a unique value of the area at the discharge plane corresponding to that at the throat; that means, unique area ratio corresponding to a particular back pressure that area ratio is known as the design area ratio corresponding to the back pressure or that back pressure corresponding to the area ratio is known as the design is the shorter pair.

Like our saturation pressure, and temperature in thermodynamics. So, this pressure is as stated with this temperature when two phrases are in equilibrium. So, a particular pressure has to be with a particular temperature it is unique couple like that. So, this with the back pressure this is the area ratio to have an undisturbed expansion in the supersonic regime with without any a reversible shock waves that is the discontinuity through which the change operation from an under expansion how we are that means, when there is an under expansion to a lower pressure to catch up the higher pressure at the outlet back pressure or from a higher pressure to a lower pressure in the ambience, that is at lower back pressure have you understood please any question? Today this is the topic that flow through convergent divergent duct isentropic flow through convergent divergent duct any question?

Thank you.